

## Standing Waves on a String

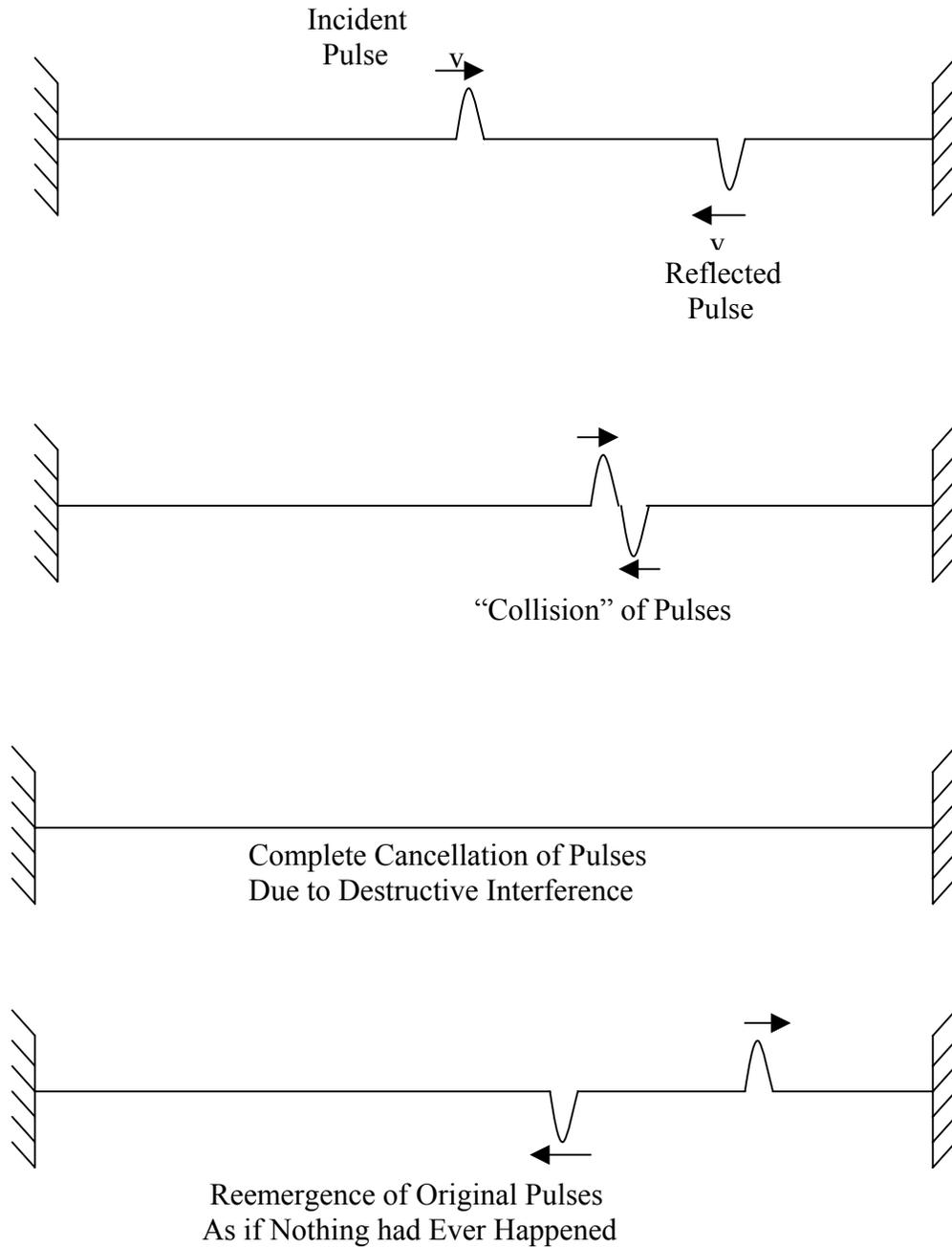
If a string is tied between two fixed supports, pulled tightly and sharply plucked at one end, a pulse will travel from one end of the string to the other. When you pluck the string you put energy into an elastic medium, and this energy travels through the medium as a transverse pulse. Transverse means that the amplitude is at right angles to the direction of propagation. The speed of the pulse through the medium, in this case the string, is a function of the properties of the string. Specifically, it is a function of the linear density of and the tension in the string. Making the string tighter and lighter increases the pulse speed, and making the string looser and heavier slows the pulse speed down. The relation between pulse speed, tension and linear density is given by the following equation:

$$v = \sqrt{\frac{T}{\mu}} \quad (\text{Equation 1})$$

where  $v$  is the pulse speed,  $T$  the tension in the string, and  $\mu$  the string's linear density.

