Device fabrication and specimen holder. We obtain our silicon nitride membrane supports from SPI company, (SPI #: 4122SN-BA) and perform subsequent processing without additional cleaning/treatment of the silicon nitride. For lithography, a bilayer of polymethyl methacrylate (molecular weights 495k and 900k dissolved to 2% in chlorobenzene, Microchem 450PMMAC2 and 950PMMAC2) is spin-coated at 6000 rpm followed by 6min hot-plate bake at 150°C, for each layer. Electron exposure (dose = ~300 µC/cm²) is performed in a Philips XL30 microscope fitted with a Nanometer Pattern Generation System (NPGS). Developing is 60 seconds in methyl-isobutyl-ketone:isopropyl-alcohol solution (Microchem M/I 1:3) without post-develop bake. Metal deposition is by electron-beam evaporation in 2 x 10⁻⁶ torr, using adjacent crystal monitor for film thickness (Ti + Pd = 2 + 28 = 30 nm). Lift-off is through immersion in acetone, followed by acetone spray. Specimen holder accommodates 400µm thick samples, with four fixed CuBe clips for in situ electrical contacts. After insertion in holder, samples are pressed against the face of the holder by a ‘backing’ clip which spans the bottom (non-membrane side) of the sample; thus, thermal anchoring to holder is through a wide area on both the top and bottom of the sample.
Hysteretic behavior of islands during thermal cycling. In our observations we note hysteresis in the melting and freezing of the individual islands. While melting of the islands proceeds uniformly, with individual islands consistently melting at the same amount of electrical power, solidification happens in a more stochastic fashion. That is to say, individual islands continue to give contrast indicative of the liquid phase despite cooling through the melting point. This supercooling has been noted in previous studies of the melting transitions of metal islands\textsuperscript{1-3}. To avoid possible complications of supercooling, all data shown are from the melting transition recorded upon device heating rather than the freezing transition. Between successive increases in bias voltage, the device is allowed to cool until all islands have returned to the solid state; this occurs rather quickly as shown in the main text.

The significance of $\alpha$ in the finite element model, as seen in equation (3). A nonzero $\alpha$, also known as the temperature coefficient of resistivity or TCR, describes the transfer of potential drop—and thus Joule heating power—from the low-T, high-$\sigma$ regions into the high-T, low-$\sigma$ regions at increased heater current. To render $\alpha$ as a fixed, dependent variable, we take advantage of the nonlinear resistance-voltage curve of our nanoscale heater wires, as shown in Figure S1. This nonlinearity is due to the self-heating of the wire and the temperature-dependent resistivity that $\alpha$ characterizes. This dependence is obtained by inverting equation (3) to give $\rho = \rho_0 (1 + \alpha \Delta T)$. It is thus strongly dependent on $\alpha$, and constraining the model to exhibit the experimentally observed curve safely gives $\alpha = 1.8 \pm 0.1 \times 10^{-3}$/K. This is similar to reported values\textsuperscript{4}, where the discrepancy with the bulk value\textsuperscript{5} of $4.2 \times 10^{-3}$/K may be attributed to grain boundary or
surface-scattering effects. Using this value, the remaining undetermined microscopic parameters are the thermal conductivity, $K$, for the metal heater wire and for the silicon nitride support membrane. As is commonly assumed for metallic systems, the thermal conductivity of the metal wire is modeled using the Wiedemann-Franz law, leaving only the silicon nitride membrane thermal conductivity undetermined.

**Silicon nitride thermal conductivity and melting point suppression in the finite element model**

As described above, the thermal conductivity of the silicon nitride membrane is left as the sole independent variable and, for simplicity, is assumed constant with temperature. The effect of the indium islands on the thermal conductivity of the silicon nitride is negligible, as we verified both by thermal modeling and by performing self-heating measurements before and after indium deposition. This thermal conductivity can then be varied in the model to find the temperature at the center of the heater wire for each of a series of applied current values. To determine the thermal conductivity, these maximum temperature values are then matched against the known heater current required to first induce melting and the known melting temperature of indium islands, \(157 \pm 10 \, ^\circ\text{C}\). Here, the uncertainty is a result of the well-characterized melting point suppression for nanoscale islands, of the size used in this study,\(^1\), which we have confirmed in separate studies using a specimen heating holder with a calibrated thermocouple. The melting temperature of each island is reproducible, and in principle could be calibrated to within 1 \(^\circ\text{C}\) or better. Instead, for simplicity, we assume that all islands have the bulk melting temperature, and treat the melting-point suppression as an experimental uncertainty. Through modeling, this translates into a $K$ value for the silicon nitride of \(3.6^{+0.5}_{-0.1} \, \text{W/m}\cdot\text{K}\).
Using this $K$, temperature maps at each heater current can be generated from the model, as shown in Fig. S2. The isothermal contours at the melting temperature (157 °C) are then assembled for each heater current value to give a composite thermal map, as shown in Fig. 2c. Other values of $K$ produce qualitatively different maps than those observed experimentally, providing further confirmation of its validity.
Figure S1: Resistance versus applied voltage curves from measurements and simulation. Larger applied voltage causes temperature to rise due to Joule heating. Resistance vs. temperature is given according to equation (3) in the main text, leaving $\alpha$ as a free parameter. Black line is experiment. Red squares are from simulation with $\alpha = 1.8 \times 10^{-3}/K$. 
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Figure S2: Simulated temperature distribution in silicon nitride membrane at 460μA applied current through Pd heater wire on surface. Isothermal contours are displayed with overlay of electrode location. Once $K$ is obtained as described above, the temperature distribution can be calculated for each applied heater current. Figure shows same area and scale as Fig. 2 of main text.
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Supporting Information References: