

Stats 117 Problem Set 5

Due: Tuesday, May 5 5:00 p.m. on Gradescope

Please show your work for each exercise. If you collaborate with someone else—this is fine—be sure to note that in your homework submission. You must each write up separate answer sets. Any starred exercise is optional: they are extra challenging theoretical exercises for further developing your mastery.

Question 5.1: A fair coin is tossed 6 times. Let X be the number of heads in the first three tosses and Y the total number of heads.

- (a) Explain why $Y = X + Z$, where $X \sim \text{Binomial}(3, \frac{1}{2})$ and $Z \sim \text{Binomial}(3, \frac{1}{2})$, and X and Z are independent.
- (b) Give the joint p.m.f. of (X, Y) . It may be easier to write this p.m.f. in terms of pairs $(x, x + z)$ where $x, z \in \{0, 1, 2, 3\}$.

Question 5.2: As in Question 5.1, a fair coin is tossed 6 times. Let X be the number of heads in the first three tosses and Y the total number of heads.

- (a) Compute $\mathbb{E}[X + Y]$.
- (b) Compute $\mathbb{E}[XY]$.

Question 5.3: John wants to plan for trick-or-treaters on Halloween. If John dresses as a ghost, the number C of children who visit his house follows a $\text{Poisson}(100)$ distribution. If John dresses as a professor, the children think he isn't celebrating Halloween, so $C \sim \text{Poisson}(10)$. If John dresses as a wolf, the children know he's celebrating Halloween but are terrified, and so $C \sim \text{Poisson}(50)$. John is equally likely to dress as a ghost, professor, or wolf. Give

- (a) The expectation $\mathbb{E}[C]$.
- (b) The variance $\text{Var}(C)$.
- (c) Compare $\text{Var}(C)$ to $\text{Var}(X)$ when $X \sim \text{Poisson}(100)$. Which is larger? Give a 1 sentence (intuitive) explanation for this.

Question 5.4: Suppose that we wish to collect baseball cards. The baseball card manufacturers, knowing that individuals most want the top players, divide players into three levels: “common” players, “uncommon” players, and “rare” players (there are 50 rare players). Suppose each pack of baseball cards contains exactly 1 rare card (which is equally likely to be any of the 50 rare players). If we open packs of cards until we've collected each rare player, how many packs of cards do we expect to open? *Hint.* Think about geometric random variables. When giving your answer, you may use that $\log n \leq \sum_{i=1}^n \frac{1}{i} \leq 1 + \log n$.