Career Concerns and Dynamic Arbitrage

Very preliminary and incomplete. Please do not circulate.

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Abstract

This paper analyzes the effect of career concerns on risky arbitrage. It presents an analytically tractable model where some fund managers can locate arbitrage opportunities, i.e., fundamentally very similar assets with a temporary difference in their price. Fund managers need operating capital from investors to exploit these opportunities. Investors judge the abilities of fund managers and decide whether to keep or fire them based on their past performance. Career concerns of fund managers distort their strategies which effect prices. In equilibrium, investors keep those arbitrageurs with higher probability who speculate on fast convergence of prices, even if the expected profit of this strategy is lower. As an effect, fund managers over invest in these strategies which increase the probability of liquidity crises: episodes with large price divergences and large aggregate losses of fund managers without any change in the fundamentals.

JEL classification: G10, G20, D5.

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1 Introduction

Trading strategies built on the price convergence of similar assets have been widely popular among hedge funds over the last decade. However, these strategies can lead to large losses when diverging prices force these hedge funds to unwind some of their positions. The spectacular near-collapse of the Long-Term Capital Management in 1998 is frequently cited as an example of this phenomenon.\footnote{For detailed analysis of the LTCM crisis see e.g. Edwards (1999), Loewenstein (2000), MacKenzie (2003). Although after the collapse of the LTCM many market participants made changes to their risk-management systems to avoid similar events, it is clear that financial markets are still prone to similar liquidity crises. A recent example is the turbulence in May 2005 connected to the price differential between General Motors stocks and bonds: “The big worry is that an LTCM-style disaster is occurring with hedge funds as they unwind GM debt/stock trade (a potential Dollars 100bn trade across the industry) at a loss, causing massive redemptions from convert arb funds, forcing them to unwind other trades, and so on, leading to a collapse of the debt markets and then all financial markets.” (Financial Times, US Edition, May 23, 2005)} In this paper, I will analyze the role of informational asymmetry between hedge funds and their investors...
in the development of such episodes. In the model, investors hire or fire hedge fund managers after assessing their abilities by their past performance. Thus, fund managers distort their strategies because of their career concerns. I show that the equilibrium price effect due to career concerns increase the probability of liquidity crises: episodes with large price divergences and large aggregate losses without any change in fundamentals.

I present an analytically tractable general equilibrium model of delegated risky arbitrage. The model is based on Kondor (2006). There are two main groups of agents: fund managers and investors. Only some fund managers have the ability to locate windows of arbitrage opportunity. These fund managers are called arbitrageurs. In such a window, temporarily there is a gap between prices of two fundamentally identical assets traded in different markets. These windows are present only during a random time interval. After a window closes, assets are traded at the same price. In effect, arbitrageurs bet on the time of the convergence of prices. When a window closes, another one opens involving a different pair of assets. Arbitrageurs need operating capital from investors to be able to exploit windows of arbitrage opportunity, because they have to collateralize their trades. For the operating capital, fund managers share the profit (or loss) with investors. The problem is that investors do not know which fund managers are arbitrageurs and which of them are quacks. Quacks cannot locate windows of arbitrage opportunity. They can only invest in negative expected-value gambles, but they can still mimic the performance of arbitrageurs in certain states. Investors observe the past performance of fund managers at the beginning of each window and decide which of them to hire, i.e., whom to provide operating capital. The decision rule of investors affects the strategies of arbitrageurs which in turn determine the probability of liquidity crises.

I emphasize two main results. First, due to the informational asymmetry, investors fire those arbitrageurs with higher probability who follow higher expected value strategies. Thus, in equilibrium investors’ decision-rule provide implicit incentives to arbitrageurs not to maximize expected trading profit in a given window. This result is due to the fact that equilibrium expected returns of different strategies contain a reputational premium or discount. A strategy of betting on a small probability event is successful infrequently. Thus, it is not very useful for arbitrageurs to signal their type, because in most states of the world these arbitrageurs do not produce a positive net return. These arbitrageurs are fired more often, which is compensated by the higher expected return (a reputational premium) associated to these strategies in equilibrium. As a result, in equilibrium each arbitrageur is indifferent whether to speculate on the larger probability event of fast convergence of prices which provides smaller expected return but larger chance of keeping the job, or speculating on the small probability event of slow convergence which provides larger expected return but smaller chance of reemployment.  

As a second main result, I show that career concerns of fund managers increase the probability of large liquidity crises. It is a result of the distorted implicit incentives provided by the decision rule of

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2This argument is related to one of the results in Dasgupta and Prat (2006). They argue that buying an asset in certain states helps informed traders to signal their type to investors, which increases the price of these assets. Thus, the price of the asset contains a reputational premium or cost. In this paper, I argue that the expected return of certain dynamic strategies also contain a reputational premium or cost. While they use this result to analyze the informational efficiency of prices of single assets, I show that the reputational premium has a systematic effect on the frequency and magnitude of liquidity crises.
investors. Because betting on early time of convergence provides a better chance for the arbitrageur to keep her job, in equilibrium more arbitrageurs follow such strategy. In effect, arbitrageurs over-invest in early stages of the window. Its effect is two fold. Because of career concerns, the gap is typically lower when the window is short and this is a high probability event. So, in this sense, most of the time career concerns help efficiency. However, time to time the window happens to be relatively long. In this case there will not be enough arbitrageurs with liquid capital who could stop the gap from a substantial increase. Thus, career concerns increase the magnitude of a liquidity crisis of given probability and – equivalently – increase the probability of a liquidity crisis of a given magnitude.

To my knowledge, this model is the first attempt to incorporate the agency problem of delegated portfolio management in a general equilibrium framework of risky arbitrage. Relatedly, this is also the first analytical model to show that career concerns increase the chance of liquidity crises, but reduce the inefficiency of prices in „normal times”. This model provides the natural bridge between two streams of the literature. First, it builds on the results of general equilibrium models of risky arbitrage (e.g., Gromb and Vayanos, 2002, Zigrand, 2004, Xiong, 2001, Kyle and Xiong, 2001, Basak and Croitoru, 2000). Second, it is naturally connected to models which analyze the effect of delegated portfolio management on traders’ decisions and asset prices in general (e.g. Dow and Gorton, 1997, Allen and Gorton, 1993, Cuoco and Kaniel, 2001, Vayanos, 2003, Berk and Green, 2004, Gümibel, 2005, Dasgupta and Prat, 2005, 2006). These papers do not consider the effect of career concerns on the activity of convergence traders and the relative prices of similar assets which is in the focus of this paper.

The closest paper to this work in spirit is the seminal paper of Shleifer and Vishny (1997) on limits of arbitrage. They also emphasize the effect of informational asymmetry between investors and arbitrageurs to arbitrageurs’ activity and the price gap. They use a two period, representative agent framework where investors cut back the operating capital of arbitrageurs by an exogenously given fraction if arbitrageurs suffer trading losses. They show that due to this exogenous decision rule related to asymmetric information, the gap will always be higher than in the full information case. Furthermore, if the gap increases in the first period, investors cut back the operating capital of arbitrageurs more, so arbitrageurs can invest less in the second period exactly when the gap is higher and the market is more profitable. In contrast, in the presented model where arbitrageurs can follow heterogeneous timing strategies, the picture is more differentiated. Most of the time asymmetric information induces arbitrageurs to keep the gap at a lower level, but for the price of larger liquidity crises in the small probability events. Furthermore, the mechanism behind this result is not based on the direct effect of instantaneous capital withdrawal from fund managers who made temporary losses. In fact, this direct channel is closed down by assumption to highlight the different mechanism. The presented results are based on the fact that strategies which would provide liquidity in crisis situations are „harder to sell” to investors, so the equilibrium expected return of these strategies will contain a reputational premium. This premium must come in the form of larger liquidity crises to compensate those managers who saved liquidity for these situations.

The structure of the rest of the paper is as follows. In the next section I introduce the model.
In section 3, I present first the benchmark equilibrium without career concerns which is followed by the presentation of equilibrium with career concerns. In section 4, I compare the two equilibria and discuss the results. Finally, I conclude.

2 A simple model of arbitrage and career concerns

There are three main groups of agents in this model: local traders, fund managers and investors. Differences in the demand functions of local traders in different markets create arbitrage opportunities. Fund managers attempt to locate and exploit arbitrage opportunities. Investors hire fund managers, provide them with operating capital and fire them if they are not satisfied with their performance. All agents are small and take prices as given. Time is continuous and infinite. Now, I present each group in detail.

