STATS 218 Homework 6

Due date: Monday, May 16

Problem 1 (Ross Ex. 8.1)

Let $Y(t) = tX(1/t)$.

(a) What is the distribution of $Y(t)$?

(b) Compute $\operatorname{Cov}(Y(s), Y(t))$.

(c) Argue that $\{Y(t), t \geq 0\}$ is also Brownian motion.

(d) Let

$$T = \inf\{t > 0 : \quad X(t) = 0\}.$$

Using (c) present an argument that

$$P\{T = 0\} = 1.$$

Problem 2 (Ross Ex. 8.2)

Let $W(t) = X\left(a^2 t\right)/a$ for $a > 0$. Verify that $W(t)$ is also Brownian motion .

Problem 3 (Ross Ex. 8.5)

A stochastic process $\{X(t), t \geq 0\}$ is said to be stationary if $X(t_1), \ldots, X(t_n)$ has the same joint distribution as $X(t_1 + a), \cdots, X(t_n + a)$ for all $n, a, t_1, \cdots, t_n$.

(a) Prove that a necessary and sufficient condition for a Gaussian process to be stationary is that $\operatorname{Cov}(X(s), X(t))$ depends only on $t - s$, $s \leq t$, and $E[X(t)] = c$.

(b) Let $\{X(t), t \geq 0\}$ be Brownian motion and define

$$V(t) = e^{-\alpha t/2} X\left(\alpha e^{\alpha t}\right)$$

Show that $\{V(t), t \geq 0\}$ is a stationary Gaussian process. It is called the Ornstein-Uhlenbeck process.