Section I: Experimental Details

Here we elaborate on the experimental details described for Fig. 1. The two-dimensional electron gas (2DEG) has density $1.5 \times 10^{11}$ cm$^{-2}$ and mobility $4.4 \times 10^6$ cm$^2$/Vs, measured at 4.2 K. This corresponds to a mean free path of 28 µm, meaning electron transport occurs in the ballistic regime in our experiment. The 2DEG is located 100 nm below the surface of the sample. A Ti/Au (10 nm / 20 nm) split-gate is patterned on the surface with electron-beam lithography. Application of a negative voltage, $V_g$, to these gates depletes the 2DEG below and defines the quantum point contact (QPC) [S1, S2]. We measure the differential conductance, $G$, across the sample through the QPC using standard lock-in techniques; we apply a small oscillating voltage, $V_{ac}$, and measure the current driven. The QPC dominates the resistance through the system, and the conductance serves as a measurement of the transmission of electrons from one side of the QPC to the other.

![Figure 1: Schematic of imaging technique. Metallic surface gates and tip (orange) create depletion regions (black) in the 2DEG (green) below. We measure the conductance $G$ across the sample. When the depletion disk is in a region of high electron flow, electrons are scattered back through the QPC (blue path), and we measure a drop in conductance $\Delta G$. The voltage across the sample $V_{sd}$ allows us to change the energy (and wavelength) of injected electrons.](image)

We use a home-built scanning gate microscope (SGM) to image electron flow [S3,S4]. The SGM is situated in a $^3$He cryostat with a base temperature of 350 mK and isolated from environmental vibrations. We use a commercial piezoresistive cantilever and atomic force microscope (AFM) tip. We evaporate a thin layer of metal (Cr/Au) onto the tip so that it is conducting at low-temperature. Using a piezoelectric scan tube, we position the tip ~30 nm above the surface of the sample, near the QPC. We apply a negative voltage to the tip, $V_{tip}$, creating a circular depletion region in the 2DEG below. When this depletion disk is in areas of electron flow, it scatters electrons. If an electron
is scattered back through the QPC, $G$ is reduced. If an electron is scattered, but not back through the QPC, $G$ is not measurably changed because most of the resistance is through the QPC. By scanning the tip and measuring the change in conductance, $\Delta G$, we can map the spatial distribution of electron flow. Images taken this way have been found to accurately represent the underlying current flow [S4, S5].

The SGM tip not only creates a depletion disk directly below in the 2DEG but also couples to the QPC with capacitance $C_t$. $C_t$ changes with the position of the tip. Therefore, as we scan the tip, $G$ also changes in a manner roughly dependent on the distance between the tip and QPC. Because we are not interested in this changing $C_t$ effect, we control $V_g$ based on tip position to account for changes in $C_t$. To accomplish this, before taking our image of electron flow, we scan the tip ~80 nm above the surface. With the tip this high, there is no depletion disk in the 2DEG below. However, $C_t$ with the tip ~80 nm above the surface is similar to that at ~30 nm because the distance between the tip and QPC is at least several hundred nanometers. Thus, we are able to measure the changing $C_t$ effect without introducing a depletion disk. While we scan the tip ~80 nm above the surface, we use feedback methods to record the gate voltage, $V_g$, which is necessary to keep $G$ constant. Then when we measure current flow by scanning the tip at a height of ~30 nm, we adjust $V_g$ using these previous measurements in order to keep $G$ constant until the depletion disk affects the system, as discussed in the imaging technique above.

**Section II: Interference Patterns At Different QPC Transmission Coefficients**

Here in Fig. 2 we present more details regarding Fig. 3 from the Rapid Communication. Panels (a) and (b) are the same as those in the Rapid Communication, but we have added (c)-(f) for more information. As a different way to visualize the interference patterns, (c) and (d) show the derivative of the imaging signal $\Delta G$ in (a) and (b) respectively. (c) shows the checkerboard pattern in (a) when the transmission coefficient of the QPC is $T = 1.0$. The “squares” of the checkerboard in (a) appear as clear locations at the bottom of (c). (d) shows that there is much less of a checkerboard pattern in (b) when the transmission coefficient of the QPC is $T = 0.5$. Instead there is a stronger ring pattern. At $T = 0.5$ the QPC conductance is quite sensitive to changes in potential at the QPC and therefore to the location of the tip. Vibrations due to mechanical resonances of the system appear as ripples at the very bottom of (b), which may resemble the checkerboard pattern. However, (d) clearly shows that the periodicity of these ripples is different from that in the checkerboard in (c). The periodicity of the ripples at the bottom of (d) along the fast-scan axis (horizontally) corresponds to the frequency of known vibrations. Additionally, because the ripples in (d) stem from vibrations of the system rather than interference of electron waves, the location of ripples changes from scan line to scan line. At $T = 0.5$ there may still be some indications of a checkerboard pattern because the mechanism responsible for the checkerboard can still occur. However, (d) shows that checkerboard is not as strong as in (c), while the ring structure is prominent.

