Supplementary Information for Coulomb Blockade in an Open Quantum Dot


1Department of Physics, Stanford University, Stanford, California 94305, USA
2Department of Applied Physics, Stanford University, Stanford, California 94305, USA
3Department of Physics, Harvard University, Cambridge, Massachusetts 02138, USA
4Department of Condensed Matter Physics, Weizmann Institute of Science, Rehovot 96700, Israel

I. QUANTUM DOT AND CHARGE SENSOR CONDUCTANCE MEASUREMENTS

We measure the large quantum dot by placing a small oscillating voltage on top of the DC bias voltage and measuring the resulting current with a DL Instruments Model 1211 current pre-amplifier and a Princeton Applied Research 124A lock-in amplifier (the circuit for the large dot is sketched in Fig. 1(a) in the main text). For the measurements of the large dot we use an oscillation frequency of 17 Hz and an excitation voltage $V_{\text{exc}}$ from 1 to 5 $\mu$Vrms. We use a separate but identically constructed circuit to measure the charge sensor. For the charge sensor measurements we use a frequency of 97 Hz and an excitation voltage of 5 $\mu$Vrms.

II. QPC CONDUCTANCE PLATEAUS

![Conductance measurements of QPC 1 (a) and QPC 2 (b) at 13 mK and 600 mK. The 600 mK plateaus have been shifted horizontally for clarity.](image)

FIG. S1. (a) Conductance measurements of QPC 1 (a) and QPC 2 (b) at 13 mK and 600 mK. The 600 mK plateaus have been shifted horizontally for clarity.

Measurements of the individual QPCs show clear conductance plateaus quantized at integer multiples of $2 e^2/h$, demonstrating that the QPCs do not have spurious resonances that could cause the observed oscillation in the dot conductance. Figure S1(a) shows the conductance of QPC 1, formed by the gates bw1 and n, with no voltage applied to any other dot gates. These data show plateaus at $G = 2 e^2/h$ and $4 e^2/h$. Figure S1(b) shows similar data for QPC 2, with voltages applied only to the gates bw2 and n. The figure also shows the results of measuring the QPCs at 600 mK. The slope of the increase in conductance between the plateaus at low and high temperatures is approximately equal, indicating that the dominant energy scale for the opening of a new mode in the QPC is greater than $\approx 50 \mu$eV.

For the measurements in the main text, the device was cooled to 4 K with a positive bias of $\approx +200$ mV applied to all gates. The positive bias “pre-depletes” the gates, preventing us from characterizing the individual QPCs in isolation even if all other gates are set to 0 V. The measurements in Fig. S1(a) and (b) have been performed after all the dot measurements reported in the paper, and following a partial thermal cycle to a temperature on the order of or greater than 100 K to reduce the effects of the positive bias voltage applied when the dot was initially cooled down to 4 K. The measurements in Fig. S1(a) and (b) were taken with a small excitation voltage and little averaging, which causes the noise on the measurement.

III. ESTIMATION OF THE REFLECTION COEFFICIENT

Through careful analysis of the temperature dependence of the average dot conductance at finite magnetic field we have determined that the reflection coefficients $r^2$ of the QPCs are on the order of 2% or less. The analysis is discussed below.
FIG. S2. Temperature dependence of the change in the ensemble-averaged dot conductance at finite magnetic field. The solid and dashed lines are fits discussed in the text.

For the work reported in this paper we have carefully tuned both QPCs to their $2 e^2/h$ plateaus so that there is one fully-transmitting spin-degenerate mode in each QPC. However, even at the optimal QPC settings there may still be a small reflection coefficient in the QPCs. This reflection coefficient affects the dot conductance, and this can be observed in the conductance at finite magnetic field where weak localization is absent. To extract the conductance at finite field, we analyze measurements of the ensemble-averaged dot conductance as a function of magnetic field like that shown by the dotted line in Fig. 4(b) of the main text. In these data we see the weak localization dip at $B = 0$, and following Huibers et al.\(^1\) we fit this dip to a Lorentzian:

$$\langle G_{\text{dot}}(B) \rangle = \langle G_{\text{dot}} \rangle_{B \neq 0} - \frac{A}{1 + (2B/B_c)^2}$$

where $\langle G_{\text{dot}} \rangle_{B \neq 0}$ is the average conductance at finite field, and $A$ and $B_c$ are the depth and width of the weak localization dip, respectively. The temperature dependence of $\langle G_{\text{dot}} \rangle_{B \neq 0}$ depends on the reflection coefficient of the QPCs.