2.1 Local traders and windows of arbitrage opportunity

Each arbitrage opportunity is based on two risky assets with the same cash flows: the $A$-asset and the $B$-asset. The only difference between assets that they are traded in different local markets: the $A$-market and the $B$-market respectively. Local traders are divided in two subgroups. $A$-traders trade only in island $A$, while $B$ traders trade only in island $B$. Local traders are identical across the two markets, except that they are subject to asymmetric and temporary demand shocks. Thus, there is an interval of random length when the demand curves of local traders across the two markets differ. This is the window of arbitrage opportunity. If local markets cleared separately, there would be a gap $g^*$ between the prices of the two assets. In each time point the window closes, i.e., the asymmetry disappears, with the constant hazard rate $\delta$. Consequently, $\tilde{t}$, the random closing time of the window is distributed exponentially.

Arbitrageurs, introduced in the next part, can reduce the size of the gap at time $t$ during the window by buying $x(t)$ units of the cheap asset and selling $x(t)$ units of the expensive asset. Preferences and optimal decision of local traders is summarized in the inverse demand function that arbitrageurs face,

$$g(t) = f(\tau(t)),$$

which shows that if the window is still open and arbitrageurs take opposite positions of the aggregate size of $\tau(t)$ than the gap between the asset prices is reduced to $g(t)$. Note, that $g(t)$ stands for the size of the gap conditional on $t < \tilde{t}$, i.e., on the window being still open. This characterizes the unconditional price pattern as well, as the unconditional size of the gap is

$$\tilde{g}(t) = \begin{cases} 
g(t) & \text{w.p. } e^{-\delta t} \\
0 & \text{w.p. } 1 - e^{-\delta t} \end{cases}.$$ 

In terms of the function $f(\cdot)$, the autarchy price gap, the gap when arbitrageurs are inactive is...
defined by

\[ g(t) = g^* = f(0). \]

Thus, if arbitrageurs are not present the gap is constant during the window and disappears when the window closes. It is a one-sided bet for any entering arbitrageur. I also assume that there is a finite position \( x_{\text{max}} > 0 \) that

\[ 0 = f(x_{\text{max}}), \]

i.e., if arbitrageurs keep selling more of the expensive asset and buying more of the cheap asset, the gap can be eliminated. Furthermore, \( f(\cdot) \) is continuous and monotonically decreasing in the interval \( x \in [0, x_{\text{max}}] \).

Our modelled world is full of similar \( A \) and \( B \) market pairs. Thus, similar windows of arbitrage opportunities keep popping up and closing. In particular, I assume that in each time point there is exactly one open window with the same structure as the previous ones. As a matter of notation, the time, \( t \), shows the time since the beginning of the given window. Values of variables in different windows will be denoted by the subscript \( s \) for window number \( s \), but only when distinction among variables in different windows is necessary.

### 2.2 Investors: the supply of capital

There is an infinite measure of potential investors, but only a finite measure of them hire fund managers and enter the market.\(^3\) Each of them has a unit of capital to invest, but none of them has the expertise to locate windows of arbitrage opportunities. This is why they hire fund managers. Each investor can hire only one fund manager at the beginning of a given window. Investors do not have the possibility to fire or hire investors within a window.\(^4\) They can only reconsider their decision when the window closes. Investors choose to hire the fund manager which looks the most able of those who are available. To judge this, they have access to the performance record of fund managers during the last window if the fund manager was hired then. As a tie-braking assumption, if a fund manager who was hired before seems exactly as able as an inexperienced one, they hire the latter. The assumption that investors do not keep records of the performance of arbitrageurs during several windows is made for simplicity. Thus, an investor who hired a fund manager in window \( s - 1 \) and observed the performance

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\(^3\)One can imagine that potential investors are indexed by the positive segment of the real line and in each window \( s \), only those indexed by a finite interval \( [0, \frac{\alpha}{\Gamma_s}] \) hire fund managers (the exact interpretation of \( \alpha \) and \( \Gamma_s \) is defined later in the text).

\(^4\)The assumption that investors cannot reconsider their decision of providing capital for a fund manager anytime can be rationalized in three ways. First it is consistent with the fact that investors of hedge funds can usually withdraw their capital only after an initial lock-up period and after advance notice. Second, hedge funds disclose their performance only in certain fixed intervals (e.g. quarterly). Thus, investors are not in the situation to assess the ability of fund managers within these intervals. (See Agarwal et al., 2004 for details on typical contracts between investors and hedge fund managers). Third, this assumption closes down the mechanism behind the results of Shleifer and Vishny (1997), where investors instantly withdraw capital from fund managers who suffer losses. Thus, it helps to highlight the effects of the mechanism presented in this paper.
of hired fund managers, \( \pi_{s-1} \), can choose an action, \( \sigma_s (\pi_{s-1}) \), of three possibilities in window \( s \). She either keeps her current fund manager (\( \sigma_s (\pi_{s-1}) = K \)) or fires her and hires the best available one (\( \sigma_s (\pi_{s-1}) = H \)) or stays out of the market (\( \sigma_s (\pi_{s-1}) = O \)). If an investor stayed out of the market in window \( s-1 \), she can choose only to hire a new manager, or stay out: \( \sigma_s (\pi_{s-1}) = H, O \).

Investors are risk-neutral. The contract between fund managers and investors is exogenously fixed: fund managers keep a \( \gamma \) share of the gross profit during their employment while \((1 - \gamma)\) share goes to the investors.\(^5\) Hence, there is no mismatch between the explicit incentives of fund managers and the preferences of investors. I make this assumption to be able to concentrate on the distortions from the implicit incentives: career concerns of fund managers.

Each investor faces a participation constraint. A potential investor in window \( s^+ \) hires a fund manager only if she is available and

\[
(1 - \gamma) \left( E(\pi_{s+1}|\pi_{s+1}) + \sum_{s=s+1}^{\infty} \Pr \left( \bigcap_{u=s+1}^{s} \sigma_u (\pi_{u-1}) = K|\pi_s \right) E(\pi_s|\pi_{s+1}) \right) \geq 1, \tag{1}
\]

where \( \pi_s \) is the gross return of this manager in window \( s \). The left hand side shows the investor’s expected share of future return if she hires the fund manager. The second term in the bracket is the expected profits in future windows weighted by the probability that the fund manager is kept until a given future window. This term shows that the investor takes into account the option value of keeping the fund manager. Investors will participate only if they at least break even in expectation. Thus, the supply of aggregate capital is infinite if the expected rate of return or gross return is larger than 1 while it is 0 if it is lower than 1.

### 2.3 Fund managers

There is an infinite measure of potential fund managers, but only a finite measure of them will be hired in each window.\(^6\) Only those fund managers are active who are hired. Each fund manager can be hired by a single investor at the beginning of a window.

Only a fixed proportion, \( \alpha \in (0, 1) \), of fund managers can locate the windows of arbitrage opportunity. I will call these fund managers arbitrageurs. Arbitrageurs instantly find the new open window when the previous one has closed. Arbitrageurs need operating capital because they have to provide collateral if they want to take positions. In particular, an arbitrageur can take a position of the size \( \frac{1}{g(t)} \) for each unit of collateral, where \( g(t) \) is the change of the value of a unit position in \( t \) if the gap remains open. This is the largest possible loss on a unit speculative position on the convergence. The position limit \( \frac{1}{g(t)} \) has the intuitive content of full collateralization of potential losses, because in any

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\(^5\)Unlike mutual fund managers, hedge funds do get a share of the profit as an incentive fee. This feature is consistent with our assumption of proportional profit sharing, even if real world hedge fund contracts are more complicated (see Agarwal et al., 2004).

\(^6\)Just as in the case of investors, one can imagine potential fund managers indexed by the positive segment of the real line and hired fund managers as a combination of finite intervals of the real line. However, as it will be shown, investors will condition their hiring and firing decision on the past performance of fund managers, so these intervals will change across windows.
interval \([t', t' + \Delta]\), if \(\{\pi (t)\}_{t' + \Delta}\) collateral is provided, the aggregate maximal loss is exactly covered by the collateral:

\[
\int_{t'}^{t' + \Delta} \pi (s) \frac{1}{g (s)} \dot{g} (s) \, ds = \int_{t'}^{t' + \Delta} \pi (s) \, ds.
\]

The assumption of endogenous position limits can also be regarded as the formalization of endogenous margin requirements or VaR constraints. The endogenous nature of the constraint plays an important role in the analysis. The larger the potential loss, the smaller the position arbitrageurs can take with their unit capital as collateral.

