Competition and Price Variation when Consumers are Loss Averse*

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Abstract We introduce consumer loss aversion into the Salop (1979) model of price competition with differentiated products. Firms face uncertain costs of production, and after observing their own cost realizations simultaneously set prices. A consumer derives “gain-loss utility” from comparing the purchase price and her satisfaction with the acquired product to her recent expectations regarding the same variables, and dislikes losses more than she likes same-sized gains. Consumers’ sensitivity to losses in money increases the price sensitivity of demand—and hence the intensity of competition—at higher relative to lower market prices, reducing or eliminating price variation in a number of senses consistent with observed pricing regularities. For any joint cost distribution, an equilibrium in which all firms always charge the same, “focal,” price exists if and only if no two possible cost realizations differ by more than a given constant. If firms’ (possibly differently distributed) idiosyncratic cost shocks have overlapping supports and sufficiently dense distributions, any equilibrium is focal. When firms face common stochastic costs, in any symmetric equilibrium the markup is strictly decreasing in cost, and the price may be constant over parts or all of the range of possible costs. Because a change in the price responsiveness of demand affects competition more when margins are high, the above tendencies are stronger in less competitive industries. Finally, because the loss in product satisfaction she would suffer makes a consumer difficult to attract from a competitor, loss aversion decreases competition and increases prices.

Keywords: Reference-dependent utility, focal price, wage rigidity, kinked demand curve, countercyclical markups, (seemingly) collusive behavior.

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1 Introduction

This paper investigates how consumer loss aversion affects oligopolistic competition. Our formal results can explain a number of widely studied regularities related to reduced price variation between products and over time. First, there are circumstances under which non-collusive firms subject to industry-wide or asymmetric idiosyncratic shocks charge an identical and sticky, “focal,” price for differentiated products.\(^1\) This challenges the blanket interpretation of focal prices as a collusive device. Second, a retailer selling a number of differentiated products often sets the same, “uniform,” price for them.\(^2\) Third, the pass-through of cost changes is smaller than in the corresponding standard model, which, combined with evidence that marginal costs move procyclically, can be viewed as implying countercyclical markups.\(^3\) Fourth, shocks are incorporated into prices to a lower extent in more concentrated (or less competitive) industries.\(^4\) Finally, our theory can be reformulated as a model of an imperfectly competitive labor market, giving rise to results on reduced wage variation analogous to those on prices above.

Our results derive from profit-maximizing firms’ reaction to consumers who—beyond the instrumental value of money—dislike paying a price that exceeds their expectation of the purchase price. Because consumers’ sensitivity to such losses makes demand more responsive at higher than at lower parts of the purchase-price distribution, competition is more intense at higher prices, decreasing or eliminating price variation. And because a change in the responsiveness of demand affects competition more when the profit margin (and hence the value of an extra consumer) is high, this tendency is stronger in more concentrated industries.

Section 2 presents our model, which builds on the Salop (1979) model of imperfect competition.

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\(^1\) For evidence of sticky prices, see Kashyap (1995), Slade (1999), and Chevalier, Kashyap and Rossi (2003). Work on focal pricing dates back to at least Hall and Hitch (1939) and Sweezy (1939), and the existence of focal prices is part of industrial-organization folklore. Knittel and Stango (2003) document focal pricing for differentiated products in credit card services, and Heidhues and Kőszegi (2005) do so for cigarettes in Hungary. In work on pricing in mail-order catalogues, Kashyap (1995, page 263) states: “[...] Orvis price-setters told me that they were matching Bean’s price moves for Hudson Bay blankets.”

\(^2\) For evidence, see McMillan (2004) and Einav and Orbach (2005).

\(^3\) For a review of evidence that marginal costs move procyclically while markups are countercyclical, see Rotemberg and Woodford (1999).

In Salop’s model, a consumer’s “taste” for a product is drawn uniformly from the circumference of a circle, where in addition \( n \) products are located equidistant from each other. A consumer’s utility from or “satisfaction” with a product is decreasing in the product’s distance from her taste. In addition, she suffers additive disutility from paying the product’s price.

Using the framework developed by Kőszegi and Rabin (forthcoming), we append these classical preferences to account for loss aversion. A consumer compares outcomes in money and product satisfaction to a “reference point” in each dimension, with losses being more painful than equal-sized gains are pleasant. We posit that a consumer’s reference point is her lagged expectations (i.e. full probabilistic beliefs) about the outcomes she is going to get. For example, if she had been expecting to spend $14.99 on a Britney Spears CD—her favorite music—she experiences a sensation of loss if she pays $18.98 instead. But she also experiences a loss if she buys a Madness CD for $14.99.\(^5\) And if she expected to pay either $14.99 or $19.99, paying $18.98 generates a mixture of two feelings, a loss of $3.99 and a gain of $1.01, with the weight on the loss increasing in the probability with which she expected to pay $14.99.\(^6\) Expectations are determined endogenously, in a personal equilibrium, by the requirement that the stochastic outcome implied by optimal behavior conditional on expectations be identical to expectations.

Consumers are in the market to purchase a single product. We assume that the \( n \) products are sold by \( n \) different firms, but our results on how the products are priced would be identical if some firms had multiple non-neighboring products—so that predictions of focal pricing below also correspond to uniform pricing of goods sold by the same firm. The firms face uncertain costs of

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\(^5\) Actual prices are taken on September 4th, 2005, from www.amazon.com. $14.99 is the retail price of both CD’s (and numerous others), while $18.98 is a typical list price.

\(^6\) There is an abundance of well-known laboratory evidence on trading behavior and risk attitudes commonly interpreted as indicating loss aversion (Kahneman, Knetsch, and Thaler 1990, Kahneman, Knetsch, and Thaler 1991, Barberis, Huang, and Thaler 2005, Rabin 2000), although not usually tied to beliefs.

Empirical evidence in the marketing and economics literatures indicates loss aversion in consumer behavior that is broadly consistent with the consumer model of this paper. Consumers seem to compare actual market prices to “reference prices” determined at least partly by “price beliefs” or expectations, and purchases are more sensitive to losses from the reference price than to gains relative to it (Erickson and Johansson 1985, Kalwani and Yim 1992, Winer 1986). Hardie, Johnson, and Fader (1993) find loss aversion in evaluations of quality as well. Consumers’ extreme dislike of price increases, especially when “conditioned by years of low inflation to expect stable prices,” has even been noted by the popular press (New York Times, January 2, 2001, in an article on weight-outs). And Courty and Pagliero (2005) find that consumers are more averse to unexpected than expected price variation—in line with our model’s premise.
production, and, after privately observing their own cost realizations, simultaneously set prices. We define a market equilibrium as a situation where all firms maximize expected profits given other firms’ behavior and consumer expectations, and consumers play a personal equilibrium correctly anticipating the distribution of prices. A focal-price equilibrium is a market equilibrium in which all firms always charge the same focal price $p^*$.

Since characterizing the full set of market equilibria for arbitrary cost distributions seems very difficult (we are unaware of general characterizations even for the standard model), we consider several more specific questions. After analyzing consumer behavior in Section 3, we begin in Section 4 by establishing a necessary and sufficient condition for a focal-price equilibrium to exist. Suppose a consumer had expected to pay $p^*$ with probability one. Because she would then assess buying at a price greater than $p^*$ as a loss in money, and buying at a price lower than $p^*$ merely as a gain in money, demand is much more responsive to unilateral price increases from $p^*$ than to unilateral price decreases from $p^*$. Due to this kink in residual demand, for a range of cost levels $p^*$ is the optimal price to charge. Thus, for any distribution of costs such that any two cost realizations of any two firms are within a given distance, a focal-price equilibrium exists. In particular, this is true when one firm has higher costs than another in all states of the world.

We next establish two properties of focal-price equilibria. First, a focal-price equilibrium is more likely to exist in more concentrated industries, where the markup and hence the value of a marginal consumer is higher. In this situation, the asymmetric demand responsiveness at $p^*$ creates a greater difference in marginal profits from price increases versus price decreases from $p^*$, and hence yields a greater range of costs for which $p^*$ is the optimal price to charge.

Second, loss aversion increases prices: a focal price is higher than even the highest possible price charged in an equilibrium of the model without loss aversion. A consumer attracted from a competitor by a price cut buys a product further from her ideal than she expected. Since she is more sensitive to the loss in product satisfaction than to the gain in money, she is difficult to attract, decreasing competition between firms.

In Section 5, we derive conditions under which any market equilibrium is a focal-price equilibrium. These conditions permit firms to have different cost distributions, but for technical reasons
require costs to be drawn independently. We establish our result in two parts. First, we provide conditions under which a firm sets a deterministic price in any equilibrium. If the consumer expected a firm’s prices to be stochastic, the sense of loss from comparing the realized price to lower possible ones would make her demand more responsive at higher than at lower prices in the firm’s anticipated distribution. If the firm’s costs do not vary much, in contradiction to equilibrium it could then increase profits either by decreasing high prices (attracting a lot of extra demand) or by increasing low prices (not losing much demand). Second, we show that if there is no firm all of whose possible costs are strictly higher than all possible costs of another firm, then firms never set different deterministic prices in equilibrium. Consider a situation where a firm H charging the highest price $p_H$ has cost no greater than a firm L charging the lowest price $p_L < p_H$. Then, firm H has a lower demand and a higher markup than firm L, which makes it more profitable for firm H to attract new demand. Furthermore, because paying $p_H$ is assessed as a loss relative to paying $p_L$, firm H faces at least as responsive demand as firm L. This means that either firm H can increase profits by lowering its price, or firm L can increase profits by raising its price.

Finally, in Section 6, we assume that firms have common stochastic marginal costs, and characterize all market equilibria with symmetric pricing strategies. This leads to a tractable model for studying price variation when conditions for focal pricing are not necessarily met, and for analyzing firms’ response to industry-wide cost shocks. Our main result is that—unlike in the standard Salop model—markups are strictly decreasing in cost, and under conditions we identify precisely, the price is sticky (and focal) in any symmetric equilibrium. Using the empirical observation that costs are strongly procyclical, this means that markups are countercyclical. As above, if a consumer expected stochastic prices, her demand would be more responsive at higher than at lower prices within the price distribution. Hence, competition is fiercer at higher prices, decreasing markups. In addition, in some regions of cost it may be that competition at higher prices is tougher to an extent that is inconsistent with firms raising their price in response to cost increases at all. In such regions, the price must be constant in cost. Because a change in the responsiveness of demand affects competition more when markups are higher, an increase in industry concentration raises both the tendency for the price to be sticky and the countercyclicality of markups when prices are
not sticky.

Our results—derived in a noncooperative static game—provide a counterpoint to the widespread informal view that reduced price variation in concentrated industries is the result of collusive behavior.\(^7\) Athey, Bagwell, and Sanchirico (2004) formalize the collusion view in a repeated price-setting game, showing (among many results) that the optimal symmetric equilibrium is often a focal-price equilibrium enforced by the threat of price war in case of a price change. While we believe this explanation is part of the story, our results provide an intuitive and in some ways more complete explanation for reduced price variation. The logic of collusion only explains why each firm should set a fixed price, not why they should set the same price. Indeed, in collusion models this is commonly assumed in some form through imposing Bertrand competition or symmetry in firms' situations or strategies. Similarly, if each colluding oligopolist sells multiple differentiated products, there is no reason for the cartel to set the same price for all these products. Furthermore, in a repeated-game setting there are typically non-symmetric equilibria that are more efficient than focal pricing (Athey and Bagwell 2001, Aoyagi 2005). In contrast, in many circumstances our model predicts focal and uniform pricing as the unique possible outcome without imposing symmetry between the firms. To emphasize this point, we argue in Section 7 that our results are robust to a number of modifications of our model, including demand asymmetries and shocks, heterogeneity in consumer preferences, and endogenous determination of the number of firms.

2 Setup

Our model is based on the Salop (1979) model of differentiated-products competition, and is behaviorally equivalent to it when there is no loss aversion ($\lambda = 1$ below). To introduce consumer loss aversion, we build on the methods of Kőszegi and Rabin (forthcoming) and Heidhues and Kőszezi (2004). By basing the reference-dependent “gain-loss utility” on classical “intrinsic utility” and fully endogenizing the reference point, this approach allows little arbitrariness in modeling choices.

\(^7\) This view is expressed, for instance, in Carlton (1989, pages 914-915) and Knittel and Stango (2003, pages 1704-1705). In addition, focal prices and reduced price variability seemed to have raised suspicions of collusion in other cases, such as the recent Sony-BMG merger case in Europe.
2.1 Consumer Behavior

We begin by describing intrinsic utility as taken from the Salop (1979) model. There is a set of consumers of mass one with differing tastes indexed by \( \chi \in [0,1] \), where \( \chi \) is uniformly distributed on a circle with perimeter one. There are \( n \geq 2 \) products denoted \( y_1, \ldots, y_n \) on the same circle equidistant from each other, where we normalize \( y_1 = 0, y_2 = 1/n, \) and so on. A consumer can buy at most one product, and to avoid unenlightening extra notation, we assume her utility from not consuming is negative infinity, so that she always does buy a product.\(^8\) For any two locations \( \chi \) and \( y \), let \( d(\chi, y) \) be their distance on the circle. The intrinsic utility of consumer \( \chi \) from buying product \( y \) at price \( p \) is \( v - t \cdot d(\chi, y) - p \), where \( k_1 = v - t \cdot d(\chi, y) \) is her intrinsic utility from or “satisfaction” with the good and \( k_2 = -p \) is her intrinsic disutility from paying its price. Like previous authors, we interpret \( \chi \) as the consumer’s “ideal variety,” and \( t \cdot d(\chi, y) \) as her disutility from consuming a product \( y \) different from her ideal. The constant \( t \) is a measure of the (intrinsic) differentiation between varieties of the product.

