Managerial Flexibility, Agency Costs and Optimal Capital Structure

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Abstract

We develop a dynamic structural model to quantitatively assess the effects of managerial flexibility and manager-specific characteristics on firms’ values, capital structures and credit risk. The model incorporates taxes and bankruptcy costs and three important dimensions on which managers affect firms: financing, effort and project selection. We implement the manager’s contract through financial securities, which leads to a dynamic capital structure for the firm consisting of inside equity, outside equity, long-term debt, and dynamic risk-free borrowing or lending (short-term debt or cash). We calibrate the baseline parameters of the model and, in particular, indirectly infer the key manager-specific parameters by matching its predictions to aggregate data. The agency costs due to managerial lack of diversification and risk aversion could substantially lower firm values and leverage ratios relative to the benchmark scenario in which managers are perfectly diversified and have no discretion. Managerial discretion in effort, project selection, and short-term debt financing have a much greater impact on the firm’s value and capital structure than discretion in long-term debt financing. Consistent with recent empirical evidence, capital structure varies widely with managerial characteristics. The firm’s long-term debt ratio and credit spread decline with the manager’s ability, and increase with her risk aversion as well as her disutility of effort, while the firm’s short-term debt (cash) ratio decreases (increases) with all three variables. The distinct components of the firm’s risk (earnings risk and project risk) have differing effects on its long-term and short-term debt. Consistent with empirical evidence, growth opportunities have a negative effect on leverage, the relation between managerial inside equity ownership and long-term (short-term) debt is negative (positive), and the relation between leverage and lagged profitability is negative in the baseline model.
1 Introduction

A growing body of empirical evidence suggests that managerial discretion and manager-specific characteristics significantly influence firms’ investment and financing decisions (Graham and Harvey, 2001, Bertrand and Schoar, 2003). However, theoretical research that analyzes the quantitative impact of managerial discretion on firms in dynamic frameworks, especially the effects of managerial lack of diversification and risk aversion, is relatively nascent. We develop a dynamic structural model to examine how undiversified, risk-averse managers’ flexibility in financing, effort and project selection affects firms’ security values and capital structures. We implement the manager’s contract through financial securities, which leads to a dynamic capital structure for the firm consisting of inside equity, outside equity, long-term debt and dynamic risk-free borrowing or lending (short-term debt or cash). We calibrate the manager-specific parameters of the model by matching its predictions to the data. Our analysis leads to the following principal findings:

- In the baseline model, the agency costs due to the manager’s lack of diversification lower firm value by over 15%, and total leverage by over 50%, relative to the benchmark scenario in which the manager is perfectly diversified and has no discretion. The manager affects the firm’s leverage largely through its short-term debt (or cash) rather than its long-term debt.

- Managerial discretion in effort, project selection, and short-term debt financing have a much greater effect on firm value than discretion in long-term debt financing.

- The firm’s capital structure varies widely with manager characteristics. The long-term debt ratio and credit spread decline with the manager’s ability, and increase with her risk aversion and disutility of effort. The firm’s short-term debt ratio declines with all three variables.

- Consistent with empirical evidence, managerial inside equity ownership is negatively (positively) related to long-term (short-term) debt, leverage declines with growth opportunities, and the relation between leverage and lagged profitability is negative in the baseline model.

- The distinct components of firm risk—project risk and earnings risk—have differing effects on long-term and short-term debt.

In our infinite horizon, continuous time framework, the cash-constrained manager of a privately held firm approaches the public debt and equity markets to obtain financing for an initial capital
investment. All long-term debt has infinite maturity, is non-callable and is completely amortized. Debt interest payments are shielded from corporate taxes. The total earnings before interest and taxes (EBIT) are affected by the manager’s unobservable effort and contractible project choices in each period. The manager’s ability and effort affect the level of earnings in each period, while her project choices influence earnings in future periods by affecting the “scale” of the firm. The manager chooses one of two possible projects in each period. The key state variable is the firm’s EBIT rate in the absence of the manager’s human capital input, which evolves as a log-normal process under either project with possibly differing drifts and volatilities.