Because of the nature of the arbitrage opportunity, in effect arbitrageurs have a timing problem. If they invest too much before the time of the convergence and the gap temporarily widens, they might lose their operating capital and be out of the market without making any profit. To simplify the analysis and to highlight the timing nature of the decision of arbitrageurs, their available strategies are restricted. Instead of allowing for any pure strategy, arbitrageurs are restricted to invest only in a single time point, but they can freely mix these simple strategies. Given that arbitrageurs are risk neutral, this is not a strong restriction. In fact, I will argue that this class of strategies are among the optimal ones in equilibrium. This restriction also simplifies the analysis to a large extent, because it limits the possible realized returns of individual arbitrageurs considerably. Formally, if the arbitrageur chooses time \(u\), \(x (t) = 0\) for \(t < u\) and \(t > u\) and \(x (u) = \frac{1}{g (u)}\). Effectively, arbitrageurs bet only on the time of the convergence. Each arbitrageur’s mixing strategy is described by a density function \(\{\mu (t)\}_{t \geq 0}\) where \(\int_{t_1 + \Delta}^{t_1} \mu (t) \, dt\) is the probability that this arbitrageur takes a maximal position at a time \(t \in [t_1, t_1 + \Delta]\). Consequently, the expected gross return of an arbitrageur in window \(s\) is

\[
E (\pi_s) = \int_0^\infty \delta e^{-\delta t} \left( \mu (t) \frac{g (t)}{\dot{g} (t)} + 1 - \int_0^t \mu (u) \, du \right) \, dt
\]

where \(\delta e^{-\delta t}\) is the value of the density function of the closing time of the window at \(t\), \(\frac{g (t)}{\dot{g} (t)}\) is her net return and the term \(1 - \int_0^t \mu (u) \, du\) is the expected capital which is not spent by time \(t\). The full expected gross return of an arbitrageur at the beginning of window \(s\) is the \(\gamma\) fraction of the sum of \(E (\pi_s)\) and expected profits from future windows weighted by the probability that she will be hired then.

The rest of fund managers, a fraction of \(1 - \alpha\), cannot locate any arbitrage opportunity. I will call these fund managers quacks. Quacks can invest only in technologies which lead to losses. In particular, if a quack is hired, she can choose to loose any \(1 - \beta (t)\) fraction of the capital provided by the investor by time \(t\) where \(1 \geq \beta (t) \geq 0\) is non-increasing in \(t\). A quack trades only, because pretending to be an arbitrageur gives her private benefit. Her utility is independent from the loss she makes, but it is an increasing function of \(Pr\left(A | \pi_{s-1} = \beta (\tilde{i})\right)\), the probability that investors consider her an arbitrageur.
given her realized gross return \( \beta (t) \) when the window closes.\(^7\)

\[ 3 \quad \text{Equilibrium} \]

As the effects of career concerns of arbitrageurs are in the focus of my analysis, I proceed as follows. First, I present the equilibrium concept. Then, as a benchmark case, I present the equilibrium without career concerns. Here, arbitrageurs maximize their expected return in each window separately without considering the effect of their decision on their career prospects. This case coincides to the equilibrium of the model presented in Kondor (2006). Then I present the equilibrium with career concerns. In section 4, I discuss the properties of the equilibrium with career concerns and compare the two cases.

\[ 3.1 \quad \text{Equilibrium concept} \]

Our equilibrium concept will combine elements of the Rational Expectation Equilibrium and the Perfect Bayesian Equilibrium. We need this combination because our agents are price-takers, the framework is dynamic and, in the same time, fund managers and investors have asymmetric information about fund managers’ type.

The equilibrium is characterized for each window \( s = 1, 2... \) by the conditional gap path \( \{g_s (t)\}_{t \geq 0} \), the aggregate strategy profile of arbitrageurs \( \{\mu_s (t)\}_{t \geq 0} \) and quacks \( \{\beta_s (t)\}_{t \geq 0} \), the aggregate measure of arbitrageurs who are hired \( \Gamma_s \) and the beliefs of investors about the type of the available fund managers given by \( \Pr (A|\pi_{s-1}) \), the probability of the given fund manager being an arbitrageur given her performance in the last window if she was hired. We will look for solutions where individual strategies, \( \mu_s (t) \), are continuous almost everywhere and \( g_s (t) \) is continuous and differentiable everywhere.

In equilibrium for each window \( s \) the following conditions hold.

1. For given beliefs \( \Pr (A|\pi_{s-1}) \) and gap \( \{g_s (t)\}_{t \geq 0} \)
   - (a) hired fund managers’ strategy is optimal in each \( t \) and
   - (b) at the beginning of each window, investors hire and fire fund managers optimally.

2. The given prices \( \{g_s (t)\}_{t \geq 0} \) and the implied expected rate of return of hiring an available fund manager
   - (a) clears the arbitrage market in each \( t \), i.e., \( f \left( \bar{\pi}_s (t) \frac{\beta_s (t)}{\lambda (t)} \right) = g_s (t) \),
   - (b) clears the capital market, i.e., fund managers hire exactly \( \Gamma_s \) measure of arbitrageurs.

\(^7\)An equivalent assumption to support the fact that quacks’ utility does not depend directly on the loss they make is that they do not lose but consume \( 1 - \beta (t) \) fraction of their capital by \( t \). If the marginal benefit of consuming capital is \( \gamma \), they will only maximize \( \Pr (A|\pi_s) \) as consuming a unit and giving it back to the investor provides the same benefit of \( \gamma \).
3. Beliefs, $\Pr(A|\pi_{s-1})$, are consistent with the optimal strategies of fund managers and Bayes’ Rule.

To simplify notation, I omit the subscript $s$ when distinction between variables in different windows is not necessary.

3.2 Equilibrium without career concerns

For this part only, let us suppose that fund managers’ reemployment is independent of their performance. In particular, let us suppose that they are rehired in the new window with a fixed probability of $q \in [0, 1)$. For reasons specified below, I assume that

$$q < \gamma.$$  \hspace{1cm} (2)

As arbitrageurs receive a fixed proportion of the profit, without career concerns each hired arbitrageur maximize her expected profit during each window and solve the following problem

$$J(v(0)) = \max_{\mu(t)} \delta e^{-\delta t} \left( \mu(t) \frac{g(t)}{g(t) + v(t)} \right) dt$$  \hspace{1cm} (3)

s.t. $\dot{v}(t) = -\mu(t)$, $v(0) = 1$

where

$$v(t) \equiv 1 - \int_0^t \mu(u) du$$

is the expected capital not lost by time $t$, which is the state variable of problem (3). The solutions of this problem are given by the differential equations

$$\delta \frac{g^b(t)}{g^b(t)} = J^b(v(t))$$  \hspace{1cm} (4)

$$\dot{J}^b(v(t)) = \delta J^b(v(t)) - \delta$$  \hspace{1cm} (5)

where the superscript $b$ denotes the equilibrium values in this benchmark case with no career concerns. The system gives the general solution of

$$J^b(v(t)) = J^b_0(v(t)) e^{\delta t} = 1 + c^b e^{\delta t}$$  \hspace{1cm} (6)

$$g^b(t) = g_\infty \frac{e^{\delta t}}{1 + c^b e^{\delta t}}$$  \hspace{1cm} (7)

where I used the fact that $J^b(v(t))$ is linear, $c^b$ is an arbitrary positive constant at this point while $g_\infty$ is the limit of the path $\{g^b(t)\}_{t \geq 0}$ as $t$ increases without bound. Remember, that $g^b(t)$ is the size of the gap conditional on $t < \tilde{t}$, i.e., the gap if the window is still open. Along this path (7) arbitrageurs
are indifferent when to make their investments. As they are indifferent, they are happy to follow any mixed strategy \( \{ \mu^b(t) \}_{t \geq 0} \). In equilibrium the individual densities \( \{ \mu^b(t) \}_{t \geq 0} \) are indeterminate, but the aggregate measure of arbitrageurs who choose to invest at each \( t \), \( \{ \overline{\mu}^b(t) \}_{t \geq 0} \), has to support the equilibrium gap path (7), i.e., has to be consistent with market clearing conditions

\[
g^b(t) = f \left( \frac{\overline{\mu}^b(t)}{g^b(t)} \right)
\]

for all \( t \). Thus a given \( c^b \) and \( g_\infty \) pins down the conditional gap path which determines \( \{ \overline{\mu}^b(t) \}_{t \geq 0} \). This in turn gives the aggregate measure of arbitrageurs who are hired, \( \Gamma^b \), by the condition

\[
\int_0^\infty \overline{\mu}^b(t) \, dt = \int_0^\infty f^{-1} \left( g^b(t) \right) g^b(t) \, dt = \Gamma^b,
\]

as individual densities has to integrate to one. The expected gross return made by each hired arbitrageur in each window is given by the value function at \( t = 0 \) by definition. From (6), it is simply

\[
J^b(v(0)) = J^b(v(0)) 1 = 1 + c^b.
\]

As the investment is one unit, \( c^b \) is the expected net return produced by a hired arbitrageur.