Based on the above intrinsic utility, we formulate consumers’ reference-dependent utility. For a riskless consumption outcome \( k = (k_1, k_2) \) and riskless reference point \( r = (r_1, r_2) \) defined over product satisfaction and money, total utility \( u(k|r) \) is composed of two additive terms: intrinsic utility introduced above, and reference-dependent “gain-loss utility” equal to \( \mu(k_1-r_1)+\mu(k_2-r_2) \). To capture loss aversion, we assume that \( \mu \) is two-piece linear with a slope of 1 for gains and a slope of \( \lambda > 1 \) for losses.

This specification incorporates three key assumptions. First, the consumer’s sense of gain or loss is directly related to the intrinsic value of the changes in question—losing something she values (e.g. $100) is more painful than losing something she does not value (e.g. a paper clip). Second, the consumer evaluates gains and losses in the two dimensions, satisfaction and money, separately. For example, if she consumes a good that costs more but is closer to her taste than expected, this is assessed as a loss in money and a gain in satisfaction—and not, for example, a single gain

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\(^8\) Our results would be identical if consumers had an option of not buying, but \( v \) below was sufficiently high (or costs and product differentiation sufficiently low) that no consumer took advantage of this option in equilibrium. In Section 7, we argue that our qualitative results on reduced price variation would survive if consumers had an outside option in addition to buying from one of the \( n \) firms.
or loss depending on total intrinsic utility relative to the reference point. This is consistent with much experimental evidence commonly interpreted in terms of loss aversion.\(^9\) Third, while money is on a different psychological dimension from any of the \(n\) products, our model also says that the \(n\) products are on the same dimension. This assumption reflects our impression that goods that compete most strongly with each other are typically hedonically substitutable; indeed, that is partly why they compete. For example, a brand of orange juice competes much more directly with another brand of orange juice than with movies.

Since we assume below that the reference point is expectations, we extend the above utility function to allow for the reference point to be a probability measure \(\Gamma\) over \(\mathbb{R}^2\):

\[
U(k|\Gamma) = \int u(k|r)d\Gamma(r). \tag{1}
\]

In evaluating \(k\), the consumer compares it to each possibility in the reference lottery. For example, if she had been expecting to pay either $15 or $20 for her favorite CD, paying $17 feels like a loss of $2 relative the possibility of paying $15, and like a gain of $3 relative to the possibility of paying $20. In addition, the higher the probability with which she expected to pay $15, the more important is the loss in the overall experience.

Having specified the utility function, we turn to modeling behavior. Suppose that the consumer has a prior \(F \in \Delta(\mathbb{R}^n_+)\) on the non-negative price vectors she might face, and also has a prior on \(\chi\). Her decision of which good to buy is made after observing the realized \(\chi\) and the realized price vector, and is described by the strategy \(\sigma : [0, 1] \times \mathbb{R}^n_+ \rightarrow \{1, \ldots, n\}\). The premise of our model is that the consumer’s reference point, and therefore also the strategy \(\sigma\), depend on her lagged rational expectations about outcomes. Specifically, her reference point is the distribution \(\Gamma_{\sigma,F}\) induced by \(\sigma\), \(F\), and her prior over \(\chi\), over vectors \((k_1, k_2)\) of product satisfaction and expenditure. To deal with the resulting interdependence between behavior \((\sigma)\) and expectations \((\Gamma_{\sigma,F})\), we use the personal-equilibrium concept from Kőszegi and Rabin (forthcoming), which requires the behavior generating

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\(^9\) Specifically, it is key to explaining the endowment effect—that randomly assigned “owners” of an object value it more highly than “non-owners”—and other observed regularities in riskless trades. The common and intuitive interpretation of the endowment effect is that owners of an object assign a high money value to it because they construe giving it up as a painful loss that counts more than the money they receive in exchange. But if gains and losses were defined over the value of an entire transaction, owners would not be more sensitive to giving up the object than to receiving money, so no endowment effect would ensue.
Definition 1. \( \sigma \) is a personal equilibrium for the price distribution \( F \) if

\[
\sigma(\chi, p) \in \arg\max_{i \in \{1, \ldots, n\}} U \left( v - t \cdot d(\chi, y_i), p_i \mid \Gamma_{\sigma,F} \right) \text{ for all } \chi \in [0,1] \text{ and } p \in \mathbb{R}^n.
\]

2.2 Market Equilibrium

The timing of our full market model is illustrated in Figure 1. Consumers first form the expectations regarding consumption outcomes that later determine their reference point. Next, firms observe their cost realizations and simultaneously set prices, aiming to maximize expected profits.\(^\text{10}\) Then, consumers observe their ideal varieties and the realized market prices, and purchase a good.

We assume that consumers’ prior on \( \chi \) is identical to the population distribution, \( U[0,1] \). Since it gives rise to the same distribution of satisfaction from each product, an equivalent model is one in which consumers know their ideal variety, but are uncertain about the positioning of the firms’ products.\(^\text{11}\) A situation where consumers have a very good idea about their ideal variety as well as the products offered corresponds to a narrow or even degenerate prior distribution on \( \chi \), and yields a different model. In Section 7, we argue that our results in Sections 4 and 6 carry over to this case unchanged, and that reasonable specifications also yield our results in Section 5.

For expositional simplicity, we assume that the \( n \) available products are produced by \( n \) different firms, with firm \( i \) producing good \( i, y_i \). In Section 7, we argue that as long as each product is

\(^{10}\) We assume profit maximization to capture our impression that firms display reference-dependent preferences far less than consumers do, and to isolate the effect of consumer loss aversion on market outcomes.

\(^{11}\) More precisely, our model is identical to one in which each consumer knows her ideal variety, and firm 1’s location is drawn from a uniform distribution on the circle, with the \( n \) firms still equidistant from each other.
owned by exactly one firm and no firm owns neighboring products, our results on how products are priced extend unchanged to situations where some or all firms produce multiple products.

Firms’ costs are jointly distributed according to $\Theta$ on the set $\prod_{i=1}^{n} [c_i, \bar{c}_i]$, where $[c_i, \bar{c}_i]$ is the smallest closed interval containing the support of firm $i$’s cost distribution. Let $c = \min \{c_i\}$ and $\bar{c} = \max \{\bar{c}_i\}$. Denote firm $i$’s pricing function by $P_i : [c_i, \bar{c}_i] \to \mathbb{R}_+$. Let $P = (P_1, \ldots, P_n)$ be the vector of pricing strategies, $F_P$ the market price distribution induced by $P$ and $\Theta$, and $P_{-i}(c_{-i}) = (P_j(c_j))_{j \neq i}$ the price vector of firms other than $i$.

**Definition 2.** The strategy profile $\{P, \sigma\}$ is a market equilibrium if

1. $\sigma$ is a personal equilibrium for the price distribution $F_P$.
2. For each $i$ and $c_i \in [c_i, \bar{c}_i]$,

$$P_i(c_i) \in \arg\max_{p_i \in \mathbb{R}_+} (p_i - c_i) \cdot \text{Prob}[\sigma(p_i, P_{-i}(c_{-i}), \chi) = i \mid c_i].$$

Our definition of market equilibrium extends Bayesian Nash equilibrium to allow for reference effects in consumer behavior. A market equilibrium needs to satisfy two conditions. First, consumers play a personal equilibrium given the correctly forecasted price distribution. For notational simplicity, our definition implicitly imposes that there is a single representative consumer, or all consumers play the same personal equilibrium. We argue in Section 7 that this does not affect our results. Second, each firm at each cost realization plays a best response to other firms’ pricing strategies, taking consumers’ expectations (and hence reference point) as given.$^{13}$

One of our goals in this paper is to investigate circumstances under which all firms charge the same price irrespective of their cost positions. We introduce a term for such a situation.

**Definition 3.** A market equilibrium is a focal-price equilibrium if there is a price $p^*$ that all firms charge with probability one. In a focal-price equilibrium, $p^*$ is the focal price.

To address what we believe is mostly a technical issue that arises in our model as well as the standard Salop model with cost asymmetry, we introduce a restricted class of market equilibria.

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$^{12}$ Without loss of generality, we restrict attention to equilibria in strategies that are pure conditional on the realized cost. In the settings we consider in this paper, no firm would randomize in equilibrium.

$^{13}$ This means that at the stage when firms’ prices are chosen, these prices do not influence consumer expectations. Firms might be able to influence consumer expectations through public commitments to prices, advertising, or other marketing activities. Analyzing these motives is beyond the scope of this paper.
**Definition 4.** A market equilibrium is an **interior equilibrium** if for any equilibrium price vector, all firms sell to a positive measure of consumers on each side of their location.

In principle, it is possible that a firm prices so low relative to its neighbor that it attracts all consumers between them. In the standard Salop model, at such a price level there is a discontinuity in the firm’s demand, as it suddenly captures all consumers on the other side of its neighbor. In our model, both convex kinks and discontinuities in demand are possible. To avoid the difficulties of dealing with such situations in equilibrium, we follow previous models with cost uncertainty (e.g. Aghion and Schankerman 2004) and focus on (what we call) interior equilibria.

### 2.3 Consumption Examples

We provide a few examples of how our model fits various shopping scenarios.

**Unpredictability of Taste:** There is a given set of beer brands available at the local pub. But a bargoer is uncertain as to which brew she will be in the mood ($\chi$) for on a particular night. Once the evening comes, she learns her mood, the prices of the beers, and decides what to drink. The same pattern applies to many types of food and entertainment consumption, where a person’s momentary taste is often unpredictable.

**Uncertainty about Taste:** A consumer is looking to buy a new car. She knows what cars are available—she knows the products offered—but does not know which one leads to the most Fahrvergnügen. Once she test-drives the cars offered, she finds out each car’s price ($p_i$) and how well it suits her ($\chi$), and decides which one to buy.

**Uncertainty about Products:** A homeowner decides to buy a new fridge. She knows exactly what combination of features she would ideally like to have (she knows $\chi$). But she does not know what is being offered in the marketplace. Upon visiting the store, she finds out what is available (she learns the products’ locations and prices), and decides which fridge to buy.

In this example, the consumer knows her ideal variety, $\chi$, but does not know the products offered. In our formal model and the previous two examples, the converse is the case. But as we have emphasized above, the two variants of the theory are in fact equivalent.

Many shopping decisions, such as buying clothes, shoes, TV’s, package holidays, etc., involve
either uncertainty about taste, uncertainty about products, or uncertainty about both.

2.4 A Reinterpretation to Labor Markets

While our formal theory is about pricing decisions, it has a reformulation to an important application, wage setting. A wage paid by a firm to an individual is for both parties’ payoffs of opposite sign as a price paid by the individual to the firm, and labor productivity is for a firm’s profits of opposite sign as a cost. With the signs of the cost and price variables flipped to become productivity and wage variables, therefore, our model becomes one of an imperfectly competitive labor market. In this reformulation, employees differ in what kind of job they would ideally like. These differences can derive, for instance, from their commuting preferences or family situation, or from nonpecuniary aspects of work. Employees always take exactly one job, and suffer disutility if it is far from their ideal one. Firms, which have positions at different points of employees’ taste spectrum, are subject to labor-productivity shocks, and after observing their own productivities simultaneously offer wages. In Appendix A, we define this wage-setting game and its correspondence with a price-setting game formally, and observe that there is a one-to-one correspondence between market equilibria in the two games where the wage-setting function is obtained as a constant plus the negative of the price-setting function. Hence, although all of our results and discussion are in the language of price setting, appropriately restated they also apply to wage setting.

3 Consumer Behavior

In this section, we derive an expression for the price responsiveness of demand. Since a firm’s incentives are in a large part determined by price responsiveness, this will be crucial for our analysis of market equilibrium.

To derive our expression, we solve for how the location of the indifferent consumer between firms 1 and 2, which is equal to firm 1’s demand on the interval \([0, 1/n]\), changes with \(p_1\) for a given \(p_2\). (In an interior equilibrium, such an indifferent consumer exists for price vectors in a neighborhood of any realized price vector.) Suppose a consumer had expected the distribution of her purchase price and the distribution of her acquired product’s distance from ideal to be \(F(\cdot)\) and
G(·), respectively, and her realized taste is \( \chi \in [0, 1/n] \). While \( F(·) \) and \( G(·) \) will be determined endogenously as part of market equilibrium in future sections, in this section we take them as given.

The consumer’s utility from buying good 1 at price \( p_1 \) is then

\[
\begin{align*}
    u_1 &= v - \chi t - p_1 \\
    &\quad - \lambda \int_0^{p_1} (p_1 - p) \, dF(p) + \int_{p_1}^{\infty} (p - p_1) \, dF(p) \\
    &\quad - \lambda t \int_0^\chi (\chi - s) \, dG(s) + t \int_{\chi}^{1/n} (s - \chi) \, dG(s).
\end{align*}
\]

The first line is intrinsic utility. The second line represents gain-loss utility in money. As captured in the first term, to the extent that the consumer expected to pay lower prices than \( p_1 \), paying \( p_1 \) is a utility-decreasing loss. But as captured in the second term, to the extent that she expected to pay higher prices, paying \( p_1 \) is a gain. The third line is gain-loss utility in product satisfaction. As with money, in as much as the consumer expected to get a product closer to her taste, good 1 is a loss, but in as much as she expected to buy a product further from her taste, good 1 is a gain.

