# The analysis of gradience in phonology: what are the right tools?

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# 1. Background

# *1.1 What sort of formal models should we consider for the analysis of gradience?*

- Some contenders not discussed here:
  - **analogical models** (Skousen 2002, Bailey and Hahn 2001, Daelemans et al. 2004)
  - **connectionist models** (Rumelhart and McClelland 1986 et seq.)
- Focus here: "Quantitatively augmented" generative models

## *1.2 Quantitatively augmented generative models*

- Rules and constraints of generative grammar cover the primary descriptive work, and are adapted to gradience by *embedding* them in a quantitative framework.
- Such frameworks are usually couched in the language of **probability**.
- I will address two such models:
  - **Stochastic Optimality Theory**
  - **Maximum Entropy** models

## *1.3 Gradient model and algorithmic learning*

- Gradient analysis is hard, and we may be able to do better with machine-learned grammars.
  - Implemented systems can comb through the data, fine-tuning the grammar with greater care than humans can.
- Grammars learned by algorithm address the long-standing goal of generative theorizing, namely to explain how acquisition is possible.

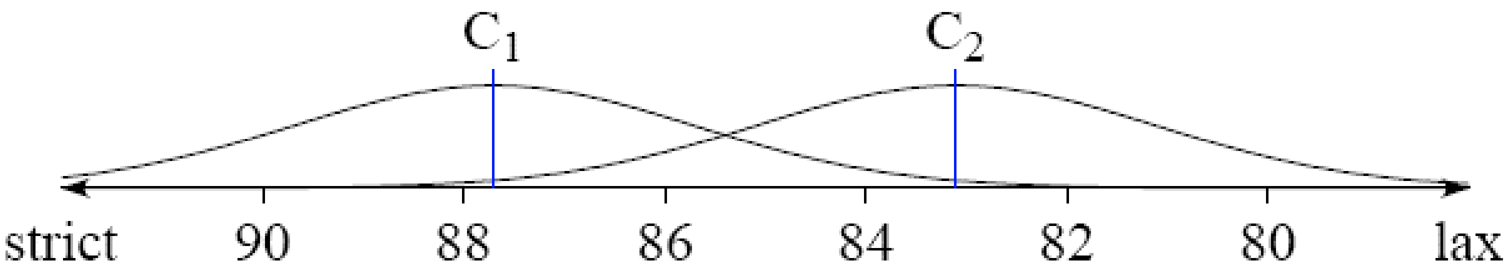
# 2. Stochastic Optimality Theory

- Refs.: Boersma (1997), Boersma and Hayes (2001)
- Basics:
  - ➢ Constraints are arranged in **ranking values** on a numerical scale, corresponding to their probability of being “ranked high.”
  - ➢ The ranking values define the means of Gaussian probability distributions, from which sampling takes place when the grammar is applied.

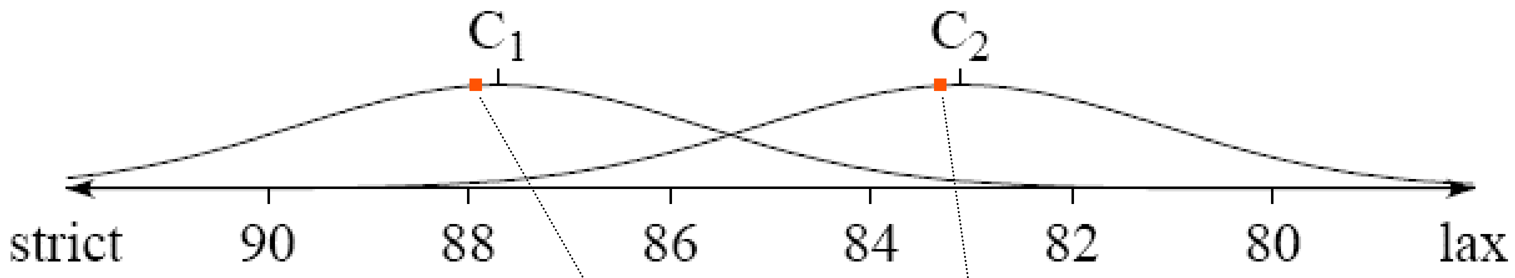
## *2.1 Example*

(taken from Boersma and Hayes 2001)

- Two constraints with distributions centered at the ranking values 87.7 and 83.1:



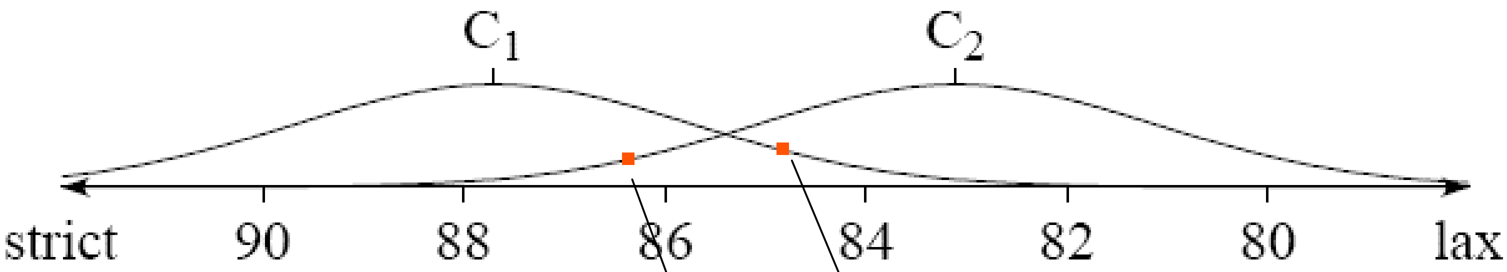
## 2.2 Sampling from the distributions and deriving a winner




- Obtain a selection point for each constraint by sampling.
- Sort constraint in descending order by selection point
- Find winner by normal methods of OT.

| /Input/ | C1 87.9 | C2 83.6 |
|---|---|---|
| ☞ Candidate1 | | * |
| Candidate2 | *! | |

## 2.3 A sample with the less-probable ranking and winner




| /Input/ | C2 85.1 | C1 83.4 |
|---|---|---|
| Candidate1 | *! | |
| ☞ Candidate2 | | * |

## *2.4 Long run behavior*

- This grammar generates Candidate1 94.8% of the time, Candidate2 5.2%.
- This is deducible analytically, or by simulation.

## *2.5 Learning Stochastic OT Grammars*

- **Starting point**
  - a constraint set
  - observed output forms with frequencies
  - suitable set of rival candidates for each input

## *2.6 Gradual Learning Algorithm (Boersma 1997)*

- Try the grammar on known input-output pairs. If it errs:
  - ➢ Incrementally raise the ranking values of all "winner-preferring" constraints
  - ➢ Incrementally lower the ranking values of all "loser-preferring" constraints.
- This has been shown in many cases to achieve good statistical matching to the learning data.

## *2.7 Other ranking algorithms for Stochastic OT*

- Maslova (to appear), Lin (2005), Wilson (2007)

# 3. Maximum Entropy grammars

- References: Eisner (2000), Johnson (2002), Goldwater and Johnson (2003), Hayes and Wilson (2007)
  - Closely related to Harmonic Grammar (Smolensky 1986, Smolensky and Legendre 2006) and more distantly to Linear Optimality Theory (Keller 2000, 2006).

## *3.1 Basics of Maximum Entropy grammars*

- Every constraint bears a *weight*, a nonnegative real number.
- The weight of a constraint specifies *a probability decrement* for candidates that violate it: "violating this constraint makes you *x* much less probable".

