Forecasting UK Inflation: Empirical Evidence on Robust Forecasting Devices

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Abstract

Forecasting inflation is fundamental to UK monetary policy, both for policy-makers and private agents. However, forecast failure is prevalent with naive devices often outperforming the dominant congruent in-sample model in forecasting competitions. This paper assesses evidence for UK annual and quarterly inflation using the theoretical framework developed by Clements and Hendry (1998, 1999) to explain the empirical findings. We build both single equation and multivariate equilibrium correction models of inflation using the automatic model selection algorithm, PcGets, and use these models along with various transformations of the models to forecast UK inflation over the period 1997-2003. Robust forecasting devices do prove useful in forecasting macroeconomic time series and they often outperform econometric models, both when there are structural breaks in the data and when the underlying process appears to be stable but with breaks in the explanatory variables. Increasing the information set does lead to improvements in forecasting performance suggesting that disaggregation can yield benefits. Finally, it is observed that much of the forecast error in the structural models is driven by the deterministic terms. Breaks in the mean of the cointegrating vector or the growth rate of the system will cause forecast ‘failure’ and results show how sensitive forecasts are to errors in these terms.

1 Introduction

Systematic mis-forecasting of economic events has led to extensive research on economic forecasting, culminating in a new theory of forecasting developed by Clements and Hendry (1998, 1999). This theory departs from the well documented scenarios in which the econometric model coincides with the Data Generating Process (DGP) in a stationary world. The forecast error taxonomy developed allows for a mis-specified model with measurement error in the data, within a non-stationary world that is subject to structural breaks. This is much more representative of the conditions in which forecasts of UK inflation are derived. This paper examines the forecasting performance of various models and forecasting rules for UK annual and quarterly inflation using the Clements and Hendry forecast theory to explain the findings of the various models and the presence of forecast failure.

We build both single equation and multivariate equilibrium correction models of inflation using the automatic model selection algorithm, PcGets. The inflation models perform well over the in-sample period from 1966-1997, incorporating most extant theories of inflation and closely matching previous models such as the annual UK inflation model of Hendry (2001). We use these in-sample models, along with various robust transformations of the models, to forecast inflation over the period 1997-2003. The transformations attempt

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to ‘robustify’ forecasts to structural breaks, and we consider various differencing devices as well as forecast pooling to overcome the problem of forecast failure.

We tentatively conclude that robust forecasting devices do prove useful in forecasting macroeconomic time series and they often outperform the dominant congruent in-sample model, both when there are structural breaks in the data and when the underlying process appears to be stable but with probable breaks in the explanatory variables. We also conclude that increasing the information set does lead to improvements in forecasting performance, suggesting that disaggregation can yield benefits. Finally, we observe that much of the forecast error in the structural models is driven by the deterministic terms. Breaks in the mean of the cointegrating vector or the growth rate of the system will cause forecast ‘failure’ and results show how sensitive forecasts are to errors in these terms.

The structure of the paper is as follows. Section 2 briefly outlines the data. Section 3 presents two models of quarterly inflation, including a single equation equilibrium correction model derived using the general-to-specific framework and a multivariate model based on various input prices and conditioning on excess demand. Section 4 assesses the 1-step and 4-step forecasting performance of these models against robust forecasting devices, ranking the models in an attempt to predict which models should forecast well. Section 5 addresses the theory of predictability, presenting evidence for this by deriving two models of annual inflation, including a lower frequency model using annual analogues and a higher frequency model using quarterly data. Section 6 assesses the forecasting performance of these annual inflation models against robust forecasting devices. Section 7 concludes.

2 Data

The data set consists of quarterly data for the UK over 1965q1-2003q2 and was derived from a number of sources detailed in the appendix. All data are seasonally adjusted and lower cases represent logarithms. The difference operator, \( \Delta_i \), is defined as \( (1 - L^i) \) where \( L \) is the lag operator. Subscripts are omitted for first differences but are indicated for fourth differences denoting the annual change. Indicator variables, given by \( I_{\text{date}} \), take the value 1 in the quarter indexed and 0 otherwise. Dummy variables combine 2 indicators taking the values -1 and +1 in the indexed quarters. The estimation sample period is usually 1966q1-1998q2, resulting in 130 observations, with 20 observations retained for forecasting extending from 1998q3-2003q2. The single equation models have a shorter estimation period from 1967q1.

The equilibrium correction models of inflation are based on a mark-up model, with excess demand pressures causing short-run cyclical movements in inflation whilst the long-run price level is determined by sectoral price levels. The main series include producer prices (\( \text{ppi} \)), import prices (\( \text{imp} \)), housing rent (\( \text{rent} \)), unit labour costs scaled for the decline in average hours (\( c^* \)), oil prices (\( \text{oil} \)), national debt (\( n \)) and external prices (\( \text{pw} \)). The short-run pressures are captured by the output gap (\( y^d \)), excess demand for unemployment (\( U^d \)), the growth rate of broad money (\( \Delta m4 \)), the short-long real interest rate spread (\( s \)) and the real effective exchange rate (\( e_r \)). A number of non-linear adjustments are included in the initial general model to capture exchange rate adjustment asymmetries. The purchasing power parity (PPP) interaction terms are given by the difference of the effective exchange rate multiplied by the lagged real exchange rate \( [\Delta e_t e_{r,t-1}] \) and the difference of the effective exchange rate multiplied by the lagged real exchange rate squared \( [\Delta e_t e_{r,t-1}^2] \) for \( j = 1, 2 \).

Figure 1, panel a records the log of the GDP deflator, unit labour costs, producer prices and world prices and panel b records the nominal effective exchange rate. Panel c records real unit labour costs and real producer prices and finally, panel d records the quarterly inflation rate.\(^1\) There is a substantial negative inflation outlier of -2.2% in 1973q2, which is most likely to be a measurement error and so an indicator

\(^1\)Figures are lettered notionally as \([a, b, c, d]\).
variable is included for this outlier.\footnote{There are also negative observations of inflation in the gross value added at basic prices deflator and the total domestic expenditure deflator for 1973q2, but we again attribute these substantial negative outliers to measurement error.} Nominal unit labour costs are adjusted for the decline in working hours by 0.0625% per quarter from 1965q1-1984q4 and for a decline of 0.03% hours per quarter from 1985q1-2003q2. This accords with the data on actual hours worked from the national statistics figures on average actual weekly hours of work. The order of integration of the price level has been extensively discussed, with Dickey-Fuller tests rarely providing conclusive evidence. The general consensus is that the price level is $I(1)$ but contains deterministic shifts which give the impression that the series is $I(2)$. Figure 2, panel a records the quarterly growth rate of producer prices, import prices, rent and unit labour costs, panel b records excess demand for goods and services derived in section 2.1, panel c records the excess demand for labour measure derived in section 2.1 and panel d records the external quarterly inflation rate.

### 2.1 Measures of Excess Demand

A measure of excess demand for goods and services is derived based on the Solow residual method using deviations from a measure of potential capacity. As the output gap is a latent variable it is notoriously difficult to measure, see Castle (2003). The Solow residual method, reported in (1), provides a good approximation as it matches the historical record of recessions and booms better than univariate statistical procedures such as the Hodrick Prescott filter.

\[
cap_t = \begin{cases} 
2.53 + 0.0026t + 0.36(k_t - wpop_t) & 1966q2 - 1980q4 \\
2.46 + 0.0033t + 0.36(k_t - wpop_t) & 1981q1 - 2003q2
\end{cases}
\]

(1)

From this measure of capacity, we calculate excess demand for goods and services as:

\[ y_t^d = y_t^{pe} - \cap_t, \]

where $y_t^{pe}$ is output per worker.

There is a substantial literature examining the importance of labour market pressures on inflation. We use a measure of excess demand for unemployment based on an equilibrium correction model following Hendry (2001). In this model, disequilibrium unemployment is based on steady state growth, with unemployment rising when the real interest rate exceeds the real growth rate and vice versa. The model describes disequilibrium unemployment as:

\[
\Delta U_{r,t} = 0.010 \Delta_4 (R_t - \Delta p - \Delta y)_t + 0.869 \Delta U_{r,t-1} + 0.008 (R_t - \Delta p - \Delta y)_{t-1} - 0.010 U_{r,t-1} - 0.005 I_{71:1} \\
+ 0.007 I_{71:2} - 0.003 I_{76:1} + 0.003 I_{90:3} + 0.003 I_{91:1}
\]

(2)

\[
R^2 = 0.860 \quad \hat{\sigma} = 0.110\% \quad SC = -13.524 \quad F_{ar}(5, 135) = 0.924
\]

\[
F_{arch}(4, 132) = 2.684 \quad F_{het}(13, 126) = 1.379 \quad \chi^2_{nd}(2) = 0.475
\]

\[
F_{reset}(1, 139) = 0.204 \quad F_{Chow}(10, 130) = 0.445 \quad T = 1966q2 - 2003q2.
\]

The model provides a reasonable fit and passes all diagnostics apart from ARCH at 5% significance.\footnote{Coefficient standard errors are shown in parentheses. $R^2$ is the squared multiple correlation, $\hat{\sigma}$ is the residual standard deviation and $SC$ is the Schwarz criterion. The diagnostic tests are of the form $F_j(k, T - l)$ which denotes an F-test against the alternative hypothesis $H_0$: $k^{th}$-order serial correlation ($F_{ar}$), $k^{th}$-order autoregressive conditional heteroscedasticity ($F_{arch}$), heteroskedasticity ($F_{het}$), the RESET test ($F_{reset}$) and parameter constancy over $k$ periods ($F_{Chow}$), and finally ($\chi^2_{nd}(2)$) represents a chi-square test for normality.} Five indicators are included in the model but do not enter into the long run solution. The resulting excess demand
for labour measure is given by:

\[ U^d_t = U_{r,t} - 0.73 (R_{t,t} - \Delta p_t - \Delta y_t). \]  

\[ (3) \]

3 Models of Quarterly Inflation

3.1 Single equation model of quarterly inflation

The initial model of \( \Delta p_t \) includes three lags of \( y^d, U^d_t, e_r, (c^* - p), s, (ppi - p), (rent - p), (imp - p), (oil - p), (n - p), R_t, \Delta ppi, \Delta rent, \Delta imp, \Delta oil, \Delta p, \Delta c^*, \Delta n, \Delta R_s, \Delta R_t, \Delta pw \), two lags of the PPP interaction terms \( \Delta e_{t-t-j}, \Delta e_{t-t-j}^2 \) and an intercept and trend. Three blip indicators were included, listed in table 1. We exclude contemporaneous covariates from the initial general model in order to reduce the possibility of reverse causation bias in the results. If some of the variables were not predetermined, a shock may cause a contemporaneous effect on inflation and other \( t \)–dated variables. For example, an exchange rate shock may impact upon import prices and inflation simultaneously, biasing the results from the inflation model. The general-to-specific framework embodied in PcGets enables all conflicting theories of inflation to be tested. PcGets automatically selects the undominated congruent model even when the formulation of the econometric relationship is not known a priori. The use of the single equation framework does require weak exogeneity in the regressors, and we relax this assumption in section 3.2 by examining a vector equilibrium correction model (VEqCM).

The initial reduction was undertaken by PcGets and a further reduction was imposed by replacing \( p_{t-1}, c^*_{t-1}, ppi_{t-1} \) and \( e_{r,t-1} \) with the mark-up, \( \pi_{t-1}^* \), reported in equation (4) and recorded in figure 7,
Figure 2: The quarterly growth rate of producer prices, import prices, rent and unit labour costs; excess demand for goods and services; excess demand for labour; the external quarterly inflation rate.

Table 1: Indicator variables for the quarterly inflation model.

<table>
<thead>
<tr>
<th>Label</th>
<th>Values</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_{(73:2,79:3)}$</td>
<td>-1 in 1973q2, +1 in 1979q3</td>
<td>Oil price shocks</td>
</tr>
<tr>
<td>$D_{(72:4,74:1)}$</td>
<td>+1 in 1972q4, -1 in 1974q1</td>
<td>Oil price shocks</td>
</tr>
<tr>
<td>$D_{(84:1,84:2)}$</td>
<td>-1 in 1984q1, +1 in 1984q2</td>
<td>Exchange rate fluctuations</td>
</tr>
</tbody>
</table>

where $pw_{L,t} = p_{W,t} - e_t$.

$$\pi_t^* = p_t - 0.70e_t^* - 0.10ppi_t - 0.20pw_{L,t} + 0.03. \quad (4)$$

The final model is reported in (5). The model represents a reasonable fit, in view of the turbulence in inflation over the period, and passes all diagnostic and constancy tests. The actual and fitted values are recorded in figure 3, along with the scaled residuals, their correlogram and residual density. The recursive coefficients, 1-step residuals and constancy tests are recorded in figure 4. The resulting model contains variables representing most theories of inflation with interpretable signs, other than the growth rate of the producer price index. This is likely to be driven by its collinearity with import prices. Both an acceleration of annual inflation term and a non-linear PPP disequilibrium term are significant. The non-linear term represents a larger impact from devaluations when there is a greater PPP disequilibrium, particularly with overvaluations. The model is similar to the annual inflation model in Hendry (2001), and both the mark-up and excess demand play an important role in the determination of inflation. Monetary terms including the growth rate of broad money and the interest rate are not retained in the model selection process.
\[ \Delta p_t = 0.064 \Delta imp_{t-1} - 0.081 \Delta ppi_{t-2} + 0.223 y^d_{t-1} - 0.128 U^d_{t-3} \]
\[ + 0.483 \Delta e_t e_{r,t-1} + 0.845 \Delta pw_{t-2} + 0.105 \Delta^2 p_{t-1} - 0.133 \pi^*_t \]
\[ + 0.049 D_{73:279:3} + 0.020 D_{72:4:74:1} + 0.016 D_{84:1:84:2} + 0.006 \]  
\[ R^2 = 0.858 \quad \widehat{\sigma} = 0.613\% \quad SC = -9.829 \]
\[ F_{ar}(5, 109) = 0.422 \quad F_{arch}(4, 106) = 0.513 \quad F_{het}(22, 91) = 0.904 \quad \chi^2_{sd}(2) = 0.926 \]
\[ F_{reset}(1, 113) = 1.311 \quad F_{Chow}(20, 114) = 0.755 \quad T = 1967q1 - 1998q2. \]

To forecast 4-steps ahead in the single equation framework we must develop a model to produce direct 4-step ahead forecasts. We use the PcGets automatic model selection procedure in which we derive a model of quarterly inflation using information lagged one year. The initial general unrestricted model (GUM) contains lags 4 to 7 of \( y^d, U^d, e_r, (c^* - p), s, (ppi - p), (rent - p), (imp - p), (oil - p), (n - p), R_t, \Delta ppi, \Delta rent, \Delta imp, \Delta oil, \Delta p, \Delta c^*, \Delta m4, \Delta n, \Delta R_s, \Delta R_t, \Delta pw \), lags 4 and 5 of \( \Delta e_t e_{r,t-j} \) and \( \Delta e_t e_{r,t-j}^2 \), and an intercept and trend. The blip indicators listed in table 1 are included. As information in periods \( t-1 \) to \( t-3 \) are excluded, residual autocorrelation is likely to bias the estimated coefficient standard errors. In order to overcome this, looser significance levels are used to select the model (using the expert users strategy in PcGets) and the autocorrelation mis-specification test is excluded from the test battery. The resulting specific model is estimated in PcGive to establish heteroskedasticity and autocorrelation consistent standard errors (HACSE) based on Andrews (1991), reported in parentheses.\(^4\)

\(^4\)In practice, further selection may be undertaken if the \( t \)-statistics from the resulting HACSE estimates result in insignificant coefficients, and we use an iterative procedure between PcGive and PcGets to determine the appropriate significance levels for selection as opposed to resorting to stepwise regression.
Figure 4: Single equation model of quarterly inflation; recursive coefficients with ±2SE, 1-step residuals and constancy tests.

\[
\Delta p_t = 0.329 \Delta p_{\text{wt},-4} + 0.086 \Delta \pi_{\text{pp},t-4} + 0.298 \gamma^d_{t-4} - 0.120 U^d_{t-7} \\
+ 0.016 \Delta \pi_{\text{oil},t-5} + 0.157 \Delta \pi_{\text{rent},t-4} - 0.060 \pi^*_t - 4 \\
+ 0.051 D_{73:2,79:3} + 0.025 D_{72:4,74:1} + 0.021 D_{84:1,84:2} + 0.0075 \\
R^2 = 0.814 \quad \bar{\sigma} = 0.698\% \quad SC = -9.597 \\
F_{\text{ar}}(5,110) = 0.884 \quad F_{\text{arch}}(4,107) = 0.990 \quad F_{\text{het}}(20,94) = 0.642 \quad \chi^2_{sd}(2) = 2.239 \\
F_{\text{reset}}(1,114) = 2.703 \quad F_{\text{Chow}}(20,115) = 0.991 \quad T = 1967q1 - 1998q2.
\]

The model is similar to the quarterly model, with a small rise in the equation standard error from 0.613\% to 0.698\% due to the reduced information set. Neither the acceleration of inflation or the PPP interaction term enter the model, implying these effects are short term, occurring within one year. The intercept is slightly higher at 0.0075 and the impact of the mark-up is much smaller as it is lagged by four periods. Even with the reduced information set, the model passes all diagnostics, including autocorrelation and heteroskedasticity, and the marginal decline in the equation standard error suggests that inflation may well correspond to an annual process because of time lags, where changes in input prices feed through to final prices slowly due to menu costs etc. The model fit is recorded in figure 5, along with the scaled residuals, their correlogram and residual density.
3.2 Multiple equation model of quarterly inflation

Our single equation analysis of quarterly inflation led to a mark-up model with an equilibrium correction mechanism involving the real exchange rate, world prices in sterling, unit labour costs and producer prices. We now generalise the model by examining a multivariate model of inflation, based on various input prices and conditioning on excess demand. The multivariate framework allows us to test the restriction of weak exogeneity that we automatically impose in the single equation framework. We do not relax the restriction of no contemporaneous covariates, which could be modelled within the autoregressive distributed lag (ADL) framework in order to avoid reverse causation bias, as discussed in section 3.1. Ideally the VAR framework would be more general to allow for cost-push, demand-pull, monetary and external factors affecting inflation but degrees of freedom restrict the number of variables we can model in the system. By conditioning on excess demand we require the output gap to be weakly exogenous and we do not include monetary variables as these are found to be insignificant in the single equation analysis.

As we wish to include a stationary explanatory variable in our analysis, i.e. the output gap, the problem of nuisance parameters arises. Rahbek and Mosconi (1999) show that the cumulated output gap should be included in the equilibrium correction mechanism, enabling the critical values for the trace test computed in Harbo et al. (1998) to be used. Without the cumulated gap in the cointegration space, the asymptotic distribution of the trace test is affected by nuisance parameters. However, this analysis shall proceed by including the lagged output gap outside the cointegration space. This is to avoid the problem of cumulating the I(0) measurement errors in the output gap to an I(1) measurement error. The standard errors associated with the output gap will be substantial as it is a latent variable, and so integrating the gap would lead to errors that could be quite misleading. The trace test statistics are used as an indication of the rank of the cointegrating vector and caution should be applied as the critical values are not exact.