Panels (e) and (f) show mechanisms that can cause checkerboard and ring interference patterns respectively. The insets are the same as the insets from the Rapid Communication, and the mechanisms are discussed there. (e) and (f) show numerical
calculations for the interference patterns generated from the different mechanisms. In both we assume the electron wavelength is that of the bulk density \( \lambda_F/2 = 32 \text{ nm} \). In (e), we allow 3 roundtrip paths of equal magnitude to interfere, as in the inset. The separation between reflection points off the QPC gates is \( d = 410 \text{ nm} \), which we estimate from a Schrödinger-Poisson calculation (SETE code). We can see that the spacing of the checkerboard is similar to that in (a) and (c). One interesting feature of this interference pattern is that the spacing between interference fringes along a line away from the QPC (i.e. along a line between green lines) can be larger than \( \lambda_F/2 \). This is due to the geometry of the 3 paths interfering and becomes more important close to the QPC. However, this geometrical effect does not fully account for our locally measured fringe spacing of 43 nm. Partial depletion due to the tip and QPC gates and density variations due to doping inhomogeneities can also contribute to this difference (see Section III below). Taking into account our larger measured fringe spacing, we estimate from our numerical calculation a lateral spacing between fringes of \( w = 47 \text{ nm} \) at the approximate location we measure \( w \) in our data. In our data, we actually measure \( w = 55 \text{ nm} \). In (f), we only allow 2 paths of equal magnitude to interfere, and we see the expected ring pattern.

**Figure 2:** Changing Transmission of QPC. (a) and (b) are the same as in the Rapid Communication except the guides to the eye for the different patterns have been removed. (c) and (d) (same color scale) show the derivative of the imaging signal \( \partial \Delta G / \partial y \) for a different visualization. (e) and (f) show numerical calculations for interference patterns generated by the different mechanisms in their respective insets. (a),(c): At \( T = 1.0 \), we observe a checkerboard pattern at the bottom of the image. (e) shows the checkerboard mechanism where reflections off both QPC gates become important close to the QPC. (b),(d): At \( T = 0.5 \), we observe a pattern with a stronger ring pattern and much less of a checkerboard. (f) shows the mechanism for rings where direct reflection from the QPC is now important.
Section III: Electron Wavelength and Density

Previous SGM experiments demonstrated that the spacing of interference fringes is an accurate measure of electron wavelength and density [S6]. In the discussion of Fig. 1 in the Rapid Communication, we refer to a measurement of an average fringe spacing of 38 nm over many fringes. We perform this measurement far from the QPC (~700 nm away and farther) and at $T = 0.5$ so that there is very little geometrical effect (discussed above in Section II). Assuming $\lambda/2 = 38$ nm implies a density of $1.1 \times 10^{11}$ cm$^{-2}$, a value ~30% lower than our measured bulk 2DEG density of $1.5 \times 10^{11}$ cm$^{-2}$. Previous SGM experiments also observed fringes that implied a density as much as 30% lower than that measured for the bulk [S3, S6]. This lower density measured from fringe spacing was attributed to partial depletion from the QPC gates and tip/cantilever. We expect gating from the QPC to have little effect on density ~700 nm away because of screening by the 2DEG (100 nm down from the surface of the sample); simulations indicate that the density ~700 nm away should be at least 95% of its non-gated value. The effects of the QPC gates can contribute to the longer local fringe spacing (43 nm) in Fig. 3 of the Rapid Communication, measured closer to the QPC (400 nm away).

Gating by the SGM tip (and supporting cantilever structure) can affect regions of the 2DEG farther away because the tip extends perpendicularly to the 2DEG wafer. This metal farther from the 2DEG is not screened as effectively. The amount of gating due to the tip at these longer distances should be relatively slowly varying with distance away from the tip. Therefore, when the tip is far from the QPC, movements of the tip do not substantially change the gating on the 2DEG between the tip and QPC. We attribute most of the difference between the density measured by fringe spacing and the density measured for the bulk to partial depletion due to the tip. Another possible contribution is that the non-gated density in this region is lower than the density measured for the bulk. We observe evidence of a non-uniform density across the 2DEG wafer.

References


