STATS 218 Homework 4

Due date: Monday, Apr 25

Problem 1 (Ross Ex. 6.1)

If $\{Z_n, n \geq 1\}$ is a martingale, show that, for $1 \leq k < n$,

$$E\left[Z_n \mid Z_1, \ldots, Z_k\right] = Z_k.$$

Problem 2 (Ross Ex. 6.2)

For a martingale $\{Z_n, n \geq 1\}$, let $X_i = Z_i - Z_{i-1}, i \geq 1$, where $Z_0 \equiv 0$. Show that

$$\operatorname{Var}\left(Z_n\right) = \sum_{i=1}^{n} \operatorname{Var}\left(X_i\right).$$

Problem 3 (Ross Ex. 6.17)

Suppose that 100 balls are to be randomly distributed among 20 urns. Let X denote the number of urns that contain at least five balls. Use Azuma-Hoeffding inequality to derive an upper bound for $P\{X \geq 15\}$.

Problem 4 (Ross Ex. 6.22)

Let $\{X_n, n \geq 0\}$ be a Markov process for which X_0 is uniform on $(0,1)$ and, conditional on X_n,

$$X_{n+1} = \begin{cases} \alpha X_n + 1 - \alpha & \text{with probability } X_n \\ \alpha X_n & \text{with probability } 1 - X_n \end{cases}$$

where $0 < \alpha < 1$. Discuss the limiting properties of the sequence X_n, $n \geq 1$.