The pulse is reflected from where it is tied to the support. When a reflection occurs from a low density medium to one of higher density (essentially infinite if the string is tied securely), the reflection inverts the pulse and it travels back down the string in the reverse direction. By Newton's third law of action and reaction, when the pulse hits the wall it exerts an upward pull on it if the pulse has a positive amplitude. The wall then exerts a downward pull on the string and the reflected pulse is inverted, giving it a negative amplitude. If the initial pulse has a positive (up) displacement and is traveling to the right, then the reflected pulse will have a negative (down) displacement and be traveling to the left.

If a series of pulses is put on the string by an oscillator, there will be a train of positive pulses traveling to the right and a train of negative pulses traveling to the left. When these pulses meet they will interfere destructively, Cancel one another out momentarily, and then reappear as if nothing had ever happened



Standing waves on a string are a result of traveling waves interfering both destructively and constructively. The nodes (places of zero amplitude) are due to destructive interference, and the antinodes (places of maximum amplitude) are due to constructive interference. When a standing wave appears, the nodes and antinodes are

fixed in place. When the conditions of the tension in the string, the linear density and the frequency of oscillations are just right, standing waves appear.

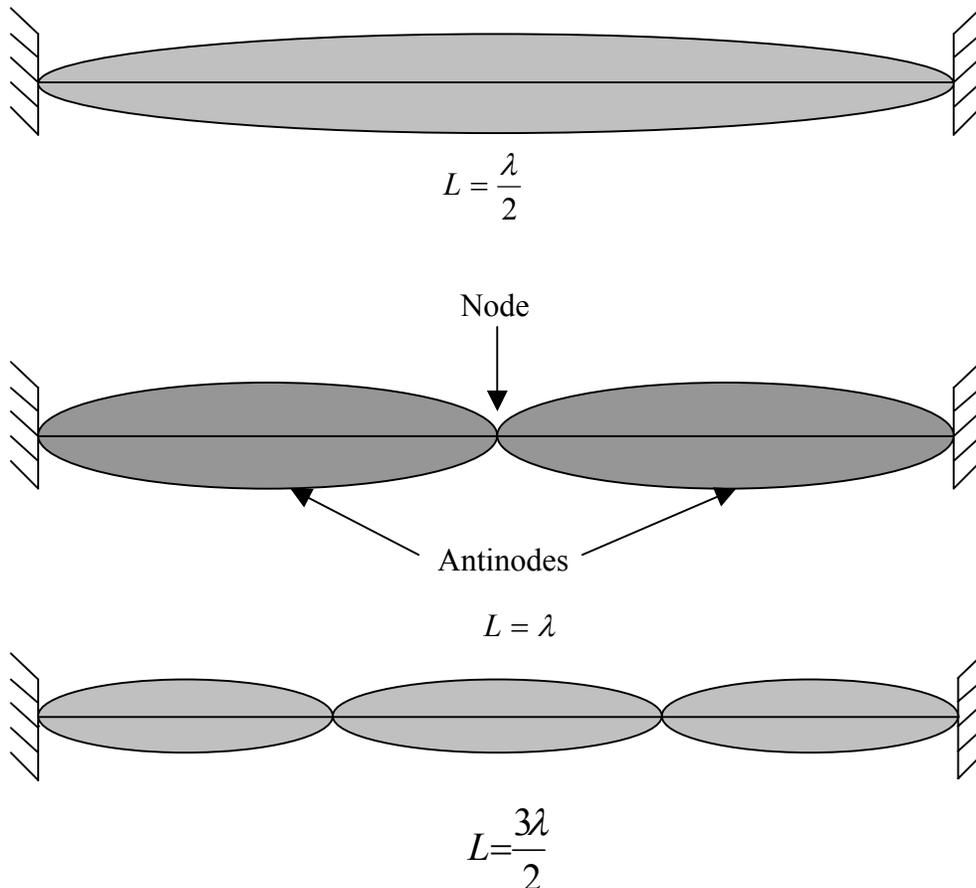
A wave has both a frequency and a wavelength that are related by the equation

$$\lambda f = v, \quad (\text{Equation 2})$$

Where  $\lambda$  is the wavelength,  $f$  the frequency, and  $v$  the velocity of the wave on the string. The wavelengths of the standing waves are fixed by the length of the string. As both ends are fixed, there must be a node at each end of the string. As the standing waves on the string are sinusoidal, the allowed number of waves on the string will be an integral number of half wavelengths, or:

$$\frac{n\lambda}{2} = L, \quad (\text{Equation 3})$$

where  $n$  is a positive integer,  $\lambda$  the wavelength, and  $L$  the length of the string. What you actually see will be something like the following illustrations.