Figure 5: Fitted and actual values, residuals, density and ACF for the EqCM model of quarterly inflation to forecast 4-steps ahead.
3.2.1 Cointegration Analysis of a Partial System

For partial cointegration analysis we adopt the framework outlined in Harbo et al. (1998). Consider a $p$-dimensional VAR with linear deterministic terms for $x_t$, where $x_t$ is a $(p \times 1)$ vector of variables at time $t$ ($t = 1, ..., T$).

$$
\Delta x_t = \Pi x_{t-1} + \sum_{i=1}^{k-1} \Gamma_i \Delta x_{t-i} + \mu + \delta t + \epsilon_t, \quad \epsilon_t \sim \text{IN}_p [0, \Sigma].
$$

(7)

The starting values, $(x_{1-k}, ..., x_0)$ are fixed, $\Gamma_i$ are $(p \times p)$ matrices and $\Pi = \alpha \beta'$ where $\alpha$ and $\beta$ are $(p \times r)$ matrices of full rank. For I(1) cointegration analysis we require the roots of the characteristic polynomial to lie on or outside the unit circle:

$$
A(z) = (1 - z) I_p - \Pi z - \sum_{i=1}^{k-1} \Gamma_i (1 - z) z^i.
$$

(8)

We also require the reduced rank condition for $x_t$ to be I(1) with $r$ cointegrating vectors given by:

$$
\text{rank} (\alpha' \Gamma \beta) = p - r,
$$

(9)

where $\alpha_\perp$ and $\beta_\perp$ are orthogonal complements defined as $[p \times (p - r)]$ matrices such that $\alpha' \alpha_\perp = 0$ and $(\alpha, \alpha_\perp)$ has full rank.

To assess the partial model we decompose $x_t$ into $y_t$ of dimension $p_1$ and $z_t$ of dimension $p_2$: $x_t' = (y_t', z_t')$. The parameter terms and error terms are decomposed similarly:

$$
\alpha = \begin{pmatrix} \alpha_y \\ \alpha_z \end{pmatrix}, \quad \Gamma_i = \begin{pmatrix} \Gamma_{y,i} \\ \Gamma_{z,i} \end{pmatrix}, \quad \mu = \begin{pmatrix} \mu_y \\ \mu_z \end{pmatrix}, \quad \delta = \begin{pmatrix} \delta_y \\ \delta_z \end{pmatrix}, \quad \epsilon_t = \begin{pmatrix} \epsilon_{y,t} \\ \epsilon_{z,t} \end{pmatrix}, \quad \Sigma = \begin{pmatrix} \Sigma_{yy} & \Sigma_{yz} \\ \Sigma_{zy} & \Sigma_{zz} \end{pmatrix}.
$$

(10)

The conditional model for $\Delta y_t$ is given by the equation:

$$
\Delta y_t = \omega \Delta z_t + (\alpha_y - \omega \alpha_z) \beta' x_{t-1} + \sum_{i=1}^{k-1} (\Gamma_{y,i} - \omega \Gamma_{z,i}) \Delta x_{t-i} + (\mu_y - \omega \mu_z) + (\delta_y - \omega \delta_z) t + (\epsilon_{y,t} - \omega \epsilon_{z,t}),
$$

(11)

where $\omega = \Sigma_{yz} \Sigma_{zz}^{-1}$. The marginal model for $z_t$ is given by:

$$
\Delta z_t = \alpha_z \beta' x_{t-1} + \sum_{i=1}^{k-1} \Gamma_{z,i} \Delta x_{t-i} + \mu_z + \delta_z t + \epsilon_{z,t}.
$$

(12)

If $\alpha_z = 0$ then the parameters in the conditional and marginal models are variation free. In this case, $\Delta z_t$ is said to be weakly exogenous and $\beta$ can be estimated efficiently from the conditional model.\(^5\)

\(^5\)Rahbek and Mosconi (1999) suggest that, rather than using the general case in which exogenous stationary regressors are added to the model given by:

$$
H(r) : \Delta y_t = \alpha \beta' y_{t-1} + \sum_{i=1}^{k-1} \Gamma_i \Delta y_{t-i} + \sum_{j=0}^{l} \Psi_j z_{t-j} + \mu + \delta t + \epsilon_t,
$$

we can extend the model to give:

$$
H^\prime(r) : \Delta y_t = \alpha (\beta' y_{t-1} + \beta' z_{t-1} + \beta' t) + \Phi z_t + \sum_{i=1}^{k-1} \Gamma_i \Delta y_{t-i} + \sum_{j=0}^{l-1} \Psi_j z_{t-j} + \mu + \epsilon_t,
$$

where $\Phi = \sum_{j=0}^{l} \Psi_j$ and $\Phi_i = - \sum_{j=i+1}^{l} \Psi_j, i = 0, ..., l - 1$. This results in a more general model as the cumulated $z_t$ appears in both the common trends and cointegrating relations. We do not adopt this approach because of the measurement errors in the cumulated output gap but it should be noted that the critical values correctly apply to this model.
Table 2: Descriptive statistics for quarterly inflation.

<table>
<thead>
<tr>
<th></th>
<th>unconditional</th>
<th>-dummies</th>
<th>1966q1-1992q4</th>
<th>1993q1-1998q2</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean</td>
<td>0.0185</td>
<td>0.0186</td>
<td>0.0209</td>
<td>0.0068</td>
</tr>
<tr>
<td>st.dev.</td>
<td>0.0154</td>
<td>0.0012</td>
<td>0.0157</td>
<td>0.0050</td>
</tr>
</tbody>
</table>

We include a constant, trend and indicator variables in the analysis. Hence, the model is specified by equation (13), in which the contemporaneous output gap is excluded for forecasting purposes. The level of the exogenous variable is included as it is an I(0) variable. The trend is restricted to lie in the cointegrating space allowing a linear trend in the cointegrating relations, whereas the constant and dummies are unrestricted. The impact of the dummies will be negligible as long as

\[ 1 = p T i = 1 D i \neq 0 \text{ for } T \neq 1. \]

Subscripts denote the corresponding parameters in equation (11).

\[
\Delta y_t = (\Pi_y, \Pi_{y,t}) \left( y_{t-1} \right) + \sum_{i=1}^{k-1} \Gamma_{yi} \Delta y_{t-i} + \sum_{i=1}^{l} \Gamma_{zi z_{t-i}} + \mu_x + \phi D_t + \epsilon_{xt}. \tag{13}
\]

### 3.2.2 Empirical Analysis

In the empirical analysis we define the initial unrestricted VAR for \( t = 1966q1-1998q2 \) as:

\[ y_t = [p_t, c_t, ppi_t, pw_t, e_t]^T. \]

The VAR is augmented by \( y_{d,t-1} \), which is I(0), and five indicator variables are included to account for special events. These are given by:

\[ D_t = [I_{74:1}, I_{79:2}, D_{73:1,75:1}, D_{73:2,79:3}, D_{83:1,92:4}]^T. \]

\( I_{74:1} \) is an impulse dummy controlling for fluctuations in world prices, producer prices and the price level arising at the time of the first oil shock. \( I_{79:2} \) is another impulse dummy accounting for the rise in VAT. \( D_{73:1,75:1} \) is a blip dummy taking the values -1,1, to account for the political effects of the Heath fiscal expansion and the Wilson-Callaghan ‘social contract’ applied to labour market bargaining. \( D_{73:2,79:3} \) is a blip dummy taking the values -1,1, accounting for oil price shocks and finally \( D_{83:1,92:4} \) takes the value 1 to control for exchange rate fluctuations following a sharp decline in the exchange rate in 1983q1 and the fall out of the ERM in 1992. Different intercepts are included for 1966q1-1992q4 and 1993q1 onwards to coincide with the switch to an inflation targeting regime. Table 2 provides some descriptive statistics of quarterly inflation to show the shift towards lower inflation during the inflation targeting regime.

The lag length for the unrestricted VAR is determined by likelihood ratio tests of model reduction, starting with four lags. Whilst eliminating the fourth lag is rejected at the 5% significance level, the test of model reduction from four lags to three lags is given by \( F(25, 361) = 1.596^* [\text{p-value}=0.037] \); balancing the fit with degrees of freedom suggests that a lag length of three would be appropriate. The F-tests on the retained regressors show that the fourth lag is marginally significant for the domestic and world price levels. A further model reduction to two lags is rejected, \( F(50, 445) = 1.654^{**} [\text{p-value}=0.005] \), and the smaller system fails many diagnostics. Tests on lags of the exogenous variable, \( y_d \), indicate that only one lag is required.

Tests for evidence of mis-specification for the unrestricted VAR are reported in table 3, with a residual analysis of the VAR recorded in figure 6. The tests are, in general, satisfactory although there is some evidence of autocorrelation in world prices. This implies the multivariate test fails residual correlation up to the fourth order at the 5% significance level. Including additional lags does not solve the problem. The residuals and their corresponding densities, correlograms and QQ-plots support the evidence for a well specified model. The recursive graphics were also examined to ensure there was no evidence of parameter change. Overall, the unrestricted VAR is congruent and we proceed with the analysis.
Table 3: Tests for mis-specification of the unrestricted VAR(3), including up to fourth order autocorrelation, fourth order ARCH, normality and heteroskedasticity. Both single and multivariate tests are reported, with p-values given in parentheses.

<table>
<thead>
<tr>
<th></th>
<th>AR(1−4) F(4,102), F(100,404)</th>
<th>ARCH(4) F(4,98)</th>
<th>Normality χ²(2), χ²(10)</th>
<th>Hetero F(32,73), F(480,863)</th>
</tr>
</thead>
<tbody>
<tr>
<td>pᵗ</td>
<td>1.801 [0.134]</td>
<td>0.937 [0.446]</td>
<td>1.047 [0.593]</td>
<td>0.981 [0.510]</td>
</tr>
<tr>
<td>uleᵗ²</td>
<td>1.890 [0.118]</td>
<td>1.194 [0.319]</td>
<td>1.591 [0.451]</td>
<td>0.947 [0.557]</td>
</tr>
<tr>
<td>ppiᵗ</td>
<td>0.908 [0.462]</td>
<td>0.612 [0.655]</td>
<td>0.768 [0.681]</td>
<td>1.383 [0.128]</td>
</tr>
<tr>
<td>eᵗ</td>
<td>0.872 [0.483]</td>
<td>1.280 [0.283]</td>
<td>3.269 [0.195]</td>
<td>0.786 [0.772]</td>
</tr>
<tr>
<td>pwᵗ</td>
<td>2.653* [0.037]</td>
<td>0.339 [0.851]</td>
<td>0.360 [0.835]</td>
<td>1.145 [0.312]</td>
</tr>
<tr>
<td>Multivariate tests</td>
<td>1.283* [0.050]</td>
<td>...</td>
<td>10.141 [0.428]</td>
<td>0.746 [0.999]</td>
</tr>
</tbody>
</table>

Table 4: I(1) cointegration analysis, reporting log-likelihoods (l), eigenvalues (λ), trace statistics (Q( r)) and p−values of the trace statistic (p-val) for all possible cointegrating ranks.