Quacks mimic arbitrageurs to pool with them. They cannot mimic those who guessed the time of the convergence right and achieved a positive net return. However, they can choose to save their capital to mimic those who decided to invest too late, i.e., who chose \( u > \tilde{t} \). Alternatively, they can lose all the capital to mimic those who invested to early at \( u < \tilde{t} \). In equilibrium, each quack must be indifferent between the two choices. The probability that a fund manager is an arbitrageur if she saves her capital and the window closes at \( \tilde{t} \) is

\[
\Pr(A|\pi_{s-1} = 1) = \frac{\Pr(\pi_{s-1} = 1|A)\Pr(A)}{\Pr(\pi_{s-1} = 1|A)\Pr(A)+\Pr(\pi_{s-1} = 1|Q)\Pr(Q)} = \frac{\left( 1 - \frac{1}{1 - \overline{\mu}} \int_0^{\tilde{t}} \overline{\mu}^b(t) \, dt \right) \alpha}{\left( 1 - \frac{1}{1 - \overline{\mu}} \int_0^{\tilde{t}} \overline{\mu}^b(t) \, dt \right) \alpha + c^b(\tilde{t})(1 - \alpha)}
\]

where \( \overline{\mu}(\tilde{t}) \) is the fraction of quacks saving their capital till \( \tilde{t} \), \( \Gamma = \int_0^\infty \overline{\mu}(t) \, dt \) is the measure of all arbitrageurs on the market and \( A \) and \( Q \) are the events that the currently hired fund manager is an arbitrageur and a quack respectively. The term in the brackets in the nominator is the proportion of arbitrageurs who bet on a time of convergence \( u > \tilde{t} \). Similarly, the probability that a fund manager is an arbitrageur if she loses her whole unit of capital and the window closes at \( \tilde{t} \) is

\[
\Pr(A|\pi_{s-1} = 0) = \frac{\Pr(\pi_{s-1} = 0|A)\Pr(A)}{\Pr(\pi_{s-1} = 0|A)\Pr(A)+\Pr(\pi_{s-1} = 0|Q)\Pr(Q)} = \frac{\alpha \frac{1}{1 - \overline{\mu}} \int_0^{\tilde{t}} \overline{\mu}^b(t) \, dt}{\alpha \frac{1}{1 - \overline{\mu}} \int_0^{\tilde{t}} \overline{\mu}^b(t) \, dt + \left( 1 - \overline{\mu}(\tilde{t}) \right)(1 - \alpha)}.
\]

\[8\]Because the indifference condition would hold in the equilibrium with general strategies of arbitrageurs too (see Kondor, 2006 for a detailed argument), the restriction that arbitrageurs have to choose a single time point to invest and mix these simple strategies does not force them to make suboptimal decisions.
In equilibrium, $\beta(t)$ is determined by the condition that expressions (8) and (9) are equal, because this makes quacks indifferent between saving or losing their unit capital. This implies the non-increasing function

$$
\bar{\beta}^h(t) = \left(1 - \frac{1}{\Gamma^h} \int_0^t \bar{\beta}^h(t) \, dt\right)
$$

and

$$
\Pr(A|\pi_{s-1} = 0) = \Pr(A|\pi_{s-1} = 1) = \alpha.
$$

Thus, the expected share of profit of an investor who hires a fund manager of unknown type is

$$
(1 - \gamma) \left[ \frac{1}{1 - q} \left( \alpha \left(1 + c^h\right) + (1 - \alpha) \beta^b \right) \right]
$$

because the fund manager is a quack with probability $(1 - \alpha)$ who produces a gross return of

$$
\beta^b = \int_0^\infty e^{-\delta t} \beta^b(t) \, dt
$$

and each fund manager is rehired with probability $q$ regardless of her performance. Given the participation constraint (1) and the free entry of investors, the equilibrium rate of $c^b$ is

$$
c^b = \frac{1}{\alpha} \left( \frac{1 - q}{1 - \gamma} - (1 - \alpha) \beta^b - \alpha \right).
$$

Condition (2) serves the only purpose to ensure that $c^b > 0$.

The only undetermined variable is the level of $g_\infty$, the limit of the conditional gap path. In Kondor (2006), I argued that in any equilibrium which is robust to the small perturbation of the inclusion of arbitrarily small holding costs, $m > 0$, $g_\infty$ must be $g^*$. The reason is that in this perturbed version, regardless of the price pattern, individual positions cannot increase above $\frac{1}{m}$. Thus, the maximum instantaneous return cannot exceed the finite amount of $\frac{g_\infty}{m}$. If $g_\infty \neq g^*$, then for any $t$ a positive measure of arbitrageurs wait with their investment until $t$ with positive probability, i.e., $\bar{\pi}^b(t) \geq f^{-1}(g_\infty) > 0$ for each $t$. However, very long windows happen very rarely, so arbitrageurs cannot be indifferent between investing at a small $t$ and waiting for a very large $t$ if the instantaneous return is bounded from above. Here, I use the same equilibrium selection mechanism which leaves us with our equilibrium summarized in the next theorem.

**Theorem 1** If fund managers do not have career concerns, because they are rehired with a constant probability $q$ regardless of their performance, the unique robust equilibrium is given by the monotonically increasing conditional gap path

$$
g^b(t) = g^* \frac{c^b e^{\delta t}}{1 + c^b e^{\delta t}}
$$
and aggregate measure of arbitrageurs investing at \( t \)

\[
\pi^b(t) = f^{-1}\left(g^b(t)\right)\dot{g}^b(t)
\]

for all \( t < \hat{t} \), where

\[
\epsilon^b = \frac{1}{\alpha}\left(\frac{1-q}{1-\gamma} - (1-\alpha)\beta^b - \alpha\right),
\]

\[
\beta^b = \int_0^\infty e^{-\delta t}\beta^b(t)\,dt
\]

\[
\bar{\beta}^b(t) = \left(1 - \frac{1}{\Gamma_b} \int_0^t \pi^b(t)\,dt\right).
\]

The decreasing function \( \bar{\beta}^b(t) \) shows the fraction of quacks who have not lost their capital by \( t \) and \( \beta^b \) is the expected return from hiring a quack.

**Proof.** The proof is a simplified version of the proof of the next theorem with straightforward modifications, so it is omitted. ■

The solid curve on Figure 1 shows the conditional gap path, \( \{g^b(t)\}_{t \geq 0} \), in the equilibrium without career concerns. The path is monotonically increasing. It shows that each time when the window remains open, the gap increases and the average arbitrageur suffer losses. The intuition behind this result is given by the indifference condition which determines the equilibrium gap path. Arbitrageurs have to be indifferent when they invest. The increasing conditional gap path implies higher reward for those arbitrageurs who are betting on a latter time of convergence. This higher reward if they are successful, compensate them for the larger risk that the window closes earlier and they miss out on the arbitrage opportunity. In Kondor (2006), I emphasized this result. I highlighted the sharp contrast with the autarchy case. When arbitrageurs are not present, the gap can never widen and the first arbitrageur could make a safe bet if she did not affect prices. By the price effect of their trades, arbitrageurs create their own losses even if their strategies are individually optimal. When the window happens to be long, the aggregate loss of arbitrageurs is large and the gap is high. This event is a liquidity crisis similar to the LTCM-crisis in September 1998. Although the fundamentals are unchanged, the level of liquid capital of arbitrageurs shrinks and the average arbitrageur liquidates her position and suffers losses.

Although the conditional gap path will be increasing in the equilibrium with career concerns as well, the equilibrium will change in a systematic way. These effects of career concerns is in the focus of the analysis.

### 3.3 Equilibrium with career concerns

Now, we return to the original set up with career concerns.

