Stats 116 Problem Set 1

Due: Wednesday, April 6, 5:00 p.m. on Gradescope

Please show your work for each exercise. If you collaborate with someone else—this is fine—be sure to note that in your homework submission. You must each write up separate answer sets. Questions are either from Ross's *A First Course in Probability* or our are home-cooked.

Question 1.1 (Ross Problem 1.7)**:**

(a) In how many ways can 3 boys and 3 girls sit in a row?

(b) In how many ways can 3 boys and 3 girls sit in a row if the boys and the girls are each to sit together?

(c) In how many ways if only the boys must sit together?

(d) In how many ways if no two people of the same sex are allowed to sit together?

Question 1.2 (Ross Problem 1.15)**:** Consider a group of 20 people. If everyone shakes hands with everyone else, how many handshakes take place?

Question 1.3 (Ross Problem 1.17)**:** A dance class consists of 22 students, of which 10 are women and 12 are men. If 5 men and 5 women are to be chosen and then paired off, how many results are possible?

Question 1.4 (Ross Theoretical Exercise 1.11)**:** The following identity is known as Fermat's combinatorial identity:

$$\binom{n}{k} = \sum_{i=k}^{n} \binom{i-1}{k-1}, \quad n \geq k.$$

Give a combinatorial argument (no computations are needed) to establish this identity. *Hint:* Consider the set of numbers 1 through n. How many subsets of size k have i as their highest numbered member?

Question 1.5 (Ross Theoretical Exercise 1.13)**:** Show that for $n > 0$ we have

$$\sum_{i=0}^{n} (-1)^i \binom{n}{i} = 0.$$

Question 1.6 (Ross Problem 2.6)**:** A hospital administrator codes incoming patients suffering gunshot wounds according to whether they have insurance (coding 1 if they do and 0 if they do not) and according to their condition, which is rated as good (g), fair (f), or serious (s). Consider an experiment that consists of the coding of such a patient.

(a) Give the sample space of this experiment

(b) Let A be the event that the patient is in serious condition. Specify the outcomes in A.

(c) Let B be the event that the patient is uninsured. Specify the outcomes in B.

(d) Give all the outcomes in the event $B^c \cup A$.

Question 1.7: In the game of bridge, 52 cards are dealt evenly between 4 players (identified by North, South, East, West; North and South form a team, as do West and East), so each receives 13 cards. Each team receives 26 cards in total. Assuming the cards are uniformly shuffled, answer the following.

(a) What is the probability that North is dealt a hand with *at least* 8 of a particular suit?

(b) How many ways are there for the team N/S (North and South) to have at least 9 hearts between them?

(c) How many ways are there for the team N/S to have at least 9 hearts *and* at least 9 spades?

(d) What is the probability that the team N/S has at least 9 hearts *and* at least 9 spades?

(e) What is the probability that the team N/S has at least 9 hearts *or* at least 9 spades?

For these questions, your answer should be in terms of ratios of binomials, e.g., as in class when we showed that the probability that a player has exactly 8 of any particular suit was

$$p = \frac{4\binom{13}{8}\binom{39}{5}}{\binom{52}{13}} \approx .005 = \frac{1}{200}.$$