To characterize the temperature dependence, we find the difference

$$\delta \langle G_{\text{dot}} \rangle = \langle G_{\text{dot}} \rangle_{B \neq 0}(T) - \langle G_{\text{dot}} \rangle_{B \neq 0}(T_0)$$

where $T_0 = 435$ mK and this quantity is plotted in Fig. S2. We see there is a very small temperature dependence of the dot conductance. There is no explicit theoretical prediction for the temperature dependence of a coherent quantum dot with one fully-transmitting spin-degenerate mode in each QPC (this is the $N = 4$ case, where $N$ is the total number of transmitting channels in both QPCs). However, for a coherent dot with $N \gg 1$ Brouwer et al.\(^2\) have calculated the temperature dependence and find that it is the same as that for an incoherent dot\(^3\) with $N \gg 1$, and is given by:

$$\delta \langle G_{\text{dot}} \rangle = c e^2 h \left[ -r^2 \ln \left( \frac{T_0}{T} \right) \right]$$  \hspace{1cm} (S1)

In this equation the reflection coefficients $r^2$ of the two QPCs are assumed to be equal and are defined by $G_{\text{QPC}} = (2 e^2/h)(1 - r^2)$. The solid red line in Fig. S2 shows the results of fitting the data to this equation, and it appears the $N \gg 1$ theoretical prediction gives a decent fit for $N = 4$. From the fit we obtain $r^2 \approx 0.02$.

Furusaki and Matveev\(^4\) have calculated the temperature dependence for an incoherent dot with $N = 4$ and found

$$\delta \langle G_{\text{dot}} \rangle = c e^2 h \left[ -4r^2 \frac{\Gamma(\frac{3}{4})}{\Gamma(\frac{1}{4})} \sqrt{\frac{\gamma U}{k_B\pi}} \right] \left( T^{-1/2} - T_0^{-1/2} \right)$$  \hspace{1cm} (S2)

In this equation $\gamma = \exp(C)$ where $C = 0.5772\ldots$ is Euler’s constant. The dashed blue line shows a fit to this equation and we obtain $r^2 \approx 0.007$. As noted, neither theory is perfectly suited to our circumstances, but these fits allow us to conclude that the reflection coefficients of the QPCs are small (on the order of 2% or less) and hence effects that are higher order in $r^2$ should be suppressed.

IV. MAGNETIC FIELD DEPENDENCE FOR FULL AND PARTIAL QPC TRANSMISSION

To explicitly demonstrate the different magnetic field dependence of Coulomb oscillations caused by coherent backscattering and those caused by the reflection coefficients of the QPCs, we intentionally induce a finite reflection
coefficient in both QPCs by making the voltage on gate n more negative. These data are shown in Fig. S3. In region B the QPCs are fully transmitting and the oscillation amplitude decreases quickly with increasing field. However, in region A where we have a finite reflection in both QPCs, we see that the oscillations depend only weakly on magnetic field.

![Graph showing dot conductance at several different magnetic fields.](image)

**FIG. S3.** Dot conductance at several different magnetic fields. For $-330 \lesssim V_n \lesssim -300$ mV (region B) the QPCs are fully transmitting while for $V_n \lesssim -330$ mV (region A) the QPCs are partially transmitting.

## V. DETERMINING THE RENORMALIZED CHARGING ENERGY

In this section, we describe how we analyze the data in Fig. 2(b) in the main text to extract the renormalized charging energy $U^* \approx 16 \mu$eV near $V_n = -315$ mV. This value is used to determine $C_{d,tot}^* = e^2/U^*$ which is an input for the fit in Fig. 3(b) of the main text.