Replacing \( p_1 \) with \( p_2 \) and \( \chi \) with \( 1/n - \chi \) in Equation (2), we get utility from buying good 2 at price \( p_2 \):

\[
\begin{align*}
    u_2 &= v - ((1/n) - \chi)t - p_2 \\
    &\quad - \lambda \int_0^{p_2} (p_2 - p) \, dF(p) + \int_{p_2}^{\infty} (p - p_2) \, dF(p) \\
    &\quad - \lambda t \int_0^{1/n} ((1/n - \chi) - s) \, dG(s) + t \int_{1/n - \chi}^{1/n} (s - (1/n - \chi)) \, dG(s).
\end{align*}
\]

Denote the indifferent consumer—for whom \( u_1 = u_2 \)—by \( x^+ \in [0, 1/n] \), and right and left limits by subscripts \( \downarrow \) and \( \uparrow \), respectively. The number \( x^+ \) is also firm 1’s demand on the interval \([0, 1/n]\). Equations (2) and (3) are differentiable with respect to \( \chi \), and right and left differentiable with respect to \( p_1 \). Hence:

\[
\left( \frac{dx^+}{dp_1} \right)_\downarrow = - \frac{\left( \frac{\partial u_1}{\partial p_1} \right)_\downarrow - \left( \frac{\partial u_2}{\partial p_1} \right)_\downarrow}{\frac{\partial u_1}{\partial \chi} - \frac{\partial u_2}{\partial \chi}} = - \frac{1}{2t} \left[ \frac{2 + (\lambda - 1)F(p_1)}{2 + \frac{\lambda - 1}{2}(G(x^+) + G(1/n - x^+))} \right],
\]

12
and \((dx^*/dp_1)_1\) is given by the expression in which \(F_1(p_1)\) replaces \(F(p_1)\) above.

The price responsiveness of firm 1’s demand derives partly from the sensitivity of the consumer’s utility to changes in the purchase price. Paying a price \(p\) is experienced as a loss relative to lower prices in the expected purchase-price distribution, and as a gain relative to higher prices in that distribution. An increase (decrease) in \(p\) is therefore counted as a loss (avoided loss) to the extent that the consumer expected lower prices, and as a foregone gain (gain) to the extent that she expected higher prices. Due to this “comparison effect,” the responsiveness of demand to price increases (decreases) is a continuously increasing function of the probability with which the consumer expected to pay lower prices, \(F(p)\) \((F_1(p))\).

The comparison effect has two important implications we will use repeatedly in the paper. First, the residual demand curve is kinked at \(p\) if and only if the purchase-price distribution has an atom at \(p\) (and otherwise it is differentiable at \(p\)). Second, the price responsiveness of demand is greater at higher prices in the purchase-price distribution.

More subtle than the effect of utility from money itself is the effect of product satisfaction on the price responsiveness of demand. The consumers who switch to product 2 when firm 1 raises its price are approximately indifferent between a product at a distance \(x^+\) from ideal and a product at a distance \(1/n - x^+\) from ideal. The more either of these options yields lower product satisfaction than expected—and hence is evaluated as a loss—the more important is the consumer’s taste in determining which product she prefers, so the lower is her price responsiveness. Since \(G(x)\) is the probability with which the consumer expected to get a product closer to her taste than \(x\), this consideration gives rise to \(G(x^+)\) and \(G(1/n - x^+)\) in the denominator in Equation (4).

In an interior equilibrium, firm 1’s total demand is composed of its demand between firms 1 and 2 (analyzed above) and its demand between firms 1 and \(n\). If \(1 - x^-\) is the indifferent consumer between firms 1 and \(n\), the right derivative of firm 1’s total demand, \(D(p_1, p_{-1})\), with respect to \(p_1\) is

\[
D_{1\uparrow}(p_1, p_{-1}) = -\frac{1}{2t} \cdot \left[\frac{2 + (\lambda - 1)F(p_1)}{2 + \frac{\lambda - 1}{2} (G(x^+) + G(1/n - x^+))}\right] - \frac{1}{2t} \cdot \left[\frac{2 + (\lambda - 1)F(p_1)}{2 + \frac{\lambda - 1}{2} (G(x^-) + G(1/n - x^-))}\right],
\]

with \(D_{1\downarrow}(p_1, p_{-1})\) being given by the expression in which \(F_1(p_1)\) replaces \(F(p_1)\) above. Whenever
firms 2 and n set the same price \( p \), we will denote the indifferent consumer on each side by \( x = x^+ = x^- \), and firm 1’s demand as a function of its price by \( D(\cdot, p) \).

### 4 Existence and Properties of Focal-Price Equilibria

We begin by establishing a necessary and sufficient condition for a focal-price equilibrium to exist. The condition allows for stochastic costs, and even for commonly known differences in (stochastic or deterministic) costs.

To derive the condition under which a focal-price equilibrium exists, we solve for the cost levels \( c_1 \) for which firm 1 does not want to deviate from a focal price of \( p^* \). Market equilibrium requires that the consumer anticipated all prices to be \( p^* \), so that she expected to spend \( p^* \) with probability 1. In addition, since (having expected equal prices) she expected to buy the product closest to her taste, the distribution \( G(\cdot) \) of the acquired product’s distance from ideal is the uniform distribution on \([0, 1/(2n)]\). Hence, \( G(x) = G(1/n - x) = G(1/(2n)) = 1 \).

Given these considerations, Equation (5) implies that \( D_1(p^*, p^*) = -1/t \). Using that \( D(p^*, p^*) = 1/n \), so long as \((p^* - c_1)/t \geq 1/n \) firm 1 cannot benefit from locally raising its price. Similarly, since \( D_{11}(p^*, p^*) = -2/(t(1 + \lambda)) \), so long as \( 2(p^* - c_1)/(t(1 + \lambda)) \leq 1/n \) firm 1 cannot benefit from locally lowering its price. Combining and rearranging these conditions, charging \( p^* \) is locally optimal if and only if

\[
p^* - \frac{t}{n} \cdot \frac{1 + \lambda}{2} \leq c_1 \leq p^* - \frac{t}{n}.
\]

(6)

In the appendix, we show that when local deviations are unprofitable, non-local deviations are also unprofitable. Therefore:

**Proposition 1.** A focal-price equilibrium exists if and only if

\[
\overline{\tau} - \underline{\tau} \leq \frac{\lambda - 1}{2} \cdot \frac{t}{n}.
\]

When there is no loss aversion, a focal-price equilibrium exists only if \( \overline{\tau} = \underline{\tau} = 0 \)—if all firms have the same deterministic cost. With loss aversion, a focal-price equilibrium can exist despite
cost differences and variation. To illustrate the difference, Figure 2 depicts the residual demand curve firm 1 faces in our model and the standard Salop model when all competitors charge price $p^*$. While the demand curve in the standard model is linear, in ours it is kinked at $p^*$. Intuitively, a price decrease of a given amount expands demand less than a price increase of the same amount reduces demand because consumers are not as attracted by a gain in money as they dislike a loss in money. Since a price increase therefore affects profits from marginal consumers more than a price decrease, and the effect of these local price changes on profits from inframarginal consumers is symmetric, for a range of cost levels neither deviation can increase profits.\footnote{Figure 2 also shows that the demand curve is convex for price decreases from $p^*$. Hence, profits are not necessarily globally concave, so the fact that $p^*$ is locally optimal does not guarantee that it is globally optimal. The proof of the proposition implies that the demand curve is not so convex that non-local price cuts are profitable.}

Proposition 1 has a number of important comparative-statics implications for when a focal-price equilibrium exists. Naturally, a focal price is easier to sustain when the range of marginal costs $\overline{c} - \underline{c}$ is smaller. Also, a focal-price equilibrium is more likely to exist when consumer loss aversion ($\lambda$) is greater. The greater is $\lambda$, the greater is the difference between a consumer’s sensitivity to price increases from $p^*$ and price decreases from $p^*$. Hence, the greater is the difference between the effects of price increases versus decreases on profits from marginal consumers, and the greater is the range of cost levels for which $p^*$ is the optimal price.

Although predictable variations in $\lambda$ are often difficult to find, one implication of the above comparative static (and our model more generally) may be that ceteris paribus, prices are less variable in consumer markets than in transactions between (presumably less loss averse) firms. Evidence in Blinder, Canetti, Lebow, and Rudd (1998) is broadly consistent with this prediction.

Most interestingly, a focal-price equilibrium is more likely to exist when market power as measured by product differentiation relative to the number of firms ($t/n$) is greater. For an intuition, consider the price $p^*$ at which a firm with cost $\overline{c}$ is just indifferent to raising its price. Then, due to a kink in demand, for a range of cost decreases it strictly prefers not to decrease its price. This range—and hence the allowed cost variation for a focal-price equilibrium to exist—is increasing in the markup $p^* - \overline{c}$, so that it is larger in less competitive industries. With a higher markup, the value of a marginal consumer is higher, so the responsiveness of demand is more important in
determining the firm’s incentives to change its price. Hence, the lower responsiveness of demand to price decreases makes the firm very reluctant to cut its price, and it will not want to do so for a greater range of cost decreases.

In addition to identifying conditions under which a focal-price equilibrium exists, Inequality (6) determines what the focal price level can be. The following proposition states this level and compares it to prices in a model without loss aversion.

**Proposition 2.** There is a focal-price equilibrium with focal price $p^*$ if and only if

$$\overline{c} + \frac{t}{n} \leq p^* \leq \overline{c} + \frac{t}{n} \cdot \frac{1 + \lambda}{2}.$$  

In the corresponding Salop model without loss aversion, the support of a firm’s interior-Bayesian-Nash-equilibrium prices is bounded above by $\overline{c} + t/n$, and this bound can only be attained if the firm has realized cost $\overline{c}$.

Proposition 2 says that in a focal-price equilibrium, consumer loss aversion leads to increased prices: even at the lowest possible cost, a firm charges a higher price than it would in the standard model at the highest possible cost. For part of the intuition, consider how a firm fares if it lowers its price below those of others. The firm attracts some consumers whose tastes are closer to the neighboring firms’ products and are compensated for this by the lower price, and who therefore consume a good that both costs and matches their taste less than expected. Since consumers are more sensitive to the loss in satisfaction than to the gain in money, lowering one’s price attracts fewer of them than without loss aversion.

In contrast, consider how a firm fares if it raises its price, which leads some of its consumers to choose other products. These consumers must either pay a higher price or get a less satisfactory product than they expected was possible. Either choice involves a loss, so the firm loses the same number of consumers as without loss aversion.\footnote{In fact, this is why in Figure 2, on the left of $p^*$ the demand curve is tangent to the corresponding standard one.} Since loss aversion decreases a firm’s incentive to lower its price and leaves a firm’s incentive to raise its price unchanged, it increases equilibrium prices.
Proposition 2 implies that if there is a focal-price equilibrium, there are generically multiple ones, with the set of possible focal prices being a closed interval. If consumers’ expectation of the price increases from \( p \) to \( p' > p \), the difference between paying \( p' \) and paying \( p \) turns from a loss to a foregone gain. Because this makes demand less responsive, firms are more willing to increase prices, within limits exactly matching the increased expectations.\(^{16}\)

Beyond a theoretical possibility, our model predicts that focal-price equilibria can exist for calibrationally non-trivial amounts of cost variation. Assuming \( \lambda = 3 \), which corresponds to the conventional assumption of about two-to-one loss aversion in observable choices (Tversky and Kahneman 1992, for example), a focal-price equilibrium exists for cost variation \( \bar{c} - c \) up to \( t/n \). Since by Proposition 2 the equilibrium markup lies in the interval \([t/n, 2t/n]\), the allowed cost variation is at least half the size of the markup.

5 Conditions for All Equilibria to be Focal

In this section, we identify sufficient conditions under which firms suppress cost shocks and adhere to focal pricing in any interior market equilibrium even though we still do not impose symmetry in firms’ cost distributions or strategies. To our knowledge, no price-setting model predicts focal prices so robustly.

We establish our main result in two parts. First, we show that if the supports of firms’ cost distributions overlap and they all set deterministic prices, they set a focal price. That is, there cannot be an equilibrium with stable but different prices. Then, we identify conditions under which all firms set deterministic prices.

**Proposition 3.** Suppose \( \cap_{i \in N} [\underline{c}, \bar{c}_i] \neq \emptyset \). If all firms set a deterministic price and either

\[
\lambda \leq 1 + \frac{2}{n-1} \left( 1 + \sqrt{1 + 2n(n-1)} \right)
\]

or \( n = 2 \), the market equilibrium is a focal-price equilibrium.

