Stats 117 Problem Set 1

Due: Monday, April 6 5:00 p.m. on Gradescope

Please show your work for each exercise. If you collaborate with someone else—this is fine—be sure to note that in your homework submission. You must each write up separate answer sets.

Question 1.1: Consider a group of 20 people. If everyone shakes hands with everyone else, how many handshakes take place?

Question 1.2: In the game of bridge, 52 cards are dealt evenly between 4 players (identified by North, South, East, West; North and South form a team, as do West and East), so each receives 13 cards. Each team receives 26 cards in total. Assuming the cards are uniformly shuffled, answer the following.

(a) What is the probability that North is dealt a hand with 8 of a particular suit?

(b) how many ways are there for North to be dealt a hand with *at least* 8 of a particular suit?

(c) What is the probability that North is dealt a hand with at least 8 of a particular suit?

(d) How many ways are there for the team N/S (North and South) to have at least 9 hearts between them?

(e) What is the probability that the team N/S (North and South) has at least 9 hearts between them?

For these questions, your answer should be in terms of ratios of binomials. So, for example, the probability that the North player has exactly 7 clubs is

$$p = \frac{1}{\binom{52}{13}} \cdot \binom{13}{7}\binom{39}{6} \approx .00882 \approx \frac{1}{113.4},$$

corresponding to there being $\binom{52}{13}$ possible hands, and $\binom{13}{7}$ ways to choose 7 clubs and $\binom{39}{6}$ ways to choose 6 of the remaining cards.

Question 1.3: In quantum physics, it is frequently important to count the number of ways that a system of particles can achieve a given energy level. Energy is quantized to some value $\varepsilon > 0$. Consider a system with n particles, each of which can have energy $0, \varepsilon, 2\varepsilon, 3\varepsilon, \ldots$. Fix a given energy level $p \cdot \varepsilon$, where $p \in \mathbb{N}$. How many ways are there for an n particle system to have energy $p \cdot \varepsilon$?

Question 1.4 (Exercise 3.7, Weather forecasting)**:** Weatherman Phil Connors believes the following about the weather tomorrow:

- $P(\{\text{rain}, \text{shine}\}) = .8$
- $P(\{\text{rain}, \text{snow}\}) = .1$
- $P(\{\text{shine}, \text{snow}\}) = .5$

Assuming that each of the individual outcomes rain, shine, and snow are distinct, do his beliefs violate the axioms of probability? Justify your answer using the axioms carefully.