Stats 117 Exam 1 — May 1, 2024

Name: _____ **SUNet ID:** _____ **ID #:** _____

- Complete the following problems. In order to receive full credit, please show all of your work and justify your answers. You do not need to simplify your answers unless specifically instructed to do so. You may use any result discussed in class or the text, but clearly state the result before using it, and verify that the hypotheses are satisfied.
- Please check that your copy of this exam contains 7 pages of exam questions, *numbered* in the upper-right, and that it is adequately stapled.

  DO NOT REMOVE ANY PAGES; if any page is missing, your exam will be considered incomplete. Incomplete exams will be assessed a 5-point penalty.
- You may use 1 piece of $8.5'' \times 11''$ paper (both sides) with formulas and other notes as a "reference sheet".
- You are allowed to use a calculator. No other electronic devices, including phones, headphones, or calculation aids, are permitted for any reason.
- **You have 2 hours.** Your organizer will signal the times between which you are permitted to be writing, including anything on this cover sheet, and to have the exam booklet open. During these times, the exam and all papers must remain in the testing room. When you are finished, you must hand your exam paper to a member of teaching staff.
- Paper not provided by course staff (apart from your own reference sheet) is prohibited. If you need extra room for your answers, use one of the blank pages provided (those pages except for the one at the end are labeled at the bottom by lower-case Roman numerals, starting with "ii"), and clearly indicate that that your answer continues there. Do not unstaple or detach pages from this exam.
- It is your responsibility to look over your graded exam in a timely manner. You have until **May 10, 5 p.m.**, to resubmit your exam for any regrade considerations; consult your section leader about the exact details of the submission process.
- Please sign the following:

> "On my honor, I have neither given nor received any aid on this examination. I have furthermore abided by all other aspects of the honor code with respect to this examination."

Signature: _____

1. (10 points) From historical data, 80% of all email is spam, and the phrase "free money" is used in 10% of spam emails. The phrase is also used in 1% of non-spam emails. A new email has just arrived which contains the phrase "free money." What is the probability that it is spam?

2. (10 points) Suppose you throw four fair dice, and let X be the number of the most frequently occurring roll. Foe example, if you roll $X(1, 2, 5, 2) = 2$ and $X(1, 4, 1, 1) = 3$. Write down the PMF of X and calculate $E[X]$.

3. (10 points) Four people, each wearing a hat, throws their hats on a table. If each person grabs a hat with their eyes closed, what is the probability at least one person got their hat back?

4. (10 points) You have two coins. One coin is a fair coin with a 0.5 probability of landing on heads. The other coin is a biased coin with a 0.25 probability of landing on heads. You pick one of these two coins at random, and begin flipping until you get 5 heads. It takes you 12 flips in order to get your 5 heads.

   (a) (5 points) What is the probability of needing 12 flips to get 5 heads?

   (b) (5 points) What is the probability that the coin you picked was the fair coin?

5. (10 points) Let $X \sim \text{Poisson}(12)$.

   (a) (6 points) Let $Y = X/4$. Is Y also Poisson? If so, with what parameter? If not, why not?

(b) (4 points) Calculate $E[Y]$.

6. (10 points) For each of the following statements, circle either TRUE (meaning, "always true") or FALSE (meaning, "not always true"), and briefly and convincingly justify your answer. 1 point for the correct choice, and 4 points for convincing justification.

   (a) (5 points) Let $X, Y \sim \text{Binomial}(12, 0.5)$ be random variables on the same sample space. Then, X and Y are the same random variable.

   Circle one, and justify below: TRUE FALSE

   (b) (5 points) Suppose $X \sim \text{Binomial}(n, p)$. Then,

   $$E[(1-p)^X] = (1-p^2)^n.$$

   Circle one, and justify below: TRUE FALSE

   Hint. Use LOTUS and the binomial theorem $\sum_{k=0}^{n} \binom{n}{k} a^k b^{n-k} = (a+b)^n$

DO NOT DETACH THIS PAGE. If you use any of this space to continue your answer, please clearly indicate the problem number(s) and **indicate on the original page of the problem that your answer continues here**.

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