\(^{16}\) Of course, firms prefer higher equilibrium prices to lower ones, and therefore have a strong incentive to manage consumers’ price expectations. Certain types of advertising and price-leadership behavior—which are outside our model—may partly serve this purpose.
For an intuition, suppose a highest-price firm H and a lowest-price firm L charge different prices $p_H$ and $p_L < p_H$, and consider the firms’ respective incentives with cost realizations $c_H$ and $c_L \geq c_H$. Then, firm H has lower inframarginal demand than firm L. Furthermore, since firm H has a higher markup than firm L, its benefit from an increase in demand is greater than firm L’s harm from a decrease in demand. And the comparison effect implies that there is a tendency for demand to be more responsive to a price decrease from $p_H$ than to a price increase from $p_L$, so that it is easier for firm H to gain demand than for firm L to lose demand. These considerations imply that in contradiction to equilibrium, either firm H can increase profits by lowering its price, or firm L can increase profits by raising its price.

This intuition, however, ignores an effect that (for $n > 2$) makes it necessary to impose Condition (7) in the proposition. A change in a firm’s price changes the distribution of marginal consumers in its two markets. As explained in Section 3, this typically changes the price responsiveness of its residual demand. If demand responsiveness changed too fast, the firm’s problem would be badly behaved in a number of ways. To rule out such possibilities, Proposition 3 above and Proposition 4 below impose restrictions on $\lambda$.

But Condition (7) is relatively weak. It only applies when $n > 2$, and it is satisfied for any number of firms whenever $\lambda \leq 1 + 2\sqrt{2} \approx 3.8$. Since the conventional assumption of two-to-one loss aversion is equivalent to $\lambda = 3$, the condition seems unproblematic.

As a second ingredient for the main result of this section, we give conditions such that all firms charge a deterministic price. Because analyzing a more general model is technically very difficult, we restrict attention to independent (idiosyncratic) cost shocks, still allowing for asymmetries in firms’ cost distributions.\(^\text{17}\)

**Proposition 4.** Suppose costs are independently distributed with $c_i \sim \Theta_i[\underline{c}, \overline{c}]$. If $38 > \lambda > 1$ and $(\overline{c} - \underline{c}) < (t/n) \cdot (3 + \lambda)/(1 + \lambda)$, there is a real number $\rho(\lambda, t, n, \overline{c} - \underline{c}) > 0$ such that if

$$\theta_i(c) \geq \rho(\lambda, t, n, \overline{c} - \underline{c})$$

\(^\text{17}\) If costs are not independent, a change in $c_i$ changes the distribution of competitors’ prices conditional on $c_i$ and hence also the distribution of marginal consumers for a given $p_i$. As explained in Section 3, this changes the price responsiveness of residual demand. While we believe this consideration would not substantially modify the comparison effect, the main force driving our result, we cannot formally analyze this more general case.
for all \( c \in [c_i, \bar{c}] \), then firm \( i \) sets a deterministic price in any interior equilibrium.

The intuition for Proposition 4 is easiest to see by first assuming that the consumer had expected firm \( i \)'s prices to be continuously distributed. Recall that due to the comparison effect, the consumer's price sensitivity at a price \( p \) is increasing in the probability with which she expected a price lower than \( p \). Hence, at least to the extent that she expected to buy from firm \( i \), her demand is more price responsive at higher than at lower prices within firm \( i \)'s price distribution. A firm with a sufficiently dense cost distribution wants to deviate from the consumer's price expectations either by decreasing its higher prices and attracting many extra consumers, or by increasing its lower prices and losing few consumers. Since this is true for any pricing strategy that varies with cost, the price in fact cannot vary with cost.

Combining Propositions 3 and 4:

**Corollary 1.** If the conditions of Propositions 3 and 4 hold, any interior market equilibrium is a focal-price equilibrium.

It is worth noting that the function \( \rho(\lambda, t, n, \bar{c} - c) \) that naturally drops out of our approximations underlying the proof of Proposition 4 is decreasing in \( t \) and increasing in \( n \), and approaches zero as \( t \to \infty \) and infinity as \( n \to \infty \). Although our conditions for all equilibria to be focal are merely sufficient (not necessary), these conditions are therefore more likely to be met in less competitive industries.

### 6 Industry-Wide Cost Shocks

In this section we fully characterize symmetric equilibria when firms always have identical marginal costs. This allows us to study, in a tractable model, price variation when conditions for a focal-price equilibrium are not necessarily met, and to illustrate that focal pricing is a special case of a more general phenomenon: reduced price variation due to loss aversion. In addition, we view this case as a natural model of oligopolists’ response to industry-wide cost shocks. We find that markups strictly decrease with cost in any market equilibrium, and that there may be regions (possibly consisting of the full support of the cost distribution) where the price is unchanging in
cost. Furthermore, markups decline faster with cost, and prices tend to be more sticky, in more concentrated industries.

Suppose firms’ common marginal cost is continuously distributed according to \( \Theta \) on \([c, \bar{c}]\). We first establish two basic properties of the symmetric market equilibria of the resulting game:

**Lemma 1.** Suppose firms have identical, continuously distributed marginal costs. In a symmetric market equilibrium, price is a continuous and non-decreasing function of marginal cost.

To understand the lemma, take costs \( c \) and \( c' \) and corresponding equilibrium prices \( p \) and \( p' > p \), and suppose that residual demand is differentiable at both \( p \) and \( p' \). Because firms use symmetric strategies, inframarginal demand is the same at the two prices (and equal to \( 1/n \)). In addition, due to the comparison effect, demand is (weakly) more responsive to price changes at \( p' \) than at \( p \), where these changes are assessed more as changes in losses rather than as changes in gains. In order for firms’ first-order conditions to be satisfied at both costs, therefore, \( c' \) must be greater than \( c \) and not be arbitrarily close to it.\(^{18}\)

We now fully characterize the set of symmetric equilibria, and then turn to a detailed discussion of the implications of this characterization. As a step toward a full analysis, we posit that for a cost \( c \), \( P(c) \) is not an atom of the market price distribution, and derive \( P(c) \). Since in a symmetric equilibrium firms set identical prices in all states of the world, consumers always choose the product closest to their taste. Hence, as in Section 4, \( G(\cdot) \) is the uniform distribution on the interval \([0, 1/(2n)]\). Furthermore, Equation (5) implies that the derivative of firm 1’s demand exists at \( P(c) \) and is equal to

\[
D_1(P(c), P(c)) = -\frac{1}{t} \cdot \frac{2 + (\lambda - 1)F(P(c))}{1 + \lambda} = -\frac{1}{t} \cdot \frac{2 + (\lambda - 1)\Theta(c)}{1 + \lambda},
\]

where \( F(P(c)) = \Theta(c) \) because \( P(\cdot) \) is non-decreasing and \( P(c) \) is not a pricing atom. For \( P(c) \) to be a profit-maximizing choice, we must have the usual first-order condition

\[
D(P(c), P(c)) + (P(c) - c)D_1(P(c), P(c)) = 0.
\]

\(^{18}\) If the price distribution has atoms at \( p \) or \( p' \), so that residual demand is not differentiable, the same argument still works by considering—instead of first-order conditions—incentives to lower one’s price from \( p' \) as compared to incentives to raise one’s price from \( p \).
Substituting Equation (8), using that \( D(P(c), P(c)) = 1/n \), and rearranging yields

\[
P(c) = c + \frac{t}{n} \cdot \frac{2 + (\lambda - 1)}{2 + (\lambda - 1)\Theta(c)} \equiv \Phi(c).
\] (9)

Expression (9) and Lemma 1 strongly restrict what a symmetric-market-equilibrium pricing function can look like. For any \( c \in [c, \bar{c}] \) that is not on a flat part of \( P(\cdot) \), \( P(c) \) is not a pricing atom, so \( P(c) = \Phi(c) \). In addition, arbitrarily close to an interior end of a flat part there are costs \( c \) for which \( P(c) \) is not a pricing atom, where again \( P(c) = \Phi(c) \). Hence, at interior ends a flat part of \( P(\cdot) \) connects continuously to \( \Phi(\cdot) \). Finally, because for \( c = \underline{c} \) Equation (8) is the left derivative of demand whether or not \( \underline{c} \) is a pricing atom, for price decreases from \( \underline{c} \) to be unprofitable we must have \( P(\underline{c}) \leq \Phi(\underline{c}) \); and by a similar argument, \( P(\bar{c}) \geq \Phi(\bar{c}) \).

Proposition 5 below establishes that the above conditions are in fact not only necessary, but also sufficient for \( P(\cdot) \) to be a symmetric-market-equilibrium pricing function. We argue here that no firm has an incentive to locally deviate from a pricing function satisfying the conditions; the appendix also considers non-local deviations. By construction, for any \( c \) that is not on a flat part of \( P(\cdot) \) there is no profitable local deviation. Also by construction, for a cost \( c \) at the low end of a flat part, there is no profitable local price decrease.\(^{19}\) Hence, there is also no profitable local price decrease for any higher cost on the same flat part. And by a symmetric argument, there are no profitable local price increases on any flat part of \( P(\cdot) \).

**Proposition 5.** Suppose firms have identical marginal costs distributed according to \( \Theta \) on \([c, \bar{c}]\). A pricing function \( P : [c, \bar{c}] \to \mathbb{R} \) is a symmetric-market-equilibrium pricing function if and only if all of the following are satisfied:

1. \( P(\cdot) \) is continuous and non-decreasing.

2. There are disjoint intervals \([f_1, f'_1], [f_2, f'_2], \ldots \subset [c, \bar{c}]\) such that \( P(\cdot) \) is constant on all \([f_i, f'_i]\) and not constant on any interval not contained in any \([f_i, f'_i]\).

3. \( P(c) = \Phi(c) \) for any \( c \notin \bigcup_i [f_i, f'_i] \).

4. \( P(\underline{c}) \leq \Phi(\underline{c}) \) and \( P(\bar{c}) \geq \Phi(\bar{c}) \).

\(^{19}\) If \( c = \underline{c} \) there is no profitable local price decrease because \( P(\underline{c}) \leq \Phi(\underline{c}) \). If \( c \) is interior, \( P(c) = \Phi(c) \), and there is no profitable price decrease because Equation (8) is the left derivative of demand.
To start identifying the implications of Proposition 5 in specific cases, suppose that $\Phi(\cdot)$ is strictly increasing. In that case, $P(\cdot)$ cannot have a flat part: because $P(c) \leq \Phi(c)$ and $P(\tau) \geq \Phi(\tau)$, a flat part cannot start at either of these points and connect continuously to $\Phi(\cdot)$; and an interior flat part cannot connect continuously to $\Phi(\cdot)$ at both ends. Hence, there are no pricing atoms, and the unique symmetric market equilibrium has $P(c) = \Phi(c)$ everywhere.

**Corollary 2.** Under the conditions of Proposition 5, if $\Phi(c)$ is strictly increasing, the unique symmetric market equilibrium has pricing strategies $P(c) = \Phi(c)$. Otherwise, a symmetric equilibrium with strictly increasing pricing strategies does not exist.

But $\Phi(\cdot)$ is not necessarily strictly increasing. Differentiating Equation (9) with respect to $c$,

$$\Phi'(c) = 1 - \frac{t}{n} \cdot \frac{(1 + \lambda)(\lambda - 1)\theta(c)}{[2 + (\lambda - 1)\Theta(c)]^2},$$

which is negative if $\theta(c)$ is very high. If $\Phi(\cdot)$ is non-increasing, then $P(\cdot)$ cannot have a strictly increasing part—where it would have to coincide with a non-increasing $\Phi(\cdot)$—so that it is constant. Hence, in these situations any symmetric market equilibrium is focal.

**Corollary 3.** Under the conditions of Proposition 5, if $\Phi(c)$ is non-increasing, any symmetric market equilibrium is a focal-price equilibrium. Otherwise, symmetric equilibria other than focal-price equilibria exist.

As with Proposition 4, the intuition for this result is easiest to see by first assuming that consumers expected firms’ prices to be strictly increasing in cost. If the density of the cost distribution is high, a small increase in $c$ implies a large increase in $F(P(c))$ and hence a large increase in the comparison effect and the corresponding price responsiveness of demand. Such a large increase in marginal revenue in response to a small increase in marginal cost is inconsistent with equilibrium: a firm can increase profits either by decreasing its higher prices and attracting substantial extra demand, or by increasing its lower prices without losing many consumers. Since this is true for any strictly increasing pricing strategy, the equilibrium price must be constant.

If $\Phi(\cdot)$ is neither strictly increasing nor non-increasing, Proposition 5 implies that market equilibria will generally consist of flat parts pasted together continuously with strictly increasing parts.
over which prices are given by $\Phi(\cdot)$. Figure 3 illustrates a non-monotonic $\Phi(\cdot)$ and possible market equilibria. For $c \in [c', c'']$ and $c \in [c'', \bar{c}]$ the pricing function cannot have a flat part, because that could not be pasted continuously with a strictly increasing part (where $P(\cdot)$ must coincide with $\Phi(\cdot)$). Hence, in these regions $P(\cdot)$ is strictly increasing and therefore equal to $\Phi(\cdot)$. Since there is a region where $\Phi(\cdot)$ is decreasing, however, the monotonicity requirement on $P(\cdot)$ implies that $P(\cdot)$ and $\Phi(\cdot)$ cannot coincide globally. Instead, the pricing function must be “ironed out” over the range where $\Phi(\cdot)$ is decreasing. Furthermore, because at the ends of a flat interval, $P(\cdot)$ connects continuously to increasing parts of $\Phi(\cdot)$, there is exactly one flat part. $P^1(\cdot)$ and $P^2(\cdot)$ are two possible market-equilibrium pricing functions.

