Sovereign Default and Asset Prices*

Sandro C. Andrade
U.C. Berkeley

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Abstract

This paper shows that the option to default on foreign debt increases the cost of capital in emerging economies. This result arises in a consumption-based asset pricing model in which the decision to default on foreign debt is endogenous. The optimal default decision implies that stock prices in emerging economies become more sensitive to real economic shocks, and therefore more volatile, as the probability of default increases. Since stock markets are not integrated internationally, the increase in volatility is compensated by an increase in the expected excess return to stocks. I calibrate the model with plausible structural parameters and show that time variation in volatility and expected return can be economically relevant. The model can be solved in closed-form, and provides testable implications connecting stock prices in an emerging market to the spread over US Treasuries at which the country’s foreign debt is traded in international markets. The model’s time-series predictions are confirmed in Brazilian data from January 1992 to October 2005. Stock market returns and changes in the sovereign yield spread are contemporaneously negatively correlated. The expected excess return and volatility of the emerging stock market increase with the sovereign yield spread. A cross-sectional implication of the model is also supported empirically: a conditional CAPM, in which the sovereign spread is the conditioning variable, explains a larger fraction of the dispersion of average excess returns across Brazilian industry portfolios than a static CAPM.

Keywords: Asset pricing, emerging markets, sovereign default, real options
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1 Introduction

What determines stock prices and the cost of capital in emerging economies? I introduce a consumption-based asset pricing model to address this question from first principles. The model is solved in closed-form, and generates time-varying expected excess return and volatility for emerging stock markets. This feature obtains despite the model’s standard aggregate consumption processes and conventional representative agent utility function. A key result of the model is that time-variation in stock market characteristics is fundamentally related to the level of the sovereign yield spread, defined as the difference between the promised yield of a dollar-denominated bond issued by the emerging country in international markets and the yield of a US Treasury bond of the same maturity. All theoretical results in the paper derive from modeling the emerging country’s option to default on foreign debt.

A calibration of the model with plausible parameters indicates that time-variation in stock market expected excess return and volatility resulting from the option to default can be economically relevant. The calibration shows that when the sovereign yield spread is zero, the expected excess return and volatility of the emerging stock market are respectively 3.84% and 12% per year. These numbers would prevail if the country did not have the option to default on its foreign debt. In contrast, when the spread is 763 basis points per year the expected excess return increases to 9.41% while the volatility rises to 35.55%. These magnitudes are consistent with the data. The median sovereign spread for Brazil from January 1992 to October 2005 was 763 basis points per year, while the average excess return and volatility of the Brazilian stock market in the same period were 7.98% and 40.67% per year respectively.

The theory shows that emerging stock markets feature "excess volatility" as a result of the option to default on foreign debt. The model-generated stock volatility when the spread is 763 basis points per year (35.55% per year) is much higher than the model-generated volatility for an otherwise identical economy without the option to default on foreign debt (12% per year). Interestingly, the 35.55% pre-default volatility is also much higher than the model-generated post-default stock volatility, which is fixed at 21% per year. The price response to real economic shocks is larger in an emerging economy than in an otherwise identical no-default economy because a shock changes the discount at which the emerging stock market is traded vis-à-vis the stock market in the no-default economy. The increase in volatility brings about an increase in expected excess return, to maintain asset markets in equilibrium.

The model produces time-series implications that are qualitatively confirmed empirically. Brazilian data shows that the stock market price and the sovereign yield spread move in opposite directions contemporaneously as the model predicts. Both are endogenously driven by real economic shocks. Data also reveals that stock market expected excess return and volatility in emerging economies increase in the level of the sovereign yield spread, again in line with model predictions.
A cross-sectional asset pricing implication of the model also finds empirical support in the data. The model produces a novel method for calculating the cost of capital of firms in emerging economies: a conditional (local) CAPM where the sovereign yield spread arises endogenously as the relevant conditioning variable. I carry out a two-pass regression methodology using Brazilian industry portfolios to evaluate the conditional CAPM’s ability to explain dispersion of average returns across industries, and compare its performance with the static CAPM. Results show that the conditional CAPM, using the yield spread as a conditioning variable, explains a larger fraction of the cross-sectional variation in average returns than the static CAPM, as the theory implies.

The following paragraphs distill the basic economic mechanisms that link the sovereign yield spread to the cost of capital in emerging markets. I start by analyzing the two essential features of emerging economies that account for such connection: the option to default on foreign debt and market segmentation.

Large levels of foreign debt create a fundamental trade-off for emerging countries. Foreign creditors cannot seize emerging market assets upon default. Thus, emerging countries can attain an immediate boost to their aggregate consumption levels by stopping to service foreign debt. However, default worsens the credit reputation of an emerging country (Eaton and Gersovitz 1981), and brings about direct retaliation from foreign creditors (Bulow and Rogoff 1989). These reduce the reneging country’s ability to grow and to share risk in the future. As a result, the emerging economy grows more slowly and with more volatility after default. The emerging country optimally balances this trade-off, by choosing a default policy that maximizes its utility. The optimal policy is to default when the local consumption level hits a lower barrier.

Market segmentation means that asset prices in emerging economies depend on local idiosyncratic conditions, rather than being set by a diversified global marginal investor. As a consequence of market segmentation, an increase in the local stock market’s volatility must be compensated by an increase in the local stock market’s expected excess return in order to keep asset markets equilibrated.

Market segmentation and the option to default on foreign debt imply that the sovereign yield spread and the cost of capital in emerging markets are fundamentally connected in equilibrium. The sovereign yield spread, set in international markets, contains information about the probability that the emerging country will default on its foreign debt. Therefore, the spread has information about the likelihood of

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1 Reinhart et al. (2003) document that the external debt to GDP ratio of developing countries near default between 1824 and 1999 was 71%.
an endogenous regime shift in local economic conditions. This regime shift is for worse, meaning that economic growth has lower mean and higher variability after default, therefore stocks in the emerging economy trade at a discount relative to an identical no-default economy. The higher the sovereign spread, the closer to default the emerging economy is, and the larger the impact of a given economic shock in the probability of default. This is because the optimal default barrier does not change with economic shocks, and real economic shocks are i.i.d. over time\(^3\). Thus, the higher the sovereign spread, the more volatile the probability of default, and the more volatile the discount at which the emerging stock market is traded vis-à-vis an identical no-default economy. The increased volatility of the discount makes stock returns more sensitive to real economic shocks as the yield spread rises. Because asset markets are segmented internationally, an increase in stock volatility in the local market brings about an increase in the expected excess return to keep the market in equilibrium\(^4\). Therefore, the cost of capital increases with the level of the sovereign yield spread. My model allows for a quantitative assessment of this relationship, guided by the first principles just described.

The relationship between equity prices in emerging economies and sovereign yield spreads - or other measures of the creditworthiness of emerging countries' foreign debt - has been empirically explored by earlier researchers. Papers such as Erb, Harvey and Viskanta (1995), Bailey and Chung (1995) and Gendreau and Heckman (2003) have previously uncovered some of the empirical time-series results documented in this paper. However, their empirical results were preliminary since data limitations prevented sharper statistical inferences.

Most importantly, the connection between the sovereign spread and the cost of capital arises endogenously in my fully-rational model without agency conflicts, whereas earlier empirical papers typically invoked exogenous political factors to explain it. Bailey and Chung (1995) use the yield spread as a measure of "political risk", defined as the possibility of adverse changes in laws and regulations in the emerging country, particularly regarding capital outflows and private property protection against governmental expropriation. Bekaert (1995) endorses the use of Institutional Investor country credit ratings – another measure of the credit quality of emerging market foreign debt - as a direct measure of "political risk", defined as the possibility of political instability and economic mismanagement in emerging countries. Bansal and Dahlquist (2002) use the yield spread as measure of "expropriation risk" in stock

\(^3\)The following thought experiment is useful. Consider flipping a coin 100 times, sequentially. Before each flip, compute the probability that Heads will come up all 100 times. Notice that this probability goes up faster as successes accumulate. After the 98th success, right before the 99th flip, the probability is 0.25. After the 99th success, the probability is 0.50, and after the 100th success the probability is 1.00. The probability of getting 100 Heads goes up by 0.25 after then 99th success, and by 0.50 after the 100th success.

\(^4\)As long as the compensation per unit of risk does not decrease too much, as it is always the case in the model.
markets, assuming that government default on foreign debt is accompanied by government expropriation of local equity assets. In contrast, I show that the yield spread and asset prices in emerging economies are connected even when incentives within the emerging economy are fully aligned, and the fully-rational local government does not expropriate shareholders or impose capital controls. The link arises endogenously as a consequence of the country’s optimal decision to default on foreign debt.

Analogously, people have typically invoked exogenous political factors to explain the stylized fact that emerging stock markets are too volatile relative to developed ones (see, for example, Morck, Yeung and Yu 2000). In contrast, my model - calibrated with plausible structural parameters - suggests that a large fraction of the "excess" volatility observed in emerging stock markets can be rationalized by endogenizing the country’s optimal decision to default on its foreign debt. The option to default on foreign debt makes emerging stock markets more sensitive to underlying real economic shocks.

Interestingly, industry practitioners already incorporate the level of the sovereign spread in cost of capital calculations for emerging markets, as documented by Harvey (2001) and Damodaran (2003). However, practitioners use an ad hoc adjustment, and their procedure is different from what my model prescribes\(^5\).\(^5\) As opposed to industry practice, the CAPM "adjustment" for the level of the sovereign spread differs from stock to stock in my model. My cross-sectional results are novel both theoretically and empirically.

1.1 Related Theoretical Papers

My model is related to Chang and Sundaresan’s (2006) equilibrium model of default. Both are set in continuous time, and feature a trade-off for a borrower. Default eliminates the burden of servicing debt, at the cost of deteriorating the borrower’s set of future consumption/production possibilities. However, the models are quite different in terms of assumptions and results. Chang and Sundaresan investigate a borrower and a lender within the same economy, while my model features outside lenders. The fixed initial amount of debt is optimally chosen in Chang and Sundaresan’s model, while it is exogenous here. Chang and Sundaresan solve their model numerically for specific parameters, while my model allows for closed-form solutions. While both models imply time-varying risk premia, the underlying economies is different. Chang and Sundaresan point out that time variation in the borrower’s effective risk aversion

\(^5\)Practitioners use the following method: regress the emerging market stock returns in US dollars on S&P 500 returns; multiply its resulting beta by the S&P 500 risk premium; and add the sovereign yield spread. The result is supposed to be the dollar excess return that rewards the holder of the emerging market stock for the risks he is taking. Godfrey and Espinosa (1996) and Lessard (1996) claim that this method captures exposure to the risks of asset expropriation and imposition of capital outflows controls. Note that the "adjustment" for the spread affects all stocks equally.
drives time variation in risk premia in their model. The mechanism is similar to the one explored in Campbell and Cochrane’s (1999) habit formation model. In contrast, time-variation in risk premia in my model is driven solely by the terms of the default trade-off, rather than by any effect operating via aggregate preferences. Because of the aforementioned modeling differences, the equity premium and the instantaneous risk-free rates can be non-monotonic in aggregate wealth in Chang and Sundaresan’s model, as opposed to in my model. Finally, I take a very different approach in solving for the optimal default boundary and asset prices.

Gibson and Sundaresan (2001) develop a continuous time model to study a country’s decision to default on its foreign debt. They focus on the dynamic game between the country and its lenders, and its impact on the pricing of sovereign debt in world markets. The authors assume that the sovereign objective function is linear in the total value of the country’s wealth. Their framework does not include consumption policies of the sovereign borrower and their impact on risk premia and local asset prices.

Kulatilaka and Marcus (1987) is an early continuous time model of endogenous sovereign default. I follow their approach of having exogenous foreign debt levels, and modeling default as an irreversible option possessed by the indebted developing country. However, in sharp contrast to the approach taken here, Kulatilaka and Marcus assume that debt service introduces a drag on the growth rate of GDP, and that the drag is increasing on the level of debt service. Moreover, these authors do not investigate asset pricing implications of the option to default.

This paper also relates to recent contributions to the macroeconomics literature. Arellano (2005) and Aguiar and Gopinath (2006) build discrete time models of the sovereign default decision. Rather than focusing on asset prices, their goal is to match empirical moments of macroeconomic variables such as consumption, output and the current account.

1.2 Related Empirical Papers


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6On their Section 4, Chang and Sundaresan (2006) write: "As the probability of default begins to increase, the agent becomes more risk averse and consume less to avoid the costliness of default."
Early empirical papers study the connection between a country’s creditworthiness and returns on its local stock market, which is the focus of my investigation. The first paper to explore this link empirically seems to be Erb, Harvey and Viskanta (1995). Others are Bekaert (1995), Bailey and Chung (1995), Bekaert et al (1997), Bansal and Dahlquist (2002) and Gendreau and Heckman (2003). These papers anticipated some of the time-series empirical results that are explored in more detail here. Domowitz, Glen and Madhavan (1998) present an empirical paper relating the Mexican short term sovereign yield spread to the short end of the term structure of interest rates in Mexican pesos.

2 Model and Calibration

My model adds endogenous foreign default to a standard Lucas-Rubinstein single-good exchange economy (Lucas 1978, Rubinstein 1976). Consider an emerging country populated by a representative agent with time separable preferences. The time discount parameter is $\rho > 0$ and the per period utility is isoelastic of parameter $A > 1$. Let $C_t$ be aggregate per capita consumption flow at time $t$. The agent’s utility is:

$$U_t \equiv E_t \int_t^\infty e^{-\rho(s-t)} \frac{C_s^{1-A}}{1-A} ds. \quad (1)$$

The country has foreign debt in the form of a perpetuity, requiring a continuous flow $\bar{X} > 0$ of fixed interest payments. As in Kulatilaka and Marcus (1987), debt is exogenous. The idea is that the main source of foreign debt is unsecured loans to the country’s government, accumulated under many different administrations and over a long period of time, possibly in sub-optimal manners$^7$.

New borrowing is not allowed, again following Kulatilaka and Marcus (1987). Recent models of default share similar exogenous liquidity constraints. Chang and Sundaresan (2006) and Gibson and Sundaresan (2001) develop continuous time models of aggregate default that rule out new borrowing too. Arellano (2005) and Aguiar and Gopinath (2006) build dynamic discrete time models of default under the assumption that emerging countries cannot raise new debt with maturity longer than one period (one year) at any price. The aforementioned liquidity constraint in continuous time is a limiting case of the one in discrete time.

The emerging country’s government taxes firms’ profits in order to service foreign debt and is responsible for the centralized default decision. The government makes

$^7$In 1825, shortly after its independence from the Portuguese, Brazil took responsibility over sterling-pound denominated sovereign bonds issued by Portugal in 1823. In 2005 dollars, this represented approximately US$ 85,000 of additional debt for each Brazilian alive in 1825. I thank Rui Pedro Esteves for this information.
the default decision using all relevant information available, and for the purpose of maximising the utility of its representative citizen. Let \( \tau \) be the endogenous default time. Taxation before default creates a gap between aggregate endowment \( Y_t \) and aggregate consumption \( C_t \):

\[
Y_t = C_t + \bar{X}, \quad 0 < t < \tau.
\]

Before \( \tau \), aggregate consumption follows a Geometric Brownian Motion (GBM) with parameters \( \mu \) and \( \sigma \). Let \( Z_t \) be a standard Wiener process on the probability space \((\Omega, F, P)\). Then

\[
dC_t = \mu C_t dt + \sigma C_t dZ_t, \quad 0 < t < \tau.
\]

There are three reasons to model consumption rather than endowment as a GBM as done above. First, to be consistent with the empirical evidence. Prasad et al. (2003) and Neumeyer and Perri (2005) document that consumption is more volatile than endowment in emerging markets. Secondly, to neutralize the effect of changes in "effective risk aversion" on risk premia. If pre-default endowment was a GBM the model would feature an effect similar to the one explored by Campbell and Cochrane (1999) and also present in Chang and Sundaresan (2006). In contrast, modeling pre-default consumption as a GBM implies that time-variation in risk premia in the model is driven solely by the terms of the default trade-off, rather than by any effect operating via preferences. Finally, this framework allows for closed-form solutions, whereas the alternative does not.