In Fig. 2(b) of the main text the diamonds are on top of a background conductance caused by Fabry-Perot interference of electrons in the big dot, making determination of the charging energy more difficult. The procedure for subtracting this background is demonstrated in Fig. S4.

Figure S4(a) shows the data from Fig. 2(b) in the main text. To isolate the background Fabry-Perot pattern, we smooth the data by averaging in gate voltage over the period of the Coulomb diamonds, $\Delta V_n \approx 3$ mV. The result of this averaging is shown in Fig. S4(b). We then subtract these averaged data from the raw data in Fig. S4(a) to isolate the oscillation. The result is shown in Fig. S4(c), with dashed white lines as guides to the eye. From the Coulomb diamonds, we find a renormalized charging energy $U^* \approx 16 \pm 4 \mu$eV at $V_n = -315$ mV.

The shape of the Coulomb diamonds is different from that typically observed in closed dots (e.g., for $V_n < -375$ mV in Fig. 2b of the main text). This unfamiliar shape occurs because the strong coupling renormalizes the total capacitance by greatly increasing the capacitance to the leads. To qualitatively understand the diamond shape, we use a simple model of the dot consisting of levels separated by $U^*$ that have a Lorentzian lineshape with width $\Gamma$. For the capacitance of the gate we use $C_n = 60$ aF and for the renormalized charging energy we use $U^* = 20$ $\mu$eV, which is within the error bar of our extraction of $U^*$ above. For the renormalized capacitance of the lead we use
FIG. S4. (a) Conductance as a function of $V_n$ at 13 mK (with charge sensor active) and $B = 0$. The QPCs each have a fully transmitting mode at $V_n = −315$ mV. (b) Result of averaging (a) over $\Delta V_n \approx 3$ mV, which is one period of the Coulomb diamonds. The averaging leaves only the background Fabry-Perot resonance. (c) Result of subtracting the background in (b) from the data in (a). These subtracted data allow us to identify Coulomb diamonds (dashed white lines are guides to the eye) and to extract a renormalized charging energy $U^*$. (d) Calculation of Coulomb diamonds using a simple model discussed in the text. The calculation qualitatively reproduces some of the features in (c).

$C_{\text{lead}}^* \approx C_{\text{d,tot}}^*/2 \approx 4000$ aF, where $C_{\text{d,tot}}^* = e^2/U^*$. We also use $\Gamma = 4 \mu eV + 0.35 eV_{ds}$, where 0.35 $eV_{ds}$ is a phenomenological term that accounts for the broadening of the states from electron dephasing at finite bias as well as for electron heating in the dot. The results of the calculation are shown in Fig. S4(d), and they qualitatively agree with the data in Fig. S4(c). The differences between the calculation and the data are not surprising given the limitations of our model: the lineshape in a strongly coupled dot is not Lorentzian, we do not account for the contribution of the level spacing to the conductance, and the level broadening is only included through a linear factor, and not through a proper model.
VI. CHARGE SENSING FITS

In this section we give a brief introduction to our charge-sensing technique and describe how we fit the charge-sensing data in the range \( V_n \leq -385 \text{ mV} \) in Fig. 3 of the main text to determine the capacitance ratio \( R_d = \frac{C_{d,CS}}{C_{sp,CS}} \approx 0.93 \pm 0.21 \). Here \( C_{d,CS} \) is the capacitance between the large dot and the charge sensor, and \( C_{sp,CS} \) is the capacitance of gate sp to the charge sensor. This ratio is used as an input for the fits in the other two \( V_n \) ranges discussed in the main text.