In combination with Equation (9), Proposition 5 has a number of important implications for symmetric equilibria. Two implications are about the level and variation in markups in our model relative to the standard one (identical to $\lambda = 1$ here). In the standard Salop model, the markup is constant in cost and equal to $t/n$. In our model, the markup is strictly greater than $t/n$ for $c < \bar{c}$, and greater than or equal to $t/n$ for $c = \bar{c}$. Hence, as in focal-price equilibria (Proposition 2), loss aversion increases the price level. The consumers a firm attracts by lowering its price experience a pure loss in product satisfaction (from choosing a product unexpectedly far from ideal), and unless $c = \bar{c}$, only some combination of gain and avoided loss in money. Hence, they are more difficult to attract than in the standard setting, decreasing competition and increasing prices.

In addition, with loss aversion markups are strictly decreasing in $c$:

**Corollary 4.** Under the conditions of Proposition 5, in any symmetric market equilibrium, $P(c) - c$ is strictly decreasing in $c$ on the support of $\Theta$.

This prediction of our theory is potentially relevant for understanding macroeconomic fluctuations. Extensive evidence reviewed by Rotemberg and Woodford (1999) indicates that costs are strongly procyclical. Hence, our model implies markups are countercyclical.\textsuperscript{20} Intuitively, recall that due to the comparison effect, consumers are more responsive to price changes at higher than at lower prices within the price distribution. Since inframarginal demand is constant across the

\textsuperscript{20} Of course, if one measures countercyclicality using the Lerner index $\frac{p}{c}$, the Salop model without loss aversion also features countercyclical markups. But that model does not feature countercyclical markups if markups are defined as $p - c$. 

23
price distribution, this means that firms compete more fiercely at higher prices, reducing markups.

A third implication of Proposition 5 is a systematic relationship between the competitiveness of the market and price variation. The more concentrated is the industry and the greater is product differentiation (the lower is \( n \) and the greater is \( t \), the lower is the derivative of \( \Phi(\cdot) \) at any \( c \). As a result, the more countercyclical are markups—the faster \( P(c) - c \) decreases with \( c \)—when price is strictly increasing in cost, and the more likely it is that any symmetric equilibrium is a focal-price one. Intuitively, when average markups are higher—as would be the case in a less competitive industry—the increased ability to attract consumers at higher prices has a greater impact on firms’ incentive to cut prices, generating markups that decrease faster in cost. If \( t/n \) is sufficiently large, the impact of an increase in demand responsiveness on firms’ incentive to cut prices is so great that firms are unwilling to raise their price at all—they charge a sticky (and focal) price.

In fact, Proposition 5 allows us to more fully describe pricing patterns for industries ranging from very competitive (\( t/n \approx 0 \)) to very uncompetitive (\( t/n \) very large). If competition is sufficiently strong, the unique symmetric market equilibrium features a strictly increasing pricing function, which is close to marginal-cost pricing if competition is very strong. At lower levels of competition, markups are higher and more countercyclical. At even lower levels of competition, the price is constant in cost near regions where the cost distribution is relatively dense, but may remain strictly increasing in cost in other regions. At very low levels of competition, the price is sticky and focal.

It is important to note that in this section identical pricing across firms was assumed, not derived. The question arises whether such identical pricing would be the outcome in an environment where idiosyncratic cost shocks also exist, but are absorbed (as in Section 5). For cases in which the cost variation is sufficiently small, Section 4 has shown that a focal-price equilibrium still exists even when there is both industry-wide and idiosyncratic cost uncertainty. More strongly, although (for reasons mentioned above) we cannot fully analyze a general model with both types of cost uncertainty, intuition developed in the last two sections suggests that in regions where both components vary little, the price will be sticky in any equilibrium.\(^{21}\) But in regions where

\(^{21}\) Again, the intuition can be seen by assuming that cost shocks are not absorbed. If costs vary little, the price distribution will be dense, so that the comparison effect implies that a small increase in the price leads to a large increase in the price responsiveness of demand. Then, a firm can increase profits either by increasing lower prices (where demand is relatively inelastic) or by decreasing higher prices (where demand is relatively elastic).
the common cost shocks are not absorbed, residual demand will be smooth, so idiosyncratic cost shocks, independent of their size, also will not be absorbed.

7 Robustness

In this section, we argue that our results are largely robust to natural variations of our model. In short, most of our results rely on the simple intuition that—due to loss aversion in money—a consumer’s sensitivity to price is increasing in the probability with which she expected to pay lower prices, and this force is not eliminated by reasonable modifications of the model.

Our definition of market equilibrium assumes that all consumers play the same personal equilibrium. Relaxing this assumption does not affect our results. In all situations in Sections 4 and 6, selection is a non-issue simply because the personal equilibrium is unique. Our proofs in Section 5 work by estimating how the responsiveness of a firm’s residual demand changes across the price distribution. Since our bounds hold for any personal equilibrium a person might be playing, they also hold if consumers play a variety of equilibria.

Because in many situations consumers are unsure either about what they want or about what is available, we have assumed a dispersed prior on $\chi$. But most of our results do not depend on this assumption. Even if $\chi$ is known perfectly, our results in Sections 4 and 6 remain unchanged.\(^{22}\) Consider when there is a focal-price equilibrium with focal price $p^*$; the logic for symmetric equilibria with industry-wide cost shocks is similar. If a firm raises its price above $p^*$, the consumers it repels choose between a loss in money and a loss in product satisfaction. If the firm lowers its price below $p^*$, the consumers it attracts experience a gain in money and a loss in satisfaction. Locally, both of these effects are the same as in our analysis above. And because the residual demand curve is two-piece linear (see Figure 2), as above a non-local deviation is unprofitable whenever a local one is.

Under reasonable refinements on personal equilibria, such as that people play the ex-ante optimal personal equilibrium, Proposition 3 also holds—although for a completely different reason.

\(^{22}\) Technically, if different consumers have different information about their preferred location, there cannot be a representative consumer. The definition of market equilibrium has to be modified accordingly to account for such heterogeneity.
than above.\textsuperscript{23} To illustrate, suppose \( n = 2 \) and firms 1 and 2 charge prices \( p_1 \) and \( p_2 < p_1 \). If a consumer plays a personal equilibrium in which she expects to buy product 2, to avoid a loss in money she strictly prefers not to take product 1 ex post. If she expects to take product 1, to avoid a loss in satisfaction she strictly prefers not to take product 2 ex post. With all consumers “locked in,” both firms want to raise their price, contradicting equilibrium.

Our methods in Proposition 4 do not extend to the case when \( \chi \) is known with certainty. The logic of our proof, however, only seems to rely on sufficiently many \textit{marginal} consumers being sufficiently uncertain about their taste that they are unsure as to which product they will buy. These consumers exhibit a similar pattern of behavior to our consumers above, so they give a firm similar incentives.

As do most applications of the Salop model, our model assumes that in competing with each other, firms’ prices affect only the allocation of demand, not its level. One way to model a market-size effect is to assume that consumers have an outside option with a randomly determined level of utility. In this case, the comparison effect makes consumers on the margin between two firms, as well as on the margin between a firm and the outside option, more responsive to price changes at higher than at lower prices in the purchase-price distribution. Hence, our qualitative results on price stability (but not necessarily our result that loss aversion increases prices) are likely to survive.

We assume above that each firm sells exactly one product. As long as no firm owns neighboring products, our results carry over unchanged to multiproduct firms.\textsuperscript{24} In interior equilibria, the incentive for locally changing one product’s price is unaffected by how many non-neighboring products a firm owns. But global deviations are weakly less profitable for a multiproduct firm because such a firm might be cannibalizing its own market.\textsuperscript{25}

\textsuperscript{23} In fact, the assumption that consumers play the ex-ante optimal personal equilibrium is much stronger than we need. It suffices to assume, for example, that consumers have an arbitrarily small amount of self-discipline in the sense that they can select ex-ante whether to impose an arbitrarily small ex-post cost on themselves if they choose a certain action deemed undesirable ex ante.

\textsuperscript{24} In this case, however, intensity of competition, which drives many of our results, is determined by the number of products rather than the number of firms (in addition to \( t \)).

\textsuperscript{25} Even if firms can own two neighboring products, all the forces behind our results are still present, so that focal pricing will often be an equilibrium, and often the only type of equilibrium. Our conditions and proofs, however, would have to account for the decreased competition between firms and for the fact that products may differ in how
The analogue of our focal-price equilibrium above for multiproduct firms is a market equilibrium in which all products have the same price. In such an equilibrium, each multiproduct firm charges the same price for its different products, so that our model predicts uniform pricing.

In our model, we assume that firms’ situations are symmetric and stable with respect to demand, while costs are possibly different and uncertain. In most industries, firms also differ in the elasticity of residual demand they face (even if all firms set the same price). One simple way to formalize such differences is to assume that marginal costs are zero, but there is a possibly different and randomly determined mass of “loyal” consumers at each of the locations 0, 1/n, . . . , (n−1)/n. Loyal consumers either buy their firm’s product, or do not consume, with their maximum willingness to pay chosen so that it is unprofitable to sell only to them. Because a change in the number of loyal consumers has similar implications for the firm’s behavior as a change in marginal cost, we confidently conjecture that all of our results and methods of proof carry over to this alternative model.26

While we have assumed that industry structure is exogenous, our model can be extended to allow for endogenous entry. Suppose industry concentration is determined by a fixed cost firms must pay to enter the market. Since the fixed cost determines the number of firms but has no impact on market equilibria given the number of firms, our qualitative results on the effect of industry concentration on market equilibria survive.

Our results on focal pricing and reduced price variation more generally (but, again, not our result on the price level) hold in a model in which consumers are loss averse only in money.27 This assumption would, in fact, substantially simplify some of our formal statements and proofs (especially those of Propositions 3 and 4). Methodologically, however, we believe that the spirit of our research—deriving insights by enriching the consumer side of industrial-organization models—requires us to use a psychologically, experimentally, and theoretically well-motivated consumer

26 The only caveat is that an increase in the number of loyal consumers leads to an increase in price, so that this model generates procyclical markups. Still, our model predicts that markups vary less than they would in the corresponding standard model.

27 In such a model, however, the money dimension is relatively more important than the good dimension: even if consumers are not loss averse (λ = 1), their demand responsiveness is twice as high as in the standard Salop (1979) model. This artificial increase in price sensitivity would lead to fiercer competition and hence reduced prices in comparison to our model and the standard model.
model rather than constructing a different (ad-hoc) model for our application. Hence, we make the far more plausible and experimentally well-motivated assumption that consumers are loss averse also in the product dimension.

The results in this paper are also robust to heterogeneity in loss aversion among consumers. Our estimation methods would have to account for such heterogeneity, but as long as there is some loss aversion in the population, the results would survive in some form.

8 Conclusion

A primitive of our model is the time at which the expectations determining the reference point are formed. For example, in the case of industry-wide cost shocks we have assumed that the reference point does not adjust to such shocks at all. In some cases—such as regulatory intervention widely publicized in advance—consumers’ expectations regarding market prices may adjust early enough to change their reference point. In this case, it is appropriate to model the intervention as a change in the entire cost distribution rather than as a shift within the same cost distribution. While Kőszegi and Rabin (2005) discuss some psychological principles related to such timing issues, in this paper we cannot formally elucidate how to determine the appropriate cost distribution in each situation.

In contrast to our results, which predict reduced price variation in a number of senses, there often seems to be excess price variation between even identical products. Such price variation seems to occur primarily in industries where consumers cannot or do not compare prices across different sellers, partly because firms deliberately make comparisons difficult. Ellison (2005) and Gabaix and Laibson (2005) identify competitive advantages of making price comparisons difficult, and the model of Spiegler (2004) suggests that this could lead to price variation. Our theory clearly applies better when prices and relevant features of products are transparent to consumers.

28 Baye, Morgan, and Scholten (2004), for example, document that for many products, different internet-based retailers charge very different prices.