## *3.2 The math relating weights to output probabilities*

- **Step 1**: for each candidate *x* for an given input:
  - Compute its violations for each constraint $C_i$.
  - For each constraint $C_i$ multiply violations $C_i(x)$ times the weight of the constraint, $w_i$.
  - Sum the result over all constraints:

$$\Sigma_i \, w_i \, C_i(x)$$

- **Step 2**: take *e* to the negative power of the sum just calculated:

$$e^{-\Sigma_i w_i C_i(x)}$$

- **Step 3**: carry out similar sums for each candidate having the same input, and sum them. Call the result Z.

$$Z = \Sigma_y \left( e^{-\Sigma_i w_i C_i(y)} \right)$$

- **Step 4**: find the fraction of Z assigned to the candidate *x* under discussion:

$$\frac{\Sigma_x \exp(-\Sigma_i w_i C_i(x))}{Z}$$

This is the probability of candidate *x*.

# *3.3 Sample grammar*

| Text | Cand. | Target freq. | Predicted freq. | ME Score | C1 | C2 | C3 |
|---|---|---|---|---|---|---|---|
| | | | | | 32.7 | 11.0 | 0 |
| Input1 | 1-1 | 1 | 0.99999993 | 11.0 | | * | |
| | 1-2 | 0 | 0.00000007 | 32.7 | * | | |
| Input2 | 2-1 | 1 | 0.9999998 | 0 | | | * |
| | 2-2 | 0 | 0.0000002 | 11.0 | | * | |

- Weights were learned by algorithm; see below.
- Example calculation, first predicted score:

$$\frac{e^{-11.0}}{e^{-32.7} + e^{-11.0}} = 0.99999993$$, the practical equivalent of 1.

## *3.4 Learning*

- There are many ways to find the weights for a MaxEnt grammar.
- I cover here the method described and used in Hayes and Wilson (2007).
- This draws heavily on Della Pietra, Della Pietra, and Lafferty (1997).

# 3.5 Criterion

- Maximize the *probability of the observed data* P(*D*), given the constraint set (maximum likelihood estimation)
  - P(*D*) is the product of the probabilities of each observed datum, i.e. $\Pi_{x \in D}$ P($x$)

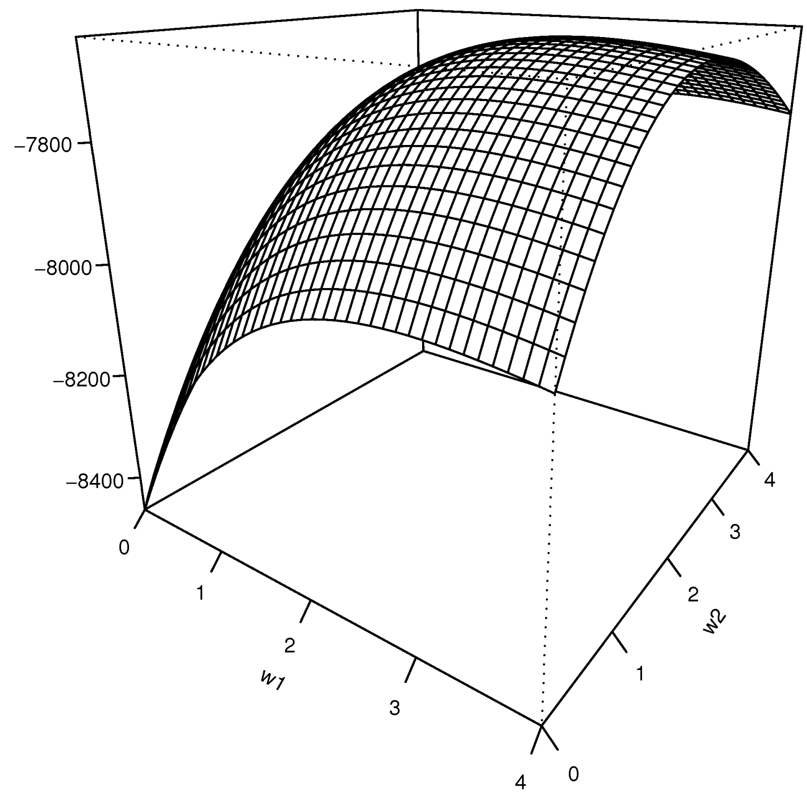
- This is a widely adopted criterion in learning theory.
- For an intuitive rationale, observe that it likewise *minimizes the probability of the unobserved data*, since probability sums to one for each input.

# *3.6 Method for maximizing probability of observed data*

- A hill-climbing search, conducted on an *n*-dimensional surface,
  - *n* = number of constraints.
  - “Altitude” is P(*D*).

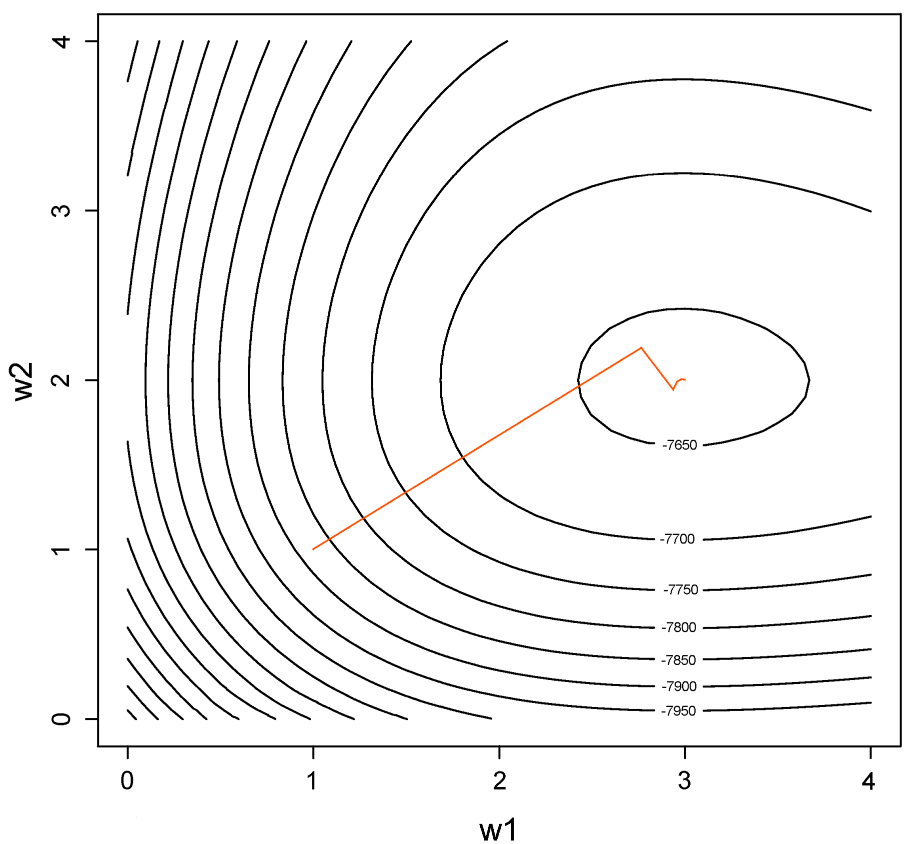
# 3.7 Example of hill-climbing

- Here, $n$ is 2, vertical axis depicts P($D$):



# *3.8 Climbing a hill stepwise: example*

- Contour map of same hill; top is at (3, 2).




| *Stage* | *Weight 1* | *Weight 2* |
|---|---|---|
| *1* | 1 | 1 |
| *2* | 2.77 | 2.19 |
| *3* | 2.94 | 1.94 |
| *4* | 2.96 | 1.99 |
| *5* | 2.98 | 2 |
| *6* | 2.99 | 2 |
| *7* | 3 | 2 |

## *3.9 How to climb*

- Climbing follows the **gradient**; i.e. vector of partial derivatives of (log) probability of observed data against individual weights ($\frac{\partial}{\partial w_i}$ log(P($D$))).

- A theorem due to Della Pietra et al. (1997) tells us how to compute the gradient: the component for each constraint is **O – E**, where
  - **O** = **observed** violation count in learning data
  - **E** = **expected** violation count (estimable from current guess for weights)

## *3.10 You won’t get lost…*

- Della Pietra et al. (1997) also demonstrate that the hill is **convex** (only one peak); hence no getting stuck in local maxima.