The default decision is irreversible. At default, and thereafter, the country stops servicing foreign debt. Foreign lenders cannot seize the country’s assets upon default. Following Eaton and Gersovitz (1981) and Bulow and Rogoff (1989), the model assumes that interest payments are to some extent enforced by the threat of expulsion from international trade and financial markets. International markets are forever shut to the reneging country\(^8\). Note that default must bring about a penalty for the emerging economy otherwise the market for sovereign debt would not be sustainable.

Barred from international markets, the country cannot take full advantage of technological innovations developed elsewhere, or use international resources to smooth out consumption over time (see Prasad et al. 2003). These constraints affect aggregate consumption. After default, consumption follows a GBM with lower mean and higher volatility - parameters \( \mu^d < \mu \) and \( \sigma^d > \sigma \). That is:

\[
dC_t = \mu^d C_t dt + \sigma^d C_t dZ_t, \quad t \geq \tau.
\]

\(^8\)Introducing an exogenous probability of re-entry would not change the qualitative implications of the model.
After default, the flow of interest payments ceases, and so does corporate taxation to service foreign debt. Therefore, consumption is equal to endowment after default. As in Arellano (2005) and Aguiar and Gopinath (2006), default is accompanied by a (small) permanent reduction in endowment flow. The parameter $\alpha$ in $(0, 1]$ governs the magnitude of the discontinuous endowment reduction:

$$Y_\tau = \alpha Y_{\tau-} + (1 - \alpha) \bar{X}.$$  \hfill (5)

The model implies that consumption flow also changes discontinuously at default$^9$:

$$C_\tau = \alpha C_{\tau-} + \bar{X}.$$  \hfill (6)

Equations (5) and (6) are key to understanding the trade-off created by the option to default. At the optimal default point the emerging country enjoys an immediate positive jump in consumption flow, since it is immediately relieved from the burden of servicing foreign debt. However, there is a (small) discontinuous permanent drop in endowment in default. More importantly, endowment and consumption rise more slowly and with more volatility after default as a result of worsened credit reputation and direct foreign creditors’ retaliation. Proposition 1 shows how the emerging country balances this trade-off. The basic intuition is that the country defines a minimum consumption barrier and defaults when consumption hits this barrier. The optimal barrier maximize the (utility) value of the country’s option to default on foreign debt.

**Proposition 1** Let $\beta$, $K$, and $K^d$ be the following parameters:

$$\beta = -\left(\frac{\mu}{\sigma^2} - \frac{1}{2}\right) - \sqrt{\left(\frac{\mu}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2 \rho}{\sigma^2}} < 0,$$

$$K = \left[ \rho + (A - 1) \left( \mu - \frac{1}{2} A \sigma^2 \right) \right]^{-1} > 0,$$

$$K^d = \left[ \rho + (A - 1) \left( \mu_0 - \frac{1}{2} A \left( \sigma_0^2 \right) \right) \right]^{-1} > K.$$

Assume that $1 < A < -\beta$ and that $\alpha$ is sufficiently close to 1. The optimal default time $\tau$ is the first time consumption hits level $B$ from above, where

$$B = \frac{z^* \bar{X}}{1 - z^* \alpha},$$  \hfill (7)

and $z^*$ is the unique value of $z$ in the interval $(0, 1)$ that solves

$$\alpha (1 - A) z^A - \beta z^{A-1} + \frac{K}{K^d} (\beta + A - 1) = 0.$$  \hfill (8)

$^9$Note that consumption flow is discontinuous. The model does not feature lumpy consumption.
Proof. See Appendix A.■

Equation (8) has an analytical solution for $A = 2, 3$ or 4. It is particularly simple to perform comparative statics exercises in these cases. From Equations (6) and (7) one can write $z^* = C_{\tau} - C_{\tau'}$. Thus, $1/z^*$ is the discontinuous percent increase in consumption flow at default. Since $z^*$ does not depend on $X$, $X$ is just a scaling parameter. It can be fixed arbitrarily without any effects on the economics of the model.

The parametric restrictions allow for an interior solution. Recall that $A$ is greater than unity by assumption. Parameter $A$ is the coefficient of relative risk aversion and the reciprocal of the elasticity of intertemporal substitution. It measures the willingness to smooth consumption over time and across states of world. If $A$ was too small, the country would never default since it would not be willing to sacrifice higher consumption growth and lower consumption variability in exchange for an immediate boost in consumption. The default option would be worth zero if $A$ was smaller or equal to unity. Similarly, $A < -\beta$ ensures that the country does not default right away. I assume that $1 < A < -\beta$ throughout the rest of the paper.

2.1 Pricing Sovereign External Debt

Asset markets are segmented from the rest of the world, i.e., the pricing kernel that prices global assets is not the same one that prices emerging market local assets. The former is endogenous to the model, while the latter is exogenously fixed. To prevent arbitrage in the model, foreign debt is the only emerging economy security that foreigners can trade on.10

Foreign creditors hold debt issued by the emerging country government. Therefore, one needs to make assumptions about the global (as opposed to local) pricing kernel in order to price emerging market foreign debt. I make two assumptions in this paper. First, the foreign interest rate $r^*$ is constant, for simplicity. Secondly, foreign investors are risk-neutral with respect to the emerging country specific risk. This assumption is plausible if the emerging economy is relatively small.

The proposition below characterizes the sovereign yield spread as a closed-form function of model parameters. Recall that emerging market debt is a perpetuity paying a continuous flow of interest $\bar{X}$ until default, and nothing thereafter. Thus, the yield spread is only defined before default, when $C_t > B$.

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10This is a reduced-form assumption. In most emerging countries, the binding constraints for foreign investment are not explicit barriers impeding foreigners to buy local assets. Implicit barriers, related to asymmetric information problems, are more important (see Stulz 2005). It is very hard to introduce these implicit barriers in a full-blown asset pricing model. Bhamra (2005) develops a model of partial segmentation due to explicit portfolio constraints.
Proposition 2 \textit{The sovereign yield spread $S_t$ is a function of the current level of consumption $C_t$, the foreign interest rate $r^*$, the optimal default barrier $B$ and the parameter $\beta < 0$ defined in Proposition 1. It is given by:}

$$S_t = r^* \frac{1}{(\frac{C_t}{B})^{-\beta} - 1}. \quad (9)$$

Moreover, the stochastic process for the sovereign yield spread is $dS_t = \eta_t (C_t) dt + \delta_t (C_t) dZ_t$, where:

$$\delta_t (C_t) = \beta \sigma r^* \frac{(\frac{C_t}{B})^{-\beta} - 1}{[(\frac{C_t}{B})^{-\beta} - 1]^2}. \quad (10)$$

\textit{Proof.} See Appendix A. $\blacksquare$

Equation (10) shows that a negative real economic shock ($dZ_t < 0$) increases $S_t$. Note that the spread approaches zero as $C_t$ grows unboundedly large, and goes to infinity as the economy approaches default. This is because the model has zero recovery in the event of default.

There is a monotonic relationship between $S_t$ and $C_t$, i.e., to each level of pre-default consumption there is one and only one associated sovereign spread. This property holds for any foreign pricing kernel in which the yield spread is a monotonic function of the emerging country’s probability of default under the physical measure. Monotonicity implies that functions of $C_t$ can be seen as functions of $S_t$. I use this argument extensively throughout the rest of Section 2.

As a parenthetical remark, note that Proposition 2 implies that the level of consumption in the emerging economy reveals the same information embedded in the sovereign spread. However, the yield spread does not suffer from practical measurement problems that plague the use of consumption in asset pricing. Rather than being measured on a quarterly or yearly basis, the sovereign spread can be instantaneously extracted from bond prices traded on liquid international markets. It is a forward-looking measure, as opposed to being computed as an average of historical data. Perhaps more importantly, the measurement of aggregate consumption in emerging economies is likely to be even more subject to errors than in developed ones, due to the scarcity of resources allocated to the task (see Wilcox 1992).

2.2 Pricing Aggregate Equity

Emerging market aggregate equity is priced by the local pricing kernel, which is endogenous to the model. The local pricing kernel is a consequence of local investors’
optimal intertemporal consumption-portfolio decisions and a market clearing condition. The kernel is proportional to the marginal utility of consumption (Rubinstein 1976).

The default time \( \tau \) is an optimal stopping time associated to the pre-default consumption process. At default consumption jumps discontinuously, and after default the stochastic process of consumption changes parameters. These features imply a non-standard pricing kernel. Let \( 1(\cdot) \) be the indicator function. The pricing kernel or discount factor of this economy is proportional to (e.g. Duffie 1996, Cochrane 2001):

\[
\Pi_t = 1_{t<\tau} \left( e^{-\rho t C_t^{-A}} \right) + 1_{t\geq\tau} \left( e^{-\rho t C_t^{-A}} \right), \forall t.
\]

Under the assumption of market segmentation, the pricing kernel of Equation (11) holds only for domestic assets. It cannot be used to price foreign assets. It is helpful to compare asset prices in this emerging economy with prices in an otherwise identical economy without the option to default on foreign debt. In particular, it is useful to define the value discount.

\textbf{Definition 1} Let \( P_t \) be the price of (unlevered) aggregate emerging market equity. Let \( P_{\text{no default}}^t \) be the price of (unlevered) equity in an otherwise identical economy without the option to default on foreign debt. Define the Value Discount before default as

\[
V_t = \frac{P_{\text{no default}}^t}{P_t}.
\]

Aggregate (unlevered) equity is a claim on the stream of aggregate after-tax endowment. Before default, corporations pay taxes equal to the amount of foreign debt service. Therefore, after-tax endowment is equal to consumption by Equation (2). After default, there are neither taxes nor debt service, and endowment is equal to consumption. Proposition 3 has closed-form solutions for the price of aggregate equity in the emerging economy, and in the otherwise identical economy without the option to default on foreign debt.

\textbf{Proposition 3} Let \( K \), \( K^d \) and \( z^* \) be as defined in Proposition 1. Let \( \theta \) be the following parameter:

\[
\theta = K^d (z^*)^{A-1} - K.
\]

Lemma A.3 shows that \( \theta < 0 \). The pre-default aggregate price-consumption ratio of the emerging economy is given by:

\[
\left( \frac{P_t}{C_t} \right) = K + \theta \left( \frac{B}{C_t} \right)^{\beta - A + 1}, 0 \leq t < \tau.
\]
The price-dividend ratio of the otherwise identical economy without the option to default on foreign debt is given by:

\[
\left( \frac{P_t}{C_t} \right)^{\text{no default}} = K, \quad \forall t .
\] (14)

The Value Discount is given by

\[
V_t = -\frac{\theta}{K} \left( \frac{B}{C_t} \right)^{-\beta_A - 1}, 0 \leq t < \tau .
\] (15)

The Value Discount is positive and monotonically increasing with the sovereign yield spread \( S_t \). It approaches zero asymptotically as \( S_t \) goes to zero. The volatility of the Value Discount increases monotonically with \( S_t \).

Proof. See Appendix A.

A Value Discount of 0.10, means that the value of corporate assets in the emerging economy is 10% smaller than what it would be if the country did not have the option to default on its foreign debt. The intuition for a positive Value Discount is as follows. If the country defaults, its economy grows more slowly and with more volatility from the moment of default onwards. Asset markets are forward looking and price-in this possibility. This reduces the current value of the flow of future endowment. The Discount is very small if the economy is far from default. The Value Discount is time-varying. For example, a negative real economic shock brings the emerging economy closer to default and thus increases the Value Discount.

The volatility of the Value Discount increases with the level of the sovereign yield spread \( S_t \). This result is because the magnitude of the effect of a given negative economic shock on the probability of default depends on how far from default the economy is. When the economy is far from default, a given shock barely increases the probability of default. On the other hand, a shock of the same magnitude has a major effect on the probability of default if the economy is already close to default. This is because the optimal default barrier is fixed and real economic shocks are i.i.d. over time, i.e., consumption follows a Geometric Brownian Motion (see Footnote 3 on page 4 for intuition). This result plays a central role in the model’s economic mechanism.

Using Proposition 2, one can write the price of unlevered equity as a closed-form function of the sovereign yield spread. Figure 1 below plots the Value Discount as a function of the sovereign spread, with the parameters discussed in Section 2.4 and summarized in Table 1.

**FIGURE 1**
Proposition 3 computes the price of unlevered equity, that is, the value of corporate assets. Stocks are levered equity, i.e., a position in equity financed by debt. For simplicity, I do not model shareholder’s limited liability and assume that the aggregate stock market is a levered position in firms’ underlying assets financed continuously at the local risk-free rate. As in similar equilibrium models, the local risk-free asset is on zero net supply and the risk-free rate is computed as a shadow price that makes the representative agent happy to hold exactly 100% of his portfolio in aggregate equity. As a result of abstracting from limited liability, leverage in this model increases equity’s expected excess return and volatility proportionately, without affecting its Sharpe Ratio. Below I define the parameter that regulates stock market leverage.

**Definition 2** Let \( \Lambda \geq 1 \) be the leverage parameter of the emerging economy stock market. The parameter \( \Lambda \) governs the aggregate debt-to-equity ratio of stock market:

\[
\Lambda = 1 + \frac{\text{Debt}}{\text{Equity}}.
\]

Proposition 4 uses the pricing formula of Proposition 3 to relate stock market returns to the sovereign yield spread, through their mutual dependence on the current level of consumption \( C_t \).

**Proposition 4** The domestic real interest rate before default is fixed at:

\[
r = \rho + A \left[ \mu - \frac{1}{2} (A + 1) \sigma^2 \right] > 0.
\]

Before default, the stochastic process for the instantaneous excess return on the aggregate stock market is given by \( dR_t = \mu_t(C_t) dt + \sigma_t(C_t) dZ_t \), where:

\[
\mu_t(C_t) = \Lambda \frac{1 + K \mu + \theta (\beta + A) \left( \mu + \frac{1}{2} (\beta + A - 1) \sigma^2 \right) \left( \frac{B}{C_t} \right)^{-\beta-A+1}}{K + \theta \left( \frac{B}{C_t} \right)^{-\beta-A+1}} - \Lambda r,
\]

\[
\sigma_t(C_t) = \Lambda \sigma \frac{K + \theta (\beta + A) \left( \frac{B}{C_t} \right)^{-\beta-A+1}}{K + \theta \left( \frac{B}{C_t} \right)^{-\beta-A+1}}.
\]

The instantaneous stock market Sharpe Ratio is given by \( \mu_t(C_t) / \sigma_t(C_t) \). Using Proposition 2, one can write \( \mu_t(C_t) \), \( \sigma_t(C_t) \) and the instantaneous Sharpe Ratio as functions of the sovereign yield spread \( S_t \).

---

11 My distinction between unlevered and levered equity is not new. Many researchers have previously modeled the aggregate stock market as a levered claim in the future endowment stream. Examples include Campbell (1986), Benninga and Protopapadakis (1990), Abel (1999) and Bansal and Yaron (2004).
Proof. See Appendix A.

Proposition 4 shows that the emerging economy’s stock market displays time-varying expected excess return, volatility and Sharpe Ratio. This result obtains despite the model’s conventional utility function and standard consumption processes, and is a consequence of modeling the option to default on foreign debt. In contrast, asset pricing models for developed markets typically require either complex preferences or complex stochastic processes in order to generate time-varying risk premia - see, for example, Campbell and Cochrane (1999) and Bansal and Yaron (2004) respectively.

Interestingly, time-variation in instantaneous volatility implies that stock market returns in emerging economies are not unconditionally normally distributed. This result obtains even though underlying real economic shocks are i.i.d. Gaussian in the model. This result could help shed light on Bekaert and Harvey’s (1997) finding that the unconditional distribution of stock returns departs more strongly from normality in emerging than in developed economies.