29 Gabaix and Laibson (2005) show that in the presence of some consumers who ignore add-on costs, firms have no incentive to make add-on prices transparent, even when it is very cheap to do so. Ellison (2005) gives natural conditions under which rational consumers who are unresponsive to an ex-ante hidden add-on price also decrease competition in transparent aspects of the product.
Figure 2: A Firm’s Residual Demand When Neighbors Charge $p^*$

For better visibility, overlapping curves are drawn as close parallel curves. The standard residual demand curve is linear. The demand curve in our model is kinked at $p^*$, and is tangent to the standard demand curve at $p^*$ for price increases. In addition, it is concave for price increases from $p^*$, and convex for price decreases from $p^*$. The demand curve in our model with $\chi$ known is two-piece linear with a kink at $p^*$, and is tangent to the demand curve with $\chi$ unknown in both directions from $p^*$. 
Figure 3: Determination of Equilibrium Pricing Functions with Industry-Wide Shocks
For better visibility, overlapping curves are drawn as close parallel curves. The solid curve $\Phi(c)$ is the solution to the first-order condition for optimal pricing assuming that $P(c)$ is not an atom of the price distribution. Since a market-equilibrium pricing function is non-decreasing and continuous, it consists of constant parts pasted together with strictly increasing parts that coincide with $\Phi(\cdot)$. Two market-equilibrium pricing functions are $P^1(c)$ and $P^2(c)$. 
Appendix A: Labor-Market Reinterpretation

Suppose employees’ “ideal jobs” are located on the unit circle, where firms’ jobs are also located. If an employee with job-taste \( \chi \) takes job \( y \), her intrinsic utility is \( v + w - t \cdot d(\chi, y) \), where \( v \) is her intrinsic (dis)utility from working, \( w \) is her wage, and \( t \cdot d(\chi, y) \) is her disutility from taking a job different from her ideal. Company \( i \)'s profit from attracting an employee is \( \psi_i - w_i \), where \( \psi_i \) is the company’s labor productivity and \( w_i \) is its wage. For any distribution of costs \( \Theta \) in the price-setting game and any constant \( \Psi \), we can define a corresponding wage-setting game in the following way. We let \( \psi_i \equiv \Psi - c_i \) for any realization \( c_i \), and require that \( w_i \leq \Psi \) for all \( i \). In the wage-setting game, employees first form expectations regarding their wage and the type of job they will get. Next, each firm’s productivity is realized and firms simultaneously set wages. Finally, employees decide which job to take (we assume employees find jobs sufficiently valuable that they always work in equilibrium). A wage-setting equilibrium is defined as a situation where employees play a personal equilibrium correctly anticipating the distribution of wages in the market, and firms maximize expected profits given other firms’ behavior and employee expectations. We have the following:

**Proposition 6.** The pricing functions \( \{P_1, P_2, \ldots, P_n\} \) are part of a pricing equilibrium if and only the wage-setting functions \( W_i : \psi_i \mapsto w_i \) defined by \( W_i(\psi_i) = \Psi - P_i(\Psi - \psi_i) \) are part of a wage-setting equilibrium in the wage-setting game.

**Proof.** Obvious.

Appendix B: Proofs

**Proof of Proposition 1.** If the condition in the proposition is satisfied, then there is a \( p^* \) satisfying \( p^* - \frac{c + \lambda \alpha}{n} \leq c \leq p^* - \frac{t}{n} \) for all \( c \in [c, \bar{c}] \). That this is a necessary and sufficient condition for local deviations to be unprofitable has been established in the text.

We now show that under the above condition, non-local deviations are also unprofitable. We start with increases in the price. First, note that the firm will never charge a price so high that it would be charging itself out of one market: if a deviating firm is charging itself out of one market, it is charging itself out of one market, it is charging itself out of both, earning zero profits. Therefore, we only need to consider deviations for which \( x \in (0, \frac{1}{2n}) \). Recall Equation 4:

\[
\frac{dx}{dp_1} = \frac{1}{27} \left[ \frac{2 + (\lambda - 1)F(p_1)}{2 + \frac{2}{n}G(x) + G((1/n) - x)} \right]
\]
Since \( F(p_1) = G(\frac{1}{n} - x) = 1 \) and \( G(x) \) is increasing in \( x \), in the range \( x \in (0, \frac{1}{2n}) \), firm 1’s demand (as a function of \( p_1 \)) is convex. This implies that if local deviations are unprofitable, non-local increases in the price are also unprofitable.

Next, we rule out the possibility that firm 1 might like to charge a price so that \( x \in [\frac{1}{2n}, \frac{1}{n}] \). In that case, Equations (2) and (3) imply that

\[
-xt - p_1 + (p^* - p_1) - \lambda t \left( x - \frac{1}{4n} \right) = - \left( \frac{1}{n} - x \right) t - p^* - \lambda t \cdot 2n \cdot \left( \frac{1}{n} - x \right) \frac{x - \frac{1}{2n}}{2} + t \cdot 2n \cdot \left( x - \frac{1}{2n} \right) \frac{x - \frac{1}{2n}}{2}.
\]

Solving for \( p_1 \) gives

\[
p_1 = p^* - \frac{1}{2n} t \left( (\lambda + 1) \left( 2x - \frac{1}{n} \right) + (\lambda - 1) \left( x - \frac{1}{4n} - nx^2 \right) \right).
\]

To show that lowering the price to \( p_1 \) is not a profitable deviation, it is equivalent to show that

\[
\frac{1}{n} (p^* - c) \geq 2x(p_1 - c) = 2x \left( p^* - c - \frac{1}{2}t\kappa \right).
\]

Rearranging and using that \( p^* - c \leq \frac{t(\lambda + 1)}{2n} \) gives that it is sufficient to show that

\[
\left( 2x - \frac{1}{n} \right) \frac{1 + \lambda}{n} \leq 2x\kappa,
\]

or equivalently

\[
(\lambda + 1) \left( 2x - \frac{1}{n} \right)^2 \geq (\lambda - 1)2x \left( nx^2 + \frac{1}{4n} - x \right) = (\lambda - 1)2x \left( 2x - \frac{1}{n} \right) \left( \frac{nx^2}{2} - \frac{1}{4} \right).
\]

This simplifies to

\[
(\lambda + 1) \left( 2x - \frac{1}{n} \right) \geq (\lambda - 1)2x \left( \frac{nx^2}{2} - \frac{1}{4} \right).
\]

Notice that in the above inequality, the left-hand side is equal to the right-hand side for \( x = \frac{1}{2n} \) and greater for \( x = \frac{1}{n} \). Furthermore, the left-hand side is linear, while the right-hand side is quadratic and convex. This implies that the left-hand side is no less for all \( \frac{1}{2n} \leq x \leq \frac{1}{n} \).

For \( n > 2 \), we are left to rule out that firm 1 undercut its rival and steals more than the entire adjacent market. We begin by ruling out deviations in which the firm captures less than two adjacent markets on each side. Let \( p_1' \) be the price at which the consumer located at \( \frac{1}{n} \) is indifferent between buying from firm 1 and buying from firm 2. This consumers utility of buying from firm 1 is

\[
v - \frac{1}{n} t - p_1' + (p^* - p_1') - \lambda t \left( \frac{1}{n} - \frac{1}{4n} \right).
\]

In case he buys from firm 2, her utility is

\[
v - p^* + \frac{1}{4n}.
\]

Thus, if the consumer is indifferent

\[
p^* - p_1' = \frac{t}{2n} \left[ 2 + \frac{3}{4} (\lambda - 1) \right].
\]

Consider the maximum price at which a local deviation is unprofitable; for this price \( p^* - c = \frac{t}{2n} [2 + \lambda - 1] \) and in this case \( p_1' - c = \frac{t}{2n} [\frac{1}{2}(\lambda - 1)] \). Thus even if firm 1 would get the entire two adjacent markets when setting \( p_1' \),
this is unprofitable as \( \frac{1}{n}(p^* - c) > \frac{1}{n}(p_1 - c) \).\(^{30}\) Obviously undercutting is (weakly) less profitable for any lower focal price.

We are left to consider the case in which \( n > 4 \), and firm 1 steals more than two adjacent markets on each side. We show that this is unprofitable because it requires firm 1 to price below marginal cost. For the consumer located at \( \frac{2}{n} \) to weakly prefer buying from firm 1 rather than firm 3, it must be that

\[
v - \frac{2}{n}t - p_1 + (p^* - p_1) - \lambda t \left[ \frac{2}{n} - \frac{1}{4n} \right] \geq v - p^* + \frac{1}{4n}.
\]

Hence, in this case \( p^* - p_1 \geq \frac{1}{4n}[1 + (\lambda - 1)] > \frac{1}{4n}[2 + \lambda - 1] \geq p^* - c \), which completes the proof.

**Proof of Proposition 2.** We have shown in the text that local deviations are unprofitable if and only if

\[
p^* - \frac{t}{n} \frac{1 + \lambda}{2} \leq c_i \leq p^* - \frac{t}{n}
\]

for all \( i \) and \( c_i \in [c_i, \overline{c}_i] \). It follows from the proof of Proposition 1 that if local deviations are unprofitable, so are global ones.

It remains to show the second part of the proposition. In the standard Salop model, for the consumer \( x \) between firms 1 and 2 who is indifferent between the two products,

\[
x = \frac{t}{n} + \frac{p_2 - p_1}{2t}.
\]

Hence, for realized cost \( c \), firm 1’s problem is

\[
\max_{p_1} \frac{p_1 - c}{2t} \left( \frac{2t}{n} - 2p_1 + E[p_2 + p_n | c] \right).
\]

This implies that

\[
P_1(c) = \frac{t}{2n} + \frac{E[p_2 + p_n | c]}{4} + \frac{c}{2}.
\]

Suppose that the supremum of prices charged by firms 1, 2, and \( n \) are \( \overline{p}_1, \overline{p}_2, \) and \( \overline{p}_n \), respectively. Suppose without loss of generality that \( \overline{p}_1 \) is the supremum of market-equilibrium prices of all firms. Then for any \( c \in [c_i, \overline{c}_i] \),

\[
P_1(c) \leq \frac{t}{2n} + \frac{\overline{p}_2 + \overline{p}_n}{4} + \frac{c}{2}.
\]

Taking the supremum of both sides implies

\[
\overline{p}_1 \leq \frac{t}{2n} + \frac{\overline{p}_2 + \overline{p}_n}{4} + \frac{\overline{c}}{2}.
\]

Rearranging gives the upper bound in the proposition.

Finally, we show that this upper bound can only be attained at \( \overline{c} \). If no firm’s price attains \( \overline{p}_1 \), we are done. Next, suppose that for a price \( c < \overline{c} \), \( P_1(c) = \overline{p}_1 \). By Inequality (12), again we are done. \( \square \)

\(^{30}\) Clearly if \( n = 3 \), the firm cannot attract two adjacent markets on each side, as there are only three local markets. Nevertheless, the upper bound on profitability we use is still valid.
Proof of Proposition 3. Posit a candidate market equilibrium in which all firms set a deterministic price and in which the highest price \( p_H \) is strictly greater than the lowest price \( p_L \). We prove that if the condition in the Proposition is satisfied, either (one of) the highest price firm(s) has a strict incentive to lower its price or (one of) the lowest price firm(s) has a strict incentive to raise its price, contradicting equilibrium.

We establish that the marginal profit of lowering the highest price is weakly greater than the marginal profit of raising the lowest price for all given cost realizations \( c \). This is sufficient because it implies that the high-price firm has a strict incentive to lower its price when it has its lowest cost realization, or the low-price firm has a strict incentive to raise its price when it has its highest cost realization (which is higher than the high-price firm’s lowest cost realization because the supports of the cost distributions overlap), contradicting equilibrium. Let \( x_{L}^{I} \) and \( x_{L}^{-} \) be one of the highest cost firm’s demands on its right and left, respectively. Define \( x_{L}^{I} \) and \( x_{L}^{-} \) similarly. We want to establish that

\[
(p_H - c) \left[ \frac{1}{2 + \frac{\lambda - 1}{2} [G(x_H^I) + G(x_H^-)]} + \frac{1}{2 + \frac{\lambda - 1}{2} [G(x_H^-) + G(x_H^I)]} \right] \times \left[ 2 + F_1(p_H)(\lambda - 1) \right]
\]

\[
\geq (p_L - c) \left[ \frac{1}{z_L^I} + \frac{1}{z_L^-} \right] \times \left[ 2 + F_1(p_L)(\lambda - 1) \right],
\]

where \( z_L^I \) and \( z_L^- \) are defined analogously to \( z_H^I \) and \( z_H^- \). For brevity, let \( \eta_H \equiv [2 + F_1(p_H)(\lambda - 1)] \) and let \( \eta_L \equiv [2 + F_1(p_L)(\lambda - 1)] \).

Notice that either \( \left( z_L^I \frac{1}{z_H^I} + z_L^- \frac{1}{z_H^-} \right) \leq \frac{1}{2} (z_H^I + z_L^-) \left( \frac{1}{z_H^I} + \frac{1}{z_L^-} \right) \) or \( \left( z_L^I \frac{1}{z_H^I} + z_L^- \frac{1}{z_H^-} \right) \leq \frac{1}{2} (z_H^I + z_L^-) \left( \frac{1}{z_H^I} + \frac{1}{z_L^-} \right) \).

We distinguish two cases depending on whether the former (Case I) or the latter (Case II) holds.