<table>
<thead>
<tr>
<th>r</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>l</td>
<td>2097.81</td>
<td>2131.37</td>
<td>2146.401</td>
<td>2154.95</td>
<td>2160.20</td>
<td>2161.30</td>
</tr>
<tr>
<td>λ</td>
<td>0.403</td>
<td>0.207</td>
<td>0.123</td>
<td>0.078</td>
<td>0.017</td>
<td></td>
</tr>
<tr>
<td>Q( r)</td>
<td>126.98**</td>
<td>59.85</td>
<td>29.78</td>
<td>12.70</td>
<td>2.19</td>
<td></td>
</tr>
<tr>
<td>p-val</td>
<td>0.000**</td>
<td>0.103</td>
<td>0.521</td>
<td>0.762</td>
<td>0.942</td>
<td></td>
</tr>
</tbody>
</table>

3.2.3 I(1) analysis

In order to determine the cointegrating rank, table 4 reports the log-likelihood values, l, the eigenvalues, λ, and the trace statistics, Q( r) for the VAR(3). Standard p−values are reported, based on Doornik (1998), although we focus on the critical values for the partial cointegration model given by Harbo et al. (1998) which are reported in table 5. The roots of the companion matrix were checked and there is no eigenvalue over 1.0 which suggests that there is no explosive root. We do not consider an I(2) analysis as inference using standard critical values cannot be made due to the partial model in which there are nuisance parameters. Hence, we work within the I(1) framework and conclude that there is one cointegrating vector.

The next step is to identify the long run structure within the model. The unrestricted factor loadings and cointegrating relations are given in (14), in which we normalise on p.

\[
\begin{pmatrix}
-0.282 \\
-0.018 \\
-0.670 \\
0.849 \\
-0.020
\end{pmatrix}
= \begin{pmatrix}
-0.873 \\
-0.094 \\
-0.050 \\
0.039 \\
-0.0012
\end{pmatrix} \begin{pmatrix}
1 \\
-0.837 \\
-0.050 \\
0.039 \\
0.0012
\end{pmatrix}.
\]

From this, we can test a number of restrictions. First, we test the restriction that the coefficient on world prices is equal to the negative coefficient on the exchange rate. The restriction is accepted; \( \chi^2(1) = 0.0107 \) [p-value=0.918]; giving world prices in sterling. We also test the restriction that the trend does not enter the cointegrating vector. This restriction is marginally accepted; \( \chi^2(1) = 3.6253 \) [p-value=0.057]; and we impose a zero coefficient on the trend. We can also undertake tests of weak exogeneity, which implies a zero coefficient on the \( \alpha \) parameter. Results are reported in table 6; we find weak exogeneity for unit labour costs.
Figure 6: Residual analysis of the unrestricted VAR, including scaled residuals, residual densities and histograms, residual correlograms and QQ plots.

<table>
<thead>
<tr>
<th>Critical Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>50%</td>
</tr>
<tr>
<td>54.8</td>
</tr>
</tbody>
</table>

Table 5: Critical values for a partial system from Harbo et al. (1998). Quantiles for \( r = 1 \) with one exogenous variable and five endogenous variables.

and world prices. Imposing all restrictions is accepted; \( \chi^2(4) = 5.5150 \) [p-value=0.238]; and the resulting restricted estimate of \( \alpha\beta' \) is given by:

\[
\alpha\beta' = \begin{pmatrix}
-0.228 \\
0 \\
-0.711 \\
0.789 \\
0
\end{pmatrix}
\begin{pmatrix}
1 & -0.925 & -0.048 & -0.024 & 0.024 \\
0.002 & 0.015 & 0.012 & ... \\
0.021 & 0.016 & 0.012 & ... \\
0.010 & 0.011 & 0.012 & ... \\
0.009 & 0.010 & 0.011 & ...
\end{pmatrix}.
\]

This is analogous to the single equation mark-up found in equation (4). The trend is insignificant in both the VAR cointegrating relation and the single equation model. The coefficient on unit labour costs is larger than in the single equation model and the coefficient on world prices in sterling is much smaller. The cointegrating vector is recorded in figure 7 along with the single equation mark-up given in equation (4). Both cointegrating relations have a zero mean for the in-sample period.

Having derived the cointegrating vector we can map the data to I(0) space to forecast inflation. In order to implement the reductions to a more parsimonious model, we use the reduced form VAR framework
Table 6: Tests for weak exogeneity. The test statistic is distributed as $\chi^2(1)$ under the assumption that the correct cointegrating rank is imposed.

<table>
<thead>
<tr>
<th>test statistic</th>
<th>$p$ - value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_t$</td>
<td>24.616</td>
</tr>
<tr>
<td>$e_t^*$</td>
<td>0.041</td>
</tr>
<tr>
<td>$ppi_t$</td>
<td>8.481</td>
</tr>
<tr>
<td>$e_t$</td>
<td>14.089</td>
</tr>
<tr>
<td>$pw_t$</td>
<td>0.578</td>
</tr>
</tbody>
</table>

Figure 7: The VAR cointegrating vector and the single equation counterpart.

in PcGets. This algorithm implements the general-to-specific selection strategy analogous to that used in the single equation model but applied to a multivariate GUM. Hendry and Krolzig (2001, ch.8) discuss generalising the Gets algorithm for VAR models. The GUM is in VEqCM form, containing 18 regressors in each equation, including two lags of the differenced variables, the lagged cointegrating vector, an intercept and five dummies. The dominant congruent model is derived, reported in equations (16) to (20). The test of model reduction from the full VEqCM to the model specification outlined is accepted; $\chi^2(55) = 68.358[p$-value=0.107]. Figure 8 records the fitted and actual values, residuals, density and ACF for quarterly inflation; we shall concentrate on this model for our forecasting purposes.

There is some evidence of autocorrelation within the system but extra lags do not help, hence we report HACSE estimates (see Andrews, 1991). If the equations are estimated independently there is no evidence of autocorrelation. However, the covariance matrix reflects collinearity between $p$, $e^*$ and $ppi$ in the residuals which is not fully modelled due to the restricted number of variables in the system. As noted earlier, degrees of freedom prevented this. The model represents a good fit. The equation standard error of 0.718% is larger than the single equation model with a standard error of 0.613%, due to the smaller variable set and restricted dynamics. However, for forecasting purposes, it is important to recall that the best in-sample model need not lead to the best forecasting model in a world of breaks. The forecast Chow test is accepted;
\[ F(100, 123) = 0.851 \text{ [p-value=0.798]} \] .

\[
\Delta p_t = 0.496 \Delta pw_{t-1} + 0.647 \Delta pw_{t-2} + 0.168 y_{t-1}^d + 0.157 c_{t-1}^* - 0.045 cv_{t-1} - 0.028 I_{74:1} + 0.043 D_{73:2.79:3} + 0.014 D_{73:1.75:1} + 0.007 (16) \\
\tilde{\sigma} = 0.718 \% \quad F_{ar}(5, 107) = 5.780^{**} \quad F_{het}(31, 91) = 0.778 \\
\chi^2_{nd} (2) = 0.685 \quad F_{arch}(4, 115) = 0.482 \quad T = 1966q1 - 1998q2.
\]

\[
\Delta c_{t}^* = 0.231 \Delta p_{t-1} + 0.338 y_{t-1}^d + 0.120 \Delta c_{t-2}^* + 1.028 \Delta pw_{t-1} - 0.108 \Delta ppi_{t-1} + 0.065 \Delta ppi_{t-2} + 0.050 D_{73:1.75:1} + 0.019 D_{73:2.79:3} + 0.007 (17) \\
\tilde{\sigma} = 1.030 \% \quad F_{ar}(5, 107) = 2.372^{*} \quad F_{het}(31, 91) = 1.452 \\
\chi^2_{nd} (2) = 0.947 \quad F_{arch}(4, 115) = 1.365 \quad T = 1966q1 - 1998q2.
\]

\[
\Delta ppi_{t} = 0.286 \Delta ppi_{t-1} + 0.341 y_{t-1}^d - 0.189 cv_{t-1} + 0.214 I_{74:1} + 0.051 D_{83:1.92:4} + 0.008 I_{023}(0.023) \\
\tilde{\sigma} = 2.617 \% \quad F_{ar}(5, 107) = 10.363^{**} \quad F_{het}(31, 91) = 4.760^{**} \\
\chi^2_{nd} (2) = 5.568 \quad F_{arch}(4, 115) = 5.281^{**} \quad T = 1966q1 - 1998q2.
\]

\[
\Delta c_{t} = 1.219 \Delta pw_{t-2} + 0.400 cv_{t-1} + 0.075 I_{79:2} - 0.113 D_{83:1.92:4} - 0.021 I_{024}(0.024) + 0.007 (18) \\
\tilde{\sigma} = 2.889 \% \quad F_{ar}(5, 107) = 5.265^{**} \quad F_{het}(31, 91) = 0.595 \\
\chi^2_{nd} (2) = 3.270 \quad F_{arch}(4, 115) = 1.238 \quad T = 1966q1 - 1998q2.
\]

\[
\Delta pw_{t} = 0.072 \Delta p_{t-2} + 0.040 y_{t-1}^d + 0.477 \Delta pw_{t-1} + 0.253 \Delta pw_{t-2} + 0.029 \Delta ppi_{t-1} + 0.009 I_{74:1} + 0.010 I_{79:2} + 0.002 I_{003}(0.003) + 0.005 (20) \\
\tilde{\sigma} = 0.315 \% \quad F_{ar}(5, 107) = 4.492^{**} \quad F_{het}(31, 91) = 1.444 \\
\chi^2_{nd} (2) = 0.754 \quad F_{arch}(4, 115) = 0.546 \quad T = 1966q1 - 1998q2.
\]

