

Statistics 191  
Introduction to Regression Analysis and Applied  
Statistics  
Practice Exam

Prof. J. Taylor

YOU MAY USE YOUR 4 SINGLE-SIDED PAGES OF NOTES  
THIS EXAM IS 14 PAGES LONG. THERE ARE 4 QUESTIONS, EACH WORTH  
10 POINTS.

I UNDERSTAND AND ACCEPT THE STANFORD UNIVERSITY HONOR CODE.

NAME: \_\_\_\_\_

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1	
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Total	

Q. 1) In order to study the relevance of different fields of study in the job market, the Stanford alumni association followed Stanford graduates in the first few years after graduation. Students were asked their starting salaries, as well as which sector (i.e. finance, technology, education) they were employed in. Within the finance field there were (among others) (20 Math & Computational Science (MCS) & 20 English graduates).

(a) The first question the researchers addressed was whether an MCS degree was worth more than a English degree in finance. After entering the data in R, they found the following:

```
> t.test(mcs, english, var.equal=T, alternative='greater')
```

Two Sample t-test

```
data: mcs and english
t = 3.718, df = 38, p-value = 0.0003228
alternative hypothesis: true difference in means is greater than 0
95 percent confidence interval:
 5509.375      Inf
sample estimates:
mean of x mean of y
 69457.52  59377.05
```

Explain the results in the output above and what conclusions the researchers can draw.

(b) A friend of your reads the results of this study and says:

*Wow! If I choose MCS as my major, the probability I'd earn more than a English grad if we both start in finance is about  $1 - 3 * 10^{-4}$ !*

Do you agree with your friend's statement? Explain.

(c) How would you obtain the same results in (a) using the `lm` command in R? APPROXIMATELY CORRECT SYNTAX IS OK HERE.

Q. 2) The study begun in Q. 1) was continued, expanding to several fields: education, finance and technology; as well as an additional degree: electrical engineering. The means in each group were (after rounding)

	Education	Finance	Technology
EE	45000	70000	80000
MCS	45000	70000	65000
English	50000	60000	50000

(a) Sketch the *interaction plot* for this model, depicting the data in the above table. From this plot, does the type of degree affect your starting salary? What about the field you begin work in? Are there any interactions? Explain.

(b) The output below results from fitting this model. What kind of a regression model is it?

```
> anova(lm(Salary~Degree*Field))
Analysis of Variance Table

Response: Salary
      Df      Sum Sq   Mean Sq  F value
Degree    2 6.2969e+09  ??????????  ????????
Field     2 1.7624e+10  ??????????  ????????
Degree:Field  4 6.7457e+09  ??????????  ????????
Residuals 171 1.4860e+10  ??????????
---
```

Make a table that includes all values overwritten with ?'s above.  
(DON'T WORRY IF YOU DON'T HAVE A CALCULATOR, FRACTIONS ARE FINE.)

- (c) How would you compute  $p$ -values for each entry of the **F value** column if you had **R**? Without explicitly computing  $p$ -values, do the  $F$  statistics support your conclusions in (a)? State the hypothesis that each **F value** is testing. **YOU CAN USE THE FACT THAT, WITH MANY DEGREES OF FREEDOM IN THE DENOMINATOR, AN  $F$  STATISTIC HAS EXPECTED VALUE 1 IF THE NULL HYPOTHESIS IS TRUE.**

Q. 3) The incidence of landslides in Northern California increases with the amount of rain that falls in any given year. Suppose we are given data of the following form, collected for each month over several years.