Case I. We rewrite Equation 13 as

\[
\eta_H \left( z_H^I + z_L^- \right) \geq \left( 1 - \frac{p_H - p_L}{p_H - c} \right) \eta_L \left( z_H^I + z_L^- \right).
\]

Equation 14 is equivalent to

\[
\eta_H \left( z_L^I \frac{1}{z_H^I} + z_L^- \frac{1}{z_H^-} \right) \geq \left( 1 - \frac{p_H - p_L}{p_H - c} \right) \eta_L \left( z_L^I + z_L^- \right).
\]

As \( \eta_H > \eta_L \) a sufficient condition for Equation (13) to hold is that

\[
\eta_H \left( z_L^I \frac{1}{z_H^I} + z_L^- \frac{1}{z_H^-} \right) \leq \frac{p_H - p_L}{p_H - c} \eta_L \left( z_L^I + z_L^- \right).
\]

Using that

\[
|z_H^I - z_L^-| = \frac{\lambda - 1}{2} \left[ G(x_H^I) - G(x_L^-) \right] - \left[ G \left( \frac{1}{n} - x_L^- \right) - G \left( \frac{1}{n} - x_H^I \right) \right],
\]

that \( g(\cdot) \) is bounded by \( 2n \), and that for all \( p < p_H \)

\[
\left| \frac{dx_L^I}{dp} \right| \leq \frac{1}{2t} \left[ 2 + (\lambda - 1)F_1(p_H) \right],
\]

we get that

\[
|z_H^I - z_L^-| \leq \frac{\lambda - 1}{2} 2n |x_H^I - x_L^-| \leq \frac{\lambda - 1}{2} 2n (p_H - p_L) \left( \frac{2 + (\lambda - 1)F_1(p_H)}{2 + \frac{\lambda - 1}{2}} \right),
\]

\]

34
and by a similar logic \(|z_H - z_L^+|\) has the same upper bound. Combining these with Equation 15 implies that it is sufficient to prove

\[
\frac{1}{p_H - c} \eta_L(z_L^+ + z_L^-) \geq (\eta_H)^2 \frac{\lambda - 1}{2} 2n \frac{1}{2} + \frac{\lambda - 1}{2} \frac{n}{2} \left( z_L^+ + z_L^- \right).
\]

Using that \(z_L^+ + z_L^- \frac{1}{z_H^+} \geq \frac{1}{2} (z_L^+ + z_L^-) \left( \frac{1}{z_H^+} + \frac{1}{z_H^-} \right)\) it is sufficient to prove

\[
\frac{1}{p_H - c} \eta_L \geq (\eta_H)^2 \frac{\lambda - 1}{2} + \frac{\lambda - 1}{2} \frac{n}{2} \left( \frac{1}{z_H^+} + \frac{1}{z_H^-} \right).
\]

(16)

Since the high-price firm’s demand is always less than or equal to \(\frac{1}{n}\), the fact that it does not want to lower its price implies

\[
1 \geq \frac{n}{2n} (p_H - c) \eta_H \left( \frac{1}{z_H^+} + \frac{1}{z_H^-} \right).
\]

Hence, a sufficient condition Equation 16 to hold is that

\[
\eta_L \geq \eta_H \frac{\lambda - 1}{2} + \frac{\lambda - 1}{2} \frac{n}{2}.
\]

For \(n = 2\), \(F(p_L) = F_1(p_H)\), so the above is satisfied for any \(\lambda > 1\). For \(n > 2\), using that \(F(p_L) \geq 1/n\) and \(F_1(p_H) \leq 1 + \lambda\), a sufficient condition for the above inequality to hold is that

\[
(4 + \lambda - 1)(2n + \lambda - 1) \geq n(2 + \lambda - 1)(\lambda - 1).
\]

Setting \(a = \lambda - 1\), this can be rewritten as

\[
0 \geq (n - 1)a^2 - 4a - 8n.
\]

Since this quadratic has one positive and one negative root, if \(a\) is positive and

\[
a \leq \frac{2}{n - 1} \left( 1 + \sqrt{1 + 2n(n - 1)} \right),
\]

the inequality is satisfied. This gives the bound in the proposition.

Case II. In this case, we rewrite Equation 13 as

\[
\eta_H \left( z_L^+ \frac{z_L^-}{z_H^+} + z_L^- \frac{z_L^+}{z_H^-} \right) \geq \left( 1 - \frac{p_H - p_L}{p_H - c} \right) \eta_L \left( z_L^+ + z_L^- \right).
\]

The remaining steps are analogous to Case I and thus omitted. 

**Proof of Proposition 4.** We begin by proving that each firm’s pricing function is continuous in cost. This fact follows from the following lemma.

**Lemma 2.** In any interior equilibrium with \(\lambda < 38\), firm i’s expected profit is single-peaked in its price for any cost realization \(c_i\).

**Proof.** Notice that (using Equation 4) profits are differentiable wherever the market price distribution does not have an atom—which is almost everywhere—and continuous. Furthermore, at prices where the profit function is not differentiable, it has a concave kink.

35
Suppose by contradiction that the profit function is not single-peaked. This implies that the profit function must have a trough. At this trough, it obviously cannot have a concave kink, so it is differentiable. To arrive at a contradiction, we prove that if firm 1’s first-order condition is satisfied at some price \( p \), profits are lower slightly to the right of \( p \).

Let the subscript 1 \( x^+ \) and \( x^- \) denote partial derivative with respect to firm 1’s price. Note that for each \( x(p_1, p_{-1}) \in \{x^-(p_1, p_{-1}), x^+(p_1, p_{-1})\} \) one has

\[
\limsup_{p_1' \searrow p_1} \frac{1}{p_1' - p_1} \cdot (x_1(p_1', p_{-1}) - x_1(p_1, p_{-1})) = -\frac{1}{2t} \left( \lambda - 1 \right) F'(p_1) + \frac{2 \left( \lambda - 1 \right)}{2 + \frac{\lambda - 1}{2} (2 + \frac{\lambda - 1}{2})} (\lambda - 1) G'(x(p_1, p_{-1})) + G'(p_1) - x(p_1, p_{-1}))
\]

\[
+ \frac{1}{2t} \left( \lambda - 1 \right) (2 + \frac{\lambda - 1}{2}) (G'(x(p_1, p_{-1})) + G'(p_1) - x(p_1, p_{-1}))
\]

\[
\leq (x_1(p_1, p_{-1}))^2 \left( \frac{\lambda - 1}{2} + 2n \right) \leq (x_1(p_1, p_{-1}))^2 \left( \frac{\lambda - 1}{2} + 2 \right).
\]

Let \( \pi(p) = (p - c)E[x^+(p, p_{-1}) + x^-(p, p_{-1})] \). We will prove that

\[
\limsup_{p_1' \searrow p_1} \frac{\pi'(p_1') - \pi(p_1)}{p_1' - p_1} < 0.
\]

This is sufficient because it shows that the derivative of the profit function is negative to the right of and sufficiently close to \( p_1 \), so that profits are smaller there.

By Equation 17, it is sufficient to prove

\[
(p_1 - c) \cdot E \left[ (x_1^-(p_1, p_{-1}))^2 + (x_1^+(p_1, p_{-1}))^2 \right] \leq \frac{\lambda - 1}{2} + 2 \cdot \frac{\lambda - 1}{2} \leq 0.
\]

To bound the above, we begin showing that I divided by II is less than or equal to \( \frac{k^2}{4k} \), where

\[
k \equiv \frac{2 + \lambda - 1}{2 + \frac{\lambda - 1}{2}}
\]

Observe that

\[
\frac{\max \{x_1^-(p_1, p_{-1}), x_1^+(p_1, p_{-1})\}}{\min \{x_1^-(p_1, p_{-1}), x_1^+(p_1, p_{-1})\}} \leq \frac{2 + \lambda - 1}{2 + \frac{\lambda - 1}{2}} = k.
\]

Now we use the following fact.

**Fact 1.** Suppose \( \tilde{a}_+ \) and \( \tilde{a}_- \) are positive random variables such that

\[
\sup \{\tilde{a}_+, \tilde{a}_-\} \leq k
\]

Then

\[
\frac{E[\tilde{a}_+ + \tilde{a}_-^2]}{E[\tilde{a}_+ + \tilde{a}_-^2]} \leq \frac{1}{2} \frac{(k + 1)^2}{4k}.
\]

**Proof.** Suppose without loss of generality that \( 1 \leq \tilde{a}_+, \tilde{a}_+ \leq k \). Since the quadratic function is convex, the ratio on the left-hand side of Inequality 19 is maximized if the support if \( \tilde{a}_+, \tilde{a}_- \) consists of the extremal values 1, k. Thus, the left-hand side of the inequality is less than or equal to the maximum of

\[
\frac{\max \{b_+^2 + 1 + b_-(k - 1)^2 + 1\}}{b_+^2 + 1 + b_-(k - 1)^2 + 1}.
\]

36
which is equivalent to maximizing
\[
\max_{b^- b^+} \frac{1}{2} \left[ \frac{k+ b^- (k^2 - 1) + 1}{(k+ b^- (k - 1) + 1)^2} \right].
\]

For brevity, let \(b = \frac{k+b^-}{2}\). Then the first order condition is satisfied if and only if
\[
(b(k - 1) + 1)^2 (k^2 - 1) - (b(k^2 - 1) + 1)2(k - 1)(b(k - 1) + 1) = 0,
\]
which yields \(b = \frac{k}{k+1}\). Substituting this into the maximant and rewriting gives the desired inequality. Hence, Inequality 18 gives
\[
(p - c) E \left[ x_1^-(p_1, p_{-1}) + x_1^+(p_1, p_{-1}) \right] \leq \frac{2}{n}.
\]

If the firm only prices its neighbors out of the market with probability zero one has
\[
(p - c) E \left[ x_1^-(p_1, p_{-1}) + x_1^+(p_1, p_{-1}) \right] \leq \frac{2}{n}.
\]

In this case, the above condition simplifies to
\[
\frac{(k + 1)^2}{4k} \frac{\lambda-1}{2 + \frac{\lambda-1}{2}} < 1.
\]

This condition holds for any \(\lambda < 38\).

From the above lemma, the following corollary is obvious.

**Corollary 5.** In an interior equilibrium with \(\lambda < 38\), the pricing function is continuous in cost for each firm.

We are now ready to prove the statement of the proposition. We prove by contradiction; suppose that there exists (at least one) firm that does not charge a deterministic price. Corollary 5 implies that there must exist a nontrivial interval of prices, each of which the firm charges for some cost. On this interval, consider a price \(p^0\) and a sequence of prices \(p^i \searrow p^0\) such that i.) none of these prices are atoms of \(F\); ii.) the pricing function \(p(\cdot)\) is differentiable at \(p^0\) with a strictly positive derivative. \(^{31}\)

Let the corresponding costs be \(c^0\) and \(c^i \searrow c^0\).

Using the same calculation as in the proof Lemma 2, we have
\[
0 < p'(c^0) < \frac{1}{2 - \frac{(k+1)^2}{4} \frac{\lambda-1}{2 + \frac{\lambda-1}{2}}}
\]

Let \(x^+(p_1, p_{-1})\) and \(x^-(p_1, p_{-1})\) be the demand of firm 1 in its two markets, as a function of its price and others’ prices. By the firm’s maximization problem,
\[
(p^i - c^i) E \left[ x_1^-(p^i, p_{-1}) + x_1^+(p^i, p_{-1}) \right] + E[x^+(p^i, p_{-1}) + x^-(p^i, p_{-1})] = 0
\]

for each \(i\), and a similar condition holds at \(p^0\).

\(^{31}\) Given our estimation in Lemma 2 (which we also use again below to bound the derivative of \(p(\cdot)\)), we can show that \(p(\cdot)\) is Lipschitz continuous. Hence, we can apply the Fundamental Theorem of Calculus to conclude that its derivative must be strictly positive on a set of positive measure.
Fix any $p_{-1}$. We find a condition under which for $x(\cdot, \cdot) \in \{x^+(\cdot, \cdot)x^-(\cdot, \cdot)\}$,

$$\limsup_{c^i \to c^0} \left[ \frac{(p' - c)x_1(p', p_{-1}) + x(p', p_{-1})}{c^i - c^0} \right] - \left[ \frac{(p' - c^0)x_1(p^0, p_{-1}) + x(p^0, p_{-1})}{c^i - c^0} \right] < 0.$$ 

This is sufficient for a contradiction because it implies that the first-order condition 20 cannot hold for all $p'$, $p^0$ (since the difference between the integrands is negative at all points, it is negative in expectation).

Notice that the above limsup is equal to

$$\lim_{c^i \to c^0} \frac{x(p', p_{-1}) - x(p^0, p_{-1})}{c^i - c^0} + \lim_{c^i \to c^0} \frac{[(p' - p^0) - (c' - c^0)]}{c^i - c^0} \cdot x_1(p^0, p_{-1})$$

$$= x_1(p^0, p_{-1}) \cdot (2p'(c_0) - 1) + (p^0 - c^0) \cdot \limsup_{c^i \to c^0} \frac{x_1(p^0, p_{-1}) - x(p^0, p_{-1})}{c^i - c^0}. \tag{21}$$

Now we work on the last term above, which is equal to

$$- \frac{p^0 - c^0}{2t} \cdot \limsup_{c^i \to c^0} \frac{1}{c^i - c^0} \left[ \frac{(\lambda - 1)(F(p') - F(p^0))}{2 + \frac{\lambda}{4} \left( \frac{\lambda}{\pi} \cdot (G(x(p^0, p_{-1})) + G((1/n) - x(p^0, p_{-1})) - G(x(p^0, p_{-1})) + G((1/n) - x(p^0, p_{-1})) - G(1/n) - x(p^0, p_{-1})) \right)}{2 + \frac{\lambda}{4} \left( \frac{\lambda}{\pi} \cdot (G(x(p^0, p_{-1})) + G((1/n) - x(p^0, p_{-1})) - G(x(p^0, p_{-1})) + G((1/n) - x(p^0, p_{-1})) - G(1/n) - x(p^0, p_{-1})) \right)} \right]$$

$$= - \frac{p^0 - c^0}{2t} \cdot \limsup_{c^i \to c^0} \frac{1}{c^i - c^0} \left[ \frac{2 + \frac{\lambda}{4} \left( \frac{\lambda}{\pi} \cdot (G(x(p^0, p_{-1})) + G((1/n) - x(p^0, p_{-1})) - G(x(p^0, p_{-1})) + G((1/n) - x(p^0, p_{-1})) - G(1/n) - x(p^0, p_{-1})) \right)}{2 + \frac{\lambda}{4} \left( \frac{\lambda}{\pi} \cdot (G(x(p^0, p_{-1})) + G((1/n) - x(p^0, p_{-1})) - G(x(p^0, p_{-1})) + G((1/n) - x(p^0, p_{-1})) - G(1/n) - x(p^0, p_{-1})) \right)} \right]$$

