Stanford Stats 116 Midterm Examination

Closed book and closed notes
Duration: 55 minutes

Spring 2021

Name: ______________________________

Student ID Number: ______________________________

By taking this exam, you agree to be bound by the Stanford Honor Code, meaning specifically in this context that you

- will not give or receive aid in examinations;
- will do your share and take an active part in seeing to it that others as well as yourself uphold the spirit and letter of the Honor Code.

Signature: ______________________________

Question M.1 (10 points)**:** Let A and B be events. The symmetric difference $A\Delta B$ is defined to be the set of all elements that are in A or B but not in both. In logic and engineering, this event is also called the XOR of A and B. Show that

$$P(A\Delta B) = P(A) + P(B) - 2P(A \cap B)$$

directly using the axioms of probability.

Question M.2: The new bridge building will house 20 Statisticians, 15 Computer Scientists and 10 Data Scientists. The second floor will have 8 offices (one person per office) and people will be randomly assigned to offices.

(a) (5 points) Find the probability that there are exactly 3 statisticians in the second floor.

(b) (5 points) Find the probability that the second floor has at least one representative from each group.

Question M.3: Susan needs to choose a password (8 characters), and the legal characters are the lower case letters a,b,c...,z (26) the uppercase letters A,B,C,...,Z (26) and the numbers 0,1,...,9 (10).

(a) (5 points) How many possibilities are there if she is required to have at least one uppercase letter in her password?

(b) (5 points) How many possibilities are there if she is required to have at least one uppercase letter and at least one lowercase letter in her password?

Question M.4 (10 points)**:** A crime is committed by one of the two suspects, A and B. Initially, there is equal evidence against both of them. In further investigation at the crime scene, it is found that the guilty party had a blood type found in 10% of the population. Suspect A does match this blood type, whereas the blood type of Suspect B is unknown. Given this new information, what is the probability that A is the guilty party?

Question M.5 (10 points)**:** We roll a fair six-sided die once. If it is an even number you win 3 times the number of the roll. If it is 1 you lose 15 and if it is something else, nothing happens (i.e. you neither win anything nor lose anything).

(a) What is the probability that you win more than 10 ?

(b) What is your expected gain/loss?

Question M.6 (10 points)**:** Let X denote the number of failures of a self-driving car test. Let's assume that $X \sim \mathsf{Poisson}(\lambda)$. What is the expected number of failures?. Show your derivation of the expected value. *Hint:* The PMF of a Poisson random variable is $P(X = x) = \frac{\lambda^x e^{-\lambda}}{x!}$, $x = 0, 1, 2, ...$