Note that $\sigma_t(C_t) > 0$ at all times, since its denominator is the price-consumption ratio, and both $(\beta + A)$ and $\theta$ are negative. Therefore, positive real economic shocks generate positive stock market returns. Since positive shocks decrease the sovereign yield spread (by Proposition 2), the model implies that stock returns and changes in the yield spread are contemporaneously negatively correlated. However, one is not causing another. Rather, both are endogenously driven by real economic shocks. In principle, there is no need to invoke exogenous political factors - such as an exogenous probability of expropriation along the lines of Cherian and Perotti (2001) and Bansal and Dahlquist (2002) - to justify the negative contemporaneous correlation between stock returns and changes in the sovereign yield spread.

Propositions 2 and 4 allow for the calculation of a "hedge ratio" between stock returns and changes in the sovereign yield spread. The model predicts that a unit change in the sovereign yield spread is contemporaneously associated to a $[\sigma_t(C_t) \div \delta_t(C_t)]$ stock return, where $\sigma_t(C_t)$ and $\delta_t(C_t)$ are given by Equations (10) and (17) respectively. I define the Hedge Ratio formally below.

**Definition 3** Define the Hedge Ratio between the sovereign yield spread and stock returns as the ratio between stock returns and changes in the yield spread. In other words, the Hedge Ratio measures the stock return associated to a contemporaneous unit change in the sovereign spread. Mathematically, the hedge ratio is given by $\sigma_t(C_t) \div \delta_t(C_t)$, where $\sigma_t(C_t)$ and $\delta_t(C_t)$ are defined in Propositions 4 and 2 respectively.
A Hedge Ratio of -1.5 means that a 10 basis points increase in the yield spread is contemporaneously associated to a aggregate stock market return of -0.15%. Since \( \sigma_t(C_t) \) and \( \delta_t(C_t) \) move over time, the Hedge Ratio is time-varying. Note that the Ratio is available as a closed-form function of the yield spread level - see Equations (9), (10) and (17). Figure 2 plots the Ratio as a function of the level of the sovereign yield spread, using structural parameters discussed in Section 2.4 and summarized in Table 1.

**FIGURE 2**

Proposition 5 further explores the relationship of stock market volatility and the sovereign yield spread. It illustrates the "excess volatility" phenomenon described in Section 1.

**Proposition 5** The stock market’s pre-default return volatility \( \sigma_t(S_t) \) increases monotonically with the level of the sovereign yield spread \( S_t \). Furthermore, the emerging economy features "excess volatility" in its stock market. More precisely, \( \sigma_t(S_t) \) is always weakly greater than the return volatility in an otherwise identical economy without the option to default on foreign debt, which is fixed at \( \sigma_{no\_default} = \Lambda \sigma \); and \( \sigma_t(S_t) \) can be substantially greater than the post-default return volatility, fixed at \( \sigma_{post\_default} = \Lambda \sigma^d \).

Proof. See Appendix A.

Figure 3 plots the instantaneous stock market volatility \( \sigma_t(S_t) \) versus the sovereign yield spread, for parameters discussed in Section 2.4 and summarized in Table 1. It also plots the levels of \( \sigma_{no\_default} \) (3 \* 0.04 = 0.12 per year) and \( \sigma_{post\_default} \) (3 \* 0.07=0.21 per year) to allow comparison with \( \sigma_t(S_t) \).

**FIGURE 3**

Figure 3 shows that the option to default on foreign debt increases the sensitivity of stock prices to real economic shocks. This effect can be economically large: the pre-default volatility \( \sigma_t(S_t) \) can be substantially higher than \( \sigma_{no\_default} \) even at moderate levels of the sovereign yield spread. The effect operates through the Value Discount. A negative economic shock takes the country closer to default, increasing the likelihood of default and thus the Value Discount. Therefore, there is one additional channel by which real economic shocks affect stock prices. The reduction in equity prices caused by a given negative shock is larger than the one that happens in a no-default economy, because of the additional price reduction due to the increase
of the Value Discount. This result provides an economic explanation for higher stock market volatility in heavily indebted emerging economies versus developed ones, even after controlling for the volatility of the underlying economic shocks.

Note that the pre-default stock market volatility is not just a time-varying weighted average of the no-default volatility $\sigma^\text{no\_default}$ and the post-default volatility $\sigma^\text{post\_default}$. Figure 3 shows that the pre-default volatility $\sigma_t(S_t)$ can be as high as 0.3866 per year when the country is very close to default, compared to 0.12 and 0.21 for $\sigma^\text{no\_default}$ and $\sigma^\text{post\_default}$ respectively. The assumption that endowment drops discontinuously at default is not solely responsible for the rise of the pre-default volatility above the post-default one. To verify this point, I change $\alpha$ from 0.89 to 1.00 - moving from an endowment loss at default of 0.078 to no loss at default - while keeping all the other parameters unchanged. In this case the pre-default volatility can be as high 0.3120, still substantially larger than $\sigma^\text{no\_default}$ (0.12) and $\sigma^\text{post\_default}$ (0.21).

Figure 3 clearly shows that the stock market’s volatility increases with the sovereign yield spread, as Proposition 5 states. This is because the volatility of the Value Discount itself increases in the sovereign spread, as shown in Proposition 3. Proposition 6 shows that the increase in risk is met by an increase in reward: the cost of capital increases with the sovereign spread $S_t$ as well.

**Proposition 6** The stock market’s pre-default expected excess return $\mu_t(S_t)$ increases monotonically with the level of the sovereign yield spread $S_t$. Moreover, $\mu_t(S_t)$ is always weakly greater than the expected return in an otherwise identical economy without the option to default on foreign debt, which is fixed at $\mu^\text{no\_default} = \Lambda a \sigma^2$.

*Proof. See Appendix A.*

Figure 4 plots the stock market’s expected excess return $\mu_t(S_t)$ versus the sovereign yield spread $S_t$, for the parameters discussed on Section 2.4 and summarized in Table 1. It also plots the levels of $\mu^\text{no\_default}$ (3 * 8 * 0.04² = 0.0384) and $\mu^\text{post\_default}$ (3 * 8 * 0.07² = 0.1176) for comparison.

**FIGURE 4**

Figure 4 clearly shows that the expected excess return on the stock market - and hence the cost of equity capital in emerging markets - increases with the level of the sovereign yield spread. Note that the expected excess return can be as high as 0.1015 per year when the economy is close to default, much higher than its counterpart in an otherwise identical no-default economy (0.0384 per year). My model, calibrated with
plausible structural parameters, shows that the option to default on foreign debt can have an economically large effect on the cost of capital in emerging markets\textsuperscript{12}.

The cost of capital rises with the sovereign yield spread to keep the asset markets of this segmented economy in equilibrium. A negative economic shock drives up stock market volatility endogenously. Local investors would move from stocks into the (local) risk-free asset if the compensation for bearing stock market risk did not increase as well.

For the set of parameters in Table 1, the Sharpe Ratio (compensation per unit of risk) decreases mildly as the sovereign yield spread increases, meaning that the expected excess return does not rise as fast as does volatility. The figure in Appendix C plots the Sharpe Ratio as a function of the yield spread. It can be shown that this result is not robust: the Sharpe ratio can go up or down with the spread, depending on parameter configurations. For example, if the relative risk aversion coefficient $A$ is small (say, 2) the Sharpe Ratio increases as the yield spread increases. Nonetheless, the time-variation in the Sharpe Ratio is economically small, and, as Proposition 6 shows, never clouds the first order effect that the expected excess return rises accompanying an increase in volatility.

\subsection{2.3 The Cross Section of Asset Prices}

The model delivers a practical method for calculating the cost of equity capital for emerging market projects. The market portfolio is instantaneously perfectly correlated with the pricing kernel of this one-factor economy, and thus mean-variance efficient at each instant of time. Hence the model implies a (local) conditional version of the Capital Asset Pricing Model. By Proposition 4, the sovereign yield spread $S_t$ fully captures time-variation in the compensation for bearing stock market risk, and can be used as a conditioning variable for the CAPM. This arises endogenously in the model, as a result of modeling the emerging country’s decision to default on foreign debt.

There are other potential conditioning variables in the model’s one-factor economy, such as the level of aggregate consumption itself (see Proposition 2). However, the sovereign yield spread is superior than consumption for practical pricing purposes. As opposed to the level of consumption, the yield spread can be extracted from bond prices that are traded on a highly liquid market. Thus, it is available in real time. Consumption, on the other hand, is measured on a quarterly or yearly basis and is an average of historical data. It is reported with a delay of months and subject to periodic, and sometimes large, revisions. Perhaps more importantly, the

\textsuperscript{12}In Figure 4, the cost of equity capital $\mu_t (S_t)$ before default is never greater than after default ($\mu^\text{no-defaul} = 0.1176$). However, for some parameter configurations the expected excess return before default rises above the expected excess return after default. For example, changing $\sigma^d$ from 0.07 to 0.05 per year while keeping the other parameter values fixed yields such result.
spread is not subject to the kind of measurement errors that aggregate consumption is, especially in developing economies (see Wilcox 1992). The spread is also a better conditioning variable than the price-dividend ratio. The spread is not affected by corporations’ dividend policies, which might depend on considerations involving the structure of taxes in the economy, and the degree of asymmetric information between corporations and outside investors.

Proposition 7 below will guide my empirical approach for estimating the conditional CAPM with real-world data.

**Proposition 7** The emerging economy features a conditional CAPM in which the sovereign yield spread $S_t$ fully captures time-variation in the compensation for bearing stock market risk. Suppose that the aggregate stock market is sub-divided in $J$ stocks. For simplicity, assume that a linear function of the sovereign yield spread $S_t$ fully captures time-variation in the market betas of individual stocks: $\beta_{jt+1}^j = \beta_0^j + \beta_1^j S_t$. Then, the unconditionally expected excess return of stock $j$ is given by:

$$E[R_{jt+1}] = \beta_0^j E[R_{t+1}] + \beta_1^j E[S_t R_{t+1}] .$$

(19)

**Proof.** See Appendix A. □

Equation (19) says that stocks that are more correlated with the market when the sovereign yield spread is high (large $\beta_1^j$) must pay a premium because they are riskier (higher beta) when risk is high (the sovereign spread is higher). Note that the unconditional correlation of these stocks with the market is not necessarily high. When there is time-variation in the compensation for bearing stock market risk the riskiness of a stock is determined by its co-variation with the market conditional on the state of the economy. In this model the sovereign yield spread tracks the state of the economy and the level of the stock market risk premium, therefore it helps to price stocks cross-sectionally.

In Section 3.2 I take Equation (19) to the data. Equation (19) can be estimated by the two-pass Fama-McBeth procedure described in detail by Cochrane (2001). I group stocks into industry portfolios to reduce sampling errors. In the first pass, the coefficients $\beta_0^j$ and $\beta_1^j$ are obtained by $J$ time series regressions of the stock portfolio $j$ excess return $R_{jt+1}$ on a constant, the market excess return $R_{t+1}$, and the interaction term $S_t R_{t+1}$. The terms $E[R_{t+1}]$ and $E[S_t R_{t+1}]$ represent the time-varying stock market risk premium, and are estimated in a single second pass regression. The second-pass regresses the average excess return across stock portfolios over the entire

---

13 This introduces additional Brownian Motions. Each of the stocks always have positive pay-offs and their sum is always equal to the aggregate stock market, which depend on the Brownian $Z_t$. See Menzly, Santos and Veronesi (2004) and Santos and Veronesi (2005) for a useful parametrization of this notion.
sample period on the coefficients $\beta_j^0$ and $\beta_j^1$ estimated in the first pass. For each stock portfolio a pricing error can be computed: it is the difference between the actual sample average excess return and its fitted value in the second-pass regression.

The static CAPM can be estimated by the same procedure described above, by omitting the interaction term $S_t R_{t+1}$ in both passes. Pricing errors associated to the static CAPM can also be computed as the residuals in the second-pass regression. If the conditional CAPM delivers smaller pricing errors than the static CAPM one can say that there is empirical evidence that the theory is useful to price the cross-section of stocks. A formal econometric test of the theory can be done by checking whether the pricing errors in the cross-sectional regression are statistically equal to zero (jointly).

The second-pass cross-sectional regression may also include a constant term\textsuperscript{14}. The inclusion of a constant term allows the computation of the adjusted $R^2$ in the cross-sectional regression. As in Jagannathan and Wang (1996), Lettau and Ludvigson (2001) and Lustig and Van Nieuwerburgh (2005), the change in the adjusted $R^2$ can be used as a measure of how much the conditional CAPM improves (or not) on the static CAPM in pricing stocks.

2.4 Parameter Calibration

This section discusses the structural parameters used to calibrate the model. Table 1 summarizes the choice of parameters. Figures 1 through 7 were created using the parameters in Table 1.

\begin{table}[h]
\centering
\textbf{Table 1}
\begin{tabular}{|c|c|}
\hline

\end{tabular}
\end{table}

The model features a representative agent with time separable isoelastic preferences. The relative risk aversion coefficient, equal to the reciprocal of the elasticity of intertemporal substitution in consumption, is $A = 8$. Mehra and Prescott (1985) argue that $A < 10$ is reasonable\textsuperscript{15}. Bansal and Yaron (2004) use a relative risk-aversion coefficient of 7.5 or 10.

\textsuperscript{14}Cochrane (2001) writes: "You can run the cross-sectional regression with or without a constant. The theory says that the constant or zero-beta excess return should be zero. You can impose this restriction or estimate a constant and see if turns out to be small. The usual trade-off between efficiency and robustness applies." Having a constant term in the second pass also helps when the risk-free rate is measured with error.

\textsuperscript{15}My choice of $A = 8$ is consistent with papers in Finance. However, it is somewhat large compared to Issler and Piqueira's (2000) point estimates using Brazilian consumption data. Their estimates of the elasticity of intertemporal substitution are close to 0.25. For annual data and a CRRA utility function, the estimate of the relative risk aversion coefficient is 4.89.
The annual time discount rate is $\rho = 0.08$, consistent with econometric estimates by Issler and Piqueira (2000) using aggregate consumption time-series. My choice of $\rho$ is much smaller than Arellano’s (2005) and Aguiar and Gopinath’s (2006). Both papers require very high impatience ($\rho$ above 0.20 per year) in order to generate non-negligible default probabilities in equilibrium.

I use $\mu = 0.020$ for the mean annual consumption growth rate before default, and $\mu^d = 0.0125$ after default. The first value is in line with the average growth rate of consumption experienced by heavily indebted emerging economies over the last two decades. Annual consumption growth volatilities are $\sigma = 0.04$ before default, and $\sigma^d = 0.07$ after default. Prasad et al. (2003) document the volatilities of total consumption growth in developing countries in the 1980s and 1990s. They report that more integrated developing countries had an average volatility of 0.038 per year, while less integrated developing countries had 0.056 per year on average. These values are consistent with the ones reported by Bekaert, Harvey and Lundblad (2005) for emerging economies before and after equity liberalization.

Note that $\mu^d < \mu$ and $\sigma^d > \sigma$ represents worsened consumption/production possibilities after default. When banned from international markets as a consequence of default, emerging countries suffer from lower average growth and higher growth volatility. The parameters calibrated above imply that the increase in growth volatility is about twice as harmful to the emerging economy as the reduction in average growth\textsuperscript{16}.

As mentioned before, $X$ is just a scaling parameter. The endogenous variables in the model adjust to any choice of $X$, such that the choice does affect any of the results reported in the paper. In other words, once one conditions on the sovereign yield spread $S_t$, the knowledge of $X$ does not provide any additional information. Parameter $\alpha$ governs the magnitude of endowment loss at default, and is fixed at $\alpha = 0.89$. Using Equation (5), this choice translates into a endowment loss at default of $\frac{(1-\alpha)B}{B+X} = 0.078$. This is close to Arellano’s (2005) alternative value of 0.080, but larger than her baseline 0.02. Hamann (2004) uses a 0.10 endowment loss-at-default. The world instantaneous real risk-free rate is $r^* = 0.0125$.

The leverage coefficient is $\Lambda = 3$, which implies an average stock market debt-to-equity ratio of 2 and average debt-to-value ratio of 0.67. This value is in line with Schmukler and Vesperoni (2001), who report an average corporate debt-to-equity ratio of 1.896 for a sample of emerging market economies in the 1980’s and 1990’s. Using a sample of 1,968 Brazilian firm-years from 1988 to 2002, Bleakey and Cowan\textsuperscript{16}To make this comparison, one needs to put these effects in the same units. This can be done by looking at the indirect utility function - see Lemma A.1 - and measuring $(\mu^d - \frac{1}{2}A (\sigma^d)^2)$ against $(\mu - \frac{1}{2}A \sigma^2)$. The reduction in the average growth rate reduces the term by 0.0075, while the increase in volatility decreases it by 0.0132.