# *3.11 Convergence*

- Since the gradient is known, and the search space is convex, the weights found are guaranteed to be optimal; i.e. to maximize P(*D*).

## *3.12 Simulations reported here*

- Carried out with a software implementation of this algorithm created by Colin Wilson; public version in progress.

# 4. Some comparisons on general grounds

## *4.1 No Harmonic bounding in MaxEnt*

- In OT, any candidate that has a strict superset of a rival’s violations *never wins*.
- Not so in MaxEnt; see below.

# *4.2 Ganging*

- **Ganging effects**: when two constraints combine to overcome the effect of one competing constraint
  - Stochastic OT permits partial, **gradient** ganging effects (see Hayes and Londe 2006, 81, for a Hungarian example)
  - Maxent also permits outright **categorical** ganging.
  - For discussion of ganging, both empirical and theoretical, see Jäger and Rosenbach (2006), Keller (2000, 2006), McClelland and Van der Wyck (2006), Pater, Bhatt and Potts (2007).

## *4.3 A point of similarity*

- Every non-stochastic OT analysis has a MaxEnt equivalent (Johnson 2002, Prince 2002), but not vice versa (Smolensky and Legendre 2006, Pater, Bhatt and Potts, 2007), so the doubt is in the area of restrictiveness, not capacity.

## *4.4 Comparing learning algorithms*

- Unlike with MaxEnt learning, the support for GLA is purely "empirical": no proof has been found that it will find the best-fit ranking values for any data pattern.
- *Nor will there ever be*. Pater (in press) has constructed a clever counterexample:
  - an insidious pattern where many of the "winner preferrers" are, for other inputs, "loser preferrers", fatally confusing the GLA.
- The MaxEnt weighting algorithm given above easily learns Pater's data pattern, as I have checked.
- The unreliability of the GLA will be a factor in the discussion below.

## *4.5 MaxEnt weighting yields great precision*

- Boersma and Hayes's (2001) Ilokano simulation, redone in MaxEnt:

| *Output* | *Target frequency* | *GLA result* | *MaxEnt* |
|---|---|---|---|
| [tawʔen] | 1/2 | .489 | 0.50000006 |
| [taʔwen] | 1/2 | .511 | 0.49999990 |
| [buːbwaja] | 1/3 | .329 | 0.33333334 |
| [bwajbwaja] | 1/3 | .337 | 0.33333328 |
| [bubwaja] | 1/3 | .334 | 0.33333329 |

- This is of no importance for modeling experimental data (which has no such precision), but very helpful for diagnosing the adequacy of constraints.

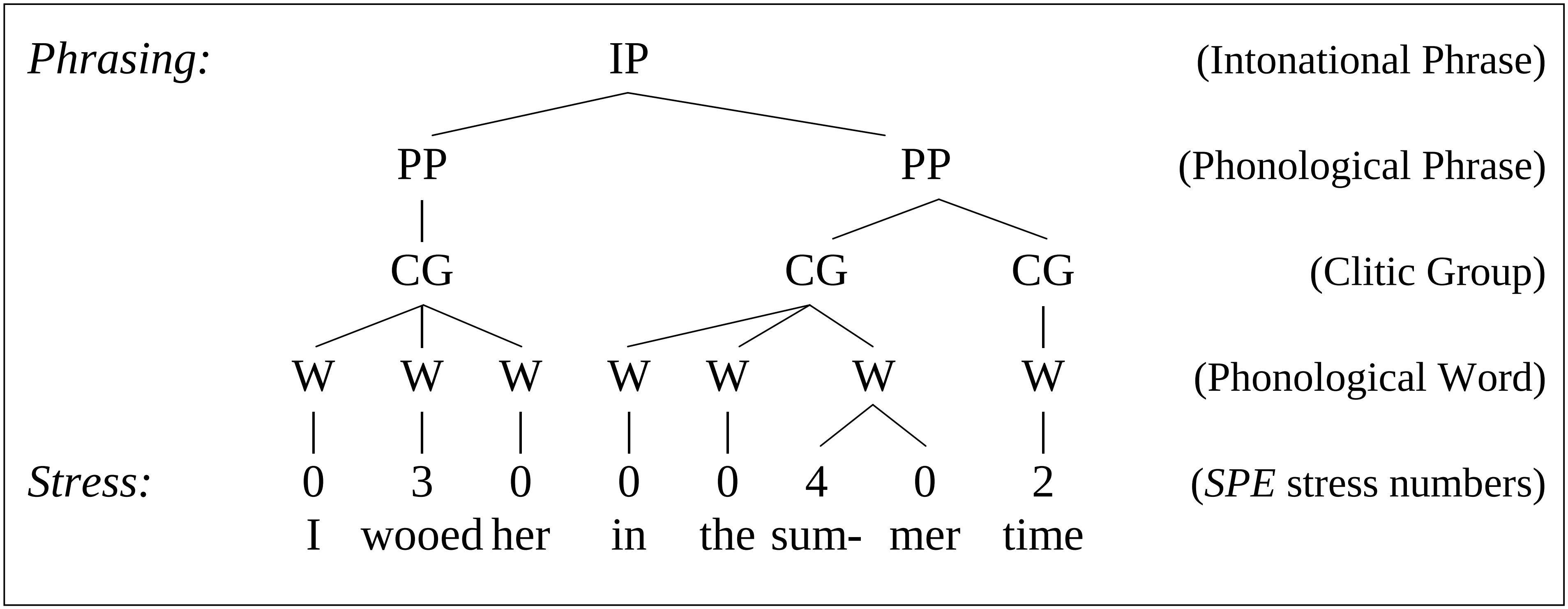
## *4.6 Remainder of this talk*

- Informal survey of my research life over the past two years—working with both GLA and MaxEnt to solve analytic problems.
- I don't yet know what the right tool for analysis of gradience is, yet, but hope that my experience is of interest.
- Cases:
    - Comparing the performance of stochastic OT/GLA with MaxEnt on a large simulation in **gradient metrics**.
    - Learning of **gradient phonotactics** (summarizing Hayes and Wilson 2007).

# 5. The textsetting problem

## 5.1 The textsetting problem

- Suppose we have a phonological representation, like this:



… and a rhythmic representation like this (Lerdahl and Jackendoff 1981)

```
    x       x       x       x
x   x   x   x   x   x   x   x
x x x x x x x x x x x x x x x x
```

- What should be the **temporal alignment of text to grid**?

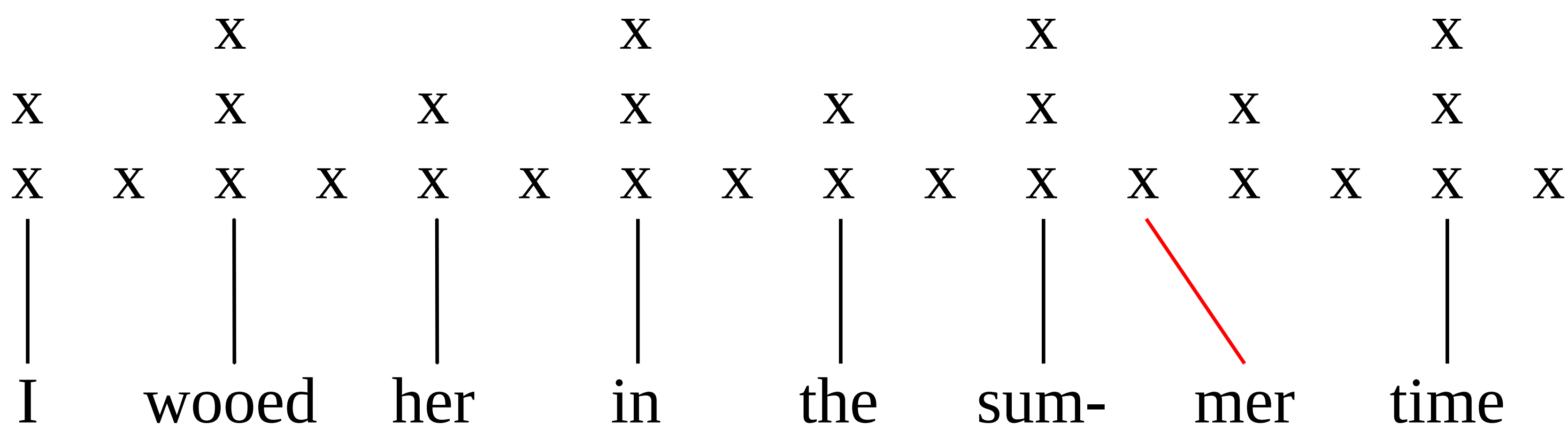
## *5.2 Possibilities I*

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | x | | | | x | | | | x | | | | x | |
| x | | x | | x | | x | | x | | x | | x | | x | |
| x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x |
| I | | wooed | | her | | in | | the | | sum- | | mer | | time | |