**Landslides:** The number of landslide events reported in the Santa Cruz mountains in that specific month.

**Rainfall:** The average rainfall in the Santa Cruz mountains in that month measured in inches.

**RainfallP:** The average rainfall in the Santa Cruz mountains in the previous month measured in inches.

(a) Consider the following R output

```
MA = lm(Landslides ~ Rainfall + RainfallP)
MA
##
## Call:
## lm(formula = Landslides ~ Rainfall + RainfallP)
##
## Coefficients:
## (Intercept)      Rainfall      RainfallP
##      0.6699         0.1065         0.0274

MB = glm(Landslides ~ Rainfall + RainfallP, family = poisson())
MB
##
## Call:  glm(formula = Landslides ~ Rainfall + RainfallP, family = poisson())
##
## Coefficients:
## (Intercept)      Rainfall      RainfallP
##      0.1807         0.0414         0.0103
##
## Degrees of Freedom: 119 Total (i.e. Null); 117 Residual
## Null Deviance:      136
## Residual Deviance: 117  AIC: 436
```

Which model would you think to be the most natural model to model the count of the number of landslide events in the Santa Cruz mountains? Explain.

- (b) In whichever model you chose, what is the effect of receiving an additional 10 inches of rain in a given month?

(c) How would you use `anova` to test whether the variable `RainfallP` is associated to `Landslides`. What is the null hypothesis in your test? The alternative?

(d) Suppose you hypothesize that there is a *supersaturation* effect. That is, when rainfall exceeds 15 inches in a given month, the relationship between `Rainfall` and `Landslides`. You decide to form a variable

$$\text{HeavyRainfall} = (\text{Rainfall} > 15)$$

Write the formula for a model that allows for a different slope and intercept in months of heavy rainfall. How many degrees of freedom does this model use to estimate the mean? (ASSUME THAT YOU HAVE RETAINED THE VARIABLE `RainfallP` FROM ABOVE.)

Gbar	N	T
30.4	7	80
32.1	4	90
36.2	9	100
33.5	3	110
38.4	11	120

Table 1: Data on averaged growth rate at different temperatures  $T$ . The column  $Gbar$  is the average of  $N$  different experiments at a fixed temperature  $T$ .

- Q. 4) A lab scientist is interested in the effect of temperature,  $T$  on the growth rate of a certain population of bacteria. For each of the temperatures in the Table 1 below, the scientist performs a number of experiments, which can be found in the column  $N$ , and records the average growth rate (averaged across all experiments of that temperature) in the column  $Gbar$  of Table 1. That is, 30.4 is the average of growth rates,  $G$ , from 7 different experiments each conducted at a temperature of 80. The scientist is interested in fitting a linear regression model for the growth rate as a function of  $T$ .

Assume that a simple linear model of the form

$$G = \beta_0 + \beta_1 T + \epsilon \tag{1}$$

is appropriate if the actual growth rates were observed (rather than  $Gbar$ , their averages over the  $N$  experiments). That is, the variance of  $\epsilon$  is constant across different values of  $T$  and is distributed as  $N(0, \sigma^2)$  independently for each experiment.

- (a) The quantities  $Gbar$  are averages over several experiments. What is the relationship between the variances of the  $Gbar$  entries in Table 1 and  $\sigma^2$  in (1)? Is their variance constant if (1) is correct?

- (b) You are given a choice between the following 3 outputs from *R* to find a confidence interval for the coefficient  $\beta_1$  in the model (1).

```
> # MODEL A
> confint(lm(Gbar~T, growth))
                2.5 %    97.5 %
(Intercept) -2.45005910 35.8900591
T            -0.01581187  0.3638119

> # MODEL B
> confint(lm(Gbar~T, growth, weights=growth$N))
                2.5 %    97.5 %
(Intercept)  0.13407264 31.3517067
T            0.03736585  0.3399493

> # MODEL C
> confint(lm(Gbar~T, growth, weights=1/growth$N))
                2.5 %    97.5 %
(Intercept) -1.07474463 40.0286748
T            -0.06427241  0.3443322
```

Which of the three models above ( $A$ ,  $B$  or  $C$ ) have unbiased estimates of  $\beta_1$  if model (1) is correct?

- (c) Which of the three models above ( $A$ ,  $B$  or  $C$ ) have valid confidence intervals for  $\beta_1$  if model (1) is correct? If a model has invalid confidence intervals, what is wrong with the confidence interval?

EXTRA SPACE

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