In the candidate equilibrium, all possible return levels are observed in some states of the nature in each window. For any time \( t \), there is a positive measure of arbitrageurs who bet on convergence
Figure 1: The curves show the conditional gap paths, \( \{g(t)\}_{t \geq 0}, \{g^b(t)\}_{t \geq 0} \), in the benchmark equilibrium (solid line) and when career concerns are present (dashed line) when the expected long-term return of an arbitrageur is the same in the two equilibria. The curves are always monotonically increasing and approach to \( g^* \). They cannot cross more than once. They cross exactly once as depicted, if the exogenous probability of rehiring an arbitrageur in the benchmark case, \( q \), is not very large and positive.

at that given \( t \). Furthermore, some fund managers always realize 0 net return while others lose their whole unit of capital. As none of the possible strategies of fund managers would lead to any other outcome, we do not have to consider out-of-equilibrium beliefs.

In this equilibrium, investors keep those arbitrageurs who manage to guess the time of the convergence right and fire all the others and the quacks. Thus, an arbitrageur maximizes the expected profit over her full time of employment and solves the following problem in each window when she is hired

\[
J(v(0)) = \max_{\mu(t)} \int_0^\infty \delta e^{-\delta t} \left( \mu(t) \left( \frac{g(t)}{g^*} + J(v(0)) \right) + v(t) \right) dt \tag{11}
\]

s.t. \( \dot{v}(t) = -\mu(t), \quad v(0) = 1 \).

The difference between problems (3) and (11) is the presence of \( J(v(0)) \) in the brackets in (11). It shows that if the window closes at time \( t \) and the arbitrageur bet on the convergence at \( t \), then she not only gets her share of the monetary profit, \( \frac{g(t)}{g^*} \), and her remaining capital \( v(t) \), but also gets the right to participate again in the next window. Note, that the value function \( J(v(0)) \) is the same in the present window (left hand side of (11)) and in the next window (right hand side of (11)). As we will see, the reason is that the profitability of each window is the same in this equilibrium.

We proceed similarly to the benchmark case. The development of the marginal value function,
$J'(v_0)$ of (11) is given by the equation

$$J'(v(t)) = \delta J'(v(t)) - \delta$$

which has the solution of

$$J'(v(t)) = 1 + ce^{\delta t}.\]$$

Thus, the expected net rate of return is $c$ as the expected return in the arbitrage sector with one unit of capital is given by

$$J(v(0)) = J'(v(0)) 1 = 1 + c. \quad (13)$$

Note that $c$ is the expected rate of return of an arbitrageur during the full span of her employment. This is in contrast to $c^b$ which is the expected rate of return in each window in the benchmark case.

After the substituting (13), problem (11) implies the equation

$$\delta \left( \frac{g(t)}{g(t)} + (1 + c) \right) = 1 + ce^{\delta t} \quad (14)$$

for the development of $g(t)$ with the general solution of

$$g(t) = g_\infty \left( \frac{ce^{\delta t}}{1 + ce^{\delta t} - \delta (1 + c)} \right)^{\frac{1}{1 - \delta (1 + c)}} \quad (15)$$

where $g_\infty$ is the limit of the conditional gap path $g(t)$.

Just as in the benchmark case, along this conditional gap path, arbitrageurs are indifferent when to invest. The individual mixed strategies $\{\mu(t)\}_{t \geq 0}$ are indeterminate but the aggregate measure of arbitrageurs who are choosing to bet on the convergence of the gap at each $t$, $\{\Pi(t)\}_{t \geq 0}$ is given by the market clearing condition

$$g(t) = f \left( \frac{\Pi(t)}{\hat{y}(t)} \right). \quad (16)$$

Note, that arbitrageurs are indifferent between available strategies only after taking into account the implications of different strategies on their career prospects. As we will see, this fact will be critical for the results.

By the same argument as in the benchmark case, a quack saves her capital for a while to pretend that she is an arbitrageur who chose a time $u > \tilde{t}$ to invest, then loses all her capital to pretend that she is an arbitrageur who invested too early. The aggregate fraction of quacks who saved their capital until $t$ is given by

$$\beta(t) = 1 - \frac{1}{\Gamma} \int_0^t \Pi(t) \, dt$$
which implies the indifference of quacks when to lose their capital as

$$\Pr(A|\pi_{s-1} = 0) = \Pr(A|\pi_{s-1} = 1) = \alpha.$$  

for all $t$.

Now let us turn for the problem of investors. First, we check which fund manager they hire if they hire any. Then we turn to investors’ participation decision.

Given the aggregate strategies of quacks and arbitrageurs, $\{\pi(t)\}_{t \geq 0}$, investors can update their probability assessment about the type of their fund manager by Bayes Rule. Investors will observe three types of fund managers hired in the previous window. In the first category, fund managers produced a positive net return in the previous window. They are the arbitrageurs with probability 1. Obviously, any investor keeps such a fund manager. There are also some fund managers who produce a 0 net return: they save all the unit capital. In the last category, fund managers lost all their capital. In both of these two groups there are both quacks and arbitrageurs, but in a proportion that

$$\Pr(A|\pi_{s-1} = 0) = \Pr(A|\pi_{s-1} = 1) = \alpha.$$  

Thus, a fund manager who was hired in the previous window but guessed the time of the convergence wrong does not look more of an arbitrageur than an inexperienced fund manager. Because of our tie-braking assumption, investors will prefer the inexperienced fund managers.\footnote{This tie-braking assumption is very reasonable, if we think of our continuous-time set-up as an approximation of a discrete-time world. In any set up where actors make decision in descreet time points, the probability that the arbitraguer guesses the time of the convergence right would be positive in any period $t$. Thus, unlike in the continuous case, $\Pr(\pi_{s-1} = 0|A) + \Pr(\pi_{s-1} = 1|A) < 1$. The fraction of quacks who keep their capital until a particular $t$ would be

$$\beta(t) = \frac{\Pr(\pi_{s-1} = 1|A)}{\Pr(\pi_{s-1} = 0|A) + \Pr(\pi_{s-1} = 1|A)}$$  

as this implies

$$\Pr(A|\pi_{s-1} = 0) = \Pr(A|\pi_{s-1} = 1)$$  

which is necessary to make quacks indifferent between keeping their capital or losing it. It is easy to check that in this case

$$\Pr(A|\pi_{s-1} = 0) = \Pr(A|\pi_{s-1} = 1) = \alpha.$$  

Therefore, investors prefer inexperienced fund managers to unsuccessful ones, just as it is in our equilibrium.}\footnote{In this model, arbitrageurs are not able to build enough reputation which would keep them employed if they are unsuccessful in one window. This is clearly an artifact of our assumption that investors use only the fund managers’ last-window performance to judge their type. Although it is a strong assumption in the technical level, the intuition of the results do not depend on it.} Arbitrageurs can keep their job only as long as they guess the time of the convergence right. This is consistent with problem (11) of arbitrageurs.
The participation decision of arbitrageurs will determine the supply of capital. An investor whose fund manager produced positive net return in window $s-1$ will decide to participate in the market if the inequality

$$1 \leq (1 - \gamma) (1 + c).$$

(17)

holds. This is participation constraint (1) given the equilibrium strategy of investors and arbitrageurs. Investors who decided to hire an inexperienced fund manager face with the participation constraint of

$$1 \leq (1 - \gamma) (\alpha (1 + c) + (1 - \alpha) \beta),$$

(18)

where

$$\beta = \int_0^\infty e^{-\delta t} \beta(t) \, dt$$

is the expected return of a quack. The two constraints are different because inexperienced fund managers might turn out to be quacks. Observe, that (18) implies (17), but the opposite is not true. This is very intuitive. If any inexperienced fund manager is hired than successful arbitrageurs are hired for sure.

The aggregate demand for capital is implied by the decision-rule of fund managers. It has two parts. Quacks will provide a flat expected return of $\beta$ regardless of the aggregate capital which they get. The demand for capital by the arbitrage sector is given by

$$\kappa^A(c) \equiv \int_0^\infty f^{-1}(g(t)) \, g'(t) \, dt = \int_0^\infty \pi(t) \, dt = \Gamma$$

(19)

where $g(t) = g_\infty \left( \frac{e^{\delta t}}{1 + ce^{\delta t}} \right)^{1 - \frac{1}{1 + c}}$. This equation can be seen as a capital demand function, because $\Gamma$ is not only the measure of arbitrageurs who provide an expected gross return of $1 + c$, but also the size of the aggregate capital of arbitrageurs corresponding to this return. The next lemma shows that there is a negative relationship between $c$ and $\Gamma$. If more arbitrageurs enter, the rate of return on the arbitrage market goes down.

