Measurement techniques

Measurements of differential conductance \( g = dI/dV_{ds} \) were performed in an Oxford TLM dilution refrigerator. (The sample is located inside the mixing chamber.) We measured \( g \) using standard ac lockin techniques (using PAR 124a with 116 preamp) at 337 Hz with a RMS excitation \( V_{ex} \) of either 1\( \mu \)V or 2\( \mu \)V, depending on temperature \( eV_{ex} \leq kT \), and measured current with a DL Instruments 1211 preamplifier. To probe nonequilibrium properties, we also added a dc voltage bias \( V_{ds} \) to the ac voltage \( V_{ex} \) through a passive circuit. Details of the electronics and filtering are contained in Ref. [S1].

Comment on other 2CK systems

In dilute rare earth/actinide alloys, thermodynamic evidence is largely compatible with 2CK, but the inability to reconcile transport data with 2CK theory has left the final state of understanding inconclusive. In the putative atomic two-level system 2CK measured by Ralph et al. in narrow metal constrictions [S2, S3] and more recently by Cichorek et al. in bulk metallic glasses [S4], the transport data nicely match the
expected scaling behavior, but a fundamental controversy persists about whether the original (near-degenerate but strongly-coupled two-level system) model Hamiltonian can even exist in a real physical system.

**Determination of $T_K$**

When the quantum dot has an odd number of electrons and the finite reservoir is not formed (e.g. Fig. 2 in Text), transport through the quantum dot displays the usual signatures of Kondo effect. The conductance as a function of temperature (e.g. Fig. 2(b) inset of the Text) matches the expected form $\tilde{g}(T)$ for a quantum dot in the Kondo regime, with the addition of a constant offset $a$:

$$g(T) = g_0 f(T/T_K) + a.$$  \hspace{1cm} (S1)

$g_0 < 2e^2/h$ reflects the intentionally-imposed asymmetry of coupling to the two conventional leads that comprise the infinite reservoir (see Asymmetry section.) The normalized temperature-dependence of conductance $f(T/T_K)$ is universal in the Kondo regime – it has no analytic form, but ranges from zero at high temperature to 1 at low temperature, with a broad logarithmic rise around $T = T_K$ [S5, S6] (the empirical form used here is given in [S6]). A conductance offset such as we observe has been seen by other experimentalists [S7], and is generically expected in the presence of potential scattering [S8]. Crucially, the main conclusions of the paper are drawn from the scaling curves from Figures 3 and 4 of the Text, which depend only on the variation in $g$ as a function of bias, not on the values we extract here for $g_0$, $a$ and $T_K$.

In Fig. S1(a), $T_K$ as a function of $sp$ is given for $c = -282 mV$ (corresponding to a single Kondo valley in Fig. 2(b) of Text). By measuring the conductance as a function of $sp$ and bias voltage $V_{ds}$ in Fig. S1(b), we observe the Kondo-enhanced density of states at the Fermi level (marked by the arrow). In Fig. S1(c), data similar to those in (a) are shown for a different strength of tunnel coupling to the right lead: $c = -244 mV$ instead of $-282 mV$. As expected, $T_K$ varies strongly across the Kondo valley in both cases, and $T_K$ is higher when the dot is more strongly coupled to the right lead ($c = -244 mV$), which increases the total $\Gamma$ of the system.

We do not have a detailed physical picture of the temperature-independent constant
offset $a$ in Eq. (S1). When we leave $a$ as a free fitting parameter, we find it does not vary much across a single valley, so we choose to hold it constant as $sp$ is varied over a single valley. At weak coupling to the right lead (coupling gate voltage $c = -282\, \text{mV}$), we find that the offset $a = 0.21e^2/h$ is constant over the Kondo valley near $sp = -230\, \text{mV}$. At stronger coupling to the right lead, for the same number of electrons in the small dot ($c = -244\, \text{mV}, sp \sim -275\, \text{mV}$) we again find a constant offset $a = 0.09e^2/h$. In Figure S3 we use the same two values of the offset determined before formation of the finite reservoir, and they work fine for collapsing the data with the finite reservoir formed. Meanwhile, $T_K$ varies substantially across a Kondo valley (Fig. S1(a) and (c)), reaching a minimum in mid-valley as expected and as observed in previous experiments [S6, S9, S10].

**Conductance of the double dot system**

In Fig. S2(a) and (b), conductance as a function of $sp$ and $bp$ is measured through the small dot, along the current path shown in Fig. S2(c). The main, broad conductance features of the small dot depend only on $sp$, as $bp$ is $> 1\, \mu\text{m}$ away and thus has a very small capacitance to the small dot. In Fig. S2(a), the gate voltage $c$ is set so that the coupling to the finite reservoir is relatively weak. In this regime, the conductance in the Kondo valleys of the small dot, at around $sp = -260\, \text{mV}$ and $-285\, \text{mV}$, is enhanced at low temperature. Gates $sp$ and $bp$ both strongly capacitively
couple to the energy of the large dot, affecting its occupancy. The diagonal stripes in
the conductance of the small dot are associated with the charge degeneracy points of
the large dot. Due to the large capacitive coupling between the two dots, adding an
electron to the large dot discretely changes the electrostatic environment of the small
dot, which changes its conductance [S11]. More complex phenomena, including SU(4)
Kondo [S12] or two channel Kondo physics [S13], may also affect the conductance near
the charge degeneracy points [S14]. We observe very weak temperature dependence in
these regimes – consistent with the exotic Kondo scenarios, but insufficiently distinctive
to clarify the relevant physics.

In Fig. S2(b), the same type of data as in (a) is shown for stronger coupling be-
tween the two dots. The Kondo valley at around \( sp = -280 \text{mV} \) has suppressed low-
temperature conductance corresponding to \( J_H > J_I \), data not shown. Suppressed low
temperature conductance has also been achieved by decreasing the coupling to either
of the two conventional leads, instead of increasing the coupling to the finite reservoir
(data not shown): what is important is the ratio between the coupling to the large
dot and the total coupling to the two conventional leads. In Fig. S2(e) conductance
as a function of \( sp \) and \( bp \) is measured through both dots in series, along the current
path shown in Fig. S2(d). The charge degeneracy points for both the large and small
quantum dots are apparent from these data, revealing the charge stability hexagon
(white hexagon superimposed as a guide to the eye).
Figure S2: Determining the charge state of the two dots.
Impact of applied magnetic field

As noted in the Text, we applied a magnetic field normal to the plane of the sample in order to deflect electron trajectories so that they cannot travel directly between the entry and the exit point contacts to the small quantum dot. Such direct paths, coexisting with resonant tunneling, give rise to Fano resonances, as seen in previous experiments on quantum dots [S15], and in measurements on the present sample at zero magnetic field. Manipulating wavefunctions with modest normal magnetic fields has been used in many other realizations of Kondo effect in quantum dots, for example the achievement of the unitary limit of transport in [S10] and tuning of Kondo coupling in a 3-terminal ring [S16].

How much impact should this field have on the Kondo physics? Due to the small g factor in GaAs dots, the Zeeman splitting is quite small at the magnetic field we applied (130 mT). Theoretically, magnetic field should be a relevant perturbation to the 2-channel Kondo state (See for example the discussion after Eq. (47) in [S17]). However, this should only be the case at very low temperatures $kT < E_{\text{Zeeman}}^2 / kT_K$. The field applied in the experiment $B = 0.13$ Tesla yields a Zeeman energy $E_{\text{Zeeman}} = 0.13 \times 25 = 3.25 \mu eV$ for GaAs. For $kT_K \approx 12 \mu eV$ in our experiment, the temperature below which the magnetic field should be relevant is $E_{\text{Zeeman}}^2 / kT_K \approx 0.8 \mu eV \approx 9.5$ mK. This effective energy scale is comparable to the base temperature of the experiments that was around $12 mK$, so for 2CK we are never in the regime where magnetic field is a relevant perturbation, with the possible exception of our very lowest temperature. The magnetic field is probably even less important than this calculation suggests, since $E_{\text{Zeeman}}$ is often lower by a factor between 1.3 and 3 in a GaAs/AlGaAs heterostructure, where the electron wavefunction leaks into the AlGaAs barrier.

In future, it would be interesting to observe the effect of a larger magnetic field, which should perturb the 2CK state. We would want to apply the field in the plane of the 2DEG, to minimize its effects on orbital states. Applying a field precisely in-plane is non-trivial. We are now setting up to do those measurements.

Note: For 1CK, which is not the main focus of the present work, Zeeman coupling is not a relevant perturbation in the renormalization group sense. Provided Zeeman
energy is substantially smaller than Kondo temperature (as it is in our experiments) it should have an effect similar to that of bias or temperature ($G = G_0 - \text{const}_1 \cdot (T/T_K)^2 - \text{const}_2 \cdot (eV_{ds}/kT_K)^2 - \text{const}_3 \cdot (g\mu_B B/kT_K)^2$), where the three constants are all of order unity. In our experiments, $g\mu_B B$ is roughly one to three times $kT_{\text{base}}$, depending on the exact g-factor. Since all the perturbations are substantially smaller than the Kondo temperature, the presence of the magnetic field should not substantially affect the scaling of conductance with temperature and bias.

**Energy dependence of 1CK data**

We have fine control over the occupancy of both the finite reservoir and the small dot with gates $bp$ and $sp$, as shown in Fig. S3(a) and more completely in S2. Conductance $g(T) \equiv g(0, T)$ at weak coupling to the finite reservoir ($J_{ir} > J_R$) fits the expected empirical form of Eq. S1, Fig. S3(b). In addition, Fig. S3(c) shows that the conductance $g(T)$ at many points in $(sp, bp)$ (Fig. S3(a)) can be collapsed onto a universal curve. The differential conductance $g(V_{ds}, T)$ of a 1CK system is further expected to follow a specific form as a function of both bias and temperature (see Asymmetry section), at an energy scale substantially below $kT_K$ [S18]:

$$\frac{g(0, T) - g(V_{ds}, T)}{T^\alpha} = \kappa \left(\frac{eV_{ds}}{kT}\right)^2,$$

where the exponent $\alpha = 2$ is characteristic of 1CK, and $\kappa = 0.82 \frac{g_0}{T_K}$. The numerical prefactor of order unity is dependent on the underlying model, numerical calculations, and proximity to the symmetric 2CK fixed point (see Comparison section), so we simply treat $\kappa$ as a free fitting parameter for each set of gate voltages. Fig. S3(d) demonstrates excellent 1CK scaling at temperatures of 12, 24, 28, and 38mK, all well below $T_K$. A nonlinear fit to the data in Fig. S3(d) yields $\alpha = 1.72 \pm 0.40$ (95% CL), consistent with $\alpha = 2$.

In Fig. S3(e-h), we demonstrate that at stronger coupling to the finite reservoir, ($J_{fr} < J_{ir}$) the small dot forms a Kondo state with the finite reservoir, as manifested by low-energy suppression rather than enhancement of conductance between the normal leads of the small dot. We must modify the form we use to fit the temperature
dependence to reflect this inversion:

\[ g(T) = g_0 \left(1 - f(T/T_K)\right) + a. \tag{S3} \]

Again temperature dependences at multiple points in \((sp, bp)\) (Fig. S3(e)) collapse onto a single normalized Kondo form \(\tilde{g}/g_0\) vs \(T/T_K\) (Fig. S3(g)), providing strong evidence that a distinct 1CK state has formed with the finite reservoir. Furthermore, using the same scaling relation as also shown in the Text and above (Eq. S2), the data again collapse onto a single (inverted) curve at low bias and temperature (Fig. S3). This matches our expectation that the large quantum dot should act as an independent screening reservoir, in the limit that its level spacing \(\Delta_{fr} < kT\). In fact, even if the level spacing is resolvable \((kT < \Delta_{fr})\), the Kondo state should be essentially unchanged as long as \(\Delta_{fr} < kT_K\) [S19].

**Scaling analysis of 1CK data**

In the main Text we demonstrated that the data we identify as reflecting a symmetric 2CK state cannot be described by a Fermi liquid scaling appropriate to 1CK. It is important to establish the converse: that the data we identify as reflecting 1CK do not follow 2CK scaling. We show this in Fig. S4, where the 1CK data presented in Fig. 3 of the Text or Fig. S3 are seen to scale as expected for 1CK and not as expected for 2CK. The expected scaling for 1CK again is

\[ \frac{g(0, T) - g(V_{ds}, T)}{T^{\alpha}} = \kappa \left(\frac{eV_{ds}}{kT}\right)^2, \tag{S4} \]

with \(\alpha = 2\) and \(\kappa = 0.82 \frac{g_0}{T_K}\) [S20]. In Fig. S4(a), we show the same 1CK scaling plot as in Fig. 4(d) of the Text.

In Fig. S4(b), we scale the same data from Fig. S4(a) as would be appropriate for 2CK behavior, i.e. with \(\alpha = 0.5\). In Fig. S4(c) and (d) we simulate idealized 1CK (Fermi liquid) data and scale them as would be appropriate for 1CK (\(\alpha = 2, (c)\)) and 2CK (\(\alpha = 0.5, (d)\)). Comparing Fig. S4(b) and (d), the simulated 1CK data deviate from perfect 2CK scaling very similarly to how the actual 1CK data deviate. Note that the qualitative behavior is the opposite of what one would expect from a breakdown of scaling when approaching some finite energy scale (e.g. \(T_K\)): curves at
Figure S3: Energy dependence of Kondo effect, with the finite reservoir formed. (a) - (d) Anti-ferromagnetic coupling to the infinite reservoir (normal leads) is stronger than coupling to the finite reservoir: $c = -282$ mV. (a) Conductance as a function of gates $sp$ and $bp$, at $V_{ds}=0$. (b) Any point in ($sp$, $bp$) such that the occupancy of the small dot is odd and that of the finite reservoir is integer shows enhanced conductance at low temperature and low bias (inset: 12 mK, grey, to 60 mK), and a temperature dependence consistent with Kondo effect. (c) The normalized dependence of conductance on temperature is uniform for different points in ($sp$, $bp$) space, while $T_K$ ranges from 50 mK to 180 mK. $\tilde{g} \equiv g - a$ is the conductance with the temperature-independent offset subtracted off. (d) Plotting a specific combination of temperature and bias collapses the data for a single point in ($sp$, $bp$) space ($T_K = 175$ mK, $g_0 = 0.75e^2/h$) onto a single V-shaped curve, corresponding to the scaling relation predicted for 1CK (Eq. (S2)). (e) - (h) Kondo effect with the finite reservoir: $c = -244$ mV. (e) Conductance as a function of $sp$ and $bp$: conductance is now suppressed rather than enhanced at low bias and temperature (cf. Fig. 3(g) of Text.) (f) Fitting the conductance as a function of temperature to the empirical form we expect for 1CK with the finite reservoir (Eq. S3) we find that $T_K$ ranges from 30 mK to 130 mK. (g) We again normalize and collapse the temperature dependence at several points in ($sp$, $bp$) onto a single curve. (h) We collapse differential conductance data like those in inset (f) (12 mK, grey, to 30 mK) at a single point in ($sp$, $bp$) onto a single inverted V-shaped curve using the same temperature-bias scaling as in (d). Deviations from perfect scaling may be related to the slightly lower Kondo temperature ($T_K = 120$ mK, $g_0 = 0.16e^2/h$.)
higher temperatures fall inside those at lower temperatures, instead of "peeling off" toward the outside above a certain bias voltage. The nonlinear fits presented in the Text quantify these observations: the best fit for $\alpha$ is $1.72 \pm 0.4$ for the 1CK data and $0.62 \pm 0.21$ for the symmetric 2CK data, clearly distinguishable from each other, and both consistent with theoretical expectations ($\alpha = 2$ and 0.5, respectively.)

**Comparison of theory and experiment for $\kappa$, 1CK scaling prefactor**

As noted in the Text, the value of $\kappa$ depends on the underlying model (Kondo effect can be derived from various different models), numerical calculations ($\kappa$ connects low-energy behavior to high-energy behavior, and no analytical results can make this link quantitatively), and proximity to the symmetric 2CK point (near the symmetric point, $T_K$ is replaced by $T_\Delta$, a measure of the asymmetry). Here we outline how to determine $\kappa$ theoretically, and we comment on the link to our experimental result.

First, a Kondo energy scale (or Kondo temperature) is only a crossover scale, so different definitions could yield values differing by some constant multiple. We want results that are independent of these initial definitions. Theoretically, the Kondo temperature is usually defined in terms of a thermodynamic quantity such as susceptibility rather than a dynamic quantity such as electrical conductance, so we must use a model to link the two. According to Costi [S21],

$$g(T) = g_0 \left(1 - \frac{\pi^4}{16} \left( \frac{T}{T_0} \right)^2 \right)^\alpha,$$

[S22] where $T_0$ is defined according to

$$\chi(T = 0) = \frac{(g \mu)^2}{4kT_0}.$$

Now

$$\kappa = g_0 \left( \frac{\pi^2}{4T_0} \right)^\alpha \frac{3}{2\pi^2}.$$

For the case $\alpha = 2$ we find

$$\kappa = g_0 \frac{3\pi^4}{32\pi^2 T_0^2} = \frac{3\pi^2}{32} \frac{1}{T_0^2}.$$
Figure S4: Scaling of 1CK
Next, we must link the thermodynamically-defined Kondo scale $T_0$ to $T_K$, defined according to $g(T_K) = g_0/2$ [S22]. Costi’s NRG calculations suggest that this link depends mildly on details of the system such as the dimensionality of the leads. For 2D leads, $T_K = 0.94T_0$. This yields $\kappa = 0.82\frac{g_0}{T_K}$, as reported in the Text. This is in rough but satisfactory agreement with our experimentally-extracted value $\kappa = 0.25\frac{g_0}{T_K}$ for both 1CK with the conventional leads and 1CK with the finite reservoir. Note that other approaches to the basic Kondo model may or may not give the same result. A. Schiller’s calculations based on an exactly-solvable model at the Toulouse limit give

$$g(T) = g_0 \left(1 - \frac{\pi^4}{48} \left(\frac{T}{T_0}\right)^2\right),$$

yielding a value of $\kappa$ three times smaller than that of the other models, and in almost perfect agreement with our experimental results. Apart from this (perhaps serendipitous) match we have no reason to believe that the exactly-solvable model at the Toulouse limit is a better description of the low-energy properties of our system than Nozières’s Fermi liquid approach.

A final complication in quantitative comparison of theory and experiment is that our measurements are not very far from the symmetric 2CK, so $T_K$ should be replaced by $T_\Delta$. It’s not clear whether $T_\Delta$ should act the same as $T_K$ at both low and high energies. Therefore, it will be interesting to perform these same analyses on a two-lead dot which exhibits simple 1CK behavior, with no link to 2CK.

**Raw data for 2CK scaling analysis**

Fig. S5 shows the raw data used in the 2CK scaling analysis. These data were obtained under conditions similar to those for the $c = -260$ mV curve in Fig. 4(e) in the Text, which shows differential conductance at widely-spaced values of the coupling gate voltage $c$. Since the parameters of the system had shifted since acquisition of the data in Fig. 4(e), the coupling gate had to be changed to $c = -258$ mV. For the scaling analysis (Fig. 4(f) of Text), to reduce the noise in the value of $g(V_{ds} = 0)$ we averaged the conductances at $-1\mu$V, 0, and $1\mu$V.

**Match of raw data to 2CK predictions**

Fig. S6 shows the 12 mK raw data used in the 2CK scaling analysis (gray curve
from Fig. S5.) A parabolic fit works only at low bias. In contrast, a square-root fit \( g(V_{ds}) = g_0 - g_1 \sqrt{V_{ds}} \), with \( g_0 \) and \( g_1 \) as fit parameters, Fig. S7) works well at intermediate bias \( (V_{ds} = 5 \text{ to } 15 \mu \text{V}) \) This crossover from quadratic to square-root behavior at bias a few times \( kT \) agrees with conformal field theory predictions for 2CK [S17, S23, S24]. This match is reinforced by the more complete scaling analysis in Fig. 4(f) of the Text.

**Asymmetry of coupling to the two conventional leads that comprise the “infinite reservoir”**

The tunnel barriers between the local site and the two conventional leads were intentionally tuned to be asymmetric, for two reasons:

a. Existing theoretical calculations for 2CK (and 1CK) give density of states in equilibrium. Our scaling measurements involve applying finite bias from one lead to the other. Understanding conductance at finite bias can require a nonequilibrium treatment [S18]. However if the coupling to the two conventional leads that form the reservoir is substantially asymmetric the Kondo resonance is pinned to the Fermi level of the strongly-coupled lead and the system is in equilibrium even for finite bias. In our setup we found \( \Gamma_{r1}/\Gamma_{r2} \approx 8 \), so this equilibrium condition is satisfied. Thus, the strong asymmetry between the couplings to the two leads means that the local site remains essentially in equilibrium with the more strongly-coupled lead, validating quantitative
Figure S6: Parabolic fit to $g(V_{ds})$ at small bias.

Figure S7: Square-root fit to $g(V_{ds})$ at intermediate bias.
comparison with predictions.

b. The symmetric 2CK state occurs when $J_1 = J_2$, which requires that $\Gamma_{fr} \approx \Gamma_{ir}$, where $\Gamma_{ir}$ is the sum of the tunnel rates to the two conventional leads. If all three tunnel barriers were equal, we would instead have $\Gamma_{fr} = 0.5\Gamma_{ir}$. It turns out to be easiest to tune the system by first matching the tunnel barrier of the finite reservoir to that of one of the open leads. With the finite reservoir open to the outside world (gate $n$ grounded) and one conventional lead fully pinched off, we maximized the two-terminal conductance between the other conventional lead and the infinite reservoir (in fact we found it could be very near $2e^2/h$, usually $\sim 1.8e^2/h$). This means $\Gamma_{fr} \approx \Gamma_{ir}$. We then slightly opened the second conventional lead to the small dot and closed off the finite reservoir from the outside world (using gate $n$). In this way, we maintained $\Gamma_{fr}$ near $\Gamma_{ir}$, as needed for $J_1 = J_2$. We felt this was the best method to ensure nearly equal $\Gamma$s and $J$s.
Bibliography


S14. More complex scenarios are possible in different parameter regimes of the same structure we study. These include 2CK in which the local twofold degeneracy is between two charge states rather than two spin states [S13], 2CK involving a mixture of spin and charge degrees of freedom [S25], and SU(4) Kondo involving both spin degeneracy on the local site and charge degeneracy in the finite reservoir [S12].


S20. This equation describes the low-energy Fermi-liquid behavior, at an energy scale substantially below $kT_K$ [S18]. For a channel-asymmetric 2CK system, the crossover scale $T_K$ is replaced by $\min\{T_K, T_\Delta\}$, where $T_\Delta$ is a measure of the channel asymmetry which goes to zero when $J_1 = J_2$ [S17].

S21. T. Costi, private communication. We have confirmed that this link between conductance and susceptibility, both at low energies, agrees exactly with results of I. Affleck, of P. Nozières, and of M. Pustilnik and L.I. Glazman.

S22. Because we have a temperature-independent conductance offset, we replace $g(T)$ with $\tilde{g}(T) \equiv g(T) - a$ in this formula, for comparison with the data on 1CK with the conventional leads (Fig. S3(d)) – the analysis is similar for 1CK with the finite reservoir, but the temperature-dependence has the opposite slope.

