Math 230A / Stat 310A –Autumn 2010

Quiz in Elementary Real Analysis

This is to give you an idea of the kind of calculations that will be considered routine and elementary in Stat 310. Your answers should consist of proofs (very brief!) or counterexamples.

In what follows, (x_n) denotes the sequence $x_1, x_2, \cdots$ of real numbers; the real numbers *do not* include $+\infty$ or $-\infty$; and (x_n) *converges* means (x_n) *converges to a real limit as* $n \to \infty$.

1. Compute

   a) $\int_0^\infty e^{-x^2/2} dx$ b) $\sum_{n=1}^\infty 1/n(n+1)$

   c) $\sum_{n=0}^\infty e^{-2n}$ d) $\lim_{N\to\infty} \sum_{n=N}^\infty e^{-n^2}$

2. True or false:

   a) $\limsup x_n > 1$ implies $x_n > 1$ for infinitely many n.

   b) $\limsup x_n \geq 1$ implies $x_n \geq 1$ for infinitely many n.

3. Let

$$A_1 = \{r : r \textit{ rational}, \textit{ and } \liminf x_n < r < \limsup x_n\}$$

$$A_2 = \{r : r \textit{ rational}, \textit{ and } \liminf x_n > r\}$$

$$A_3 = \{r : r \textit{ rational}, \textit{ and } \limsup x_n < r\}$$

   State a simple condition involving the three sets above, that is equivalent to "(x_n) converges to a finite value".

4. Suppose $|x_n - x_{n+1}| < \epsilon_n$. Say whether (x_n) converges, in the case

   a) $\epsilon_n = 1/\sqrt{n}$

   b) $\epsilon_n = 1/n^2$

5. Any convergent sequence of integers has a very striking property. What is it?

6. Let f be a real valued function defined on the real line, and suppose f is positive, strictly increasing, and bounded. Suppose also that f is continuous on the rationals. True or false: f is continuous on the whole real line.

7. Consider the set of all infinite binary sequences. Is this set countable or uncountable? Prove your answer.