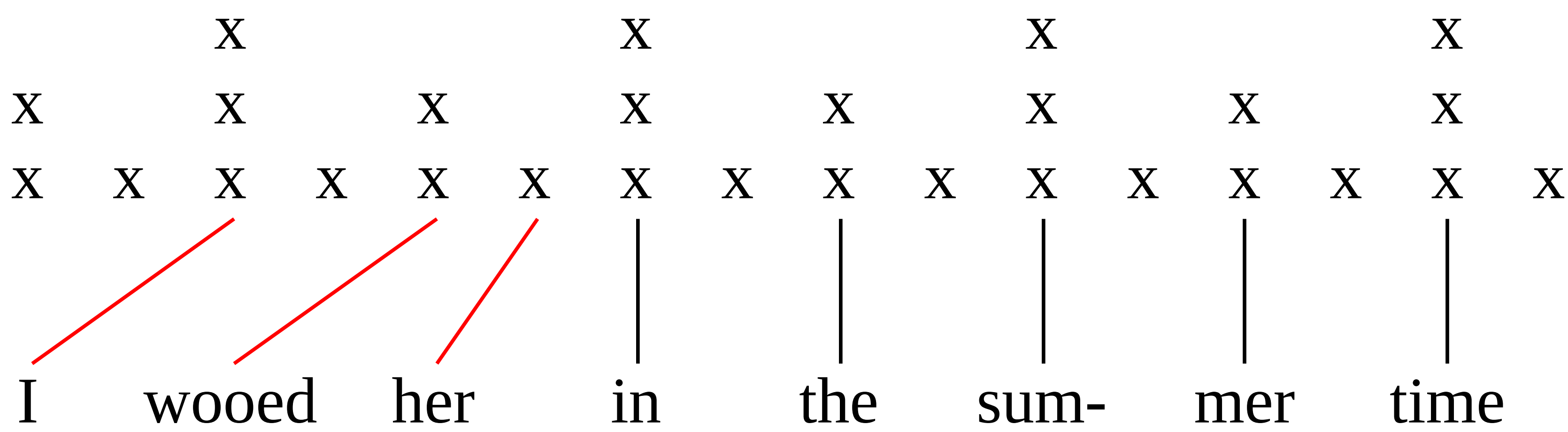
## *5.3 Possibilities II*

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | x | | | | x | | | | x | | | | x | |
| x | | x | | x | | x | | x | | x | | x | | x | |
| x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x |
| I | | wooed | | her | | in | | the | | sum- | | mer | | time | |

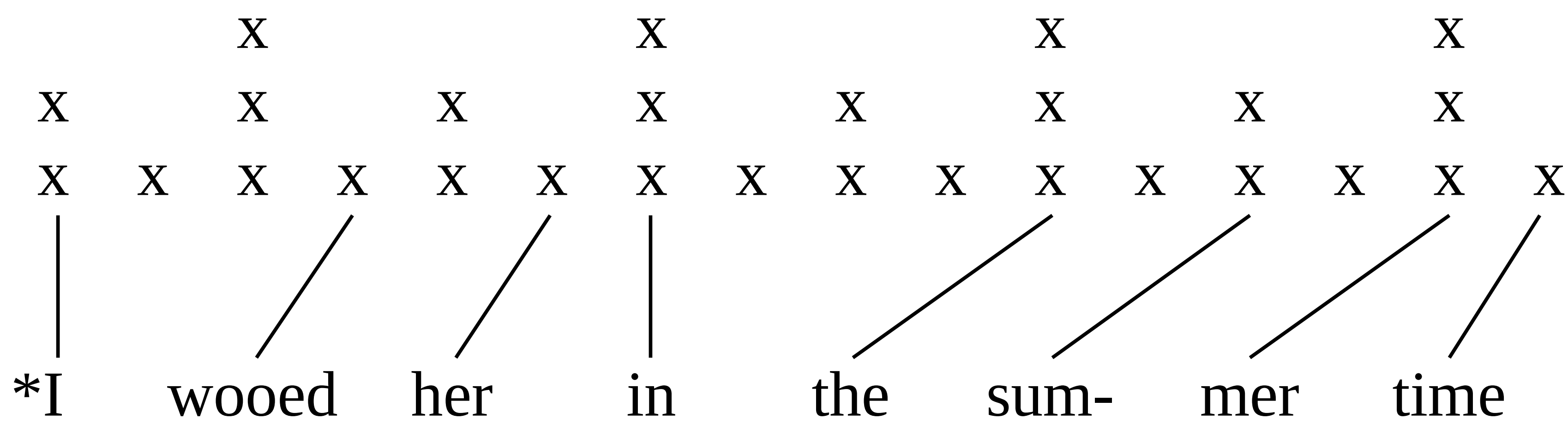
## 5.4 Possibilities III



## *5.5 Possibilities IV*



## *5.6 Not a possibility: one ill-formed textsetting*



## *5.7 The textsetting problem*

- People align texts to grids fluently—e.g. when they sing new verses to songs. How do they do it? How do they judge the well-formedness of settings?

- Textsetting is one problem in the field of **metrics**
  - see Halle and Keyser 1969, Kiparsky 1975, and much later work

- It is also a canonical area for **gradient analysis**:
  - usually multiple possibilities, which vary in preference
  - but also thousands of forms that must be fully excluded.

## *5.8 Previous work*

- A rule-based analysis:  Halle and Lerdahl (1993)
- An empirical study, with chanted settings elicited from nine native speaker consultants:  Hayes and Kaun (1996)
- A non-stochastic OT analysis, covering only "consensus" settings of the Hayes/Kaun corpus:  Hayes (in press)
- A preliminary stochastic OT grammar, learned with "easy", prefiltered data:  Hayes (2005)

## *5.9 My long-term plan for the study of textsetting*

- The theories of phonology and metrics will provide an appropriate **constraint set**.
- All the rest should follow from the choice of **framework**, particularly the learning algorithm.
- Exposure to different kinds of input data will result in differing textsetting **styles** or **dialects**, each the result of different stochastic rankings/weightings.

# *5.10 Learning simulations*

- Data taken from the Hayes/Kaun corpus (426 4-beat lines)
- Goal was to replicate the frequencies with which the consultants selected settings.
  - Hence values range from 0 (0/9) to 1 (9/9)
- Tools used:
  - Stochastic OT/GLA
  - MaxEnt

# *5.11 Constraints employed*

- These are the best constraint set I could devise for non-stochastic analysis (improving slightly on Hayes, in press)
- They serve three basic functions:

| *Match stress to rhythm* | *Regulate duration* | *Demarcate line division* |
|---|---|---|
| REGULATE SW<br>REGULATE SM<br>REGULATE MW<br>DON'T FILL W<br>MATCH LEXICAL STRESS<br>*MISMATCHED σ ˈσ]$_P$<br>FILL STRONG | RESOLUTION<br>STRONG IS LONG | *LAPSE<br>DON'T FILL 16<br>DON'T FILL 1 |

# *5.12 These constraints aren't bad*

- Applied to 364 lines with "consensus" votes, using nonstochastic OT to predict the most-selected scansion:
  - 267 successes, 90 misses, 7 ties
  - = about 3/4 correct

## *5.13 Training data*

- All 3592 textsettings (933 distinct) used by any consultant
- All 7,069 "contender" settings (Riggle 2004): those which, in OT, win or tie on at least one ranking.
- These two categories overlap heavily, but 408 attested settings (213 distinct) textsettings were not contenders.
- 40,000 other candidates, randomly selected from the ~4,000,000 logical possibilities.

