Stats 117 Final Exam — June 10, 2024

Name: _____ **SUNet ID:** _____ **ID #:** _____

- Complete the following problems. In order to receive full credit, please show all of your work and justify your answers. You do not need to simplify your answers unless specifically instructed to do so. You may use any result discussed in class or the text, but clearly state the result before using it, and verify that the hypotheses are satisfied.
- Please check that your copy of this exam contains 9 pages of exam questions, *numbered* in the upper-right, and that it is adequately stapled.

  DO NOT REMOVE ANY PAGES; if any page is missing, your exam will be considered incomplete. Incomplete exams will be assessed a 5-point penalty.
- You may use 1 piece of $8.5'' \times 11''$ paper (both sides) with formulas and other notes as a "reference sheet".
- You are allowed to use a calculator. No other electronic devices, including phones, headphones, or calculation aids, are permitted for any reason.
- **You have 3 hours.** Your organizer will signal the times between which you are permitted to be writing, including anything on this cover sheet, and to have the exam booklet open. During these times, the exam and all papers must remain in the testing room. When you are finished, you must hand your exam paper to a member of teaching staff.
- Paper not provided by course staff (apart from your own reference sheet) is prohibited. If you need extra room for your answers, use one of the blank pages provided (those pages except for the one at the end are labeled at the bottom by lower-case Roman numerals, starting with "ii"), and clearly indicate that that your answer continues there. Do not unstaple or detach pages from this exam.
- It is your responsibility to look over your graded exam in a timely manner. You have until **June 14, 5 p.m.**, to resubmit your exam for any regrade considerations; consult your section leader about the exact details of the submission process.
- Please sign the following:

> "On my honor, I have neither given nor received any aid on this examination. I have furthermore abided by all other aspects of the honor code with respect to this examination."

Signature: _____

1. (10 points) David Copperfield has two identical looking coins in his pocket – one is fair, and one has 2/3 probability of coming up heads. David picks a coin without knowing which – each is equally likely to be picked.

   (a) (3 points) The coin is tossed twice, coming up heads both times. Given this information, what is the probability that the chosen coin is the fair coin?

   (b) (3 points) Are the events "the first toss of the chosen coin is heads" and "second toss of the chosen coin is heads independent?" Explain.

(c) (4 points) Suppose David asks you to wager \$10; David tosses the chosen coin twice. If the two tosses are the same, you win \$25; if the two tosses are different, you get \$0. Should you take the bet?

2. (10 points) Let $X \sim \text{Poisson}(3)$.

   (a) (4 points) Find $E[2^X]$. Simplify.

   (b) (6 points) Suppose $Y \sim \text{Poisson}(3)$ and that X and Y are independent. Consider $T = X + Y$; you are given that $T \sim \text{Poisson}(6)$. Find the conditional distribution of X given $T = n$; i.e. $P(X = k \mid T = n)$.

   Remark. Your answer should be a named distribution.

3. (10 points) Let $X \sim \text{Uniform}(-1, 1)$ and let $Y = X^2$.

   (a) (7 points) Compute $\text{Cov}(X, Y)$.

   (b) (3 points) Are X and Y independent?

4. (10 points) Gene, Dennis, and Anav go for lunch. They decide who will pay the check by each flipping a coin; the "odd person" will pay. For example, if the three tosses are HTH, the person that flipped tails will pay.

   If all three flips are the same, they make another round of flips, and will continue to do so until there is an odd person.

   (a) (5 points) What is the probability that exactly 3 rounds of flips are made?

   (b) (5 points) What is the expected number of flips?

5. (10 points) Let $X \sim \text{Uniform}(0, 1)$ and define $Y = \ln\left(\frac{X}{1-X}\right)$.

   (a) (5 points) Show that X and $1 - X$ have the same distribution.

   (b) (5 points) Compute $E[Y]$ *without using integrals.*

6. (10 points) The joint PDF of X and Y is

$$f(x,y) = \begin{cases} xe^{-(x+y)}, & x > 0, y > 0 \\ 0, & \text{otherwise} \end{cases}.$$

(a) (5 points) Compute $\text{Var}(X)$.

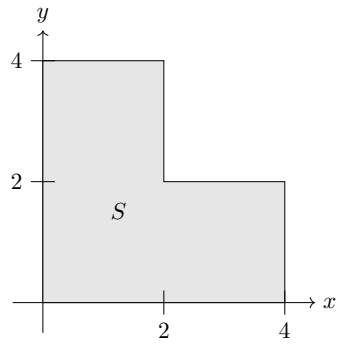
(b) (5 points) Are X and Y independent?

7. (10 points) The weekly gravel demand X (in tons) is exponentially distributed with mean of 8 tons. Zelda, the owner of a gravel pit, wants to sell as much gravel as he can, but he can produce at most 6 tons of gravel per week. Let Y denote the tons of gravel sold per week by Zelda.

   (a) (5 points) Find the CDF of Y.

   (b) (5 points) Compute $E[Y]$.

8. (10 points) Let S be the shaded region below.



The joint PDF of X and Y is given by

$$f_{X,Y}(x,y) = \begin{cases} c, & (x,y) \in S \\ 0, & \text{otherwise} \end{cases}.$$

(a) (5 points) Find c.

(b) (5 points) Find $P(Y > 2X)$.

DO NOT DETACH THIS PAGE. If you use any of this space to continue your answer, please clearly indicate the problem number(s) and **indicate on the original page of the problem that your answer continues here**.

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