The sensitivity of our quantum dot charge sensor is \( 1.7 \times 10^{-4} \, e/\text{Hz}^{1/2} \) referenced to the detector. This is a factor of three less sensitive than Berman et al.\(^5\) and Duncan et al.\(^5\). To increase our charge sensitivity we average together multiple charge-sensing measurements taken over the same range of \( V_n \). One potential difficulty in averaging together the data is that background charge fluctuations in the donor layer can cause small shifts of the Coulomb blockade peaks in \( V_n \). When the QPCs are in the tunneling regime and the charge-sensing signal is large, it is easy to identify these shifts and correct for them by aligning the charge sensor response between successive traces. However, when the QPCs are more transmitting, the charge-sensing signal is small and detecting these shifts is difficult. This is where we take advantage of our ability to simultaneously measure charge and transport. The Coulomb oscillations are clearly visible in transport over the entire range of QPC transmission, and we can use our transport measurements to identify the small shifts caused by the background charge. By measuring the background-induced shifts from our transport data, we are able to correctly align the charge-sensing data and average the data together. The number of traces we average depends on the strength of the charge-sensing signal. In the region where the QPCs are fully transmitting and the charge-sensing signal is weakest, we average 300 traces in order to increase our sensitivity by a factor of 17.

![Fig. S5](image_url)  
**Fig. S5.** (a) Small dot Coulomb blockade peak used for charge sensing. We convert the change in the conductance \( \Delta G_{CS} \) of the charge sensor into an effective voltage change \( V_{\text{eff}} \). (b) Simultaneous measurement of charge-sensing signal \( V_{\text{eff}} \) (left axis) and transport (right axis) in the large dot, when the conductances of both QPCs are less than \( 2 \, e^2/h \).

Figure S5 shows how we analyze the change in the conductance of the charge sensor. When we make the voltage on a dot gate less negative, this increases the charge on the large dot. The conductance of the adjacent charge sensor is affected by the electric fields from both the gate and the additional charge on the large dot. We denote the resulting change in the charge sensor conductance \( \Delta G_{CS} \). Figure S5(a) shows how we convert \( \Delta G_{CS} \) into an effective voltage change \( V_{\text{eff}} \), which if applied to the gate sp would produce the same \( \Delta G_{CS} \). When an electron is added to the large dot, we expect a decrease in \( V_{\text{eff}} \). This is illustrated by the data in Fig. S5(b), which are taken in the regime where the conductances of both QPCs are less than \( 2 \, e^2/h \) and we have well-defined Coulomb blockade. The measurement of \( G_{\text{dot}} \) (black trace, right axis) shows a clear Coulomb blockade peak. A simultaneous measurement of the charge sensor (blue trace, left axis) shows that \( V_{\text{eff}} \) initially increases as \( V_n \) is made less negative because of the capacitance between the gate and the charge sensor. However at the value of \( V_n \) where an electron is added to the dot, there is a sharp decrease in \( V_{\text{eff}} \). To highlight the correspondence between the decrease in \( V_{\text{eff}} \) and the peaks in \( G_{\text{dot}} \), we study the derivative \( D = dV_{\text{eff}}/dV_n \).

To fit the charge-sensing data in Fig. 3 in the main text we use a model similar to that in Berman et al.\(^7\) illustrated in Fig. S6(a). In this diagram, \( C_{sp,CS} \) and \( C_{n,CS} \) are the capacitances of gates sp and \( n \) to the charge sensor, respectively. \( C_{n,d} \) is the capacitance of gate \( n \) to the large dot, \( C_{d,CS} \) is the capacitance between the dot and the charge sensor, and \( C_{d,\text{tot}} \) is the renormalized total capacitance of the large dot, related to the renormalized charging energy by \( U^* = e^2/C_{d,\text{tot}}^* \). Based on this model for two quantum dots\(^8\) (the large dot and the charge sensor), we can derive the dependence of \( D = dV_{\text{eff}}/dV_n \) on the capacitances, giving rise to the equation

\[
D = R_n + R_d \frac{C_{n,d} - e \, dN_d/dV_n}{C_{d,\text{tot}}^*} \tag{S3}
\]