# 5.14 Results (weights, ranking values)

| *Constraint* | *MaxEnt* | *GLA* |
|---|---|---|
| REGULATE SW | 14.02 | 106.0 |
| FILL STRONG | 12.28 | 108.0 |
| DON'T FILL16 | 4.78 | -1100.1 |
| *LAPSE | 4.07 | -1099.4 |
| DON'T FILL W | 3.81 | -3272.6 |
| DON'T FILL 1 | 2.08 | -3275.4 |
| MATCH LEXICAL STRESS | 1.97 | -1102.1 |
| RESOLUTION | 1.72 | -3278.7 |
| *MISMATCHED σ 'σ]$_P$ | 1.67 | -959.8 |
| REGULATE SM | 1.61 | -3274.8 |
| REGULATE MW | 0.94 | -3276.4 |
| STRONG IS LONG | 0.91 | -3328.9 |
| MATCH RISING LEX. STRESS | 0.14 | 49.8 |

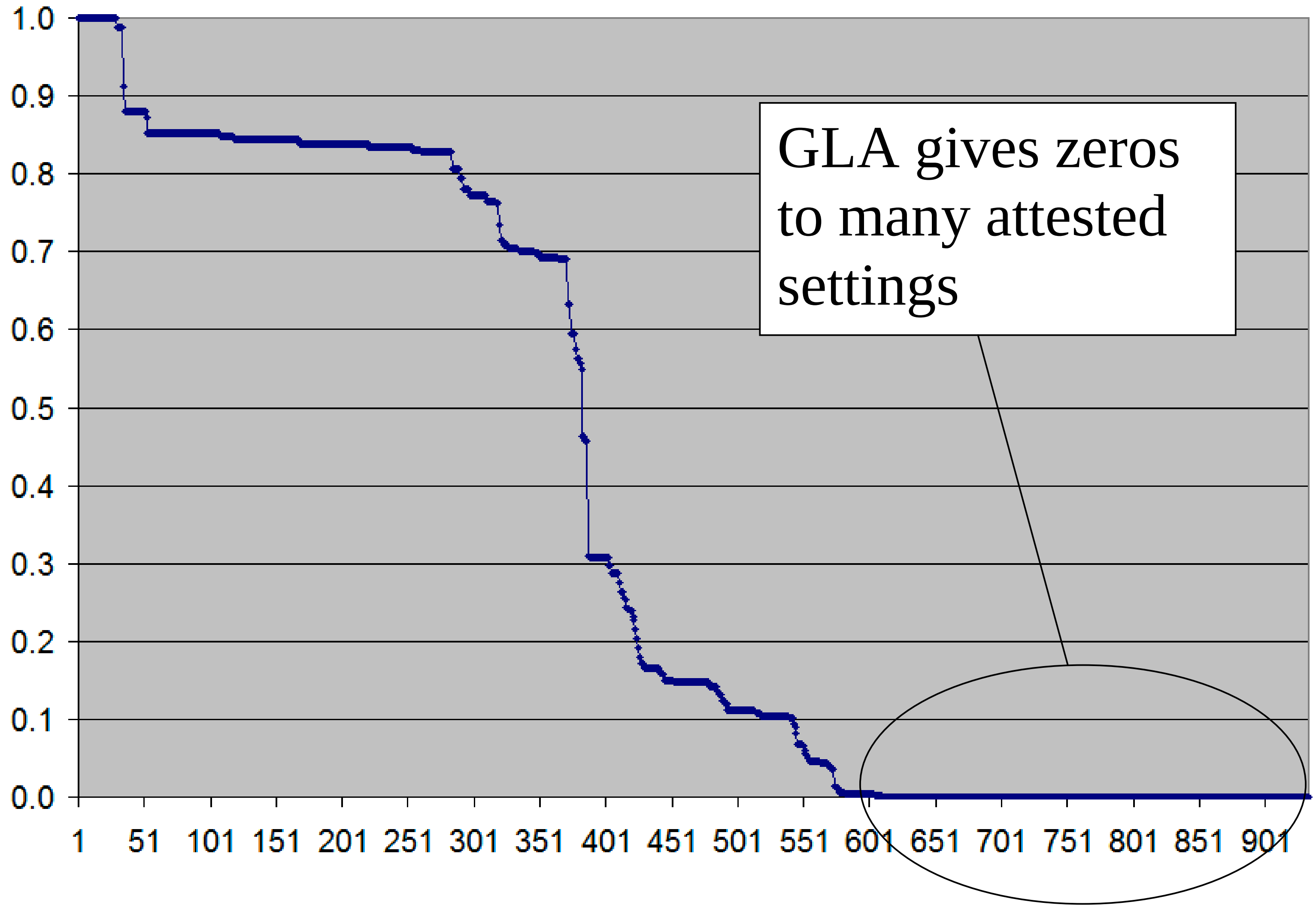
# *5.15 Comparison of grammar effectiveness*

- Correlation coefficients, all predicted frequencies vs. all observed, for two models. Not too bad, and also very similar!

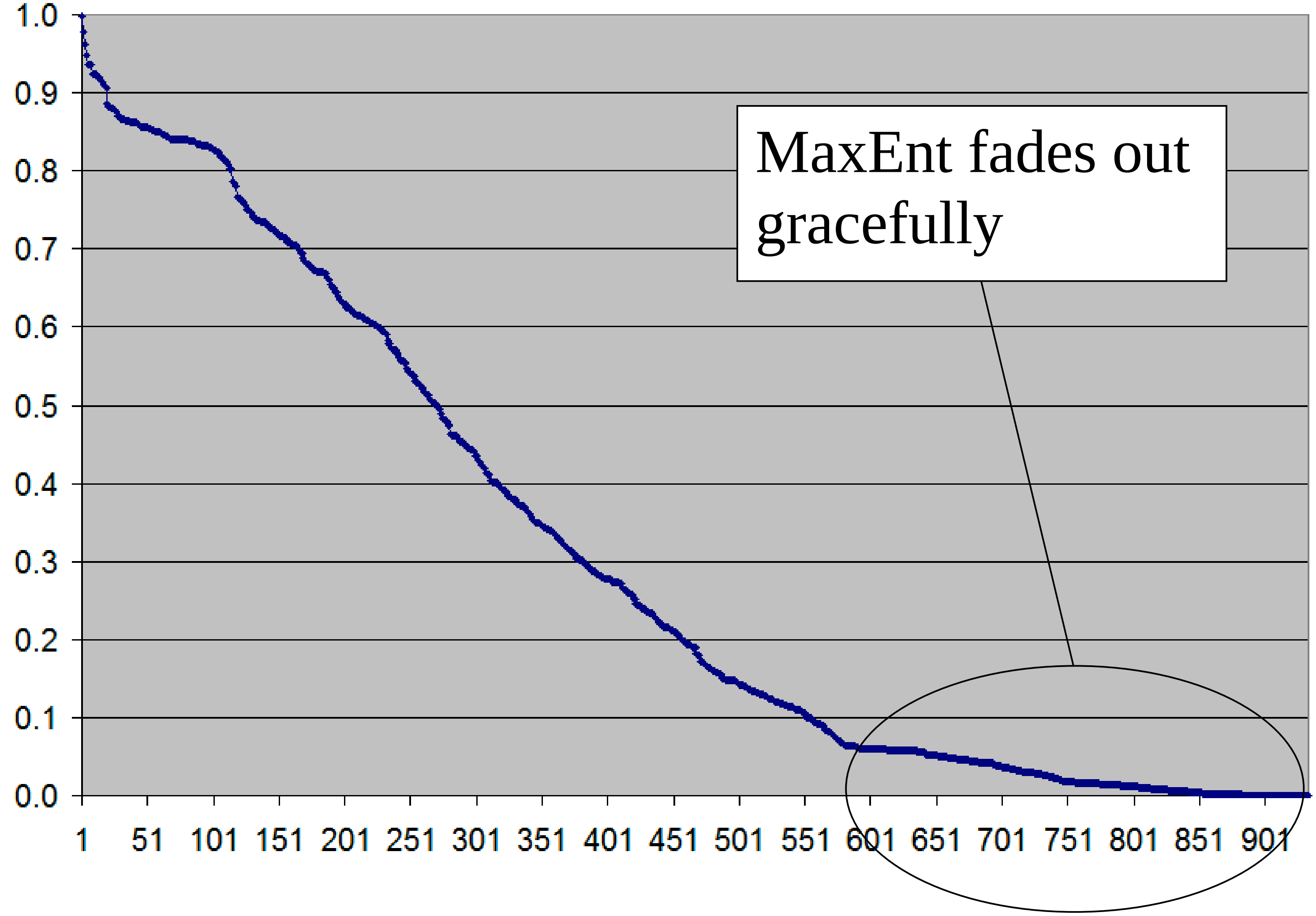
| | |
|---|---|
| MaxEnt grammar: | r = 0.843 |
| Stochastic OT grammar: | r = 0.841 |