**Lemma 1** The aggregate capital demand function of the arbitrage sector,

$$\kappa^A(c) \equiv \int_0^\infty f^{-1}(g(t)) \frac{\partial g(t)}{\partial t} \, dt = \Gamma$$

where $g(t) = g_\infty \left( \frac{e^{\delta t}}{1 + ce^{\delta t}} \right)^{1 - \frac{1}{1 + c}}$ is downward sloping, i.e., $\frac{\partial \kappa^A(c)}{\partial c} < 0$.

**Proof.** The proof is in the appendix. ■

I use the downward sloping capital demand function to argue that in equilibrium some inexperienced fund managers are always hired. Thus, the equilibrium rate of return on the arbitrage market,
\[
c = \frac{1}{\alpha} \left( \frac{1}{1-\gamma} - (1-\alpha) \beta - \alpha \right)
\]
given by (18). This is shown in the next lemma.

**Lemma 2** In equilibrium, (17) holds as a strict inequality and the participation constraint (18) is binding.

**Proof.** The proof is in the appendix.

The only undetermined variable is \( g_{\infty} \). Just as in the benchmark case, the equilibrium is required to be robust for the small perturbation of the presence of holding cost, so \( g_{\infty} = g^* \).

I summarize the properties of the equilibrium in the following Theorem.

**Theorem 2** There is an equilibrium where in each window, the monotonically increasing conditional gap path is given by

\[
g(t) = g^* \left( \frac{ce^{\beta t}}{1 + ce^{\beta t} - \delta (1+c)} \right) \frac{1}{1-e^{\beta \delta t}}
\]

where

\[
c = \frac{1}{\alpha} \left( \frac{1}{1-\gamma} - (1-\alpha) \beta - \alpha \right)
\]

\[
\beta = \int_0^\infty e^{-\delta t} \beta(t) \, dt
\]

\[
\beta(t) = \left( 1 - \frac{1}{\Gamma} \int_0^t \pi(t) \, dt \right).
\]

Furthermore, the equilibrium aggregate strategies are given by

\[
\pi(t) = f^{-1}(g(t)) \dot{g}(t)
\]

for all \( t < \hat{t} \). The measure of arbitrageurs who are hired in each window is

\[
\Gamma = \int_0^\infty \pi(t) \, dt.
\]

Beliefs of investors whose fund manager made a gross return of \( \pi_{s-1} \) are

\[
\Pr(A|\pi_{s-1}) = \begin{cases} 
\alpha & \text{if } \pi_{s-1} = 0 \\
\alpha & \text{if } \pi_{s-1} = 1 \\
1 & \text{if } \pi_{s-1} > 1 
\end{cases}
\]

and investors keep their fund managers only if \( \pi_{s-1} > 1 \). Otherwise they will be indifferent whether
to hire an inexperienced fund manager or stay out of the market. If the previous window closed at $\tilde{t}$, then $\Gamma - \bar{\mu}(\tilde{t})$ measure of arbitrageurs will decide to enter.

**Proof.** Proof is in the appendix. ■

The dashed line in Figure 1 shows the main properties of the equilibrium conditional gap path, $(g(t))_{t \geq 0}$. It is apparent that this gap path is also monotonically increasing, so arbitrageurs create losses in this case as well. However, the two paths do not coincide. In the case depicted in Figure 1, career concerns imply a lower gap in the early stages of the window and higher gap if the window lasts sufficiently long. The intuition behind this fact and the implications are discussed in the next part.

## 4 Discussion

Although the gap paths in the equilibria with and without career concerns looks similar, career concerns result in systematic distortions in individual strategies and the distribution of returns. I will concentrate on two important implications. First, I show that in equilibrium arbitrageurs who follow strategies with high expected return are fired with larger probability. Thus, higher expected returns are penalized by the market for operating capital. Second, I show that if we compare the equilibrium with career concerns to the equilibrium with no career concerns when arbitrageurs provide the same expected return during their employment, liquidity crises of the same magnitude happen more frequently when career concerns are present.

To facilitate the discussion, let us fix an arbitrary interval length $\Delta > 0$ and consider the case when arbitrageurs follow atomistic strategies defined as follows.

**Definition 1** An arbitrageur follows the $t_1$ atomistic strategy, if she chooses a particular time $t_1$ and follows the mixed strategy where $\mu(t) = \frac{1}{\Delta}$ if $t \in [t_1, t_1 + \Delta]$ and $\mu(t) = 0$ otherwise.

In this case each arbitrageur chooses a $\Delta-$interval and follows a mixed strategy with equal weights on each point of this interval. If the window closes before this interval, the arbitrageur will miss out on the arbitrage opportunity while if the window closes after $t_1 + \Delta$, she loses all of her capital. If the window closes within the interval, the arbitrageur makes positive net return with positive probability. In particular, the expected return in a window $s$ of an arbitrageur following a $t_1$ atomistic strategy conditional on the closing time of the window $\tilde{t}$, $\pi_{s,t_1}(\tilde{t})$, is

$$
\pi_{s,t_1}(\tilde{t}) \equiv E(\pi_s|\tilde{t},t_1) = \left\{ \begin{array}{ll}
1 & \text{if } \tilde{t} < t_1 \\
\frac{1}{\Delta} \frac{g(\tilde{t})}{g(t)} + \frac{t_1 + \Delta - \tilde{t}}{\Delta} & \text{if } \tilde{t} \in [t_1, t_1 + \Delta] \\
0 & \text{if } \tilde{t} > t_1 + \Delta
\end{array} \right..
$$

Note, that any mixed strategy $\mu(t)$ can be constructed from the building blocks of atomistic strategies, if $\Delta$ is small enough. Because individual strategies are undetermined in equilibrium, atomistic strategies can serve as equilibrium strategies. Thus, we do not lose any intuitive content by focusing
on atomistic strategies only. Considering the distribution of the expected return $\pi_{s,t_1} (\tilde{t})$ of atomistic strategies instead of more complicated mixed strategies or the return distribution of pure strategies also simplifies the discussion. It helps us to explicitly focus on the consequences of betting on the convergence in different time points without facing the difficulty of interpreting values of a density function instead of probabilities.

4.1 Career concerns and the market for operating capital

The distorting effect of career concerns is very apparent, if we consider the relationship between the expected return of a strategy followed by an arbitrageur and her chance of keeping her job in equilibrium.

Just as the benchmark equilibrium, the equilibrium with career concerns is also determined by the indifference condition that each arbitrageur has to be indifferent when to invest. Choosing any time interval will provide the same expected return. However, when career concerns are present arbitrageurs do not consider only the expected return from the current window. They consider also the expected return from future windows weighted by the probability that they will still be hired. For example, the expected return from a $t_1$ atomistic strategy is

$$
1 - e^{-\delta t_1} + \int_{t_1}^{t_1+\Delta} e^{-\delta t} \left( \frac{1}{\Delta} \frac{g(t)}{g(\tilde{t})} + \frac{t_1 + \Delta - \tilde{t}}{\Delta} \right) dt + \left[ \frac{e^{-\delta t_1} - e^{-(t_1+\Delta)\delta}}{\Delta} J'(\tilde{t}) \right]
$$

where the first term is the expected return from the current window, while the second term is the expected return from future windows. The term $e^{-\delta t_1} - e^{-(t_1+\Delta)\delta}/\Delta$ is the probability that the arbitrageur can keep her job in the next window. As the sum is the same for any $t_1$ to ensure the indifference of arbitrageurs in equilibrium, if one part increases the other part must decrease. Arbitrageurs trade off the current expected return and the probability that they will be kept. This implies the following Proposition.

**Proposition 1** The larger the expected return of a strategy in the current window, the larger the chance that an arbitrageur following this strategy will be fired.

As the probability $e^{-\delta t_1} - e^{-(t_1+\Delta)\delta}/\Delta$ is decreasing in $t_1$, earlier strategies provide smaller expected return and larger chance of survival for an arbitrageur than late strategies. An arbitrageur can signal her type easier with early strategies, because short windows are more frequent and she will be successful more often.