There is a substantial impact from external prices feeding through to domestic inflation with both lags corresponding to a coefficient of approximately unity. This may well be reflecting collinearity with domestic prices as well, as no lagged dependent variables are retained. The output gap has a 17% effect, so is an important source of inflation. The cointegrating vector enters with an impact of 5%, and so the exchange rate, unit labour costs and producer prices all enter significantly. An intercept is not retained, suggesting that there is no evidence of autonomous inflation. The output gap feeds into unit labour costs and producer prices with a greater impact than inflation, at 34% for both impacts, suggesting that excess demand pressures are recognised more fully at the intermediate stage. The cointegrating vector does not enter into the equation for unit labour costs, reflecting the evidence found for weak exogeneity. External inflation has a substantial impact on unit labour costs, with a coefficient of approximately one. The exchange rate equation contains the lagged external inflation rate and the mark-up which is difficult to explain, but an equation standard error of 2.89% suggests that the model is poor. The exchange rate is a notoriously difficult variable to model and often a random walk is used, although the RW produced an inferior model for this sample.
4 Quarterly Inflation Forecasts

To examine the problems arising from forecasting using the VEqCM, we shall consider a simplified model of equation (13). As we found no evidence of a trend in the cointegrating relation we shall set $\Pi_{y,t} = 0$. We shall also exclude the lagged dependent variables and the exogeneous I(0) variables for expositional clarity by setting $\Gamma_{y_i} = 0$ for $i = 1, \ldots, k - 1$ and $\Gamma_{z_i} = 0$ for $i = 1, \ldots, l$. Finally, we shall exclude dummies by setting $\phi = 0$. Considering the joint vector of variables, $x_t$ (i.e. we map back from $y_t$ to $x_t$), we can represent (13) as:

$$
\Delta y_t = \Pi_y y_{t-1} + \mu_x + \epsilon_{xt} \\
= \Delta x_t = \alpha \beta' x_{t-1} + \tau + \epsilon_t
$$

where $\Pi_y = \alpha \beta'$, $\mu_x = \tau$, and $\epsilon_{xt} = \epsilon_t$. We can rewrite the DGP as:

$$
\Delta x_t = \gamma + \alpha (\beta' x_{t-1} - \mu) + \epsilon_t.
$$

It is clear from equation (23) that breaks occurring in the deterministic terms can arise through $\mu$, where $E [\beta' x_t] = \mu$, or via breaks in the unconditional growth rate of the system, $\gamma$, where $E [\Delta x_t] = \gamma$. In order to detect where forecast errors arise, we need to distinguish between changes in $\mu$ and $\gamma$. Clements and Hendry (1999) outline the forecast errors and variances of equation (23) when a break occurs in $\mu$, $\gamma$ or $\alpha$, along with the analogous forecast errors and variances for a variety of forecasting devices. The importance of the deterministic terms must be emphasized. For example, if equation (22) was estimated, where $\tau = \gamma - \alpha \mu$, the 1-step forecasts for the VEqCM have a root mean square forecast error of approximately twice that of the forecast errors for (23). The success of robust forecasting devices highlight the presence of breaks in the deterministic terms.

---

6 A third possibility for deterministic breaks arise from shifts in $\alpha$, the adjustment coefficients. We abstract from this case in the following analysis.
4.1 Forecasting methods

Estimating a model over \( t = 1, \ldots, T \), with a forecast horizon of \( t = T + 1, \ldots, T + H \), the forecast in \( T + h \) is given in equation (24), where \( I_T \) is the information set at time \( T \), \( \theta_T \) is the set of estimated model parameters at time \( T \) and the forecast is a function \( h \) steps ahead, \( \psi_h \). The resulting forecast error in period \( T + h \) is given in equation (25).

\[
\tilde{x}_{T+h|T} = \psi_h \left( I_T, \hat{\theta}_T \right) \quad (24)
\]

\[
e_{T+h|t} = x_{T+h|T} - \tilde{x}_{T+h|T}. \quad (25)
\]

In order to compare the accuracy of forecasts we shall examine the bias and efficiency (captured by the mean absolute error) of the forecasts derived from each model. Combining these criteria leads to the mean square forecast error (MSFE), and we report the square root of the MSFE (RMSFE).

\[
ME = E \left[ e_{T+h|T} \right]. \quad (26)
\]

\[
MAE = E \left[ |e_{T+h|T}| \right]. \quad (27)
\]

\[
MSFE = E \left[ e_{T+h|T} e'_{T+h|T} \right] = V \left[ e_{T+h|T} \right] + E \left[ e_{T+h|T} \right] E \left[ e_{T+h|T} \right]. \quad (28)
\]

As forecast accuracy rankings can change as the forecast horizon changes (based on MSFE), multi-step forecasts are also examined for 4-steps ahead. Whilst multi-step forecasts will not be immune from structural breaks, they may capture long memory effects not contained in the 1-step forecasts. In order to forecast more than one step ahead in a multivariate framework, either an ‘iterated’ 1-step estimator or a direct \( h \)-step estimator can be used. The iterated 1-step forecast is most common, defined as:

\[
\tilde{y}_{T+h} = x'_{T} \tilde{\beta}^h, \quad (29)
\]

\[
E \left[ (y_{T+h} - \tilde{y}_{T+h}) | y_T \right] = \left( \beta^h - E \left[ \tilde{\beta}^h \right] \right) y_T. \quad (30)
\]

Equation (30) gives the average conditional error. It is assumed that the estimators and the latest observations are approximately independent.

The direct \( h \)-step estimator is non-recursive in that all information needed to derive an \( h \)-step forecast is available at time \( T \). The forecast is obtained by regressing \( y_T \) on the regressors lagged \( h \) periods. The estimator is given as:

\[
\tilde{\beta}_h = \arg \min_{\beta_h} \sum_{t=h}^{T} (y_t - x'_{t-h} \beta_h) (y_t - x'_{t-h} \beta_h)'. \quad (31)
\]

Hence, in comparison to equations (29) and (30) the forecasts and average conditional errors are given as:

\[
\tilde{y}_{T+h} = x'_{T} \tilde{\beta}_h, \quad (32)
\]

\[
E \left[ (y_{T+h} - \tilde{y}_{T+h}) | y_T \right] = \left( \beta^h - E \left[ \tilde{\beta}_h \right] \right) y_T. \quad (33)
\]

The relative forecast accuracy of the two multi-step forecasts depends upon the accuracy of the estimators, \( \tilde{\beta}^h \) and \( \tilde{\beta}_h \). Chevillon and Hendry (2004) find that the iterated 1-step forecasts are preferable when the model is well specified for both stationary and I(1) processes. However, in the case of a mis-specified model for a non-stationary DGP, or if negative residual serial correlation or deterministic shocks are unaccounted for, direct multi-step estimation may lead to more accurate forecasts. The key factor is the size of the drift.

---

7 Another common method of assessing forecast accuracy is Mean Absolute Percentage Error but this is not reported for quarterly inflation as it is measured as \( 100 \frac{H}{H} \sum_{t=1}^{H} \left| \frac{y_{t+h} - y_t}{y_t} \right| \), which can be positive infinity for \( y_t = 0 \). Two of the forecast horizon realizations of quarterly inflation are zero.
this gets bigger, the benefits of the direct multi-step forecasts outweigh the iterated 1-step procedure. Only the direct \( h \)-step forecast can be used in the single equation framework.

The forecasting models examined include the EqCM and VEqCM defined in equations (5) and (16) respectively, the difference of the EqCM and VEqCM in which the coefficients from equations (5) and (16) are imposed, \( \Delta \text{EqCM} \) and \( \Delta \text{VEqCM} \) equations that exclude the I(-1) double differenced terms, i.e. they just include the difference of the cointegrating vector, a differenced VAR based on a five year rolling average growth rate (denoted DV), the difference of the DV, i.e. a random walk (denoted DDV), an AR(3) model and a longer period difference given by \( \Delta x_{T+i|T+i-1} = \frac{1}{4} \sum_{j=1}^{4} \Delta x_{T+i-j} = \frac{1}{4} \Delta x_{T+i-1} \). The pooled forecast is also computed.

Using (23) as the in-sample DGP, the VAR in differences is given by:

\[
\Delta x_t = \gamma + \xi_t.
\]

This will be mis-specified unless the cointegrating rank is 0. In practice, a five year rolling average is used rather than \( \gamma \) because of the regime changes over the entire in-sample period.\(^8\) The double differenced VAR (DDV) is given by:

\[
\Delta^2 x_t = \zeta_t
\]

and this will track inflation by one quarter. As differencing lowers the degree of the polynomial in time we can eliminate shifts in trend and location shifts, reducing them to impulses and blips by differencing. A further advantage of differencing is that the forecast retains the structural information. We can rewrite (35) as:

\[
\Delta x_t = \Delta x_{t-1} + \zeta_t = \gamma + \alpha (\beta' x_{t-2} - \mu) + \epsilon_{t-1} + \zeta_t
\]

and so the structural model is included in the forecast (albeit with a one period lag) and the only extra cost is an additional error term, \( \zeta_t \).

Another adaptive device that may be used is differencing the VEqCM. The reasoning behind the method is that shifts in the mean are the most problematic for forecasting. If there occurs a shift in the equilibrium mean that is unaccounted for, forecasts will be adjusting to the old mean and will therefore be off target for the entire adjustment period. Differencing equation (23) leads to:

\[
\Delta x_t = \Delta x_{t-1} + \alpha \beta' \Delta x_{t-1} + \Delta \epsilon_t = (I + \alpha \beta') \Delta x_{t-1} + \nu_t,
\]

which is the first difference of the initial VAR with the rank restrictions from cointegration imposed. Alternatively, writing equation (37) as:

\[
\Delta^2 x_t = \alpha \beta' \Delta x_{t-1} + \nu_t
\]

shows that the double differenced VAR can be augmented by \( \alpha \beta' \Delta x_{t-1} \). As the device differences the mean, a shift in \( \mu \) will imply the forecast will fail in the period following the break but will then correct as \( \Delta \mu = 0 \) in subsequent periods. Hence, as in the case of the DDV, a differenced VEqCM or EqCM will robustify forecasts to deterministic shifts. On a note of caution, unnecessary differencing will lead to increased uncertainty which may increase the MSFE.