- Nevertheless there is reason to think the maxent model is doing better:
  - the Stochastic OT/GLA model **assigns zero probability to too many settings**.

# *5.16 GLA predicted frequencies, sorted descending, for all settings volunteered by consultants*



# *5.17 MaxEnt predicted frequencies, sorted descending, for all settings volunteered*



# *5.18 Why does Stochastic OT/GLA gives zeros to so many attested settings?*

- Clearest answer: **ranking errors**
  - Some constraint pairs must be given **very close ranking values**, because they jointly determine common patterns of free variation.
  - But the GLA assigns them **very distant** values, corresponding to strict ranking.

# *5.19 Example of stochastic OT grammar failure*

- A common type of free variation (Hayes, 2005) requires free ranking of
  - STRONG IS LONG (give more time to strong beats)
    RESOLUTION (give little time to non-final stressed syllables)
- Tableau follows.

# *5.20 Tableau: a common kind of free variation*

```
     x       x          x        x
x    x   x   x    x     x    x   x
x  x x x x x x x  x  x  x x  x x x x
|  | |   |   | |  |     |    |   |
```

```
     x       x          x        x
x    x   x   x    x     x    x   x
x  x x x x x x x  x  x  x x  x x x x
|  | |   |   |    |  |  |    |   |
```

| | RESOLUTION | STRONG IS LONG |
|---|---|---|
| Such a pret-ty **sto**ry you soon shall hear | | * |
| Such a pret-ty **sto-** ry you soon shall hear | * | |

## *5.21 Ranking failure*

- The GLA placed these constraints very far apart:

| | |
|---|---|
| RESOLUTION | -3278.7 |
| STRONG IS LONG | -3328.9 |

(~ 50 units), and thus couldn't derive the second free variant.

- Since this variation pattern is common, this is a major source of the error of assigning too many zeros.

# *5.22 MaxEnt grammar does ok with these lines*

```
Line                                              Probability

     x       x          x        x
x    x   x   x    x     x    x   x
x  x x x x x x x  x  x  x x  x x x x
|  | |   |   | |  |     |    |   |
Such a pret-ty  story you   soon shall hear           .273
     x       x          x        x
x    x   x   x    x     x    x   x
x  x x x x x x x  x  x  x x  x x x x
|  | |   |   |    |  |  |    |   |
Such a pret-ty  sto-  ry you soon shall hear          .123
```

## *5.23 Another possible problem*

- The problem just noted was a problem with the GLA, trying to find the right ranking.
- But is there a Stochastic OT grammar that works *at all*??
- There would be none, if **harmonically bounded** candidates should be able to emerge with positive (though non-maximal) scores.
- Various constraint-based theories (Keller 2001, 2006; Coetzee 2006) do permit harmonically bounded candidates to win; MaxEnt is among them.

# *5.24 Harmonically bounded candidates in MaxEnt*

- MaxEnt allows harmonically bounded winners, though never with the highest frequency.
  - Simplest example, with just one constraint:

| Input | Cand. | Predicted Freq. | ME Score | C1 |
|---|---|---|---|---|
| | | | | 2.2 |
| Input | Cand1 | 0.9 | 0 | |
| | Cand2 | 0.1 | 2.2 | * |

## *5.25 Do harmonically bounded candidates win in textsetting?*

- About 11% of the settings volunteered by the consultants are not in the OT factorial typology of the constraint set.
- MaxEnt grammar gives most of these modest scores, averaging 0.04.
- Superficial implication: nonoptimal candidates can win, supporting MaxEnt.
- But this is *extremely tentative*—the problem could lie with the constraint set.

## *5.26 Summarizing*

- So far, MaxEnt is emerging as the better tool for my analytic purpose, due to:
  - Greater accuracy
  - Perhaps, its indulgence of harmonically bounded candidates

- But before drawing any conclusions we should try to learn more by
  - trying other stochastic ranking algorithms
  - exploring more possibilities for the constraint set.

# 6. Phonotactic learning with MaxEnt

## *6.1 My project with Colin Wilson*

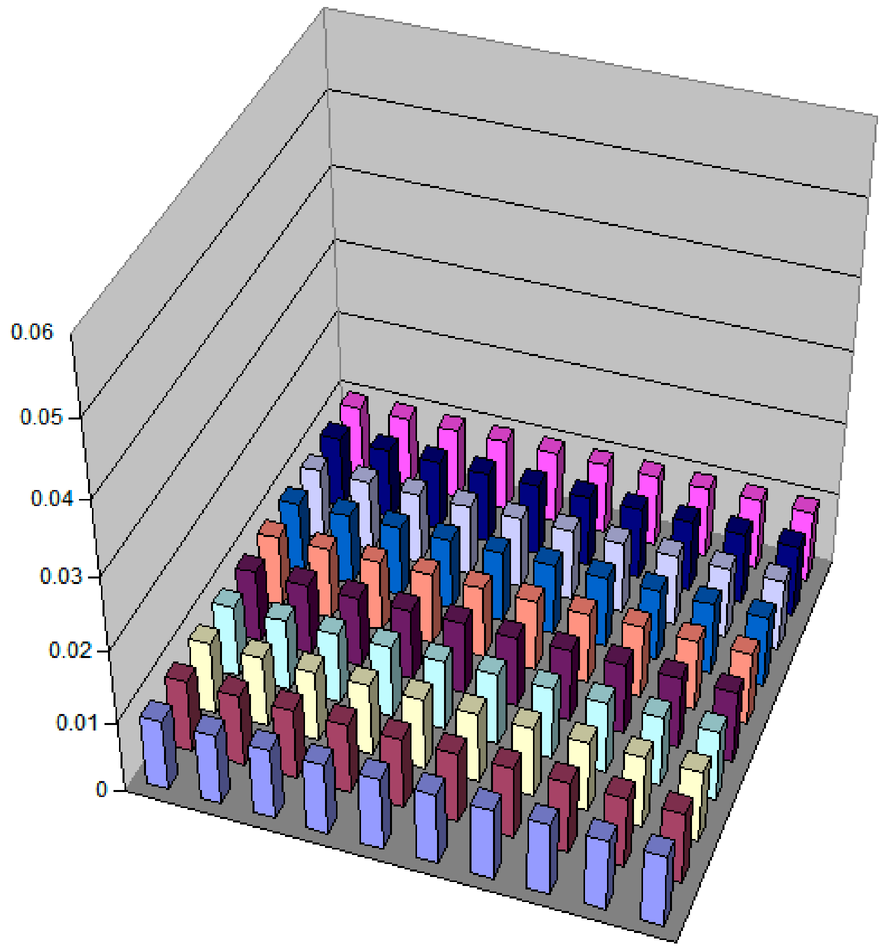
- We seek to produce an automated system that, examining representative phonological forms from languages will
  - learn a set of phonotactic constraints
  - weight them under the principles of MaxEnt
  - make accurate gradient predictions about the phonotactic well-formedness of any novel form

# 6.2 *Two kinds of probability distribution in phonology*

- Assume an input, and find the probability of possible corresponding outputs.
- What we do: assign probability **to all forms**.
  - Any one form will have an ultra-low probability, but the differences that exist among the ultra-low can be large and meaningful.
  - The problem of $\infty$ (unbounded string lengths) can be dealt with, for example by limiting the strings to (roughly) the length of those found in the learning data.