Note that the negative association between the expected return of strategies followed by each arbitrageur an her career prospect is consistent with the positive relationship between her career prospect and realized return. Arbitrageurs are kept if they make a positive net return and fired otherwise. Thus, the presented model is consistent with the positive flow-performance relationship documented in the literature.\(^{11}\)

\(^{11}\)Chevalier and Ellison (1997) document positive flow-performance relationship in the mutual fund industry, while
As the intuition behind the result is fairly simple, it should generalize to markets with other structures. If some strategies are better signalling devices than others, the expected return of these strategies will decrease in equilibrium. Strategies which are „easier to sell” to investors will be less profitable.

A similar result is presented in Dasgupta and Prat (2005). They analyze the effect of career concerns to the price of a single asset. They show that the price can deviate from the expected fundamental value of the asset even in their risk-neutral framework, because buying some assets in certain states help informed traders to signal their type. Thus asset prices contain a reputational premium or cost. The result in Proposition 1 shows that expected return on dynamic strategies also contain a reputational premium or discount. In the next subsection, I show that the distribution of this premium has systematic implications on the probability and magnitude of liquidity crises.

4.2 Career concerns and the frequency of liquidity crises

In this subsection, I present the effect of career concerns on the size and frequency of liquidity crises. To concentrate on this effect only, I compare the gap paths \( g(t) \) and \( g^b(t) \) when hired arbitrageurs provide the same expected return during their employment, i.e.,

\[
\frac{c^b}{1 - q} = c.
\]

Naturally, the comparison depends on the size of \( q \), the probability of reemployment in the benchmark case, as it is described in the following proposition.

**Proposition 2** For any \( q \), \( \lim_{t \to \infty} (g^b(t) - g(t)) = 0 \). Furthermore, there is a critical level of \( \varepsilon^0 > 0 \) that

1. For \( q = 0 \), \( g(t) < g^b(t) \) for all \( t \),
2. if \( 0 < q < \min(\varepsilon^0, \gamma) \), the curves \( g(t) \), \( g^b(t) \) intersect exactly once in a point \( t^* \) and \( g(t) > g^b(t) \) for all \( t > t^* \) and \( g(t) < g^b(t) \) for all \( t < t^* \) and
3. if \( \gamma > \varepsilon^0 \) and \( \gamma > q > \varepsilon^0 \), then \( g(t) > g^b(t) \) for all \( t \).

**Proof.** The proof is in the appendix.

The relative position of the conditional gap paths depends on the difference \( c - c^b \), which depends on the constant \( q \), the probability of reemployment in the benchmark case. The first point of the proposition refers to the case when the probability of reemployment is 0 in the benchmark equilibrium. In this case, the expected return of arbitrageurs in a single window of the benchmark case, \( c^b \), must be equal then the long term return with career concerns, \( c \). This implies that if we compare the expected return of arbitrageurs in a single window, it must be larger in the benchmark case. This is in line with the result that the gap is larger in the benchmark case for all \( t \). The third case refers to the opposite

Agarwal et al. (2004) presents similar results for the hedge fund industry.
situation. When $q$ is very large, arbitrageurs are employed for a long time in the benchmark case. Thus, their expected return in a single window is much smaller in the benchmark case to keep the long term expected returns the same. This is in line with the smaller gap in the benchmark case.

The most interesting comparison is provided by the second case of Proposition 2. When $q$ is in the intermediate range, not only the long term expected returns of arbitrageurs are the same in the two equilibria, but expected returns during a single window are similar as well. This is the case, in which we can observe the effect of career concerns the most clearly. This case is depicted on Figure 1. As it is apparent, with career concerns the gap is smaller in short windows, but larger in long windows. This is closely related to the result in Proposition 1. The expected return of the strategy of betting on long windows has to increase relative to the benchmark case, to compensate arbitrageurs following this strategy for the smaller chance that they can keep their job. The opposite is true for the expected return of the strategy of betting on fast convergence. Thus, when career concerns are present, the competition of arbitrageurs for investors keeps the gap in a low level most of the time. However, in the aggregate arbitrageurs save less liquidity for the event of a long window, because being successful in the small probability events of long windows does not help too much to impress investors. Accordingly, the gap increase drastically when these small probability events realize. This is the premium to compensate those arbitrageurs who are ready to speculate on long windows, i.e., provide liquidity in these small probability events. The following proposition shows, this result in the fact that for large crises, the magnitude of aggregate loss of arbitrageurs with career concerns stochastically dominates the same loss in the benchmark case. Crises of the same size happen more often with career concerns, and – equivalently – if we compare two events happening with the same probability, the crisis with career concerns will be larger. This is true in both the first and second cases of Proposition 2.

**Proposition 3** If $q < \min \left( \varepsilon^b, \gamma \right)$ there is a critical $L^*$ that the aggregate loss of all arbitrageurs who did not choose to invest exactly in $\tilde{t}$ with career concerns stochastically dominates the same loss in the benchmark case in the first order sense given that this loss is larger than $L^*$. Formally,

$$\Pr(\{\int_0^t \tilde{\pi}(u) du \leq L \mid \int_0^t \pi(u) du > L^*\} < \Pr(\{\int_0^t \tilde{\pi}^b(u) du \leq L \mid \int_0^t \pi^b(u) du > L^*\})$$

for all $L \in (L^*, \Gamma)$.

**Proof.** The aggregate loss of arbitrageurs at $t$ if $t < \tilde{t}$ is

$$\int_0^t \tilde{\pi}(u) du = \int_0^t f^{-1}(g(u)) \dot{g}(u) du = F^{-1}(g(t)) - F^{-1}(g(0))$$

with career concerns and

$$\int_0^t \tilde{\pi}^b(u) du = F^{-1}(g^b(t)) - F^{-1}(g^b(0))$$

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with no career concerns. As \( \frac{\partial F^{-1}(g(t))}{\partial g(t)} = f^{-1}(g(t)) > 0 \), losses are increasing in \( g(t) \) and \( g^b(t) \) and decreasing in \( g(0) \) and \( g^b(0) \). Both in the first and second cases of Proposition 2, \( g^b(0) > g(0) \). As

\[
\lim_{t \to \infty} g(t) = \lim_{t \to \infty} g^b(t) = g^*,
\]

it is either true that for any \( L \)

\[
\Pr \left( \int_0^t \pi(u) \, du \leq L \right) < \Pr \left( \int_0^t \pi^b(u) \, du \leq L \right)
\]

or there must be a \( t^0 \) that for any \( t > t^0 \),

\[
\int_0^t \pi^b(u) \, du < \int_0^t \pi(u) \, du
\]

Defining

\[
L^* = \int_0^{t^0} \pi(u) \, du = \int_0^{t^0} \pi^b(u) \, du
\]

gives the result as the probability of longer windows is smaller. ■

Results in this section are also connected to the “copy-cat problem”. It was a popular view among practitioners after the near-collapse of the LTCM that the extent of the crisis was related to the widespread similarities between the positions of LTCM and other hedge-funds and investment banks. (See e.g. Edwards, 1999, MacKenzie, 2003) Thus, the natural trading partners of LTCM were hurt in the same time as LTCM by adverse shocks, so when troubled hedge funds tried to liquidate some of their positions, the markets for these assets dried up. This exacerbated the crisis. The results of this section suggest that career concerns imply inefficient similarities among arbitrageurs’ portfolios as an equilibrium consequence of arbitrageurs’ individually optimal decisions. Those strategies which help arbitrageurs to signal their abilities are more popular even if they provide smaller expected return. Proposition 3 shows that this mechanism indeed increases the chance and magnitude of liquidity crises.

5 Conclusion

I presented a general equilibrium model of delegated risky arbitrage. In the model two types of fund managers compete for the capital of investors who are uninformed about the type of fund managers. Some fund managers can locate windows of arbitrage opportunities: pairs of fundamentally very similar assets traded temporarily at different prices. These are the arbitrageurs. They can make profit from betting on the time of convergence. Other fund managers cannot locate these windows, but they can mimic the outcome of arbitrageurs when arbitrageurs are unlucky. Investors observe the performance of fund managers and update their beliefs about fund managers’ type. Based on their beliefs they hire and fire arbitrageurs.

I highlighted two main results. In this set-up, arbitrageurs who speculate on the fast convergence
of prices are fired with smaller probability, because they are successful more often. Consequently, they can signal their type more often. However, because of this advantage of this strategy, the competition of arbitrageurs drive down the corresponding expected return. Thus, in equilibrium, there is a negative association between the career prospects of an arbitrageur and the expected return of the strategy she follows. Relatedly, the second main result is that liquidity crises are typically more frequent when career concerns are present. The reason is that there are too many arbitrageurs are betting on the fast convergence of prices, so the arbitrage sector as a whole do not save enough liquidity for the rare events of long price discrepancy. Hence, if it happens, prices will diverge to a large extent and the aggregate loss of arbitrageurs will be large.