\(^8\)The question of an optimal in-sample period for developing a forecasting model is one that needs to be addressed. The trade-off between a large sample period and the increased probability of structural breaks implied that a five year rolling average delivered the best forecasts out of a range of sample periods tested.
4.2 Forecasting results

A comparison of the forecast results of the models considered for the period 1998q3-2003q2 is provided in table 7. This records the 1-step and 4-step forecast errors, absolute forecast errors and RMSFEs of the various forecasting models using equations (26) to (28). First, assessing the 1-step forecasts, figure 9a shows a reasonable performance of the VEqCM for quarterly inflation in which all of the realised outcomes lie within the ±2\(\hat{\sigma}_f\) error bars. The model does overpredict inflation over 2000 and 2001. The average prediction during 2000-2001 was 3.2%pa compared to an actual average inflation rate of 2.1%pa. However, the forecast error variance is not substantial (note that all graphs are on the same scale for comparison) and the RMSFE of 0.55% is fairly good. The \(\Delta\)VEqCM, imposing the coefficients from equation (16) are recorded in figure 9b along with \(\Delta\)VEqCM\(_{d}\) which is the \(\Delta\)VEqCM excluding all double differenced terms. Differencing the entire VEqCM will result in I(-1) terms, \(\Delta^2X_{t-i}\), which are likely to inflate the forecast variance as the terms will exacerbate measurement error, and this is reflected in the MAE of the \(\Delta\)VEqCM. Hence, we also consider excluding the difference of the short run dynamics, which will still result in robust forecasting models but will exclude the noise that inflates the error variance.

Comparing these results to two other robust forecasting devices, figures 9c and 9d record the forecasts from the DDV and DV respectively. The DV is based on a 5-year rolling average mean growth rate as opposed to the sample mean (which yields a RMSFE of 1.31%). Both adaptive devices perform well, with mean errors of 0.03%. The DDV tracks inflation by one quarter, demonstrating that not only should the DDV perform well when there are breaks to adjust to, but the device also performs well when there are no breaks as the method is tracking a stable series. The DV does particularly well as inflation has been very stable over the forecast horizon, suggesting that if breaks cannot be preempted and the series is stationary, a forecast based on the mean growth rate does perform well. In contrast, if inflation were I(1), as in the 1970s and 1980s, this forecasting device would not be as successful as a further differencing using the DDV device.

Examining the performance of the single equation model, figure 10a records the forecasts from the EqCM. The RMSFE of 0.54% is comparable to that of the VEqCM. Differencing the EqCM, as shown in figure 10b, does not yield any improvement as the variance is inflated, but removing the short run dynamics does improve the forecasts. Both adaptive devices given by the AR(3) model in figure 10c and \(\Delta_4X_{T+i-1}\) in figure 10d forecast particularly well. The adaptive devices result in a much smoother forecast which outperforms all models. The pooled forecast performs well if forecast errors offset each other.

Figures 11 and 12 record the corresponding forecasts for one year ahead quarterly inflation. Again, the smoothed forecast from annual inflation and the DV predict inflation well, showing the relative stability of inflation over the forecast period and the pooled forecast is particularly good. There is a more marked improvement in the differenced equilibrium correction models excluding the double differenced terms at the 4-step horizon. Almost all models forecast better out-of-sample than the VEqCM performs in-sample, with an equation standard error of 0.72% which can be directly compared to the RMSFE.

Table 8 examines the 20 quarter ahead inflation forecasts for the VEqCM against a powered up AR(1) model given by:

\[
\Delta p_t = \alpha + \beta \Delta p_{t-1} + \epsilon_t \\
\Delta p_{T+h|T} = \sum_{i=0}^{h-1} \alpha_i \beta^i + \beta^h \Delta p_T. \tag{39}
\]

The RMSFE of the VEqCM for 20 periods ahead of 0.48% is excellent, reflecting the relative stability of inflation over the forecast period and the lack of deterministic trend in the VEqCM model. Two AR(1) models are examined; the first is estimated over the full sample and the second is estimated from 1992 onwards. Estimating over the full sample period results in a RMSFE of 3.5 times that of the smaller sample, demonstrating the extent of structural change over the sample period. If the model is a good representation
Figure 9: 1-step forecasts of quarterly UK inflation from the VEqCM, ΔVEqCM and ΔVEqCMβ, DDV and DV (5yr rolling average).

Figure 10: 1-step forecasts of quarterly inflation from the EqCM, ΔEqCM and ΔEqCMβ, AR(3) and $\frac{1}{4}\Delta_4 p_t$. 
Figure 11: 4-step forecasts for quarterly inflation derived from the VEqCM, ΔVEqCM and ΔVEqCM, β, EqCM, ΔEqCM and ΔEqCM, β models.

Figure 12: 4-step forecasts for quarterly inflation derived from the DDV, DV (5yr rolling average), AR(3) and $\frac{1}{4}\Delta_4 p_t$. 
<table>
<thead>
<tr>
<th>Horizon</th>
<th>1-step ME</th>
<th>4-step ME</th>
<th>1-step MAE</th>
<th>4-step MAE</th>
<th>1-step RMSFE</th>
<th>4-step RMSFE</th>
</tr>
</thead>
<tbody>
<tr>
<td>VEqCM</td>
<td>0.06</td>
<td>-0.20</td>
<td>0.45</td>
<td>0.47</td>
<td>0.55</td>
<td>0.58</td>
</tr>
<tr>
<td>ΔVEqCM</td>
<td>-0.06</td>
<td>-0.18</td>
<td>0.61</td>
<td>0.63</td>
<td>0.69</td>
<td>0.73</td>
</tr>
<tr>
<td>ΔVEqCMₜ</td>
<td>-0.03</td>
<td>-0.08</td>
<td>0.46</td>
<td>0.42</td>
<td>0.54</td>
<td>0.51</td>
</tr>
<tr>
<td>DV₅yr</td>
<td>-0.03</td>
<td>-0.03</td>
<td>0.34</td>
<td>0.34</td>
<td>0.41</td>
<td>0.40</td>
</tr>
<tr>
<td>DDV</td>
<td>-0.03</td>
<td>0.01</td>
<td>0.47</td>
<td>0.40</td>
<td>0.55</td>
<td>0.49</td>
</tr>
<tr>
<td>EqCM</td>
<td>-0.08</td>
<td>-0.38</td>
<td>0.42</td>
<td>0.60</td>
<td>0.54</td>
<td>0.75</td>
</tr>
<tr>
<td>ΔEqCM</td>
<td>-0.03</td>
<td>-0.01</td>
<td>0.68</td>
<td>0.65</td>
<td>0.79</td>
<td>0.73</td>
</tr>
<tr>
<td>ΔEqCM₀</td>
<td>-0.03</td>
<td>-0.02</td>
<td>0.46</td>
<td>0.38</td>
<td>0.55</td>
<td>0.47</td>
</tr>
<tr>
<td>AR(3)</td>
<td>-0.24</td>
<td>-0.53</td>
<td>0.39</td>
<td>0.55</td>
<td>0.47</td>
<td>0.66</td>
</tr>
<tr>
<td>1/₅Δ₄P₀⁻⁻⁻⁻</td>
<td>0.01</td>
<td>0.02</td>
<td>0.34</td>
<td>0.35</td>
<td>0.40</td>
<td>0.43</td>
</tr>
<tr>
<td>Pooled</td>
<td>-0.09</td>
<td>-0.12</td>
<td>0.37</td>
<td>0.37</td>
<td>0.43</td>
<td>0.43</td>
</tr>
</tbody>
</table>

Table 7: Summary of quarterly inflation forecasting results for 1-step and 4-step ahead forecasts over 1998q3-2003q2. Figures reported as percentages, based on equations (26), (27) and (28).

<table>
<thead>
<tr>
<th>Dynamic Forecasts (20 periods)</th>
<th>ME</th>
<th>MAE</th>
<th>RMSFE</th>
</tr>
</thead>
<tbody>
<tr>
<td>VEqCM</td>
<td>-0.183</td>
<td>0.400</td>
<td>0.480</td>
</tr>
<tr>
<td>AR(1) powered (1966q1-1998q2)</td>
<td>-1.187</td>
<td>1.187</td>
<td>1.254</td>
</tr>
<tr>
<td>AR(1) powered (1992q1-1998q2)</td>
<td>-0.069</td>
<td>0.324</td>
<td>0.387</td>
</tr>
</tbody>
</table>

Table 8: Summary of quarterly inflation forecasting results for dynamic forecasts over 20 period forecast horizon. Figures reported as percentages, based on equations (26), (27) and (28).

of the economy and the structure of the economy remains relatively unchanged, then forecast accuracy should decline as the forecast horizon increases because innovation errors accumulate and predictability falls. However, if models are mis-specified and unanticipated shifts occur, particularly in the deterministic terms, classical forecast theory does not hold. In this case, forecast failure can easily arise and it may be possible that forecast accuracy increases with the forecast horizon. Hence, even though inflation is I(0) over the forecast horizon, breaks may well be causing forecast failure, as demonstrated by the 20-step forecasts outperforming most 1-step and 4-step forecasts considered in table 7.

### 4.3 Ranking of forecasting models

As well as assessing the models on the MSFE criterion, this section aims to rank the models in terms of the closest absolute forecast to actual inflation. Further, the models are examined to see if there is any autocorrelation in which models perform best (and worst) over the forecast horizon with a view to leading to a more informed decision regarding which models to use. The models are also tested for how well they predict sign changes in inflation, although the relative stability of inflation over the forecast horizon implies that this is less relevant than in periods in which inflation is closer to an I(1) process.

On RMSFE and MAE criteria, the smoothed DDV given by $\frac{1}{₅}\Delta₄P₀⁻⁻⁻⁻$ and the DV based on a five year rolling average are preferred over the 1-step horizon and the DV and pooled forecast perform best over the 4-step horizon. A question worth addressing is which of these models delivers the closest forecast at each forecast period. To do this, table 9 ranks the top two forecasting models based on the absolute forecast error at every horizon. The worst model is also reported. Although the table is rather cumbersome to read, it is clear that no model systematically outperforms the other models over the forecast period and likewise, no model systematically has the largest absolute errors. The magnitude of the errors is fairly similar across
models and there is a lot of fluctuation with regard to which models deliver the smallest absolute errors. Forecast accuracy appears to be rather volatile, with forecasting models being ranked both ‘best’ and ‘worst’ over the forecast period. Most models tend to perform well on some occasions and poorly on others and so we cannot draw conclusions regarding a systematic ranking.

