Stats 116 Problem Set 5

Due: Wednesday, May 4 5:00 p.m. on Gradescope

Please show your work for each exercise. If you collaborate with someone else—this is fine—be sure to note that in your homework submission. You must each write up separate answer sets. Questions are either from Ross's *A First Course in Probability* or our are home-cooked special sauce.

Question 5.1 (Ross problem 5.17)**:** The salaries of physicians in a certain specialty are approximately normally distributed. If 25% of them earn less than \$180,000 and 25% earn more than \$320,000, approximately what fraction earn

(a) less than \$200,000?

(b) between \$280,000 and \$320,000?

Question 5.2 (Ross problem 5.31)**:**

(a) A fire station is to be located along a road of length $A < \infty$. Suppose that fires occur uniformly on the interval $[0, A]$. Where should we locate the fire station to minimize the expected distance from the fire? That is, choose a to minimize

$$\mathbb{E}[|X - a|]$$

when $X \sim \mathsf{Uni}[0, A]$.

(b) Now instead suppose that $A = \infty$, so that the road is infinitely long. If the distance of the fire from point 0 is exponentiall distributed with rate λ, i.e., $X \sim \mathsf{Exp}(\lambda)$, at what location a should we place the fire station to minimize $\mathbb{E}[|X - a|]$?

Question 5.3 (Ross theoretical exercise 5.2)**:** Assume that Y has a density f and that $\int |x|f(x)dx < \infty$. (You don't need to really use the finite integral condition, but one does need it to be fully rigorous.) Show that

$$\mathbb{E}[Y] = \int_0^\infty P(Y \geq y)dy - \int_0^\infty P(Y \leq -y)dy.$$

Hint: Show that both

$$\int_0^\infty P(Y \leq -y)dy = -\int_{-\infty}^0 xf(x)dx \quad \text{and} \quad \int_0^\infty P(Y \geq y)dy = \int_0^\infty xf(x)dx.$$

Question 5.4 (Ross theoretical exercise 5.6)**:** Use the result of Question 5.3, that is, that if Y is a nonnegative random variable then $\mathbb{E}[Y] = \int_0^\infty P(Y \geq t)dt$, to show that if $X \geq 0$ is a nonnegative random variable then

$$\mathbb{E}[X^n] = \int_0^\infty nx^{n-1}P(X \geq x)dx. \tag{5.1}$$

Hint: Begin with the identity $\mathbb{E}[X^n] = \int_0^\infty P(X^n \geq t)dt$ and make the change of variables $t = x^n$. *Cultural note:* The result (5.1) does not actually rely on X having a density, and remains true for *any* nonnegative random variable X.