Although the presented model focuses on the complicated problem of delegated portfolio management in a general equilibrium set up, it is analytically very tractable. This is a unique feature in the similar literature. This gives the hope that the model will be applicable to further problems in the field. I suggest two tasks for further research. First, in this model the aggregate capital of arbitrageurs is the same in each window regardless what happened in the past. In reality, we can observe that after a crisis, price gaps tend to remain high for a while. It would be an important exercise to analyze the equilibrium fluctuations of arbitrage capital with the help of an extension of the work presented in this paper. Second, it is clear that the effect of career concerns in this model are related to the fact that investors can observe the past performance of fund managers in regular intervals (at the beginning of each window). Thus, the strength of the effect is naturally related to the frequency of information disclosure about funds’ performance. Therefore, this model is a potential tool to assess the welfare effects of different regulatory regimes for hedge funds.

References


Appendix

Proof of Lemma 1. First note that

$$\int_0^\infty f^{-1}(g(t)) \frac{\partial g(t)}{\partial t} \, dt = F^{-1}(g^*) - F^{-1}(g_0)$$

where $F^{-1}(\cdot) = \int f^{-1}(g) \, dg$, because $\frac{\partial F^{-1}(g(t))}{\partial t} = f^{-1}(g(t)) \frac{\partial g(t)}{\partial t}$. Thus, we can write

$$\frac{\partial \kappa^A(c)}{\partial c} = \frac{\partial \kappa^A(g_0(c))}{\partial g_0} \frac{\partial g_0}{\partial c}$$

Furthermore,

$$\frac{\partial \kappa^A(g_0(c))}{\partial g_0} = - \frac{\partial F^{-1}(g_0)}{\partial g_0} = -f^{-1}(g_0) < 0.$$
Also note, that \( g_0 = g(0) = g^* \left( \frac{c}{1+c - \delta(1+c)} \right)^{\frac{1}{1-\delta(1+c)}} \). Hence,

\[
\frac{\partial g_0}{\partial c} = g^* \left( \frac{c}{1+c - \delta(1+c)} \right)^{\frac{1}{1-\delta(1+c)}} \frac{\delta \left( \ln \left( \frac{c}{1+c - \delta(1+c)} \right) \right)}{(1-\delta + \delta c)^2} c(1+c) - \delta(1+c).
\]

The sign of the derivative depends on the term \( \delta \left( \ln \left( \frac{c}{1+\gamma(1+\theta)} \right) \right) c(1+c) + 1 - \delta(1+c) \). The minimum of this term for \( c > 0 \) is at \( c = \frac{1-\delta}{\delta} \) when it is 0. Thus the term is always non negative and \( \frac{\partial g_0}{\partial c} > 0 \).

**Proof of Lemma 2.** At the beginning of the first window all investors can hire only inexperienced fund managers. Investors will participate in the market and hire a fund manager, if (18) holds. Because of the free entry of investors, the equilibrium \( c \) in the first window will be

\[
c_1 = \frac{1}{\alpha} \left( \frac{1}{(1-\gamma)} - (1-\alpha) \beta - \alpha \right).
\]

This determines a \( \Gamma_1 \) by (19). Let us suppose that the first window closed at \( \tilde{t} \). In the second window, if (18) did not hold, the maximum measure of investors who would hire an investor would be \( \mu(\tilde{t}) \), as this is the measure of investors who decide to keep their current arbitrageurs. However, because of lemma 1, \( \Gamma_1 > \mu(\tilde{t}) \) implies that the equilibrium rate of return of arbitrageurs implied by (19) and consistent with a measure of arbitrageurs smaller or equal than \( \mu(\tilde{t}) \) is larger than \( c_1 \). Thus, investors prefer to hire inexperienced fund managers, which is a contradiction. Consequently, the rate of return is equal to \( c_1 \) in the second window, and with the same argument in all subsequent windows as well.

**Proof of Theorem 2.** Because the main steps of the proof are discussed in the main text, here I only give the draft and specify those details which are not discussed in the main text.

It is clear that if beliefs of investors are such that they keep only those arbitrageurs who produce positive net returns than the problem of arbitrageurs is given by (11) with interior solutions given by the differential equations of (12) and (14). We have to check whether there are corner solutions. In a corner solution, there is a time \( T \), that by \( T \) all arbitrageurs have made their maximal investment in all states of the world, thus arbitrageurs cannot take positions. Hence, if such a period existed, \( \overline{\pi}(t) = 0 \) and \( g(t) = g^* \) for \( t \geq T \) as in autarchy. But observe, that the position limits with unit capital goes to infinity as \( t \) increases, so the expected return of playing a mixed strategy with positive \( \mu(t) \) in an interval of \( [t_1, t_1 + \Delta] \) increases without bound as \( t_1 \) increases. Which is in contradiction with \( \overline{\pi}(t) = 0 \) for all \( t \geq T \) as it would imply that arbitrageurs do not save capital for times with arbitrarily large return. Thus we have only interior solutions given by expression (15). As discussed in the main text, only \( g_\infty = g^* \) gives a solution which is robust for the inclusion of arbitrarily small
trading cost. It is also easy to check that (15) is monotonically increasing as

\[
\frac{\partial g(t)}{\partial t} = g^* \delta \left( \frac{c(1 + ce^{\delta t} - \delta(1 + c))}{1 + ce^{\delta t} - \delta(1 + c)} \right) > 0.
\]

Given the conditional gap path, the aggregate measure of arbitrageurs investing in each time \( t \), \( \{\pi(t)\}_{t \geq 0} \), is given by (24) and the equilibrium measure of arbitrageurs who enter the market, \( \Gamma \), is (25). A fraction of \( 1 - \Theta(t) \) quacks defined in (23) lose their capital by \( t \) as otherwise they were not indifferent among their available strategies. Bayes Rule gives the equilibrium beliefs (26) which are consistent with the equilibrium strategies of arbitrageurs and quacks. With these equilibrium beliefs, investors keep arbitrageurs only if they make positive net return which closes the argument. The last step is to determine \( c \), which must be given by (21), because of Lemma 2.

For the proof of Proposition 2, first we have to prove the following lemma.

**Lemma 3** There is a threshold \( t^0 = \max \left( 0, \frac{1}{\delta} \ln \frac{\delta(1+c)}{(c-\delta)} \right) \) that if \( t > (\leq) t^0 \) then \( \frac{g(t)}{g^*(t)} > \frac{g^*(t)}{g(t)} \).

**Proof.** The result is a straightforward consequence of equations (4) and (14).

**Proof of Proposition 2.** Note, that Lemma 3 implies that if there is a \( t^+ \) where \( g(t^+) = g^n(t^+) \) and \( \dot{g}(t^+) > \dot{g}^n(t^+) \) then \( t^+ < t^0 \) where \( t^0 \) is defined in 3. Similarly, if there is a \( t^- \) that \( g(t^-) = g^n(t^-) \) and \( \dot{g}(t^-) < \dot{g}^n(t^-) \) then \( t^- > t^0 \). Consequently, if there is an intersection where \( g(t) \) crosses \( g^n(t) \) from below, there cannot be an intersection at a smaller \( t \) where \( g(t) \) crosses \( g^n(t) \) from above.

An other implication of Lemma 3 is that if there is any \( t^{++} \) that \( g^n(t^{++}) > g(t^{++}) \) there must be a \( t^+, t^+ > t^{++} \), where \( g(t) \) crosses \( g^n(t) \) from below. The reason is that if there is no such \( t^+ \) then \( g^n(t) > g(t) \) for \( t > t^{++} \) including all \( t > t^0 \). Thus, Lemma 3 implies \( \dot{g}^n(t) > \dot{g}(t) \) for all \( t > t^0 \) and

\[
g^n(t) - g(t) = g^n(t^0) - g(t^0) + \int_{t^0}^{t} (\dot{g}^n(u) - \dot{g}(u)) \, du
\]

increases for all \( t > t^0 \). But this is in contradiction with \( \lim_{t \to \infty} g(t) = \lim_{t \to \infty} g^n(t) = g^* \).

These two implications of Lemma 3 imply that if \( g^n(0) > g(0) \) then there will be exactly one intersection where \( g(t) \) crosses \( g^n(t) \) from below and if \( g^n(0) < g(0) \) there will not be any intersections.