Stats 117 Problem Set 1

Due: Monday, April 6 5:00 p.m. on Gradescope

Please show your work for each exercise. If you collaborate with someone else—this is fine—be sure to note that in your homework submission. You must each write up separate answer sets. Any starred exercise is optional: they are extra challenging theoretical exercises for further developing your mastery.

Question 1.1: Consider a group of 24 people. If everyone performs a square dance with each other group of three people (four people belong in a square in a square dance), how many dances occur?

Question 1.2: In the game of bridge, 52 cards are dealt evenly between 4 players (identified by North, South, East, West; North and South form a team, as do West and East), so each receives 13 cards. Each team receives 26 cards in total. Assuming the cards are uniformly shuffled, answer the following.

(a) What is the probability that North is dealt a hand with 9 of a particular suit?

(b) how many ways are there for North to be dealt a hand with *at least* 9 of a particular suit?

(c) What is the probability that North is dealt a hand with at least 9 of a particular suit?

(d) How many ways are there for the team N/S (North and South) to have at least 11 hearts between them?

(e) What is the probability that the team N/S (North and South) has at least 11 hearts between them?

For these questions, your answer should be in terms of ratios of binomials. So, for example, the probability that the North player has exactly 7 clubs is

$$p = \frac{1}{\binom{52}{13}} \cdot \binom{13}{7}\binom{39}{6} \approx .00882 \approx \frac{1}{113.4},$$

corresponding to there being $\binom{52}{13}$ possible hands, and $\binom{13}{7}$ ways to choose 7 clubs and $\binom{39}{6}$ ways to choose 6 of the remaining cards.

Question 1.3: A fair six-sided die is rolled 10 times. How many ways are there for each face to appear an even number of times?

Question 1.4 (Exercise 3.7, Weather forecasting)**:** Weatherman Phil Connors believes the following about the weather tomorrow:

- $P(\{\text{rain}, \text{shine}\}) = .9$
- $P(\{\text{rain}, \text{snow}\}) = .15$
- $P(\{\text{shine}, \text{snow}\}) = .4$

Assuming that each of the individual outcomes rain, shine, and snow are distinct, do his beliefs violate the axioms of probability? Justify your answer using the axioms carefully.