given in the main text, with \( R_n = C_{n,CS}/C_{sp,CS} \) and \( R_d = C_{d,CS}/C_{sp,CS} \).
Figure S6(b) shows a simultaneous measurement of the charge-sensing signal and conductance for \( V_n < -385 \text{ mV} \) (magnification of the data in Fig. 3a of the main text). In this region, a calculation of the individual QPC conductances based on measurements of the dot conductance shows that one of the dot QPCs is partially transmitting, while the other is in the tunneling regime. For this configuration, theoretical work by Schoeller and Schön\(^9\) and Grabert\(^10\) predict \( dN_d/dV_n \). We fit the data in Fig. S6(b) to these theoretical predictions using equation S3. For these fits, we input \( C_{n,d} \approx 58 \text{ aF} \) estimated from the spacing of the Coulomb Blockade peaks and \( U^* \approx 115 \mu \text{eV} \) estimated from the Coulomb diamonds in Fig. 2(b) of the main text. For the Schoeller and Schön fits we input \( g = G_{\text{QPC1}} + G_{\text{QPC2}} \approx 0.8 \text{ e}^2/\hbar \) estimated from the calculation of the QPC conductances (the fits do not depend sensitively on this estimate). To estimate \( R_n \), we use the fact that in the region \(-330 \text{ mV} < V_n < -300 \text{ mV} \) when both QPCs are open, \( dN_d/dV_n \) is close to its classical value \( C_{n,d}/e \), with small perturbations from the residual charge quantization. So the average charge-sensing signal in this region is \( \langle D \rangle \approx R_n \), and from the data we estimate \( R_n \approx 0.008 \). This value is close to what we expect: from measurements of the charge sensor response to voltage changes on individual gates, we measure \( C_{n,CS} = 0.08 \text{ aF} \) and \( C_{sp,CS} = 12 \text{ aF} \), giving \( R_n = 0.007 \).

Using these values as inputs we fit the data in Fig. S6(b). The solid blue line shows a simultaneous fit to both the charge-sensing data and the transport data from the theory of Schoeller and Schön, a fit also shown in Fig. 3(a) of the main text. The solid black line shows a fit to the theoretical prediction of Grabert. The theoretical predictions agree well with the data, and from these and other fits, we extract \( R_d \approx 0.93 \pm 0.21 \). Using \( C_{sp,CS} = 12 \text{ aF} \) estimated from measurements of the charge sensor, we obtain \( C_{d,CS} \approx 11 \text{ aF} \).

The different fits return a temperature \( T \approx 54 \pm 20 \text{ mK} \), which is significantly higher than our electron temperature of 13 mK. The high temperatures extracted from the fits are caused by the Coulomb blockade peaks being broadened by the back-action of the charge sensor on the large quantum dot\(^11\). When an electron tunnels on and off the charge sensor, it “gates” the large dot and acts like a fluctuating gate voltage that affects the energy of the large dot. The

---

FIG. S6. (a) Schematic of the large quantum dot and charge sensor, showing the capacitances relevant to determining the charge-sensing signal as described by Eqn. S3. (b) Simultaneous measurement of transport and charge sensing. The solid and dashed lines are fits discussed in the text.
fluctuations of the large dot energy have energy scale \((C_{d,CS}/C_{d,tot}) (e^2/C_{s,tot}) = (C_{d,CS}/C_{s,tot}) U^* \approx 8 \mu eV\), where \(U^* = 115 \mu eV\) in this gate voltage range, and \(C_{s,tot} \approx 150 \text{ aF}\) is the total capacitance of the small dot. These fluctuations broaden the Coulomb Blockade peaks, and this broadening appears as an increased temperature in our fits: the energy scale of 8 \(\mu eV\) converts (via Boltzmann’s constant) to a temperature of 93 mK, which is on the order of the broadening that we obtain in the fits. We note that as we increase the conductance of both QPCs in the large dot, the renormalized charging energy \(U^*\) decreases and the energy scale of the back-action on the large dot grows smaller. When both QPCs are fully transmitting, \(U^* \approx 16 \mu eV\) and the energy scale of the back-action is \(\approx 1 \mu eV\), which is on the order of our temperature of 13 mK.

* samasha@stanford.edu
† Present address: Hitachi GST, San Jose, CA 95135
‡ Present address: Solyndra, Fremont, CA 94538