STATS 218 Homework 8

Due date: Monday, May 30

Problem 1 (Grimmett Ex. 13.7.2)

Let W be a standard Wiener process. Fix $t > 0, n \geq 1$, and let $\delta = t/n$. Show that $Z_n = \sum_{j=0}^{n-1} \left(W_{(j+1)\delta} - W_{j\delta} \right)^2$ satisfies $Z_n \to t$ in mean square as $n \to \infty$.

Problem 2 (Grimmett Ex. 13.7.3)

Let W be a standard Wiener process. Fix $t > 0, n \geq 1$, and let $\delta = t/n$. Let $V_j = W_{j\delta}$ and $\Delta_j = V_{j+1} - V_j$. Evaluate the limits of the following as $n \to \infty$:

(a) $I_1(n) = \sum_j V_j \Delta_j$,

(b) $I_2(n) = \sum_j V_{j+1} \Delta_j$,

(c) $I_3(n) = \sum_j \frac{1}{2} \left(V_{j+1} + V_j \right) \Delta_j$,

(d) $I_4(n) = \sum_j W_{\left(j+\frac{1}{2}\right)\delta} \Delta_j$.

Problem 3 (Grimmett Ex. 13.12.7)

Let $X_0, X_1, \ldots$ be independent $N(0,1)$ variables, and show that

$$W(t) = \frac{t}{\sqrt{\pi}} X_0 + \sqrt{\frac{2}{\pi}} \sum_{k=1}^{\infty} \frac{\sin(kt)}{k} X_k$$

defines a standard Wiener process on $[0, \pi]$.