QUAL – DAY 2
Friday, January 6th

General Physics II
Quantum Mechanics II
Statistical Mechanics
Electromagnetism
1. At the end of World War II the US published a document acknowledging the existence of the Manhattan project and giving whatever information was deemed safe to give away. In particular there was the sequence of three photographs of the Trinity Test, the first nuclear explosion detonated at the White Sands desert not far from Alamogordo, New Mexico. The device was an implosion-type plutonium weapon, more advanced than the bomb that destroyed Hiroshima a few weeks later. It was exploded from a tower and the photos show the fireball at different times, together with, in one photogram, a ruler with the linear scale. This turned out to be a
dumb move: a Soviet physicist, by the name of Sedov, saw the photos and with some dimensional analysis estimated in a few minutes the order of magnitude of the energy released in the explosion that was supposed to be secret!

a) Find the “Sedov solution” to the problem and, knowing that the density of air is 1.3 kg/m³ and 1 kton of TNT releases \( \sim 5 \times 10^{12} \) J, figure out the yield (in ktons) of the Trinity test. (In case the copy did not come out sharp the ruler in the photograms is 100 m long (they should have used feet to confuse the Soviets!), and the time on the second frame is 0.016 s.)

b) Do you expect your answer to be an overestimate or an underestimate of the real yield of the device? Why?

2. Consider the free oscillations of a drop of liquid. This is somewhat unfamiliar to us because we live on the earth where gravity makes “free drops of liquid” fall down pretty fast... but I have seen a movie shot by some astronaut where they try to catch a big drop of water while in orbit and, indeed, the drop does pulsate (b.t.w. I have always thought this to be a reckless exercise... what if the water would get into some instrument !!!).
oscillating drop

a) Use dimensional analysis to find the period of oscillations. This is a famous problem, it was solved first by Lord Rayleigh and published in Nature 95 (1915) 66 (you may find it interesting to look this old paper up in the library; it is missing from the physics library but, apparently, Green has it).

b) Use your solution to compute the ratio between oscillation frequencies for drops of water and mercury. (Densities are 1 g/cm³ and 13.6 g/cm³ for water and mercury. Surface tensions are 0.073 N/m and 0.487 N/m for water and mercury.)
Quantum Mechanics 2

A particle is incident on a barrier (formed by the y-axis) with momentum $k_1$ and energy $E = \frac{h^2 k_1^2}{2m}$, at an angle $\alpha$, and is partially reflected with momentum $k_2$ at angle $\beta$, and partially penetrates to the region $x > 0$, moving there with momentum $k_3$ at angle $\gamma$. The potential energy of the particle is $V = 0$ to the left of the barrier (at negative $x$), and $V_0$ to the right of the barrier (at positive $x$).

The wave function of the particle is

$$
\Psi = e^{ik_1x} + R e^{ik_2x} \text{ at } x < 0 ,
$$

$$
\Psi = T e^{ik_3x} \text{ at } x > 0 .
$$

1) Find angles $\beta$ and $\gamma$ and momenta $k_2$ and $k_3$ as functions of $k_1$, $V_0$ and $\alpha$.

2) Find the reflection and transmission coefficients $R$ and $T$.

3) Describe all cases where a complete reflections occurs (for $V_0 > 0$ and for $V_0 < 0$).
A typical grad student office can be approximated by a cube one meter on a side. Imagine that there are $N$ gas molecules in this office in thermal equilibrium. Ignore interactions between molecules. Construct a three dimensional rectilinear coordinate system with origin at the center of the cube. What is the probability that at a given time all the molecules will be in the octant $x > 0, y > 0, z > 0$? Explain your reasoning. Imagine that you can measure these positions once a second. For approximately what $N$ will the molecules be in this octant once during the current age of the universe? Do you think it is necessary to keep a gas mask in your office to guard against this eventuality?

Suppose the office is connected by tube to a reservoir so that the number of atoms $N$ can fluctuate around an average value. On general grounds what is the $N$ dependence of the size of these fluctuations. Ignoring coefficients, estimate the value of $N$ for which the size of fluctuations is approximately one percent of the mean.

Consider a system described by the canonical ensemble. Here the energy $E$ fluctuates around a mean value $< E >$. Show that the mean squared fluctuation $< (E - < E >)^2 >$ is given by

$$< (E - < E >)^2 >= -\beta^2 \log Z/\partial \beta^2 = k_B T^2 C_V$$

where $Z$ is the canonical partition function, $T$ is the temperature, $k_B$ is Boltzmann’s constant, $\beta = 1/k_B T$, and $C_V$ is the heat capacity.

Using this formula give a qualitative argument why the root mean squared energy fluctuations are small compared to the average energy if the system is large.
Electromagnetism

A solid rod of metal with length $L$, radius $a$, and conductivity $\sigma$, is placed inside of a solenoid that generates a magnetic field $B(t)$ that varies with time at a rate $dB/dt$. The magnetic field is parallel to the axis of the metal rod as shown in the diagram below.

(a) In terms of the variables given above, determine the rate of energy dissipation in the metal rod. (Ignore the effects of skin depth.)

(b) Suppose the rod in part (a) is divided up into $N$ rods, each of length $L$, with the same total volume. What is the rate of energy dissipation in terms of $N$ and the answer found in part (a)?

(c) Now suppose $B(t) = B_o \sin(\omega t)$. Estimate how high the frequency $\omega$ can be and have the approximation made in part (a) remain valid.