# *6.3 Graphic Illustration*

- Imagine the space of conceivable forms (here, just 100) to have equal *a priori* probability:



- A set of weighted phonological constraints penalizes various subsets. Here are four (schematic) ones, with their weights:

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | * | * | * | * | * | * | * | * | * | * |
| 2 | * | * | * | * | * | * | * | * | * | * |
| 3 | * | * | * | * | * | * | * | * | * | * |
| 4 | | | | | | | | | | |
| 5 | | | | | | | | | | |
| 6 | | | | | | | | | | |
| 7 | | | | | | | | | | |
| 8 | | | | | | | | | | |
| 9 | | | | | | | | | | |
| 10 | | | | | | | | | | |

Weight: 3

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | * | * | * | * | | | | | | |
| 2 | * | * | * | * | | | | | | |
| 3 | * | * | * | * | | | | | | |
| 4 | * | * | * | * | | | | | | |
| 5 | * | * | * | * | | | | | | |
| 6 | * | * | * | * | | | | | | |
| 7 | * | * | * | * | | | | | | |
| 8 | * | * | * | * | | | | | | |
| 9 | * | * | * | * | | | | | | |
| 10 | * | * | * | * | | | | | | |

Weight: 4

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | | | | | | | * | * | * | * |
| 2 | | | | | | | * | * | * | * |
| 3 | | | | | | | * | * | * | * |
| 4 | | | | | | | * | * | * | * |
| 5 | | | | | | | * | * | * | * |
| 6 | | | | | | | * | * | * | * |
| 7 | | | | | | | * | * | * | * |
| 8 | | | | | | | * | * | * | * |
| 9 | | | | | | | * | * | * | * |
| 10 | | | | | | | * | * | * | * |

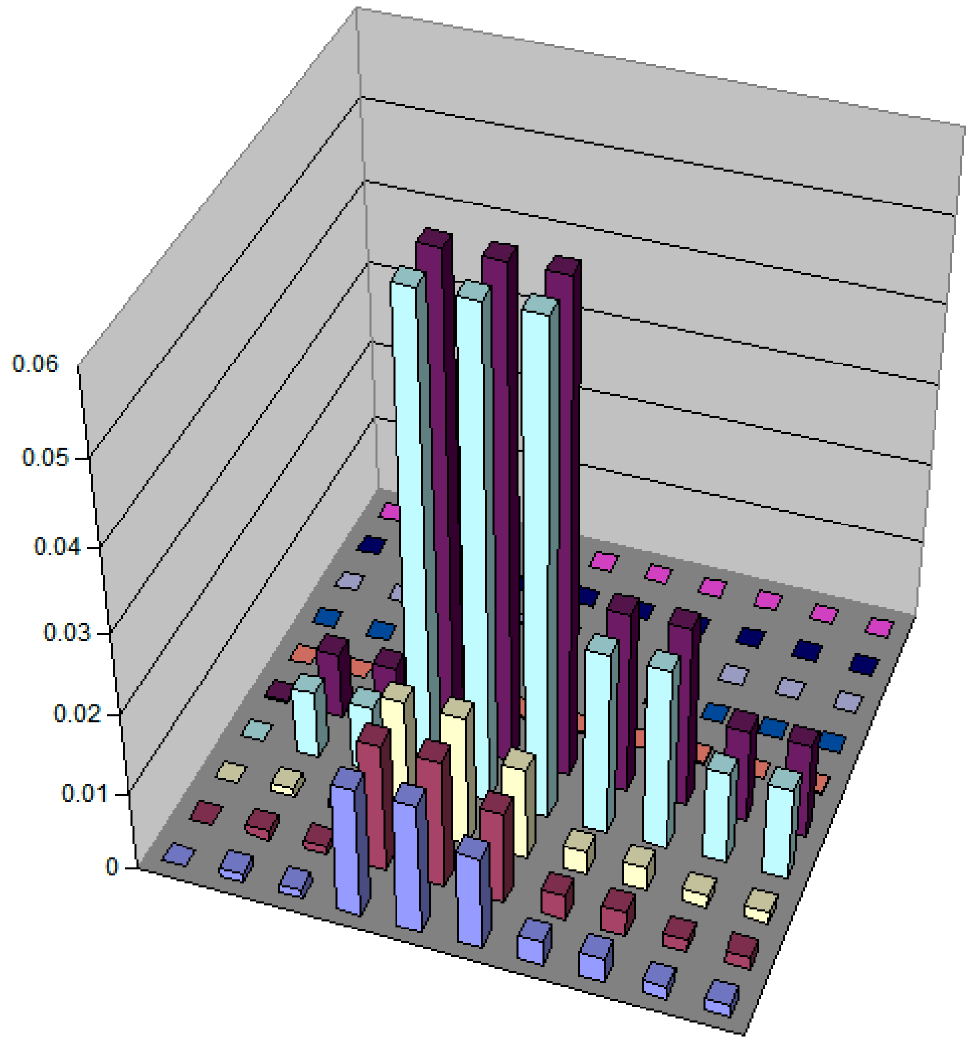
Weight: 2

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | | | | | | | | | | |
| 2 | | | | | | | | | | |
| 3 | | | | | | | | | | |
| 4 | | | | | | | | | | |
| 5 | | | | | | | | | | |
| 6 | | | | | | | | | | |
| 7 | | | | | | | | | | |
| 8 | | | | | | * | * | * | * | * |
| 9 | | | | | | * | * | * | * | * |
| 10 | | | | | | * | * | * | * | * |

Weight: 1

## *6.4 Apply the MaxEnt formula*

- This was: $\frac{\Sigma_x \exp(-\Sigma_\mathrm{i} \mathrm{w}_\mathrm{i} \mathrm{C}_\mathrm{i}(x))}{\mathrm{Z}}$ ; slide 19 above)
- We will obtain a probability for every form.
- With other constraints not shown here added in, the graph of probability now looks like the next slide.




- Probability has been reassigned, gradiently, to a small subset of all possible forms.

## *6.5 Other Hayes/Wilson agenda items*

- Eschew a UG that has all the constraints in it; instead learn them; using a much more modest UG as starting point.
- Test the learnability implications of phonological theories (e.g. autosegmental, metrical): do they make systems learnable that would otherwise not be?

## *6.6 Sample simulation: English Onsets*

- Training set, from the CMU Online Pronouncing Dictionary:

  k 2764, r 2752, d 2526, s 2215, m 1965, p 1881, b 1544, l 1225, f 1222, h 1153, t 1146, pr 1046, w 780, n 716, v 615, g 537, dʒ 524, st 521, tr 515, kr 387, ʃ 379, gr 331, tʃ 329, br 319, sp 313, fl 290, kl 285, sk 278, j 268, fr 254, pl 238, bl 233, sl 213, dr 211, kw 201, str 183, θ 173, sw 153, gl 131, hw 111, sn 109, skr 93, z 83, sm 82, θr 73, skw 69, tw 55, spr 51, ʃr 40, spl 27, ð 19, dw 17, gw 11, θw 4, skl 1

# 6.7 Grammar fabricated: 23 constraints

| *Constraint* | *Wght* |
|---|---|
| 1. *[+son,+dors] | 5.64 |
| 2. *[+cont,+voice,–ant] | 3.28 |
| 3. *[^–voice, +ant, +strid][–approx] | 5.91 |
| 4. *[ ][+cont] | 5.17 |
| 5. *[ ][+voice] | 5.37 |
| 6. *[+son][ ] | 6.66 |
| 7. *[–strid][+cons] | 4.40 |
| 8. *[ ][+strid] | 1.31 |

| *Constraint* | *Wght* |
|---|---|
| 9. *[+lab][^+approx, +cor] | 4.96 |
| 10. *[–ant][^+approx, –ant] | 4.84 |
| 11. *[+cont,+voice][ ] | 4.84 |
| 12. *[–cont,–ant][ ] | 3.17 |
| 13. *[ ][–back] | 5.04 |
| 14. *[+ant,+strid][–ant] | 2.80 |
| 15. *[+spread][^+back] | 4.82 |
| 16. *[+cont,+voice,+cor] | 2.69 |

| *Constraint* | *Wght* |
|---|---|
| 17. *[+voice][^+approx, +cor] | 2.97 |
| 18. *[+cont, –strid][^+approx, –ant] | 2.06 |
| 19. *[ ][^–cont, –voice, +lab][+cons] | 3.05 |
| 20. *[ ][+cor][^+approx, –ant] | 2.06 |
| 21. *[+cont,–strid] | 1.84 |
| 22. *[+strid][–ant] | 2.10 |
| 23. *[–cont, –voice, +cor][^+approx, –ant] | 1.70 |

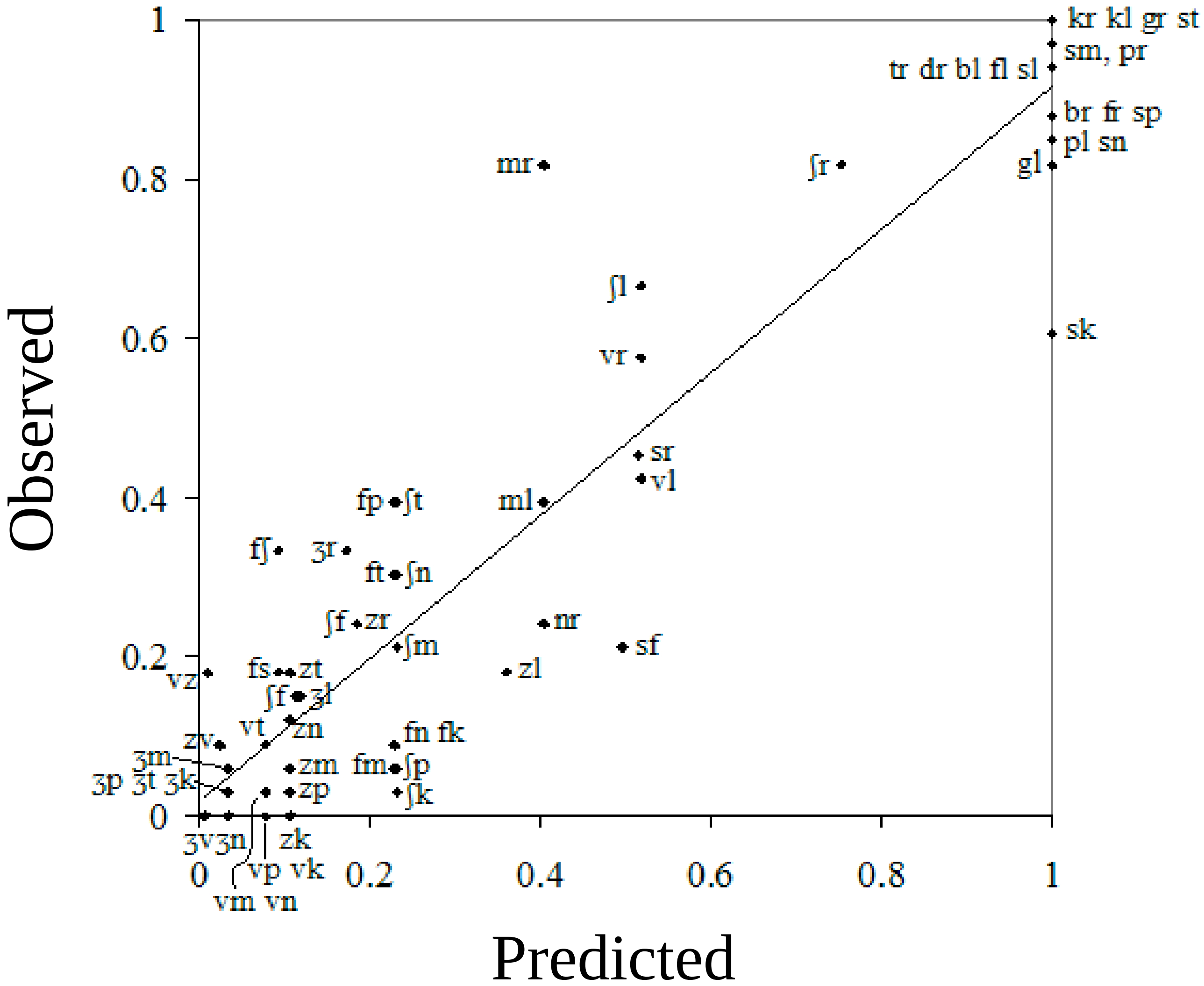
## *6.8 Testing the grammar*

- Experimental data from Scholes (1966)
- 33 subjects rated 66 monosyllabic nonce words, with ordinary syllable rhymes; independent variable was the onset.

## 6.9 Results

- Predictions of our grammar correlate well with the Scholes data, r = 0.946
- This outperforms all other approaches we tried for comparison (e.g. Coleman and Pierrehumbert 1997, *n*-gram model from Mohri 2002, Allauzen et al. 2005)

# *6.10 Scattergram: rescaled model predictions vs. Scholes data*



## *6.11 Larger scale work*

- We have analyzed the complete phonotactics of Wargamay (Australian, Dixon 1981), showing that we can fully cover at least the simpler phonotactic systems of languages.

# *6.12 Some phonotactic learning algorithms using Optimality Theory*

- Hayes 2004, Prince and Tesar 2004, Jarosz 2006
- This work assumes the standard OT approach of the **Rich Base**: a legal form is one that can derived from any input.
- Why must things be done this way? [or, why do I think this…] Because OT is based inherently on a comparison of alternatives.

# *6.13 Why the OT/Rich Base scheme may be inappropriate to phonotactic learning*

- Problems of **search space size**: we aren't just rating the forms, but any form as derived from any underlying representation.

- Previous work seem to suffer from this:

  - Hayes (2004), Prince and Tesar (2004): idealize to non-gradient learning
  - Jarosz (2006): makes a gradient system the goal, and uses the same basic strategy (maximum likelihood estimation) as Hayes/Wilson. But search space is the full set of rankings (factorial in size).

- All three: examples are schematic, not real-language.

## *6.14 Rating forms in isolation goes out on a limb*

- The idea of assigning probability just to forms (as opposed to the outputs for an input) raises many further questions—e.g., how to relate phonotactics to alternations.

# 7. Conclusions

## *7.1 Analysis of gradience in phonology cannot be taken as peripheral*

- Gradient phenomena are pervasive.
- The question of how to analyze gradience quickly moves us into the question of choice of framework, with major implications for nongradient phonology.

# *7.2 Some possible reasons for favoring a Maximum Entropy approach*

- Accuracy and trustability of its affiliated weighting algorithm.
- Perhaps: ability to assign modest probabilities to harmonically bounded candidates
- Ability to form phonotactic grammars without the use of the Rich Base principle and its accompanying search-space problem

Thank you

- Comments and afterthoughts to: bhayes@humnet.ucla.edu

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