Stat 218 – List of topics treated

1. **Convergence of random variables (in probability, almost sure, in distribution)**
2. **Renewal processes:**
   (a) Definition
   (b) Law of large numbers and central limit theorem.
   (c) Renewal theorems (key, Blackwell, elementary)
   (d) Wald's lemma
   (e) Excess life, current life.
3. **Discrete-time martingales:**
   (a) Conditional expectation.
   (b) Definition of martingales (and sub- super-). Examples with random walks and Markov chains.
   (c) Azuma-Hoeffding inequality.
   (d) Concentration of chromatic number of random graphs.
   (e) Martingale convergence theorem (for UI sub-martingales, plus special cases.)
   (f) Application to Wright-Fisher model of population genetics.
   (g) Kolmogorov's inequality.
   (h) Stopping times, stopped martingales, optional stopping theorem
   (i) Application to hitting times.
   (j) Application to SGD (Robbins-Monro).
4. **Brownian motion and diffusions:**
   (a) Definition of Brownian morion (BM).
   (b) BM as limit of random walks.
   (c) Continuous time martingales.
   (d) Stopped martingales.
   (e) Distribution of hitting times.
   (f) Reflection principle.
   (g) Diffusion processes: definition.
   (h) Forward and backward diffusion equations.
   (i) Applications: Fokker-Planck equation, Ornstein Uhlenbeck process.
   (j) Transition probabilities determine the distribution of diffusions.