The gray areas in the illustrations above depict the blurred vertical motion of the string as it vibrates. However, a standing wave is a static, unmoving pattern. Even though a node is perfectly motionless, though, energy is flowing through it as a standing wave consists of interfering traveling waves. Equations 1 and 2 are both for the velocity of the wave on the string, but to mathematically model the standing wave, we eliminate the horizontal velocity.

$$v = \sqrt{\frac{T}{\mu}} \qquad v = f\lambda$$
$$\sqrt{\frac{T}{\mu}} = f\lambda \qquad \text{(Equation 4)}$$

Equation 4 is the equation you will use in this lab to investigate standing waves.

In investigating the standing wave system, you will use Equation 4 to predict the linear density of the string using a graphical method. However, it is a good idea to have a rough idea of what the linear density of the string is before using Equation 4. The string used is a highly elastic bungee cord, and will stretch quite a bit. Take the cord and weigh it with a pan balance. Record the mass, and measure its length. The mass divided by the length is the linear density of the cord. But, as the cord will be stretched quite a bit when a weight is hung from one end, the linear density you find using Equation 4 will be less than what you calculated using the scale and a ruler. *The directly calculated linear density using the scale and a ruler is an upper limit, and the linear density found using standing waves will be less because of the stretching.*

Equation 4 will be linearized, and the slope of the line will be the linear density.

$$\sqrt{\frac{T}{\mu}} = f\lambda$$

Both the tension in the cord and the linear density are constant, whereas both the frequency and the wavelength of the standing wave patterns are variable. You will be able to vary the frequency by turning the knob on the frequency generator, so the frequency will be the independent variable and plotted on the x-axis. The wavelength is

the dependent variable and will be plotted on the y-axis.

$$\lambda = \sqrt{\frac{T}{\mu}} \left(\frac{1}{f}\right) \quad \text{(Equation 5)}$$

The slope of the line is  $\sqrt{\frac{T}{\mu}}$ , and knowing that the tension in the cord is the weight hanging from its end, the linear density can be calculated.



### **Experiment:**

Take the bungee cord and run it through the hole on top of the pylon sticking up from the speaker cone. Tie the cord to the wall, or to the vertical ring stand support. Slide the speaker as close to the tied end as you can get it. Tie a weight (0.5kg-1kg) to the other end of the cord, and suspend the weight from the pulley. Hook the speaker to the audio amplifier/signal generator using the low ohm outputs.

Take a length of stretchable bungee cord and weigh it on a pan balance. Now measure its *unstretched* length. Calculate the mass per unit length ( $\mu$ ) by dividing the mass in kilograms by the length in meters. The result is the upper limit of the cord's linear density that you'll find using Equation 5. Measure the length  $L$  of the stretched bungee cord. Measure from the pylon on top of the speaker cord to the top of the pulley.

Turn on the generator and set it for a 1 Hertz sine wave output. Notice the vertical oscillations of the speaker cone attached to the cord. Increase the frequency of the signal generator until you see the *second* resonance (first overtone), then lower the frequency until you find the fundamental with only one antinode. The reason for doing this is that it's easy to mistake the false resonances one sees on the cord for the fundamental. By going initially to the first overtone or second resonance (which is shown in the photograph above), you have an unambiguous reference point. Record the frequency of the fundamental, and then increase the frequency until you see the second resonance (or first overtone) again and record the frequency. Record the frequencies for the first six resonances. As the number increases, the amplitude of the antinodes will decrease until it becomes hard to detect the resonances.

Plot the inverse of the frequency on the x-axis, against the wavelength on the y-axis. It should be a straight line with a positive slope. From the slope of the graph find the linear density of the *stretched* cord, and compare it with the linear density of the unstretched cord. The value from your graph should be less than the value calculated from the unstretched cord.