This analysis is conditional on the sample examined. Inflation is relatively stable over the period and all the forecast errors are of a similar magnitude. During a period in which inflation is more volatile there may well be some systematic rankings. Identifying why particular models perform well would be the logical step in this case.

### 4.4 Forecasting correct sign changes

Another important question to address when examining forecasting models is whether the models correctly forecast signs and changes in signs. Over the forecast period, quarterly inflation was positive other than on one occasion when inflation was negative (2000q2) and on two occasions was zero (1999q4 and 2001q3). None of the models correctly forecast the negative inflation observation. However, negative quarterly inflation in 2000q2 was just -0.1% and so forecasting signs is not very informative for inflation. Most models always predict positive inflation, other than the differenced EqCM and VEqCM models, and the DDV predicts one negative inflation observation in the quarter following 2000q2. Given the Bank of England’s inflation target, predicting the sign of inflation is not an informative model criterion.

A more informative indication of the models relative forecast accuracy would be to examine how well the models forecast a change in the sign of inflation, i.e. the difference of inflation. Over the 20 forecast observations, quarterly inflation rose in 8 periods and fell in 12 periods. There is no systematic movement in inflation, as can be seen in figure 13b which records the first difference of quarterly inflation.
Figure 13: Quarterly inflation, the change in quarterly inflation and the ability of the forecasting models to predict the sign change in inflation for the 1-step and 4-step forecasts.

13d record the number of observations in which each forecasting model correctly predicted the sign change in inflation. Both the 1-step and 4-step forecasts suggest that for these models, the chance of predicting a rise or fall in inflation is not systematically correct. Again, we need to apply the caveat that the results are sample specific and could just be due to the nature of inflation over the forecast horizon.

Having looked at the forecasting models in terms of their rankings based on absolute errors and which models predict sign changes, we may be rather negative over the conclusions. There is no systematic ‘good’ model and none seem to forecast sign changes correctly on average. However, we need to bear in mind the small magnitude of errors. A key issue given the similarity of forecast errors between models is whether any of the models would have led to the Bank of England missing the inflation target band. Issues such as the time lag, forecasts conditional on the path of interest rates, the balance of uncertainties etc. also need to be addressed.

5 Models of Annual Inflation

Forecasting requires predictability, where a process \( y_t \) is defined as predictable with respect to an information set, \( I_{t-1} \), over \( T \) if \( D_{yt} (y_t|I_{t-1}) \neq D_{yt} (y_t) \) for \( \forall t \in T \). Hence, we require \( y_t = \phi (I_{t-1}, \nu_t) \) where \( \nu_t \) is unpredictable, see Clements and Hendry (2005). Predictability is relative to the information that is used. If we forecast from a reduced information set, i.e. use \( J_{t-1} \subset I_{t-1} \) to predict \( y_t = \tilde{\nu}_t (I_{t-1}) \), we will obtain less accurate but unbiased predictions. Unpredictability is not invariant to the data frequency used and so temporal disaggregation cannot lower the predictability of \( y_t \). This implies that as lower frequency data is a subset of higher frequency data, we should obtain more accurate predictions forecasting annual inflation.
using quarterly data as opposed to annual data, although both forecasts should be unbiased.

In order to test the theory we derive two models of annual inflation. We examine the simple case where
\( y_t = \beta_t (I_t - 1) + \nu_t \), for a model in which annual inflation is forecasted using a single equation dynamic model based on general-to-specific methodology. The first model we examine is the annual analogues of quarterly data, requiring information in fourth differences and lagged four periods only to emulate annual data. This is our lower frequency model. We then develop a model of annual inflation using quarterly data, representing the higher frequency model and examine the improvement in forecasting performance when the information set is increased.

5.1 Annual analogue model of annual inflation

The initial model of \( \Delta_4 p_t \) includes lags 4-7 of \( y^d, U^d, e_t, (c^* - p), s, (ppi - p), (rent - p), (imp - p), (oil - p), (n - p), (n\_r), \Delta_4 ppi, \Delta_4 rent, \Delta_4 imp, \Delta_4 oil, \Delta_4 p, \Delta_4 c^*, \Delta_4 m, \Delta_4 q, \Delta_4 r, \Delta_4 r_t, \Delta_4 pw \), two

PPP interaction terms given by \( \Delta e_t e_{t-1} \) and \( \Delta e_t e_{t-j}^2 \) for \( j = 4, 5 \) and an intercept and trend. Five indicator variables are included given by:

\[ D_t = [I_{72.4}, I_{73.2}, I_{74.1}, I_{84.1}, I_{84.2}] \]

and a year long dummy variable was included for 1979q3-1980q2. To derive a model of annual inflation in annual analogues using quarterly data autocorrelation must be corrected for. To still use the general-to-specific framework embodied in PcGets, the standard errors need to be adjusted as least squares will be inefficient. The first order autocorrelation is of magnitude 0.7, with highly significant second and third order autocorrelation. As a rough guide, we can adjust the t-values that are selected in PcGets by a factor

\[ \sqrt{\frac{1 + \rho}{1 - \rho}} \]

Hence, we shall initially retain variables with a t-statistic greater than 4.76.\(^9\) To refine the selection using a more rigorous adjustment for autocorrelation, the model is then estimated in PcGive and tested down using heteroskedasticity and autocorrelation consistent standard errors based on Andrews (1991).

The resulting model is reported in (42), with HACSEs reported in parentheses. The model fails autocorrelation, as expected, and the RESET test of model specification which may be an indication of some non-linearity. The model also fails the forecast Chow test, but out-of-sample fit is not a criterion on which to base in-sample model selection as this would bias the forecasting results. The model contains most theories of inflation found in Hendry (2001), although no role for money growth or the interest rate spread is found. The mark-up is given in equation (41).

\[ \pi_t^* = (p_t - 0.58 c_t^* - 0.34 p_{\Delta t} - 0.07 p_{\Delta t^2}) + 4.12 \]  

(41)

The corresponding graphics including the model fit, residuals and autocorrelation function are given in figure 14 and the recursive are recorded in figure 15.

\[
\begin{align*}
\Delta_4 p_t &= -0.333 \Delta_4 p_{t-5} + 0.801 y^d_{t-4} + 0.258 \Delta_4 c^*_{t-5} - 0.341 U^d_{t-6} \\
&+ 0.092 \Delta_4 rent_{t-6} + 0.035 \Delta_4 oil_{t-4} + 0.640 \Delta_4 pw_{t-4} \\
&- 0.325 \pi^*_{t-4} - 0.058 I_{74.1} - 0.033 I_{84.1} + 0.043 D_{79:3:80:2} + 0.026 \\
\text{R}^2 &= 0.951 \quad \sigma = 1.202\% \\
\text{SC} &= -8.476 \\
F_{arch}(5, 106) &= 6.599** \\
F_{het}(19, 91) &= 1.328 \\
\chi^2_{19.91} &= 4.40 \\
F_{reset}(1, 110) &= 19.169**
\end{align*}
\]

---

\(^9\)This can be done by adjusting the probabilities for the t-tests in the expert users strategy option in PcGets.
Figure 14: Model of annual inflation using quarterly data in annual analogues; model fit, residuals, their density and correlogram.

Figure 15: Model of annual inflation using quarterly data in annual analogues; recursive coefficients with ±2SE, 1-step residuals and constancy tests.
5.2 Quarterly model of annual inflation

A model of annual inflation using quarterly data is developed in order to compare the forecasts with (42). The model is selected using the same methodology in which first differences are included in the GUM, reflecting the higher frequency data. Again, we only use lags dated $t - 4$ and previous in order to forecast 1-year ahead. The model is reported in equation (43), with standard errors adjusted for autocorrelation and heteroskedasticity in parentheses.

\[
\Delta_4 p_t = 0.711 u_t^d + 0.609 \Delta c_{t-4}^d - 0.405 U_{t-7}^d + 0.021 \Delta o\ell_{t-7} \\
+ 0.258 \Delta ppi_{t-4} + 1.528 \Delta pw_{t-5} - 0.279 \pi_{t-5}^* - 0.022 I_{73:2} \\
- 0.041 I_{74:1} + 0.060 D_{79:3,80:2} + 0.079 D_{75:2,75:3} + 0.032 \\
(0.056) \\
(0.080) \\
(0.048) \\
(0.005) \\
(0.043) \\
(0.323) \\
(0.037) \\
(0.003) \\
(0.006) \\
(0.003) \\
(0.006) \\
(0.003) \\
(0.003)
\]

\[R^2 = 0.952 \quad \hat{\sigma} = 1.189\% \quad SC = -8.497\]

\[F_{ar}(5, 106) = 1.531 \quad F_{arch}(4, 103) = 0.191 \quad F_{het}(18, 92) = 1.098 \quad \chi^2_{ad}(2) = 5.862\]

\[F_{reset}(1, 110) = 4.766* \quad F_{Chow}(20, 110) = 1.513 \quad T = 1967q4 - 1998q2.\]

There is a slight improvement in fit moving to the higher frequency data, with an equation standard error of 1.189% as opposed to 1.202% for the annual model. The corresponding graphics including the model fit, residuals and autocorrelation function are given in figure 16 and the recursives are recorded in figure 17.

5.3 Quarterly inflation model used to forecast 1-year ahead inflation

An alternative rule that we investigate is based on deriving a model of annual inflation using quarterly data and fixing the estimated coefficients to forecast 4-steps ahead as opposed to 1-step ahead. If the DGP is given as:

\[
\Delta_4 x_t = \gamma + \alpha (\beta' x_{t-1} - \mu) + \nu_t, \tag{44}
\]
we use the forecasting rule given by:

$$\Delta_4 x_{T+4|T} = \hat{\gamma} + \hat{\alpha} \left( \beta x_{T-1} - \hat{\mu} \right).$$  \hfill (45)

Whilst this is a mis-specified model, we can interpret the forecasting rule as:

$$\Delta_4 x_{T+4|T} = \Delta_4 x_T - \tilde{\nu}_t,$$

and so the forecasting rule is the DDV excluding the estimated error term. The DGP can be augmented by lagged dependent variables. As well as protecting against breaks via the DDV component, all available information up to time $T$ is used to develop the dominant, congruent in-sample model. However, to forecast one year ahead we lose the quarterly information. If we develop an in-sample model based on the annual forecasts we are losing relevant in-sample information but by using these coefficients lagged by one year we are making the implicit assumption that the exogenous variables have the same impact lagged one quarter as they do lagged one year.