21
(2004) report a mean debt-to-value ratio of 0.518. The authors report a debt-to-value standard deviation of 0.357. It is not trivial to compute confidence intervals for the debt-to-value ratio because observations are not independent.

3 Empirical Results

The model in Section 2 links stock prices in an emerging economy to its sovereign yield spread. This section tests implications of the model using Brazilian data. Whenever feasible, I check not only qualitative predictions but also quantitative ones, using the structural parameters calibrated in Section 2.4 and summarized in Table 1. Let me first describe the data.

Sovereign yield spreads are extracted from prices of dollar-denominated bonds issued by the Brazilian central government in international markets. The yield spread is the difference between the bond’s promised yield and the yield of a US Treasury bond of the same maturity. I use yield spreads calculated by J.P. Morgan. The bank calculates spreads for several sovereign borrowers on a daily basis and reports an average yield spread for each country, weighted across its various international bonds. This weighted average is called the Emerging Market Bond Index (EMBI) Sovereign Yield Spread. Brazil, Mexico and Venezuela have the longest uninterrupted EMBI time-series available, dating back to January 02 1992. The average maturity of Brazilian sovereign bonds in the EMBI Index changes over time. Although the average maturity does not drift too much, its time variation introduces some noise in my empirical procedures. For example, if the average maturity falls, one should expect a decline in the sovereign yield spread that is not matched by a contemporaneous decrease in stock prices.

For aggregate stock returns, I use local currency returns on the Brazilian IBOVESPA Stock Index. The Index includes reinvested dividends. Excess returns are computed using the local overnight SELIC interest rate, provided by the Central Bank of Brazil. This rate represents the overnight cost of borrowing funds in local currency for a healthy bank, or for the central government.

\footnote{There are actually two EMBI indexes with slight methodological differences. The pure EMBI, described by Vandersteel (1995), has country level data available since January 02 1992, and was discontinued in June 28 2002. The EMBI+, described by Kim (2004), has country level data available since January 02 1998 onwards. I use the EMBI until December 31 1997 and the EMBI+ thereafter.}

\footnote{The SELIC rate is the best candidate for a risk-free rate in Brazil, but is not perfect. The country has a large state-owned bank (BNDES) which disburses long-term loans at lower rates than the SELIC. As a result, the average zero-beta rate in Brazil is likely to smaller than the SELIC rate.}
3.1 Comparing Levels

This sub-section provides a simple assessment of the model’s ability to match the main features of stock market data in the period of January 1992 to October 2005. I do not pursue a formal econometric test of the model\textsuperscript{19}.

Panel A in Table 2 documents that the median sovereign yield spread from January 1992 to October 2005 was 763 basis points per year. In Panel B I use the formulas and calibration from Section 2 to compute stock market characteristics associated with a spread of 763 basis points per year. Panel B compares the main moments of Brazilian stock market returns in the sample period and the results generated by the model side by side.

\textit{TABLE 2}

The model-generated expected excess return is reasonably close to the data. Panel B shows that the average monthly excess return on the Brazilian stock market from January 1992 to October 2005 was 0.0798 (annualized). Note that the uncertainty associated with this estimate is large because of a low signal-to-noise ratio and a relatively short time-series. When the yield spread $S_t$ is at 763 basis points, Equations (9) and (17) give a expected excess return of 0.0941.

The model also does a reasonably good job in matching the volatility of stock returns observed in the data. Panel B shows that the volatility in the Brazilian stock market within each calendar month was 0.4067 (annualized) from January 1992 to October 2005 was. The two standard-deviation confidence interval is ±0.0452. When the spread is at 763 basis points (median spread in the sample period), Equations (9) and (18) yield a return volatility of 0.3555 per year. The model, calibrated with plausible parameters, can rationalize higher volatility in emerging markets without invoking exogenous political factors. As previously discussed, the option to default on foreign debt increases the sensitivity of stock prices to real economic shocks because a given shock changes the discount at which stocks are traded vis-à-vis an identical no default economy.

The model does not match the empirical average Hedge Ratio. Panel B shows that from January 1992 to October 2005 a 10 basis points increase in the sovereign yield spread was on average matched by a contemporaneous –0.431% stock return. In contrast, Equations (9), (10) and (18) give a Hedge Ratio of just –0.5883, meaning that a 10 basis points increase in the spread is related to a –0.059% stock return.\textsuperscript{19}

\textsuperscript{19}Such a test would probably lead to the formal rejection of the model. One reason for the likely rejection is the fact that the null hypothesis that the Brazilian sovereign yield spread has a unit root is rejected in the data. This is at odds with the model, which implies that the spread is non-stationary.
This result indicates that although the model gets the correct sign of the Hedge Ratio it is not able to fit its average value. Relative to model predictions, daily stock returns tend to be too large relative to contemporaneous sovereign yield spread changes.

Incidentally, the one-month risk-free rate generated by the model (Equation 16) is 0.1824. This is surprisingly close to the average real risk-free rate in Brazil from January 1992 to October 2005, which was 0.1743.

3.2 Time-Series Implications

This sub-section checks whether the model time-series implications are confirmed in the data.

**Realized equity returns: the Hedge Ratio.** Propositions 2 and 4 show that changes in the sovereign yield are contemporaneously negatively correlated with stock market returns. The regression results in Table 3 show that Brazilian data from January 1992 to October 2005 confirm this implication.

Table 3 reports results of two types of regressions. First, I regress the daily stock market’s excess return on a constant and the contemporaneous change in the sovereign yield spread. This regression is mis-specified according to the model, since Equation (10) and (18) show that the Hedge Ratio changes with the level of the spread. The second type of regression is less subject to this problem because it includes an interaction term to control for changes in the level of the spread. The interaction term is the product of the change in the yield spread and the log of the level of the spread measured in basis points. The log is used because Figure 2 shows that the effect is non-linear and concave. Table 4 has results for the full sample, as well as for two time sub-samples.

**TABLE 3**

The first type of regression shows that equity returns and changes in sovereign spreads are strongly negatively correlated contemporaneously as the theory predicts. Table 3 reports that the average empirical Hedge Ratio from January 1992 to October 2005 is −4.3064. The Table also shows that the Hedge Ratio decreases (in absolute value) with the level of the yield spread as the model predicts (see Figure 2). The coefficient on spread changes is negative (−34.5778) while the coefficient on the interaction term is positive (4.2286).

When calibrated with the parameters discussed in Section 2.4, the model does not match the magnitude of the Empirical Hedge Ratio. Figure 5 plots the empirically fitted Hedge Ratio against the level of the sovereign spread using the regression result
in Table 3. The Empirical Hedge Ratio is \( \hat{\beta}_1 + \hat{\beta}_2 \ln(S_t \times 10,000) \), where \( \hat{\beta}_1 \) and \( \hat{\beta}_2 \) are the full-sample regression coefficients in Table 3. This Empirical Hedge Ratio is compared to the Theoretical Hedge Ratio, which is calculated with the structural parameters of Section 2.4. The Theoretical Hedge Ratio is the same as in Figure 2. Figure 5 shows that the model gets the correct sign of the Hedge Ratio and the fact that it decreases with the level of the sovereign spread. But the Empirical Hedge Ratio is considerably larger than the Theoretical Hedge Ratio.

**FIGURE 5**

Results in Table 3 hold in time sub-samples. Appendix D reports the Spearman (rank) correlation between stock returns and contemporaneous yield spread changes. The correlation is strongly negative and one can comfortably reject the null hypothesis that the two series are independent.

**Volatility of equity returns:** Proposition 5 states that the higher the probability of foreign default, the more volatile are stock returns. Regression results in Table 4 show that this prediction of the model is confirmed empirically using Brazilian data from January 1992 to October 2005.

In Table 4 I regress the stock volatility level on a constant and the log of the contemporaneous level of sovereign yield spread. The log function is used because Figure 3 shows that the model implies that volatility is roughly linear in the log of the yield spread. To check the robustness of my results, I use two different ways to measure volatility. Panel A uses the conditional volatility fitted by a GARCH(1,1) model estimated with daily data and the full sample (see Engle 1982 and Bollerslev 1987). Panel B uses monthly data and volatility is the standard deviation of daily excess returns within each calendar month.

**TABLE 4**

Panels A and B of Table 4 indicate that stock volatility and spreads are strongly positively correlated as the theory predicts. The higher the spread, the higher tends to be the stock market’s volatility. Results also hold for time sub-samples.

I use Figure 6 to check whether the magnitude of the coefficients reported in Table 4 agree with model predictions, using the parameters discussed in Section 2.4. Figure 6 compares the model-generated relationship between volatility and yield spread to regression results. The model-generated volatility as function of the spread is given by Equation (18) and illustrated in Figure 3. On the other hand, the empirical volatility is \( \beta_0 + \beta_1 \ln(S_t \times 10,000) \), where \( \beta_0 \) and \( \beta_1 \) are the full-sample coefficients.
reported in Table 4 Panel B. Note that, as opposed to Figure 3, the sovereign spread is not in Log scale in Figure 6.

**FIGURE 6**

Theoretical curve in Figure 6 is outside the band for the relevant range of the sovereign yield spread, which is between the 5% (404) and 95% percentile of the data (1, 437). This is just a re-statement of the fact that the model-generated volatility when the spread is at 763 (median Brazilian spread in the period) is only 0.3555 whereas the average volatility in the period is 0.4067. More interestingly, Figure 6 shows that the relationship between stock market’s volatility and the sovereign yield spread is stronger in the data than in the model, at least when the model is calibrated as in Section 2.4. The volatility changes more drastically with the spread than what the model predicts.

Appendix E reports Spearman (rank) correlations between volatility and the sovereign yield spread. Results confirm that the time-series are strongly positively correlated. The null of independence can be comfortably rejected. Appendix F has results for yet another way to measure stock volatility. In Appendix F I do not impose the GARCH(1,1) structure on daily volatility volatility, but measure volatility each day using Parkinson’s (1980) range estimator (one day sample only). Results confirm that the volatility increases with the level of the yield spread as the model predicts.

**Expected equity returns:** Proposition 6 states that the stock market’s expected excess returns increases with the level of the sovereign spread. In other words, a high spread today predicts a high excess return going forward. Regression results in Table 5 confirm this implication of the model using Brazilian data from January 1992 to September 2005.

Table 5 tests the null hypothesis of no predictability using three different time horizons. Realized excess returns are computed looking at one, two and three calendar months ahead, and regressed on the lagged log of the sovereign yield spread. To explore the (limited) dataset as efficiently as possible, I pool Brazilian data with data from Mexico and Venezuela. These are the only two other emerging markets with EMBI data available uninterruptedly since Jan 02 1992.

**TABLE 5**

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20 The log function is used because Figure 4 indicates that the expected excess return is roughly linear in the log of sovereign yield spread when the spread is in the range observed in January 1992 to October 2005.

21 Panel B of Table 6 only uses Mexican data up to February 2002. At that point in time the country received investment grade status by Standard and Poor’s. Results are qualitatively the same if the full sample for Mexico is used.
Results in Table 5 show that the coefficient on the lagged sovereign yield spread is positive in all the three forecasting regressions as the model predicts. The higher the spread, the higher tends to be the stock market’s excess return going forward. The evidence of predictability is statistically significant for all horizons. Note that the magnitude of the coefficient on the log lagged spread is similar for all periodicities. Appendix G has predictability regressions using Brazilian data only. Results there also show that the coefficients on the log lagged spread are similar to the ones in Table 5 for all forecasting horizons.

I use Figure 7 to check whether the magnitude of the coefficients reported in Table 5 is consistent with model predictions, when the model is calibrated as in Section 2.4. Figure 7 compares the model-generated relationship between expected excess returns and yield spread given by Equation (17) with the relationship resulting from the regression in Table 5. The empirical expected excess return as function of the spread is \( \beta_0 + \beta_1 \log(S_{t+10,000}) \), where \( \beta_0 \) and \( \beta_1 \) are the monthly coefficients reported in Table 5. Note that, as opposed to Figure 3, the sovereign spread is not in Log scale in Figure 6.

\begin{figure}[h]
\centering
\caption{Figure 7 illustrates the fact that it is hard to detect time-variation in expected excess returns. The 95% confidence band around the Empirical point estimate is very wide. The Theoretical curve is within the band for the relevant range of the sovereign yield spread, which is between the 5% (403) and 95% percentile of the data (1,437). But the point estimate suggests that relationship between return volatility and sovereign yield spread is stronger in the data than what the model predicts, at least when the model is calibrated as in Section 2.4.}
\end{figure}

3.3 Cross-Sectional Implication

In this section I use returns on industry portfolios to estimate the Conditional CAPM Equation (19) empirically. I use a Fama McBeth two-pass regression methodology, described in Cochrane (2001). This empirical approach has been used by Lettau and Ludvigson (2001) and Lustig and Van Nieuwerburgh (2005), among others. Results are consistent with model predictions: the conditional CAPM - using the sovereign yield spread as a conditioning variable - does a better job explaining dispersion of average returns across industry portfolios than the static CAPM.

To perform the two-pass methodology one needs to start by arbitrarily choosing test portfolios\(^{22}\). I use monthly excess returns on 24 test portfolios: twelve industry

\(^{22}\)Research by Daniel and Titman (2005) shows that empirical results of recent asset pricing models depend crucially on the choice of the test portfolios.
portfolios and twelve managed portfolios based on the industry portfolios and the sovereign yield spread. I use Dow Jones Total Market (DJTM) Industry Portfolios in Brazil, at the Supersectoral Level\textsuperscript{23}. Returns are in local currency, and computed each calendar month starting in January 02 1992 and ending in September 30 2005. Following Cochrane (1996) and Lettau and Ludvigson (2001), the exposure of a managed portfolio to the original portfolio is controlled by the sovereign yield spread level. A leveraged long position is formed when the yield spread is unusually large. When the spread is unusually low the portfolio is levered down. A short position is formed when the spread is very low. More precisely, the excess return of the managed portfolio of industry \( j \) at time \( t \) is:

\[
R_{j}^{(\text{managed})} = (1 + S_{t-1}^{*}) R_{t}^{j},
\]

where \( S_{t-1}^{*} = \frac{S_{t-1} - \overline{S}}{\sigma_{S}} \), and \( \overline{S} \) and \( \sigma_{S} \) are respectively the unconditional mean and standard deviation of the yield spread.

The first pass of the procedure has \( J \) time-series regressions. Each test portfolio excess return \( R_{t}^{j} \) is separately regressed on a constant, the contemporaneous market return \( R_{t} \), and the product between the market return and the lagged spread \( R_{t}S_{t-1}^{*} \). I standardize the spread following the advice in Ferson et al.(2003). The first pass regressions yield \( \beta_{0}^{j} \) and \( \beta_{1}^{j} \) for each stock portfolio \( j \) The second pass has one cross-section regression: the average excess return across the test portfolios \( \overline{R} \) is regressed on the betas estimated in the first pass regression. The \( \lambda_{1} \) and \( \lambda_{2} \) coefficients estimated on the the second pass refer to the stock market risk premium terms \( E[R_{t+1}] \) and \( E[S_{t}R_{t+1}] \) of Equation (19). When estimating the static CAPM I omit the term \( R_{t}S_{t-1}^{*} \) in both the first and second passes. My theory implies that this omission will reduce the model’s ability to fit the data.

Results are in Table 6. I report two sets of results: including a constant in the second pass or not. In the regressions including a constant I report the \( R^{2} \) and the adjusted \( R^{2} \) Table 6 has two types of t-stats: corrected for heteroskedasticity; and corrected for heteroskedasticity and for generated regressors using Shanken’s (1992) correction. As Lettau and Ludvigson (2001) point out, Jagannathan and Wang (1998) show that the Fama-McBeth procedure does not necessarily overstate the precision of standard errors if conditional heteroskedasticity is present. Hence, it is useful to report conventional t-statistics as well.