Now, notice that one of $G(x(p^0, p_{-1})) - G(x(p', p_{-1}))$ and $G(x(1/n - p^0, p_{-1})) - G(1/n - x(p', p_{-1}))$ is greater than or equal to zero, and since $G(s) - G(s') \leq 2n(s - s')$ for any $s > s'$, the other one is greater than or equal to $-2n \left[ x(p^0, p_{-1}) - x(p^0, p_{-1}) \right]$. Using also $G(x(p^0, p_{-1})) + G(1/n - x(p^0, p_{-1})) \geq 1$, this implies that the above is less than or equal to

$$(p^0 - c^0)p'(c^0) \left( x_1(p^0, p_{-1}) \frac{\lambda - 1}{\lambda + 1} F'(p^0) + (x_1(p^0, p_{-1}))^2 \frac{(\lambda - 1)n}{2 + \frac{\lambda}{4}} \right).$$

Substituting into Expression 21 and using that $|x_1(p^0, p_{-1})| \leq \frac{\frac{\lambda + 1}{2\sqrt{\lambda}}}{\frac{\lambda}{2} + \frac{\lambda}{2}} = \frac{\lambda}{2}$ implies that it is sufficient to prove

$$1 + k^2 \frac{\lambda - 1}{\lambda + 1} \frac{n}{2t} (p^0 - c^0)p'(c^0) \leq 2p'(c^0) + (p^0 - c^0)p'(c^0) \frac{\lambda - 1}{\lambda + 1} F'(p^0).$$

Using that $F'(p^0) \geq D(p^0) \frac{\theta(c^0)}{p'(c^0)}$ and that

$$p^0 - c^0 = \frac{D(p^0)}{-D'(p^0)} \leq \frac{2n}{2t + 1 + \lambda} = \frac{2n}{n}(1 + \lambda),$$

the above becomes

$$1 + k^2 (\lambda - 1)p'(c^0) \leq 2p'(c^0) + (p^0 - c^0)D(p^0) \frac{\lambda - 1}{\lambda + 1} \theta(c^0). \tag{22}$$

38
To finish our proof, we put a bound on the firm’s profits \((p^0 - c^0)D(p^0)\). In a market equilibrium, no firm charges a price less than \(c\), so firm 1’s profits are at least as much as it would make if both of its neighbors charge \(c\) with probability one. If firm 1 also charges \(c\), its demand in each of its two markets is \(\frac{1}{2t}\). Now

\[
|x_1(p_1, p_{-1})| \leq \frac{1}{2t} \cdot \frac{1 + \lambda}{2 + \frac{\lambda}{2}} = \frac{k}{2t}.
\]

This implies that a sufficient condition for the firm to be able to sell profitably is

\[
\overline{c} - \underline{c} < \frac{1}{2t} = \frac{t}{n} k.
\]

Furthermore, if this is the case, its profits are at least

\[
2(p - c) \left(\frac{1}{2n} - \frac{k}{2t}(p - \overline{c})\right) = (p - c) \left(\frac{1}{n} - \frac{k}{t}(p - \overline{c})\right).
\]

Maximizing this expression with respect to \(p\) and setting \(c = \overline{c}\) gives

\[
(p^0 - c^0)D(p^0) = \frac{k}{4t} \left(\frac{1}{n} k - (\overline{c} - \underline{c})\right)^2 = \frac{k}{4 n^2} \left(\frac{1}{k} - \gamma\right)^2,
\]

where \(\gamma \equiv (\overline{c} - \underline{c})/(t/n).

Now we have two cases.

Case I: \(k^2(\lambda - 1) \leq 2\). In this case, a sufficient condition for Inequality 22 to hold is

\[
\frac{t}{n^2} \theta(c^0) \geq \frac{\lambda + 1}{\lambda - 1} \frac{4k}{1 - k \gamma}.
\]

Case II: \(k^2(\lambda - 1) > 2\). Then, substituting our bound for \(p'(c^0)\) into Inequality 22 and rearranging gives that a sufficient condition is

\[
\frac{t}{n^2} \theta(c^0) \geq \frac{4k}{1 - k \gamma} \cdot \frac{(1 + \lambda) k^2 - (k+1)^2}{2 - \frac{k+1}{k^2} \cdot \frac{(k+1)^2}{4}}.
\]

This completes our proof.

**Proof of Lemma 1.** We begin with proving continuity. Suppose by contradiction that \(c^0 \rightarrow c\) but \(P(c^0) \rightarrow P(c)\). Then, since the pricing function is obviously bounded, we can choose the sequence so that \(P(c^0)\) converges; let \(P(c^0) \rightarrow P^* \neq P(c)\). Furthermore, suppose that \(P^* > P(c)\); the proof for the other case is analogous.

Since \(P(c^0)\) is optimal when the marginal cost is \(c^0\), a firm cannot benefit from marginally lowering its price. Using Equation 4 to express the firm’s marginal profit from lowering its price, this implies that

\[
\frac{1}{2n} - (P(c^0) - c^0) \cdot \frac{2 + (\lambda - 1)F_1(P(c^0))}{1 + \lambda} \geq 0.
\]

(23)

Similarly, since \(P(c)\) is optimal when the marginal cost is \(c\), a firm cannot benefit from marginally raising its price. Using Equation 4, this implies that

\[
\frac{1}{2n} - (P(c) - c) \cdot \frac{2 + (\lambda - 1)F_1(P(c))}{1 + \lambda} \leq 0.
\]

(24)

Subtracting Inequality 23 from Inequality 24 gives

\[
(P(c^0) - c^0) \cdot \frac{2 + (\lambda - 1)F_1(P(c^0))}{1 + \lambda} - (P(c) - c) \cdot \frac{2 + (\lambda - 1)F_1(P(c))}{1 + \lambda} \leq 0.
\]

39
The limit of the left-hand side of this inequality as \( i \to \infty \) is positive, a contradiction.

Next, we prove by contradiction that \( P(c) \) is non-decreasing. Suppose that \( c' > c \) and \( P(c') < P(c) \). Since \( P(c) \) is optimal when the marginal cost is \( c \), a firm cannot benefit from marginally lowering its price. As above, this implies that

\[
\frac{1}{2n} - (P(c) - c) \cdot \frac{2 + \lambda (\lambda - 1) F_1(P(c))}{1 + \lambda} \geq 0.
\]

Similarly, since \( P(c) \) is optimal when the marginal cost is \( c \), a firm cannot benefit from marginally raising its price. Therefore,

\[
\frac{1}{2n} - (P(c') - c') \cdot \frac{2 + \lambda (\lambda - 1) F_1(P(c'))}{1 + \lambda} \leq 0.
\]

Subtracting Inequality 25 from Inequality 26 gives

\[
(P(c) - c) \cdot \frac{2 + \lambda (\lambda - 1) F_1(P(c))}{1 + \lambda} - (P(c') - c') \cdot \frac{2 + \lambda (\lambda - 1) F_1(P(c'))}{1 + \lambda} \leq 0,
\]

a contradiction. \( \square \)

**Proof of Proposition 5.** We first show that any symmetric equilibrium pricing function satisfies the above properties. Property 1 follows from Lemma 1. Since \( P(\cdot) \) is increasing and continuous, \( P^{-1}(p) \) is a closed interval for any \( p \) on the range of \( P(\cdot) \). Let \( p_1, p_2, \ldots \) be the (at most countable) set of prices \( p_i \) such that \( P^{-1}(p_i) \) is a non-trivial interval, and let \([f_i, f_i'] = P^{-1}(p_i)\). These \([f_i, f_i'] \) satisfy Property 2 by construction. Also, for any \( c \not\in [f_i, f_i'] \), \( P(c) \) is not an atom of the pricing distribution, so a firm’s demand is differentiable, and hence \( P(c) \) must satisfy Equation 9. This implies that Property 3 holds. Notice that \( D_{i1}(P(c), P_{-1}(c)) = -\frac{2}{1 + \lambda} \), so firm 1 does not want to decrease its price at \( c \) only if \( (P(c) - c) \frac{1}{1 + \lambda} \leq \frac{1}{n} \), which implies the first part of Property 4. Also, \( D_{i1}(P(c), P_{-1}(c)) = -\frac{1}{n} \).

So for raising the price marginally to be unprofitable, we must have \( (P(\overline{c}) - \overline{c}) \frac{1}{1 + \lambda} \geq \frac{1}{n} \), which implies the second part of Property 4.

We now argue that if \( P(\cdot) \) satisfies the properties in the Proposition, it is an equilibrium pricing strategy. Notice that for any \( c \not\in [\underline{c}, \overline{c}], c \not\in [f_i, f_i'] \), demand is differentiable from the right. Since \( P(c) = \Phi(c) \) for all such \( c \), our analysis in the text implies that there is no profitable local price increase. We are left to consider non-local price increases. Analogously to Proposition 1, since the demand curve is concave for price increases, the result is immediate.

Now for any \( c \not\in [\underline{c}, \overline{c}], c \not\in [f_i, f_i'] \), demand is differentiable from the left. Furthermore, since \( P(c) = \Phi(c) \) for all such \( c \), our analysis in the text implies that local price decreases are unprofitable. We now consider non-local price decreases.

The proof mirrors the proof of Proposition 1. Suppose the realized cost is \( c \), so that the firm’s price in the posited equilibrium is \( P(c) \). At this price, consumers’ marginal utility in money from a price decrease is \( 2 + \lambda (\lambda - 1) F_1(P(c)) \).

We will use that as the price decreases, this marginal utility in money also decreases.

We first rule out the possibility that firm 1 might like to charge a price \( p_1 \) so that the indifferent consumer is \( x \in [\frac{\underline{c}}{2n}, \frac{\overline{c}}{2n}] \). Equating Expressions (2) and (3), setting \( p_2 = P(c) \), and replacing the difference in money utilities,

\[
P(c) - p_1 + \left[ -\lambda \int_0^{P(c)} (p_1 - p) \ dF(p) + \int_{P(c)}^{\infty} (p - p_1) \ dF(p) \right] = \left[ -\lambda \int_0^{P(c)} (P(c) - p) \ dF(p) + \int_{P(c)}^{\infty} (p - P(c)) \ dF(p) \right],
\]
This completes the proof that non-local price decreases are unprofitable. For the consumer located at \( \frac{1}{n} \), it is also not profitable to lower it for \( i, f \notin [f, f'] \). For any \( c \notin [f, f'] \), we have \( P(\tilde{c}) = P(c) = P(f') \). Since it is not profitable to lower the price at \( f \), it is also not profitable to lower it for \( c \), and since it is not profitable to raise the price for \( f' \), it is also not profitable to raise it for \( c \).

We are left to prove that there are no profitable deviations for \( \tilde{c} \) and \( \bar{c} \). Our analysis of non-local deviations above (which only used that \( P(\tilde{c}) = \Phi(\tilde{c}) \)) implies that for \( P(\tilde{c}) = \Phi(\tilde{c}) \), there is no profitable deviation. Now suppose that \( P(\tilde{c}) < \Phi(\tilde{c}) \). Demand responsiveness to price decreases from \( P(\tilde{c}) \) is then the same as when \( P(\tilde{c}) = \Phi(\tilde{c}) \). Hence,
with the markup being lower, the incentive to lower the price is smaller than for \( P(c) = \Phi(c) \), so there is no profitable price decrease. Next, we deal with price increases from \( c \). Since \( P(c) < \Phi(c) \), we consider two cases. First, suppose that \( P(c) \) is a constant \( p^* \). Then, using that by Property 4 in the proposition \( \Phi(c) \geq p^* \geq \Phi(c) \), and Equation 9, the condition in Proposition 2 is satisfied. Hence, \( p^* \) is a market-equilibrium focal price. If \( P(c) \) is not constant, there is a largest interval \( [c, f_1'] \) for which it is constant, and where \( f_1' < c \). In this case, our argument in the previous paragraph applies. Finally, a similar argument works for price deviations from \( c \).

Proof of Corollary 2. Suppose by contradiction that there is a constant interval \( [f_1, f_1'] \). By Conditions 3 and 4 of Proposition 5, we must have \( P(f_1) \leq \Phi(f_1) \). But by the same two conditions, we must also have \( P(f_1') \geq \Phi(f_1') \), which is impossible since \( \Phi(\cdot) \) is strictly increasing on the interval while \( P(\cdot) \) is constant.

Proof of Corollary 3. We first prove by contradiction that if \( \Phi(c) \) is weakly decreasing, then any symmetric equilibrium is a focal-price one. Suppose the price is not deterministic. Then, by the continuity of the pricing function, there are cost levels \( c, c' > c \) such that \( P(c) \) and \( P(c') \) are not atoms of the price distribution. Thus, for these cost levels, the chosen price must satisfy Equation 9. Using that \( \Phi(c) \) is strictly decreasing, this means that \( P(c') < P(c) \), contradicting that the pricing function is non-decreasing.

If \( \Phi(c) \) is not weakly decreasing, then there are obviously non-constant \( P(\cdot) \) satisfying Proposition 5.

Proof of Corollary 4. The statement is true on both the constant and strictly increasing parts of the pricing function.

References