We derive a model of annual inflation by imposing a coefficient of unity on $\Delta_3 p_{t-1}$ and so we are effectively modelling quarterly inflation. The selection process retained the lagged dependent variables and a test of imposing the restriction of $[\Delta p_{t-4} = \Delta p_{t-5} = \Delta p_{t-6} = 1]$ is accepted; $F(1, 110) = 0.438$ [p-value=0.509]. Having derived the quarterly model of annual inflation, the coefficients are fixed and the model is re-

Figure 17: Model of annual inflation using quarterly data; recursive coefficients with ±2SE, 1-step residuals and constancy tests.
Table 10: Forecast performances of models for 4-quarter ahead annual inflation. Figures reported as percentages, based on equations (26), (27) and (28).

<table>
<thead>
<tr>
<th>Model</th>
<th>ME</th>
<th>MAE</th>
<th>RMSFE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ann EqCM</td>
<td>0.11</td>
<td>1.96</td>
<td>2.08</td>
</tr>
<tr>
<td>Qu EqCM</td>
<td>-0.53</td>
<td>1.26</td>
<td>1.58</td>
</tr>
<tr>
<td>Qu ΔEqCMβ</td>
<td>0.11</td>
<td>0.87</td>
<td>1.02</td>
</tr>
<tr>
<td>DDV1yr</td>
<td>0.12</td>
<td>0.79</td>
<td>0.93</td>
</tr>
<tr>
<td>AR(4) (lags 4-7)</td>
<td>-1.62</td>
<td>1.62</td>
<td>1.84</td>
</tr>
<tr>
<td>AR(1) (powered up)</td>
<td>-0.71</td>
<td>0.91</td>
<td>1.11</td>
</tr>
<tr>
<td>Qu EqCMin sample</td>
<td>0.02</td>
<td>0.53</td>
<td>0.64</td>
</tr>
<tr>
<td>Pooled Forecast</td>
<td>-0.34</td>
<td>0.60</td>
<td>0.77</td>
</tr>
</tbody>
</table>

Estimated on the variables lagged one year. The resulting model is given in equation (47).

\[ \Delta_4 p_t = 1.000 \Delta_3 p_{t-4} + 0.208 y_{t-4}^d + 0.194 \Delta c_{t-4}^* - 0.134 U_{t-4}^d \\
+ 0.109 \Delta imp_{t-5} + 0.414 \Delta pw_{t-7} - 0.021 \pi_{t-5}^* + 0.018 I_{T2:4} \\
- 0.046 I_{73:2} - 0.030 I_{74:1} + 0.045 I_{79:3} + 0.018 I_{84:2} + 0.007 \]

\[ \tilde{\sigma} = 3.240\% \quad SC = -6.859 \]

\[ F_{ar}(5,118) = 49.075^{**} \quad F_{het}(19,103) = 9.973^{**} \quad \chi^2_{ind}(2) = 7.774^{**} \]

\[ F_{Chow}(20,123) = 0.0386 \quad T = 1967q4 - 1998q2. \]

The model is a poor fit and is clearly mis-specified, failing many diagnostics. However, the model need not be a good in-sample model for it to perform well out-of-sample as we shall see in the next section.

### 6 Annual Inflation Forecasts

We now assess the forecasting performance of the annual inflation models derived above against various forecasting devices for the period 1998q3-2003q2. Table 10 reports the 1-year ahead forecast errors, absolute forecast errors and RMSFEs of the forecasting models based on equations (26) to (28). The models include the annual analogue derived in equation (42), the quarterly inflation model derived in equation (43) and the differenced quarterly model excluding double differenced terms. Other models include a DDV forecasting one year ahead given by \( \Delta_4 x_{T+4|T} = \Delta_4 x_T \), an AR(4) model that gives direct forecasts as the regressors are lagged four periods, and AR(1) model in which the forecasts are derived by powering up the lagged coefficient and the model derived in equation (47) in which the model is a quarterly EqCM model in-sample but the coefficients are fixed for the 4-quarter lagged regressors to deliver direct 4-step ahead forecasts.

Figure 18 records the forecasts from the models outlined above. The quarterly model forecasts outperform those from the annual model, suggesting that there are gains to be made from moving to higher frequency data. Time disaggregation is beneficial in this context. However, the concern of increased noise may be a problem with higher frequency data in some cases and caution must be applied when choosing the optimal data frequency. The AR(4) model has a substantial bias and again the question of the optimal in-sample period is relevant. Both DDV models perform particularly well with a RMSFE of 0.93% and when the estimated error term is excluded the RMSFE falls to 0.64%. This is likely to be capturing the relative stability of inflation over the forecast horizon, but the method would also robust to large structural breaks in inflation as well. Pooling is also shown to be beneficial, indicating that no model encompasses the others.
Figure 18: 1-year ahead forecasts of annual inflation from the annual EqCM and the quarterly EqCM, the DDV, the AR(4) and the in-sample quarterly model.

7 Conclusion

In conclusion, the analysis has highlighted the importance of deterministic terms when forecasting and particular attention should be paid to $\mu$ and $\gamma$. Shifts in the mean of the cointegrating vector or the unconditional growth rate of the system will lead to forecast failure. However, it is often difficult to identify where breaks are occurring in the data. Pooling is shown to be successful in many situations and this could be seen as an ‘insurance policy’ type forecast rather than selecting a particular forecasting method ex ante.

We find that there are clear benefits to disaggregation. In this paper we have looked at data frequency but Hendry and Hubrich (2004) also look at disaggregation across variables and space. Moving to higher frequency data should enable breaks to be picked up sooner and so the forecasting method could switch to the robust forecasting device faster.

We have used the automatic model selection procedure embodied in PcGets to select forecasting models, both for single equation models and to select a more parsimonious representation of the VEqCM model. Whilst the program is currently designed to select linear models, we have tested for some non-linearity by including purchasing power parity interaction terms in the GUM. Following this methodology we can select non-linear inflation models by including non-linearities and interaction terms in the set of potential regressors. Castle and Hendry (2005) look at selection of non-linear models by including various transformations such as squares and cross products in the GUM. Hendry and Krolzig (2003) demonstrate that PcGets can handle more variables than observations, thus ensuring selection of non-linear models is feasible in applications such as this, where we already have 75 regressors. A future area of research is to investigate the forecasting performance of non-linear inflation models.

The empirical example of UK quarterly and annual inflation demonstrates the success of adaptive forecasting devices outlined in the Clements and Hendry (1998, 1998) theory of forecasting. Whilst there is no obvious break, such as in the case of UK Money Demand due to the Banking Act of 1984, there is a clear
regime shift throughout the 1990s in which inflation is reduced to levels previously seen in the 1950s. This follows the fall out of the ERM and the subsequent switch to inflation targeting. The move to Central Bank independence in 1997 has brought about low and stable inflation, and whether permanent or transitory, robust forecasting devices are picking up this behaviour, whereas congruent in-sample models are still correcting to ‘the old’ equilibrium. Hence, the application of the forecasting theory to inflation does yield the results predicted by the theory.

8 References


### 9 Data Appendix

\[ Y_t \quad \text{Gross Domestic Product: chained volume measures: Seasonally adjusted. [NS, ABMI]} \]
\[ P_t \quad \text{Domestic Product (Expenditure) at market prices deflator: Seasonally adjusted. [NS, YBGB]} \]
\[ Gva_{dt} \quad \text{Gross Value Added at basic prices: chained volume measures: Seasonally adjusted. [NS, ABMM]} \]
\[ M_{4t} \quad \text{Nominal broad money stock (end period), £million. [NS, AUYN]} \]
\[ R_{ST,t} \quad \text{Three-month treasury bill rate. [DS, UKGBILL3]} \]
\[ R_{LT,t} \quad \text{Yield on 20-year gilts. [DS, UKGBOND]} \]
\[ N_t \quad \text{Public sector net debt, £million. [NS, BKQK]} \]
\[ W_{pop,t} \quad \text{Population aged 16-59/64, '000s. [NS, YBTFT from 1992. Pre-1992, EPG, DEG, EG]} \]
\[ Emp_{pt} \quad \text{Total number in employment, aged 16+, '000s. [NS, MGRZ from 1992. Pre-1992, EPG, DEG, EG]} \]
\[ NH_{t} \quad \text{Average actual weekly hours of work (all workers in main & 2nd job). [NS, YBUV from 1992. Pre-1992, EPG, DEG, EG]} \]
\[ PPI_t \quad \text{PPI manufacturing input - raw materials. [DS, UKOPP029F]} \]
\[ IMP_t \quad \text{Import price index. [DS, UKIMPPRCF]} \]
\[ RENT_t \quad \text{Actual rentals for housing + Imputed rentals for housing, £million. (Seasonally adjusted using X11). [NS, ADFT+ADFU]} \]
\[ C_t \quad \text{Unit labour cost index for the whole economy. [NS, LNNL]} \]
\[ K_t \quad \text{Net capital stock for the whole economy excluding dwellings sector, £million. [BoE]} \]
\[ I_t \quad \text{Total gross fixed capital formation, constant price, £million. [NS, NPQT]} \]
\[ E_t \quad \text{£ Effective Exchange Rate index. [DS, UKXTW.NF]} \]
\[ HO_t \quad \text{M0 wide monetary base (end period), £million. Seasonally adjusted. [NS, AVAE]} \]
\[ PW_t \quad \text{OECD Consumer Price Index. [DS, OCICP009F]} \]
\[ OIL_t \quad \text{World Market Price of crude petroleum. [DS, WD176AAZA]} \]