\textit{TABLE 6}

\textsuperscript{23} Return data for Dow Jones Total Market Industry Portfolios in Brazil are available since January 1992. Dividends are re-invested in the portfolios. Dow Jones portfolios in Brazil are tilted towards large stocks. The data can be downloaded from Dow Jones website for free.
Results in Table 6 are consistent with model predictions. The regressions with a constant shows that the static CAPM explains about 30% of the cross-sectional variation in average returns across the 24 test portfolios. On the other hand, the conditional CAPM explains about 50% of the variation. Accounting for time-variation in the stock market risk premium does seem to improve the pricing of stock portfolios. The static CAPM has pricing errors 20% higher than the conditional CAPM. The risk premium coefficients $\lambda_1$ and $\lambda_2$ are statistically significant before accounting for uncertainty in the $\beta_0^j$ and $\beta_1^j$ coefficients estimated in the first pass. However, the Shanken correction drastically reduces the t-statistics. This is a weakness of my empirical results, and is a result of using relatively short time-series in the first pass regressions. Appendices H and I report two-pass regressions results with bi-monthly and quarterly rather than monthly data. Qualitative results are the same.

Note that the estimated zero-beta rates $\lambda_0$ in Table 6 are negative and statistically insignificant at conventional levels, in contrast to typical cross-sectional asset pricing studies using U.S. data (see Lettau and Ludvigson 2001, for example). The regressions without a constant show that imposing the model-implied constraint $\lambda_0 = 0$ reduces the estimates of $\lambda_1$ substantially, in both the static CAPM (from 0.0245 to 0.0151) and the conditional CAPM (from 0.0147 to 0.0080). The average market premium $\lambda_1$ becomes closer to the unconditional point estimate reported on Table 2, Panel B (0.0798 per year is 0.0067 per month). Note that the conditional CAPM produces $\lambda_1$ estimates much closer to 0.0067 in the regressions with and without a constant. Therefore, the conditional CAPM is likely to perform better than the static CAPM in time-series asset pricing tests.

The conditional CAPM implied by the model produces economically relevant differences in the cost of capital across industries. The following example helps illustrating the results in Table 6, and provides a taste of how much time-variation in market betas and in the market risk premium affect stocks cross-sectionally. Appendix J lists the $\beta_0^j$ and $\beta_1^j$ coefficients estimated in the first pass of Table 6. Stocks in the Basic Resource Industry Portfolio have an unconditional market beta of 0.64, while the unconditional beta of Banks stocks is 0.73. If the unconditional risk premium in the Brazilian stock market is 8% per year, the static CAPM predicts a cost of equity capital of 5.1% and 5.8% per year (on top of the risk-free rate) for Basic Resources and Banks respectively. However, Bank stocks are substantially riskier than Basic Resources stocks than what the comparison of unconditional betas suggests. We have: $\beta_1^{Banks} = 0.096$ while $\beta_1^{Basic\ Resource} = -0.151$, meaning that Bank stocks co-vary more strongly with the market when the Sovereign Yield Spread $S_t$ is high, as opposed to Basic Resources stocks. Using a coefficient $\lambda_1 = 0.0130$ as in Table 6 gives that cost of equity capital for Banks and Basic Resources is respectively 7.3% and 2.8% per year.
4 Conclusion

I have presented a consumption-based asset pricing model to study how the option to default on foreign debt affects asset prices in emerging economies. The model allows for closed-form solutions. A key result of the model is that both the volatility and the expected excess return of emerging stock markets increase with the level of the sovereign yield spread. The economic intuition is as follows. As a result of the option to default on foreign debt, stock prices in emerging economies trade at a discount relative to identical no-default economies. This is because default brings about a negative regime change: after default there is lower average economic growth and higher economic growth volatility. The sovereign spread has information about the country’s probability of default, and therefore about the likelihood of an endogenous regime change in local economic conditions. When the yield spread is high, the economy is close to default and the impact of a given real economic shock on the probability of default is larger, since the optimal default barrier does not change with economic shocks and economic shocks are i.i.d. over time. Therefore, the probability of default is more volatile at higher spreads, which causes the discount at which the emerging stock market trades vis-à-vis an identical no-default economy to be more volatile at higher spreads. The higher volatility of the discount causes higher volatility in stock prices at higher levels of the sovereign yield spread. Since asset markets are segmented internationally, an increase in stock market volatility must be accompanied by an increase in its expected excess return to keep local asset markets in equilibrium. The model allows for a quantitative assessment of how the option to default on foreign debt affects asset prices in emerging economies.

A calibration of the model with plausible structural parameters suggests that the time-variation in the emerging stock market’s expected excess return and volatility is economically relevant. In a no-default economy, the stock market expected excess return and volatility are respectively 3.84% and 21% per year. As a result of the option to default on foreign debt, the expected excess return and volatility increase to 9.41% and 35.55% per year, when the sovereign yield spread is at 763 basis points per year.

The model produces time-series implications that are empirically supported. Stock returns and changes in the sovereign yield spread are contemporaneously negatively correlated. The stock market’s volatility increase with the level of the sovereign yield spread. A high sovereign yield spread forecasts a high stock market’s excess return in the future, which indicates that the stock market’s expected excess return increases with the yield spread. However, the model, when calibrated with the structural parameters in Section 2.4, cannot match the time-series magnitudes observed in the

\[24\text{See Footnote 3 on page 4 for "statistical intuition".}\]
\[25\text{The median Brazilian yield spread in January 1992 to October 2005 was 763 basis points per year. In the same period, the stock market average excess return and volatility were 9.48% and 40.67% respectively.}\]
Stock returns are too large relative to contemporaneous changes in the yield spread. The increase in stock market volatility and expected excess return with the level of the spread appears to be stronger in the data than what the model predicts.

Future research might investigate if extending the model improves its ability to match the magnitudes estimated in time-series regressions. A short list of possible extensions follows. Duffie and Epstein’s (1992) stochastic differential utility separates relative risk aversion from the reciprocal of the elasticity of intertemporal substitution, which would provide one additional degree of freedom in choosing parameters. Following Collin-Dufresne and Goldstein (2001), the foreign interest rate could be stochastic, and correlated with the domestic consumption process. The foreign debt continuous coupon could be stochastic as well. Finally, an exogenous Poisson re-entry process after default could be introduced. Some of these extensions require giving up on closed-form solutions.

The model delivers a novel method for calculating the cost of capital in emerging markets: a conditional (local) CAPM in which the sovereign spread is the relevant conditioning variable. Stocks that co-vary more strongly with the market when the sovereign yield spread is high are riskier, and therefore command a premium. I use a two-pass Fama-McBeth regression to show that the conditional CAPM has smaller pricing errors than the static CAPM and explains a larger fraction of the dispersion of average returns across Brazilian industry portfolios. I also show that accounting for time-variation with the sovereign yield spread of both the stock market risk premium and of stock betas may induce large deviations from the static CAPM paradigm. These results are in line with model predictions.

Finally, this paper suggests that it might be unnecessary to invoke exogenous political factors – presumably driven either by complex agency problems in local politics or by irrationality of political leaders – to account for high volatility in emerging stock markets. Likewise, these exogenous political factors might not be necessary to explain the link between sovereign yield spreads and equity prices in emerging countries. Both phenomena arise endogenously in a simple model in which incentives within the emerging country are perfectly aligned and a fully-rational and benevolent local government optimally chooses to default on foreign debt.
References


Appendix A: Proofs

**Lemma A.1:** Suppose consumption follows a GBM with fixed parameters \( \mu \) and \( \sigma \). Define \( F(C_t, \mu, \sigma) \equiv E_t \int_t^\infty e^{-\rho(s-t)} \frac{C_s^{1-A}}{1-A} ds \). Then:

\[
F(C_t, \mu, \sigma) = \frac{1}{1-A} KC_t^{1-A},
\]

where

\[
K = \left[ \rho + (A-1) \left( \mu - \frac{1}{2} A \sigma^2 \right) \right]^{-1}.
\]

**Proof.** See Merton (1971).

**Lemma A.2:** Suppose consumption follows a GBM with fixed parameters \( \mu \) and \( \sigma \), and that \( C_t > B \). Let \( \tau = \min \{ s : C_s = B \} \). Define \( H(C_t, B, \mu, \sigma) \equiv E_t e^{-\rho(\tau-t)} \). Then

\[
H(C_t, B, \mu, \sigma) = \left( \frac{B}{C_t} \right)^{-\beta},
\]

where

\[
\beta = -\left( \frac{\mu}{\sigma^2} - \frac{1}{2} \right) - \sqrt{\left( \frac{\mu}{\sigma^2} - \frac{1}{2} \right)^2 + 2 \frac{\rho}{\sigma^2}} < 0.
\]


**Proof of Proposition 1**

The problem of the government at time \( t \) is:

\[
\sup_\tau E_t \int_t^\infty e^{-\rho(s-t)} \frac{C_s^{1-A}}{1-A} ds
\]

This is a traditional optimal stopping problem in a GBM framework. It is well known that the optimal \( \tau \) is associated with the first time consumption hits an optimal barrier \( B \). That is, \( \tau = \min \{ s : C_s = B \} \). For an interior solution, it is obvious that the optimal barrier is below the current level of consumption. Note that the stochastic processes for \( C_t \) are different before and after \( \tau \). I use different integration dummies in each phase to help the reader keep track of this fact. Breaking up the integral in a phase before default and a phase after default, one can write:

\[
E_t \int_t^\infty e^{-\rho(s-t)} \frac{C_s^{1-A}}{1-A} ds = E_t \int_t^\tau e^{-\rho(u-t)} \frac{C_u^{1-A}}{1-A} du + E_t \int_\tau^\infty e^{-\rho(v-t)} \frac{C_v^{1-A}}{1-A} dv
\]

Note that the pre-default term in Equation 14 can be written as:

\[
E_t \int_t^\tau e^{-\rho(u-t)} \frac{C_u^{1-A}}{1-A} du = E_t \int_t^\infty e^{-\rho(u-t)} \frac{C_u^{1-A}}{1-A} du - E_t \int_\tau^\infty e^{-\rho(u-t)} \frac{C_u^{1-A}}{1-A} du
\]

The strong Markov property of the GBM (see Shreve 2004) implies:
Substituting (24) and the equations above into (23) yields:

\[
E_t \int_{\tau}^{\infty} e^{-\rho(u-t)} \frac{C_{1-A}^{1-A}}{1-A} \, du = E_t \int_{\tau}^{\infty} e^{-\rho(\tau-t)} \frac{C_{1-A}^{1-A}}{1-A} \, du
\]

\[
E_t \int_{\tau}^{\infty} e^{-\rho(u-t)} \frac{C_{1-A}^{1-A}}{1-A} \, dv = E_t \int_{\tau}^{\infty} e^{-\rho(\tau-t)} \frac{C_{1-A}^{1-A}}{1-A} \, dv
\]

Note that Equation (25) involves expressions defined in Lemmas A.1 and A.2. Using the functions defined in these Lemmas reduce the problem of the government to:

\[
\max_B F(C_t, \mu, \sigma) + H(C_t, B, \mu, \sigma) \left[ F(\alpha B + \bar{X}, \mu^d, \sigma^d) - F(B, \mu, \sigma) \right]
\]  

(26)

The first term in Equation (26) is the indirect utility in the no-default economy. The second term represents the utility value of the default option. Clearly, this value is positive for the optimal \(B\). The optimal default barrier \(B\), associated with the optimal stopping time \(\tau\), maximizes the utility value of the option to default. Setting the derivative of Equation (26) with respect to \(B\) equal to 0 defines the optimal \(B\) as function of model parameters. Equations (7) and (8) have the final result of such computation. From Equation (7):

\[
z^* = \frac{B}{\alpha B + \bar{X}} = \frac{C_{\tau-}}{C_{\tau}}
\]  

(27)

Therefore, the optimal solution \(z^*\) has to be in the interval \((0, 1)\). This is directly related to the trade-off that the option to default allows. The defaulting country enjoys an immediate positive jump in consumption flow from \(B\) to \(\alpha B + \bar{X}\). Note that endowment is reduced from \(B + \bar{X}\) to \(\alpha B + \bar{X}\) at default. After default, however, consumption (and endowment) grow more slowly and with more volatility.

Given the parametric constraints, Equation 8 allows for a unique solution \(z^*\) in \((0, 1)\). For simplicity, first suppose that \(\alpha = 1\). Equation 8 has a solution \(z^*\) in the interval \((0, 1)\) because it is continuous in \(z\), its left hand side is strictly negative for \(z = 0\) and strictly positive for \(z = 1\). The first derivative with respect to \(z\) of the left hand side of Equation 8 is equal to zero if and only if \(z = 0\) or \(z = \frac{\beta}{\alpha} < 0\). Therefore the left hand side is a monotonic function of \(z\) in the interval \((0, 1)\), and there is only one \(z^*\) in \((0, 1)\) that solves Equation 8. Finally, note that Equation 8 is continuous in \(\alpha\). Therefore, if \(\alpha\) is close enough 1, the equation stills admits one and only one solution \(z^*\) in the interval \((0, 1)\).

Some algebra reveals that the second order condition for a maximum is satisfied. The second derivative of the utility function at the optimal \(B\) is:

\[
\frac{\overline{X}}{C_{\tau^*}} B^{-\beta-A-2} z^* (\alpha z^* + \beta) < 0
\]

The inequality holds because \(z^*\) is in the interval \((0, 1)\), \(\alpha\) is in \((0, 1)\) and \(\beta < -1\).
Proof of Proposition 2
Emerging market’s foreign debt is a perpetuity paying a continuous flow of interest \( \bar{X} \) until \( \tau \), and nothing thereafter. Let \( Q^* \) be the risk-neutral measure prevailing in global markets. The price \( F_t \) of the emerging market perpetuity is:

\[
F_t = \mathbb{E}_{Q^*}^t \int_t^\tau \bar{X} e^{-r^* (u-t)} du, \forall t < \tau.
\]

By assumption \( \bar{X} \) and \( r^* \) are constants. Since foreign investors are risk-neutral with respect to emerging market’s specific risk, the dynamics of \( C_t \) is the same under \( Q^* \) and under the physical measure. Therefore:

\[
F_t = \bar{X} \mathbb{E}_t \int_t^\tau e^{-r^* (u-t)} du.
\]

Write the integral above as a difference between the integral from \( t \) to \( \infty \) and the integral from \( \tau \) to \( \infty \). Using the strong Markov property as in Proposition 1, and the fact that \( r^* \) is constant, yields:

\[
F_t = \bar{X} \left[ \int_t^\infty e^{-r^* (u-t)} du - \mathbb{E}_t e^{-\rho (\tau-t)} \int_\tau^\infty e^{-r^* (u-\tau)} du \right]
\]

Solving the integrals and using Lemma A.2 gives:

\[
F_t = \bar{X} \left[ 1 - \left( \frac{B}{C_t} \right)^{-\beta} \right].
\]

The perpetuity’s promised yield is defined as \( y_t = \frac{\bar{X}}{F_t} \). The yield of a risk-free perpetuity is \( r^* \). Thus, the sovereign yield spread \( S_t = y_t - r^* \) is given by Equation (9). Applying Ito’s Lemma to Equation (9) gives Equation (10).

Lemma A.3: Let \( K, K^d \) and \( z^* \) be as defined in Proposition 1. Let \( \theta = K^d (z^*)^{A-1} - K \). Then \( \theta < 0 \).

Proof. This is a consequence of the option to default having positive utility value. Note that:

\[
\theta = K^d \left( \frac{\alpha B + \bar{X}}{B} \right)^{1-A} - K
\]

\[
= \frac{1-A}{B^{1-A}} \left( \frac{B}{C_t} \right)^{-\beta} \left( \frac{B}{C_t} \right)^{-\beta} \left[ \frac{1}{1-A} K^d (\alpha B + \bar{X})^{1-A} - \frac{1}{1-A} KB^{1-A} \right]
\]

\[
= \frac{1-A}{B^{1-A}} \left( \frac{B}{C_t} \right)^{-\beta} H(C_t, B, \mu, \sigma) \left[ F(\alpha B + \bar{X}, \mu^d, \sigma^d) - F(B, \mu, \sigma) \right] < 0.
\]

The inequality holds because \( H(C_t, B, \mu, \sigma) \left[ F(\alpha B + \bar{X}, \mu^d, \sigma^d) - F(B, \mu, \sigma) \right] > 0 \) for the optimal \( B \) (see Equation 26 in Proposition 1) and \( A > 1 \).
Proof of Proposition 3

The price of unlevered equity is the value of the future after-tax endowment stream. The basic pricing equation (see Duffie 1996 or Cochrane 2001, for example) is:

\[ P_t \Pi_t = E_t \int_t^\infty \Pi_s C_s ds, \forall t \]

Using the definition of the pricing kernel \( \Pi_t \):

\[ P_tC_t^{-A} = E_t \int_t^\infty e^{-\rho(a-t)} C_s^{1-A} ds. \tag{28} \]

First consider the case \( 0 \leq t < \tau \). Following exactly the same steps of Proposition 1 gives:

\[ P_tC_t^{-A} = (1 - A) \left\{ F(C_t, \mu, \sigma) + H(C_t, B, \mu, \sigma) \right\} \left\{ F(\alpha B + \overline{X}, \mu^d, \sigma^d) - F(B, \mu, \sigma) \right\}. \tag{29} \]

Using the definitions of \( F(\cdot) \) and \( H(\cdot) \) in Lemmas A.1 and A.2, and dividing both sides by \( C_t^{-A} \) gives:

\[ P_t = \left[ K + \theta \left( \frac{B}{C_t} \right)^{-\beta-A+1} \right] C_t. \tag{30} \]

After default, when \( t \geq \tau \), the model is a standard Lucas-Rubinstein economy with a CRRA representative agent. By Lemma A.1, the price-dividend ratio is fixed at \( K^d \). Since \( A > 1 \), equity prices go up discontinuously at default:

\[ \left( \frac{P_{\tau^-}}{P_{\tau}} \right) = (z^*)^{A-1} < 1. \]

This is a well-known feature of the Lucas-Rubinstein economy with CRRA preferences and relative risk aversion greater than unity. Default is bad news for consumption growth. After default, real interest rates fall by so much that equity prices actually go up.

The identical economy without the option to default is also a standard Lucas-Rubinstein one. Foreign debt is serviced forever. By Lemma A.1, the price dividend ratio is fixed at \( K^d \). Therefore, the value discount for \( 0 \leq t < \tau \) is:

\[ V_t = \frac{P_t^{no\_default}}{P_t} - P_t = -\frac{\theta}{K} \left( \frac{B}{C_t} \right)^{-\beta-A+1} > 0. \tag{31} \]

Recall that \( 1 < A < -\beta \) for an interior solution. Therefore, the value discount is a strictly decreasing function of \( C_t \) and, by Proposition 2, a strictly increasing function of the sovereign yield spread \( S_t \). Equation (31) shows that discount approaches zero asymptotically as \( S_t \) goes to zero.

Applying Ito’s Lemma to Equation (31) gives that the volatility of the Value Discount is equal to \( -\frac{\theta}{K} (\beta + A - 1) \left( \frac{B}{C_t} \right)^{-\beta-A+1} \). Since \( 1 < A < -\beta \), the volatility is strictly decreasing in \( C_t \). By Proposition 2, the volatility of the Value Discount is strictly increasing in the sovereign yield spread \( S_t \).
Proof of Proposition 4

The instantaneous (pre-default) risk-free rate implied by the pricing kernel in Equation (11) is constant and given by Equation (16) (see Duffie 1996). Although the pricing kernel of Equation (11) is non-standard, the instantaneous interest rate is the same as in a standard Lucas-Rubinstein economy. This result is because consumption follows a diffusion, hence its path is continuous everywhere and "it takes time" to reach the default barrier.

By definition, the instantaneous excess return of unlevered equity is:

$$dR_t^U = \frac{dP_t + C_t dt}{P_t} - rdt.$$ 

Consider the case \(0 < t < \tau\). Applying Ito's Lemma to \(P_t\) of Equation (13) and substituting above gives the stochastic process for the excess return in unlevered equity. Stocks represent a levered position in corporate assets, where each unit of own capital is augmented by \(1 \Lambda\) units of capital borrowed at the risk-free rate. Therefore, \(dR_t = \Lambda dR_t^U\) and Equations (17) and (18) hold. ■

Proof of Proposition 5

The post-default volatility and the volatility in the otherwise identical economy without the option to default follow directly from the definition of instantaneous return, and the application of Ito's Lemma to Equations (14). Equation (18) has the stock market instantaneous volatility \(\sigma_t\) as a function of \(C_t\), for \(0 < t < \tau\). The sign of its derivative is given by:

$$\text{sign} \left\{ \frac{\partial \sigma_t}{\partial C_t} \right\} = \text{sign} \left\{ \theta K (\beta + A - 1) \left( \frac{B}{C_t} \right)^{-\beta - A + 1} \frac{1}{C_t^2} \right\} < 0.$$

The pre-default volatility \(\sigma_t\) increases when \(C_t\) decreases. By Proposition 2, the expected excess return on the stock market increases with the sovereign yield spread \(S_t\).

The post-default expected excess return and the expected excess return in the otherwise identical economy without the option to default follow directly from the definition of instantaneous return, and the application of Ito's Lemma to Equations (14). Equation (17) has the stock market's instantaneous expected excess return \(\mu_t\) as a function of \(C_t\), for \(0 < t < \tau\). The sign of its derivative is given by:

$$\text{sign} \left\{ \frac{\partial \mu_t}{\partial C_t} \right\} = \text{sign} \left\{ K (\beta + A - 1) \left( \mu + \frac{1}{2} [\beta + A] \sigma^2 \right) - 1 \right\} < 0.$$

To verify the inequality, use the the definition of \(K\), he parametric restriction \(1 < A < -\beta\) and the fact that \(\rho = \frac{1}{2} \beta^2 \sigma^2 + (\mu - \frac{1}{2} \sigma^2)\). By Proposition 2, the stock market’s expected excess return increases with the yield spread \(S_t\).

The pre-default expected excess return \(\mu_t\) achieves its minimum \(\Lambda A \sigma^2\) when \(C_t \rightarrow \infty\). Therefore, the expected excess return is always (weakly) greater in the economy...
with the default option. The pre-default expected excess return achieves its maximum
\[ \Lambda^{1+Ku+\theta(\beta+A)(\mu+x(\beta+A-1)s^2)} \] when \( C_t \to B. \]

**Proof of Proposition 7**

Equations (4) and (11) show that the return to the aggregate stock market is perfectly (negatively) correlated with the pricing kernel. Therefore, the market portfolio is conditionally mean-variance efficient (see Cochrane 2001). Therefore, for any stock \( j \) with
\[ \beta^j_{t+1} = \text{Cov}_t[R^j_{t+1}, R_{t+1}] / \text{Var}_t[R_{t+1}], \]
there exists a random variable \( \epsilon^j \) such that:
\[ R^j_{t+1} = \beta^j_{t+1} R_{t+1} + \epsilon^j_{t+1}, \] (32)

where
\[ E[\epsilon^j_{t+1}] = E[\epsilon^j_{t+1} R_{t+1}] = E[\epsilon^j_{t+1} S_t] = 0. \]

Substituting \( \beta^j_{t+1} = \beta^0_j + \beta^j_1 S_t \) in the Equation above and taking unconditional expectations (using the Law of Iterated Expectations) yields Equation (19). Similar assumptions and derivations appear in Ferson and Schadt (1996) and Ferson, Sarkissian and Simin (2005).
Appendix B: The Real Term Structure of Interest Rates

Appendix B derives in closed-form the solution for the (domestic) real term structure of interest rates. I present this derivation for completeness and future reference: the empirical part of the paper does not test model implications involving the term structure due to lack of good quality data. The model implies that the shape of the real term structure depends on the level of sovereign yield spread \( S_t \). Interestingly, the model generates rich dynamics for the term structure despite featuring conventional preferences and consumption processes.

Let \( B^T_t \) be the time \( t \) price of a zero-coupon bond that pays one unit of the good at time \( T > t \). Before default, the instantaneous real interest rate is fixed. Nonetheless, the shape of the real term structure before default depends on \( S_t \). The instantaneous risk-free rates before and after default are constant, and given respectively by (see, for example, Duffie 1996):

\[
 r = \rho + A \left[ \mu - \frac{1}{2} (A + 1) \sigma^2 \right] > 0 \\
 r^d = \rho + A \left[ \mu^d - \frac{1}{2} (A + 1) \left( \sigma^d \right)^2 \right] < r .
\]

Although the pricing kernel of Equation (11) is non-standard, the instantaneous interest rate is the same as in a standard Lucas-Rubinstein model. This is because consumption follows a diffusion, hence its path is continuous everywhere and "it takes time" to reach the default barrier.

The price of the zero-coupon bond \( B^T_t \) is given by:

\[
 B^T_t = E_T C_T .
\]

Using the pricing kernel in Equation (11):

\[
 B^T_t C^{-A}_t = e^{-\rho(T-t)} E_t C^{-A}_T .
\]

(33)

After default the economy is a standard Lucas-Rubinstein one. Thus the real term structure is flat for \( t > \tau \). Consider the case \( 0 \leq t < \tau \). Let \( f_t(\tau) \) be the time \( t \) density function of the default time \( \tau \) (i.e., \( f_t \) is the density of the first passage time \( \tau \) to \( B \) from \( C_t \)). Then \( \int_t^\tau f_t(\tau) d\tau \) is the time \( t \) probability that default will occur in the interval \((t, T)\). From Equation (32):

\[
 B^T_t C^{-A}_t = e^{-\rho(T-t)} E_t \left[ C^{-A}_T \mid \text{no default until } T \right] \left( 1 - \int_t^T f_t(\tau) d\tau \right) \\
 + e^{-\rho(T-t)} \int_t^T E_t \left[ C^{-A}_T \mid \text{default at } \tau \in (t, T) \right] f_t(\tau) d\tau
\]

(34)

Note that:

\[
 E_t \left[ C^{-A}_T \mid \text{no default until } T \right] = C_t^{-A} e^{-A[\mu - \frac{1}{2} (A + 1) \sigma^2](T-t)}
\]

and

\[
 E_t \left[ C^{-A}_T \mid \text{default at } \tau \in (t, T) \right] = (\alpha B + \lambda) C^{-A} e^{-A[\mu^d - \frac{1}{2} (A + 1) (\sigma^d)^2](T-\tau)} .
\]

(35)
Therefore, plugging (34) in (33):

\[
B_t^T C_t^{-A} = e^{-p(T-t)} C_t^{-A} e^{-A \left[ \mu - \frac{1}{2}(A+1)\sigma^2 \right](T-t)} \left( 1 - \int_t^T f_t(\tau) d\tau \right) + e^{-p(T-t)} \int_t^T (\alpha B + \bar{X})^{-A} e^{-A \left[ \mu - \frac{1}{2}(A+1)\sigma^2 \right](T-\tau)} f_t(\tau) d\tau
\]

Following Rubinstein and Reiner (1991) and Leland and Toft (1996):

\[
B_t^T = e^{-r(T-t)} (1 - F_t^T) + \left( \frac{\alpha B + \bar{X}}{C_t} \right)^{-A} e^{-rA T} e^{\mu T} G_t^T,
\]

where:

\[
G_t^T = \left( \frac{C_t}{B} \right)^{-a+x} N[q_1(t, T)] + \left( \frac{C_t}{B} \right)^{-a-x} N[q_2(t, T)];
\]

\[
q_1(t, T) = -\ln \left( \frac{C_t}{B} \right) - x\sigma^2(T-t) - \frac{a}{2} \sigma^2 \ln \left( \frac{C_t}{B} \right) + x\sigma^2(T-t)
\]

\[
a = \frac{\mu - \frac{1}{2}\sigma^2}{\sigma^2} ; \quad x = \frac{\sqrt{a^2\sigma^4 - 2(\rho - \rho_0)\sigma^2}}{\sigma^2};
\]

and

\[
F_t^T = G_t^T |_{x=a}.
\]

One can use Proposition 2 to transform functions of \( C_t \) in functions of \( S_t \).
Table 1
Model Parameters

Table summarizes model parameters discussed in Section 2.4. Time unit is one year. Figures 1 through 7 and Basic Model Results on Panel B of Table 2 use the parameters in Table 1. Note that the debt service parameter is irrelevant for the plotting Figures 1 through 7.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>Relative risk aversion coefficient</td>
<td>8</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Time preference discount</td>
<td>0.08</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Pre-default average consumption growth</td>
<td>0.02</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Pre-default consumption growth volatility</td>
<td>0.04</td>
</tr>
<tr>
<td>$\mu^d$</td>
<td>Post-default average consumption growth</td>
<td>0.0125</td>
</tr>
<tr>
<td>$\sigma^d$</td>
<td>Post-default consumption growth volatility</td>
<td>0.07</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Governs endowment loss at default</td>
<td>0.89</td>
</tr>
<tr>
<td>$r^*$</td>
<td>Foreign real interest rate</td>
<td>0.0125</td>
</tr>
<tr>
<td>$\Lambda$</td>
<td>Leverage parameter (1+debt-to-equity ratio)</td>
<td>3</td>
</tr>
<tr>
<td>$\bar{X}$</td>
<td>Debt service parameter</td>
<td>any $\bar{X} &gt; 0$</td>
</tr>
</tbody>
</table>
Table 2
Descriptive Statistics and Basic Model Results

Panel A has daily data descriptive statistics for the Brazilian sovereign yield spread (EMBI Index) in basis points per year. Panel B has descriptive statistics for Brazil’s IBOVESPA Stock Index. Returns are in local currency and annualized. The local interest rate is the SELIC overnight rate accumulated within each month. I use the IPCA Inflation Index to compute the real risk-free rate - results are robust to using the IGPM Inflation Index. The Hedge Ratio measures the contemporaneous co-movement of stock returns and changes in the yield spread. The Average Hedge Ratio in the data is a result of the regression reported on Table 3. Standard errors are robust to heteroskedasticity and robust to autocorrelation when there is evidence of serial correlation in residuals. The sample period for both Panels starts 02-Jan-1992 and ends 30-Sep-2005. Panel B includes results generated by the model with parameters calibrated in Section 2.4 and summarized in Table 1.

Panel A: Descriptive Statistics of Brazilian Sovereign Yield Spreads (EMBI) in basis points per year (daily data)

<table>
<thead>
<tr>
<th>Yield Spread (S_t)</th>
<th>5% Ptile</th>
<th>25% Ptile</th>
<th>Median</th>
<th>75% Ptile</th>
<th>95% Ptile</th>
<th># obs.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>404</td>
<td>592</td>
<td>763</td>
<td>997</td>
<td>1,437</td>
<td>3,308</td>
</tr>
</tbody>
</table>

Panel B: Descriptive Statistics and Model Results for Brazilian Stock Returns and Real Risk Free Rate

<table>
<thead>
<tr>
<th>Variable</th>
<th>DATA</th>
<th>MODEL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stock Market average excess return (monthly, annualized)</td>
<td>0.0798 0.1350 165</td>
<td>0.0941</td>
</tr>
<tr>
<td>Stock Market average excess return volatility (daily within month, annualized)</td>
<td>0.4067 0.0226 165</td>
<td>0.3555</td>
</tr>
<tr>
<td>One-month real risk-free rate (monthly, annualized)</td>
<td>0.1743 0.0158 165</td>
<td>0.1824</td>
</tr>
<tr>
<td>Average Hedge Ratio (daily)</td>
<td>-4.3064 0.3637 3306</td>
<td>-0.5883</td>
</tr>
<tr>
<td>Value Discount</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

0.0869
Table 3 presents regressions of daily IBOVESPA Stock Index excess returns on changes in the sovereign yield spread (EMBI Index). It also presents regressions in which the spread change is interacted with the log of the Spread level in basis points. Data are daily, and returns are in local currency. The full sample period starts 02-Jan-1992 and ends 30-Sep-2005. The first sub-sample ends 30-Dec-1998, and the second sub-sample starts 04-Jan-1999. Standard errors are corrected for heteroskedasticity. There is no evidence of serial correlation in residuals.

\[ R_t = \beta_0 + \beta_1 (S_t - S_{t-1}) + \beta_2 (S_t - S_{t-1}) \ln(10,000 \times S_t) + \epsilon_t \]

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>constant (( \beta_0 ))</td>
<td>0.0003 (0.58)</td>
<td>0.0003 (0.48)</td>
<td>0.0002 (0.52)</td>
</tr>
<tr>
<td>(t-stat)</td>
<td>0.0000 (0.02)</td>
<td>0.0002 (0.29)</td>
<td>-0.0001 (0.31)</td>
</tr>
<tr>
<td>Change in Yield Spread (( \beta_1 ))</td>
<td>-4.3064 (-11.84)</td>
<td>-5.2673 (-8.45)</td>
<td>-3.3878 (-7.09)</td>
</tr>
<tr>
<td>(t-stat)</td>
<td>-34.5767 (-8.70)</td>
<td>-24.3053 (-5.64)</td>
<td>-37.7605 (-11.26)</td>
</tr>
<tr>
<td>Change in Yield Spread * Ln(10,000*Yield Spread) (( \beta_2 ))</td>
<td>-4.2284 (7.47)</td>
<td>-2.7186 (1.76)</td>
<td>4.7035 (10.48)</td>
</tr>
<tr>
<td>(t-stat)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sample size</td>
<td>3306</td>
<td>1681</td>
<td>1625</td>
</tr>
<tr>
<td>Std. errors</td>
<td>White</td>
<td>White</td>
<td>White</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>0.2430</td>
<td>0.2433</td>
<td>0.2849</td>
</tr>
<tr>
<td></td>
<td>0.2800</td>
<td>0.2504</td>
<td>0.3644</td>
</tr>
</tbody>
</table>
Table 4
Volatility of Equity Excess Returns and Contemporaneous Sovereign Yield Spread

Panels A and B present regressions of the volatility of excess returns on the IBOVESPA Stock Index on the log of the sovereign yield spread (EMBI Index). Returns are in local currency. Panel A has daily data. The daily volatility is constructed from a GARCH (1,1) model estimated with daily data for the full-sample. The daily volatility is annualized by multiplying by the square root of 250. Panel B has monthly data. In each calendar month, volatility is defined as the standard deviation of daily excess returns within the month. The daily volatility is annualized by multiplying by square root of 250. The full sample period starts 02-Jan-1992 and ends 30-Sep-2005. The first sub-sample ends 30-Dec-1998 and the second sub-sample starts 04-Jan-1999. Standard errors are corrected for heteroskedasticity and serial-autocorrelation.

\[ V_t = \beta_0 + \beta_1 \ln(S_t * 10,000) + \epsilon_t \]

Panel A: Daily data - \( V_t \) is daily GARCH(1,1) volatility fitted in full sample period (annualized)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant (( \beta_0 )) (( t )-stat)</td>
<td>-0.4189 (2.65)</td>
<td>-0.9077 (3.24)</td>
<td>-0.1042 (1.26)</td>
</tr>
<tr>
<td>Ln (10,000 *Yield Spread) (( \beta_1 )) (( t )-stat)</td>
<td>0.1267 (5.24)</td>
<td>0.2110 (4.92)</td>
<td>0.0685 (5.43)</td>
</tr>
<tr>
<td>Sample size</td>
<td>3307</td>
<td>1682</td>
<td>1625</td>
</tr>
<tr>
<td>Std. errors</td>
<td>Newey-West</td>
<td>Newey-West</td>
<td>Newey-West</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>0.0724</td>
<td>0.1219</td>
<td>0.1004</td>
</tr>
</tbody>
</table>

Panel B: Monthly data - \( V_t \) is standard deviation of daily excess returns within month \( t \) (annualized)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant (( \beta_0 )) (( t )-stat)</td>
<td>-0.7597 (1.90)</td>
<td>-1.1126 (1.92)</td>
<td>-0.5966 (1.56)</td>
</tr>
<tr>
<td>Ln (10,000*Yield Spread) (( \beta_1 )) (( t )-stat)</td>
<td>0.1756 (2.88)</td>
<td>0.2430 (2.78)</td>
<td>0.1363 (2.28)</td>
</tr>
<tr>
<td>Sample size</td>
<td>165</td>
<td>84</td>
<td>81</td>
</tr>
<tr>
<td>Std. errors</td>
<td>Newey-West</td>
<td>Newey-West</td>
<td>Newey-West</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>0.0857</td>
<td>0.1379</td>
<td>0.1200</td>
</tr>
</tbody>
</table>
Table 5 presents regressions of the excess return on Stock Market Indexes on the lagged log of the corresponding sovereign yield spread (EMBI Index). The regression pools data from three countries: Brazil, Mexico and Venezuela. Stock indexes are IBOVESPA (Brazil), IPC (Mexico) and Caracas SE Index (Venezuela). Returns are in local currency, and calculated with three different periodicities: monthly, every two months and quarterly. Returns do not overlap. Standard errors are robust to heteroskedasticity and clustered by time. The sample begins 02-Jan-1992 and ends 30-Sep-2005. For Mexican data the sample period stops when the country received investment grade status from Standard and Poor’s (Feb-2002).

\[ R_{t+1} = \beta_0 + \beta_1 \ln(10,000 \times Y_{t}) + \epsilon_t \]

<table>
<thead>
<tr>
<th></th>
<th>Monthly (annualized)</th>
<th>Every two months (annualized)</th>
<th>Quarterly (annualized)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Constant ((\beta_0))</strong></td>
<td>-2.3293 (-2.24)</td>
<td>-2.6687 (-2.58)</td>
<td>-2.9023 (-2.71)</td>
</tr>
<tr>
<td><strong>Lagged (\ln(10,000\times\text{Yield Spread})) ((\beta_1))</strong></td>
<td>0.3580 (2.28)</td>
<td>0.4150 (2.67)</td>
<td>0.4566 (2.78)</td>
</tr>
<tr>
<td><strong>Sample size</strong></td>
<td>452</td>
<td>225</td>
<td>151</td>
</tr>
<tr>
<td><strong>Std. errors</strong></td>
<td><em>White + time cluster</em></td>
<td><em>White + time cluster</em></td>
<td><em>White + time cluster</em></td>
</tr>
<tr>
<td><strong>(R^2)</strong></td>
<td>0.0138</td>
<td>0.0338</td>
<td>0.0504</td>
</tr>
</tbody>
</table>
Table 6
Two-Pass Regression: Conditional CAPM versus Static CAPM

Table 6 presents results of the second-pass cross-sectional regression in a two-pass Fama-McBeth regression methodology. In the first pass the betas are estimated with 24 time series regressions using monthly returns of the 24 test portfolios. The second pass is a single cross-sectional regression: average excess returns across 24 test portfolios are regressed on their portfolio betas estimated in the first pass regression. Returns are calculated in local currency in each calendar month. The test portfolios are 12 Dow Jones Total Market industry portfolios for Brazil, augmented by 12 managed portfolios based on the industry portfolios and on the lagged sovereign yield spread, as described on Section 4. Two sets of t-stats are presented: robust to heteroskedasticity; and robust to heteroskedasticity and corrected for generated regressors (Shanken 1992). The sample period starts 02-Jan-1992 and ends 30-Sep-2005.

\[
R_i^j = a_j + \beta_0^j R_t + \beta_1^j R_{t-1}^{} + \varepsilon_i^j
\]

first-pass regression:

\[
\bar{R}^j = \lambda_0 + \lambda_1 (\hat{\beta}_0^j) + \lambda_2 (\hat{\beta}_1^j) + \alpha_i^j
\]

second-pass regression:

<table>
<thead>
<tr>
<th>Without constant in 2nd Pass</th>
<th>With constant in 2nd Pass</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>CAPM</strong></td>
<td><strong>Cond. CAPM</strong></td>
</tr>
<tr>
<td><strong>Constant (( \lambda_0 ))</strong></td>
<td>-</td>
</tr>
<tr>
<td>([t-stat])</td>
<td>-</td>
</tr>
<tr>
<td>([t-stat])</td>
<td>-</td>
</tr>
<tr>
<td><strong>Market (( \lambda_1 ))</strong></td>
<td>0.0151</td>
</tr>
<tr>
<td>([t-stat])</td>
<td>[6.01]</td>
</tr>
<tr>
<td>([t-stat])</td>
<td>(1.49)</td>
</tr>
<tr>
<td><strong>Market * Lagged Yield Spread (( \lambda_2 ))</strong></td>
<td>-</td>
</tr>
<tr>
<td>([t-stat])</td>
<td>-</td>
</tr>
<tr>
<td>([t-stat])</td>
<td>-</td>
</tr>
<tr>
<td><strong>Std. errors (.( j ))</strong></td>
<td>White</td>
</tr>
<tr>
<td><strong>Std. errors (.)</strong></td>
<td>White + Shanken</td>
</tr>
<tr>
<td><strong>R^2</strong></td>
<td>-</td>
</tr>
<tr>
<td><strong>Adjusted R^2</strong></td>
<td>-</td>
</tr>
<tr>
<td><strong>Square root MSE</strong></td>
<td>0.0095</td>
</tr>
<tr>
<td><strong># Obs</strong></td>
<td>24</td>
</tr>
</tbody>
</table>

50
Figure 1 plots the emerging economy’s aggregate equity Value Discount versus the sovereign yield spread (see Definition 1 and Proposition 3). A Value Discount equal to 0.10 means that the value of corporate assets in the emerging market is 10% lower than it would otherwise be if the country did not have the option to default on its foreign debt. The parameters used to plot the figure are discussed in Section 2.4 and summarized in Table 1. Note that the sovereign yield spread is measured in basis points per year and plotted in log scale.
Figure 2 plots the Hedge Ratio in the emerging economy versus the sovereign yield spread (see Definition 2 and Propositions 2 and 4). A Hedge Ratio of -1.5 means that a 10 basis points increase in the yield spread is contemporaneously associated with a stock market excess return of -0.15%. The parameters used to plot the figure are discussed in Section 2.4 and summarized in Table 1. Note that the sovereign yield spread is measured in basis points per year.
Figure 3 plots the volatility in the emerging economy’s stock market versus the sovereign yield spread (see Proposition 4). For the purpose of comparison, the no-default and post-default volatility levels are also plotted. The no-default volatility level is the one that prevails in the otherwise identical no-default economy, and is fixed at $3 \times 0.04 = 0.12$ per year. The post-default volatility level is fixed at $3 \times 0.07 = 0.21$ per year. The parameters used to plot the figure are discussed in Section 2.4 and summarized in Table 1. Note that return volatilities are per year, and the sovereign yield spread is measured in basis points per year and is plotted in log scale.
Figure 4 plots the expected excess return in the emerging economy’s stock market versus the sovereign yield spread (see Proposition 4). For the purpose of comparison, the no-default and post-default expected excess return levels are also plotted. The no-default expected excess return level is the one that prevails in the otherwise identical no-default economy, and is given $3 \times 8 \times (0.04)^2 = 0.0384$ per year. The post-default level is $3 \times 8 \times (0.07)^2 = 0.1176$ per year. The parameters used to plot the figure are discussed in Section 2.4 and summarized in Table 1. Note that expected excess return is per year, and the sovereign yield spread is measured in basis points per year and is plotted in log scale.
Figure 5
Empirical Hedge Ratio versus Theoretical Hedge Ratio

Figure 5 plots the Hedge Ratio in the emerging economy versus the sovereign yield spread (see Definition 2 and Propositions 2 and 4). A Hedge Ratio of -1.5 means that a 10 basis points increase in the yield spread is contemporaneously associated to an aggregate stock market return of -0.15%. The Theoretical Hedge Ratio is as in Figure 2. It is calculated using the structural parameters discussed in Section 2.4 and summarized in Table 1. The Empirical Hedge Ratio is calculated with the output of the full-sample regression on Table 3. Note that the sovereign yield spread is measured in basis points per year. The Figure includes a 95% confidence band for the Empirical Hedge Ratio.
Figure 6
Empirical versus Theoretical Relationship between Stock Volatility and Sovereign Yield Spread

Figure 6 plots the emerging economy’s stock market volatility versus the sovereign yield spread. The Theoretical curve is given in Proposition 4, and calculated with the structural parameters discussed in Section 2.4 and summarized in Table 1. The theoretical curve is the same as in Figure 3 (but in regular rather than log scale). The Empirical curve is constructed with the output of the full sample regression in Table 4, Panel B. The Figure includes a 95% confidence band for the Empirical Volatility.
Figure 7
Empirical versus Theoretical Relationship between Stock Expected Excess Return and Sovereign Yield Spread

Figure 7 plots the emerging economy’s expected excess return on the stock market versus the level of the sovereign yield spread. The Theoretical curve is given in Proposition 4 and calculated with the structural parameters discussed in Section 2.4 and summarized in Table 1. The Theoretical curve is the same as Figure 4 (but in regular rather than log scale). The Empirical is constructed with the output of the monthly regression in Table 5. The Figure includes a 95% confidence band for the Empirical Expected Excess Return.
Appendix C
Stock Market Sharpe Ratio

The figure plots the emerging economy’s stock market Sharpe Ratio versus the sovereign yield spread (see Proposition 4). For purpose of comparison, the no-default and post-default Sharpe Ratio levels are also plotted. The no-default Sharpe Ratio is the Sharpe Ratio that prevails in the otherwise identical no-default economy, and is fixed at $8 \times 0.04 = 0.32$ per year. The post-default Sharpe Ratio is fixed at $8 \times 0.07 = 0.56$ per year. The parameters used to plot the figure are discussed in Section 2.4 and summarized in Table 1. Note that the Sharpe Ratio is per year, and the sovereign yield spread is measured in basis points per year plotted in log scale.
Appendix D
Equity Excess Returns and Contemporaneous Changes in Sovereign Yield Spread

Panel B presents Spearman (rank) correlations between IBOVESPA Stock Index excess returns in local currency and changes in the yield spread of dollar-denominated Brazilian sovereign bonds (EMBI Index). Data are daily, and returns are in local currency. The full sample period starts on 02-Jan-1992 and ends in 03-Oct-2005. The first sub-sample ends in 30-Dec-1998, and the second sub-sample starts in 04-Jan-1999.

Spearman correlation between daily IBOVESPA excess returns and contemporaneous changes in Brazilian sovereign yield spread

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Spearman’s Rho</td>
<td>-0.4988</td>
<td>-0.4713</td>
<td>-0.5398</td>
</tr>
<tr>
<td>p-value of H0: variables are independent</td>
<td>&lt; 0.0001</td>
<td>&lt; 0.0001</td>
<td>&lt; 0.0001</td>
</tr>
<tr>
<td>Sample size</td>
<td>3306</td>
<td>1681</td>
<td>1625</td>
</tr>
</tbody>
</table>
Appendix E
Volatility of Equity Excess Returns and Contemporaneous Sovereign Yield Spread

The table presents Spearman (rank) correlations between stock market return volatility and the level of the sovereign yield spread. Data are daily. The daily volatility is fitted with a GARCH (1,1) model estimated for the full-sample, and annualized by multiplying by the square root of 250. The full sample period starts on 02-Jan-1992 and ends in 03-Oct-2005. The first sub-sample ends 30-Dec-1998, and the second sub-sample starts 04-Jan-1999.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Spearman’s Rho</td>
<td>0.3046</td>
<td>0.3690</td>
<td>0.2716</td>
</tr>
<tr>
<td>P-value of H0: variables are independent</td>
<td>&lt; 0.0001</td>
<td>&lt; 0.0001</td>
<td>&lt; 0.0001</td>
</tr>
<tr>
<td>Sample size</td>
<td>3307</td>
<td>1682</td>
<td>1625</td>
</tr>
</tbody>
</table>
Appendix F
Volatility of Equity Excess Returns and Contemporaneous Sovereign Yield Spread – Alternative Volatility Measurement

Panel A presents regressions of daily volatility of on IBOVESPA Stock Index excess returns in local currency on the log of the yield spread of dollar-denominated Brazilian sovereign bonds. The daily volatility is calculated using Parkinson’s’ (1980) formula, and then multiplied by square root of 250. The full sample period starts in 27-Aug-1993 and ends in 30-Sep-2005. The first sub-sample ends in 30-Dec-1998, and the second sub-sample starts in 04-Jan-1999. Standard errors are corrected for heteroskedasticity and serial-autocorrelation (ten lags). Panel B presents Spearman (rank) correlations between squared IBOVESPA Stock Index excess returns in local currency and changes in the yield spread of dollar-denominated Brazilian sovereign bonds. The sample definitions are the same as in Panel A.

\[ V_t = \beta_0 + \beta_1 \ln(S_t \times 10,000) + \varepsilon_t \]

Panel A: Regressions of daily Parkinson volatility in IBOVESPA excess returns (times \(\sqrt{250}\)) on Brazilian sovereign yield spread

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>constant (t-stat)</td>
<td>-0.4972 (-2.92)</td>
<td>-0.9971 (-3.09)</td>
<td>-0.3149 (-2.44)</td>
</tr>
<tr>
<td>Ln (10,000*Yield Spread) (t-stat)</td>
<td>0.1188 (4.55)</td>
<td>0.2044 (4.13)</td>
<td>0.0840 (4.20)</td>
</tr>
<tr>
<td>Sample size</td>
<td>2904</td>
<td>1279</td>
<td>1625</td>
</tr>
<tr>
<td>Std. errors</td>
<td>Newey-West</td>
<td>Newey-West</td>
<td>Newey-West</td>
</tr>
<tr>
<td>(R^2)</td>
<td>0.0549</td>
<td>0.0960</td>
<td>0.0621</td>
</tr>
</tbody>
</table>

Panel B: Spearman correlation between daily Parkinson volatility in IBOVESPA excess returns (times \(\sqrt{250}\)) and Brazilian sovereign yield spread

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Spearman’s Rho</td>
<td>0.2689</td>
<td>0.3445</td>
<td>0.2408</td>
</tr>
<tr>
<td>p-value of H0: variables are independent</td>
<td>&lt; 0.0001</td>
<td>&lt; 0.0001</td>
<td>&lt; 0.0001</td>
</tr>
<tr>
<td>Sample size</td>
<td>2904</td>
<td>1279</td>
<td>1625</td>
</tr>
</tbody>
</table>
Appendix G

Expected Equity Excess Returns and Contemporaneous Sovereign Yield Spread

Panel A presents regressions of the excess return on the IBOVESPA Stock Index on the lagged log of the sovereign yield spread (EMBI Index). Panel B presents equivalent pooled regressions for three countries: Brazil, Mexico and Venezuela. Stock indexes are IBOVESPA (Brazil), IPC (Mexico) and Caracas SE Index (Venezuela). Returns are in local currency, and calculated with three different periodicities: monthly, every two months and quarterly. Returns do not overlap. In Panel A standard errors are robust to heteroskedasticity. In Panel B standard errors are robust to heteroskedasticity and clustered by time. The sample begins 02-Jan-1992 and ends 30-Sep-2005. For Mexican data the sample period stops when the country received investment grade status from Standard and Poor’s (Feb-2002).

\[ R_{t+1} = \beta_0 + \beta_1 \ln(S_i \cdot 10,000) + \varepsilon, \]

<table>
<thead>
<tr>
<th></th>
<th>Monthly (annualized)</th>
<th>Every two months (annualized)</th>
<th>Quarterly (annualized)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant ((\beta_0)) ((t-stat))</td>
<td>-2.3241 (-1.17)</td>
<td>-2.2852 (-0.98)</td>
<td>-2.8551 (-1.25)</td>
</tr>
<tr>
<td>Lagged Ln(10,000*Yield Spread) ((\beta_1)) ((t-stat))</td>
<td>0.3587 (1.21)</td>
<td>0.3529 (1.02)</td>
<td>0.4436 (1.28)</td>
</tr>
<tr>
<td>Sample size</td>
<td>165</td>
<td>82</td>
<td>55</td>
</tr>
<tr>
<td>Std. errors</td>
<td>White</td>
<td>White</td>
<td>White</td>
</tr>
<tr>
<td>(R^2)</td>
<td>0.0084</td>
<td>0.0113</td>
<td>0.0231</td>
</tr>
</tbody>
</table>
Two-Pass Regression: Conditional CAPM versus Static CAPM – Returns every two months

Table 6 presents results of the second-pass cross-sectional regression in a two-pass Fama-McBeth regression methodology. In the first pass the betas are estimated with 24 time series regressions using returns every two months of the 24 portfolios. The second pass is a single cross-sectional regression: average excess returns across 24 test portfolios are regressed on their portfolio betas estimated in the first pass regression. Returns are calculated in local currency in each calendar bi-month. The test portfolios are 12 Dow Jones Total Market industry portfolios for Brazil, augmented by 12 managed portfolios based on the industry portfolios and on the lagged sovereign yield spread, as described on Section 4. Two sets of t-stats are presented: robust to heteroskedasticity; and robust to heteroskedasticity and corrected for generated regressors (Shanken 1992). The sample period starts 02-Jan-1992 and ends 30-Aug-2005.

First-pass regression:

\[ R_t^j = \alpha + \beta_0 R_t + \beta_1 R_t S^*_{t-1} + \varepsilon_t \]

Second-pass regression:

\[ \bar{R} = \lambda_0 + \lambda_1 (\hat{\beta}_0) + \lambda_2 (\hat{\beta}_1) + \alpha \]

<table>
<thead>
<tr>
<th>Without constant in 2nd Pass</th>
<th>With constant in 2nd Pass</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>CAPM</td>
</tr>
<tr>
<td>Constant (( \lambda_0 ))</td>
<td></td>
</tr>
<tr>
<td>[t-stat]</td>
<td></td>
</tr>
<tr>
<td>(t-stat)</td>
<td></td>
</tr>
<tr>
<td>Market (( \lambda_4 ))</td>
<td>0.0431</td>
</tr>
<tr>
<td>[t-stat]</td>
<td>[7.22]</td>
</tr>
<tr>
<td>(t-stat)</td>
<td>(1.57)</td>
</tr>
<tr>
<td>Market * Lagged Yield Spread (( \lambda_2 ))</td>
<td>-0.0374</td>
</tr>
<tr>
<td>[t-stat]</td>
<td>[6.25]</td>
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<tr>
<td>(t-stat)</td>
<td>(1.56)</td>
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<tr>
<td>Std. errors (.</td>
<td>)</td>
</tr>
<tr>
<td>Std. errors (.)</td>
<td>White + Shanken</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>-</td>
</tr>
<tr>
<td>Adjusted ( R^2 )</td>
<td>-</td>
</tr>
<tr>
<td>Square root MSE</td>
<td>0.0230</td>
</tr>
<tr>
<td># Obs</td>
<td>24</td>
</tr>
</tbody>
</table>
Appendix I
Two-Pass Regression: Conditional CAPM versus Static CAPM – Quarterly Returns

Table 6 presents results of the second-pass cross-sectional regression in a two-pass Fama-McBeth regression methodology. In the first pass the betas are estimated with 24 time series regressions using quarterly returns of the 24 portfolios. The second pass is a single cross-sectional regression: average excess returns across 24 test portfolios are regressed on their portfolio betas estimated in the first pass regression. Returns are calculated in local currency in each calendar quarter. The test portfolios are 12 Dow Jones Total Market industry portfolios for Brazil, augmented by 12 managed portfolios based on the industry portfolios and on the lagged sovereign yield spread, as described on Section 4. Two sets of t-stats are presented: robust to heteroskedasticity; and robust to heteroskedasticity and corrected for generated regressors (Shanken 1992). The sample period starts 02-Jan-1992 and ends 30-Sep-2005.

First-pass regression: \( R_t^j = \alpha_j^0 + \beta_j^0 R_t + \beta_j^1 R_t S_{t-1}^* + \epsilon_j^i \)

Second-pass regression: \( \overline{R^i} = \lambda_0 + \lambda_1 (\hat{\beta}_0^j) + \lambda_2 (\hat{\beta}_1^j) + \alpha_i^j \)

Without constant in 2nd Pass | With constant in 2nd Pass
--- | ---
<table>
<thead>
<tr>
<th>CAPM</th>
<th>Cond. CAPM</th>
<th>CAPM</th>
<th>Cond. CAPM</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Constant (( \lambda_0 ))</strong></td>
<td>-</td>
<td>-</td>
<td>-0.0398</td>
</tr>
<tr>
<td>[t-stat]</td>
<td></td>
<td></td>
<td>[-3.16]</td>
</tr>
<tr>
<td>(t-stat)</td>
<td></td>
<td></td>
<td>(-3.03)</td>
</tr>
<tr>
<td><strong>Market (( \lambda_1 ))</strong></td>
<td>0.0506</td>
<td>0.0326</td>
<td>0.0860</td>
</tr>
<tr>
<td>[t-stat]</td>
<td>[8.53]</td>
<td>[6.36]</td>
<td>[7.46]</td>
</tr>
<tr>
<td>(t-stat)</td>
<td>(1.25)</td>
<td>(0.81)</td>
<td>(2.06)</td>
</tr>
<tr>
<td><strong>Market * Lagged Yield Spread (( \lambda_2 ))</strong></td>
<td>-</td>
<td>0.0432</td>
<td>-</td>
</tr>
<tr>
<td>[t-stat]</td>
<td></td>
<td>[6.49]</td>
<td></td>
</tr>
<tr>
<td>(t-stat)</td>
<td></td>
<td>(1.16)</td>
<td></td>
</tr>
<tr>
<td><strong>Std. errors ([.j])</strong></td>
<td>White</td>
<td>White</td>
<td>White</td>
</tr>
<tr>
<td><strong>Std. errors ([.\text{.}])</strong></td>
<td>White + Shanken</td>
<td>White + Shanken</td>
<td>White + Shanken</td>
</tr>
<tr>
<td><strong>( R^2 )</strong></td>
<td>-</td>
<td>-</td>
<td>0.6203</td>
</tr>
<tr>
<td><strong>Adjusted ( R^2 )</strong></td>
<td>-</td>
<td>-</td>
<td>0.6034</td>
</tr>
<tr>
<td><strong>Square Root MSE</strong></td>
<td>0.0273</td>
<td>0.0194</td>
<td>0.0244</td>
</tr>
<tr>
<td><strong># Obs</strong></td>
<td>24</td>
<td>24</td>
<td>24</td>
</tr>
</tbody>
</table>
Appendix J  
Results of First-Pass Regression on Table 6

The table presents the betas estimated in the first pass regression of Table 6, for the 24 portfolios. Twelve of the portfolios are Dow Jones Total Market Index industry portfolios, at the Supersectoral Level. Dow Jones Total Market in Brazil are tilted towards large stocks. The market portfolio used to estimate the betas is the broad IBOVESPA Stock Index. Excess returns are in local currency. The sample period starts 02-Jan-1992 and ends 30-Sep-2005.

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>( \beta_0 ), (t-stat)</th>
<th>( \beta_1 ), (t-stat)</th>
<th>( \beta_0 ), (t-stat)</th>
<th>( \beta_1 ), (t-stat)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oil and Gas</td>
<td>1.08 , (15.44)</td>
<td>-</td>
<td>1.08 , (15.40)</td>
<td>0.014 , (0.20)</td>
</tr>
<tr>
<td>Chemicals</td>
<td>0.47 , (8.51)</td>
<td>-</td>
<td>0.47 , (8.48)</td>
<td>-0.018 , (-0.33)</td>
</tr>
<tr>
<td>Basic Resources</td>
<td>0.64 , (12.11)</td>
<td>-</td>
<td>0.64 , (12.35)</td>
<td>-0.151 , (-2.91)</td>
</tr>
<tr>
<td>Construction and Materials</td>
<td>0.48 , (7.91)</td>
<td>-</td>
<td>0.48 , (7.88)</td>
<td>-0.009 , (-0.14)</td>
</tr>
<tr>
<td>Industrial Goods and Services</td>
<td>0.64 , (7.82)</td>
<td>-</td>
<td>0.64 , (7.90)</td>
<td>0.042 , (0.51)</td>
</tr>
<tr>
<td>Food &amp; Beverage</td>
<td>0.59 , (10.55)</td>
<td>-</td>
<td>0.59 , (10.52)</td>
<td>0.027 , (0.47)</td>
</tr>
<tr>
<td>Personal and Household Goods</td>
<td>0.71 , (9.48)</td>
<td>-</td>
<td>0.71 , (9.48)</td>
<td>-0.090 , (-1.20)</td>
</tr>
<tr>
<td>Utilities</td>
<td>1.20 , (13.61)</td>
<td>-</td>
<td>1.20 , (13.61)</td>
<td>0.090 , (1.02)</td>
</tr>
<tr>
<td>Consumer Services</td>
<td>0.74 , (10.23)</td>
<td>-</td>
<td>0.74 , (10.21)</td>
<td>0.035 , (0.48)</td>
</tr>
<tr>
<td>Telecommunications</td>
<td>1.05 , (30.51)</td>
<td>-</td>
<td>1.05 , (30.51)</td>
<td>0.038 , (1.12)</td>
</tr>
<tr>
<td>Banks</td>
<td>0.73 , (14.66)</td>
<td>-</td>
<td>0.73 , (14.80)</td>
<td>0.096 , (1.96)</td>
</tr>
<tr>
<td>Financial Services</td>
<td>0.67 , (11.36)</td>
<td>-</td>
<td>0.67 , (11.32)</td>
<td>0.029 , (0.48)</td>
</tr>
<tr>
<td>Oil and Gas – managed</td>
<td>1.09 , (8.64)</td>
<td>-</td>
<td>1.10 , (13.10)</td>
<td>1.21 , (14.38)</td>
</tr>
<tr>
<td>Chemicals – managed</td>
<td>0.45 , (6.21)</td>
<td>-</td>
<td>0.45 , (6.52)</td>
<td>0.27 , (4.00)</td>
</tr>
<tr>
<td>Basic Resources – managed</td>
<td>0.49 , (4.96)</td>
<td>-</td>
<td>0.49 , (5.11)</td>
<td>0.29 , (3.06)</td>
</tr>
<tr>
<td>Construction and Material - managed</td>
<td>0.47 , (6.18)</td>
<td>-</td>
<td>0.47 , (6.66)</td>
<td>0.39 , (5.43)</td>
</tr>
<tr>
<td>Industrial Goods and Services - managed</td>
<td>0.67 , (5.73)</td>
<td>-</td>
<td>0.68 , (6.13)</td>
<td>0.53 , (4.76)</td>
</tr>
<tr>
<td>Food &amp; Beverage – managed</td>
<td>0.62 , (6.18)</td>
<td>-</td>
<td>0.62 , (7.94)</td>
<td>0.79 , (10.09)</td>
</tr>
<tr>
<td>Personal and Household Goods - managed</td>
<td>0.63 , (6.28)</td>
<td>-</td>
<td>0.63 , (6.92)</td>
<td>0.53 , (5.80)</td>
</tr>
<tr>
<td>Utilities – managed</td>
<td>1.29 , (7.32)</td>
<td>-</td>
<td>1.30 , (9.24)</td>
<td>1.36 , (9.70)</td>
</tr>
<tr>
<td>Consumer Services – managed</td>
<td>0.76 , (6.96)</td>
<td>-</td>
<td>0.77 , (8.53)</td>
<td>0.80 , (8.92)</td>
</tr>
<tr>
<td>Telecommunications – managed</td>
<td>1.08 , (11.51)</td>
<td>-</td>
<td>1.08 , (23.75)</td>
<td>1.04 , (22.91)</td>
</tr>
<tr>
<td>Banks – managed</td>
<td>0.82 , (6.92)</td>
<td>-</td>
<td>0.83 , (10.26)</td>
<td>1.11 , (13.76)</td>
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<td>Financial Services - managed</td>
<td>0.70 , (6.04)</td>
<td>-</td>
<td>0.71 , (7.69)</td>
<td>0.91 , (9.86)</td>
</tr>
</tbody>
</table>