Outside investors are risk-neutral. As in DeMarzo and Fishman (2006), the manager also has linear inter-temporal preferences. In contrast with DeMarzo and Fishman (2006), however, her subjective discount rate is stochastic that reflects her costs of bearing firm-specific risk (see Alvarez and Jermann, 2000). She receives dynamic incentives through explicit contracts contingent on the firm’s earnings. As in Aghion and Bolton (1992) and Zwiebel (1995), we consider an “incomplete contracting” framework in which only single-period contracts are enforceable. Further, as in Fudenberg et al (1990), Aghion and Bolton (1992), and studies in the “signalling” literature (see Chapter 6 of Tirole, 2006), the manager has the bargaining power and offers the contract to shareholders in each period. The contracts must be incentive compatible for the manager and satisfy shareholders’ dynamic participation constraints, which evolve with the firm’s performance.

Debt is serviced entirely as long as the firm is solvent by the dilution of equity if necessary. Bankruptcy is declared endogenously when the value of equity falls to zero. The firm’s future earnings after bankruptcy are lowered by direct bankruptcy costs as well as indirect costs arising from imperfections in the firm’s product market (e.g. relationships with customers and suppliers) that are external to the manager-firm relationship. The manager continues to operate the firm after bankruptcy and also effectively incurs personal bankruptcy costs because her compensation is tied to the firm’s earnings. All structural parameters of the model are common knowledge.

We characterize the rational expectations equilibrium in which the firm’s capital structure and the manager’s dynamic compensation, effort and project choices are endogenously determined. The manager chooses the firm’s capital structure to maximize her expected utility rationally anticipating debt tax shields and bankruptcy costs. Shareholders and bondholders value their respective claims rationally anticipating the effects of the manager’s actions on the firm’s earnings.
The manager’s optimal effort and project choices depend on her project tradeoff—an analytical relation among her risk aversion, the projects’ risks, and their drifts—that determines her subjective valuation of the firm’s projects. Depending on the sign of the project tradeoff, the manager could either increase or decrease risk in financial distress. Moreover, the fact that the manager dynamically alters the firm’s projects causes her payoff structure to be non-linear in the firm’s earnings, which is consistent with observed compensation structures.

We implement the manager’s contracts through financial securities, specifically, inside equity and dynamic risk-free borrowing and lending. The firm’s resulting capital structure is dynamic and consists of inside equity, outside equity, long-term debt, and short-term risk-free debt (or cash). The different components of the firm’s capital structure play complementary roles. The firm’s long-term debt structure primarily reflects the tradeoff between debt tax shields and (the manager’s personal and firm-level) bankruptcy costs. The inside equity stake and the firm’s short-term debt provide optimal incentives to the risk-averse manager. In particular, the manager’s inside equity stake declines with her risk aversion and disutility of effort because an increase in either of these variables increases the costs of risk-sharing between the manager and outside investors.

To investigate the effects of the various dimensions of managerial discretion—financing, project selection, and effort—we analyze three benchmark scenarios.

- **No Discretion in Long-Term Debt Financing**: The manager enjoys discretion in project selection and effort, but chooses the firm’s long-term debt structure to maximize economic value (the value of total earnings including the manager’s stake less her disutility of effort).

- **No Discretion in Long-Term Debt Financing or Project Selection**: The manager has discretion in effort, but chooses the long-term debt structure and projects to maximize economic value.

- **No Discretion in Financing, Project Selection or Effort (the “First Best” Scenario)**: The manager is perfectly diversified and chooses the long-term debt structure, projects and effort to maximize economic value.

We numerically derive the manager’s capital structure choices in the actual and benchmark scenarios. To determine the baseline values of the model parameters for our analysis, we follow studies such as Hennessy and Whited (2005) and Strebulaev (2007) by directly setting the values

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1If the firm has positive short-term risk-free debt, it has negative cash and vice versa.
of those parameters for which there is guidance from previous empirical research. We indirectly infer the baseline values of the remaining manager-specific parameters—risk aversion, ability and disutility of effort—by matching the firm’s long-term debt ratio, cash ratio, and the manager’s inside equity stake to their average values in the data.

In the baseline model, the agency costs due to the manager’s lack of diversification and risk aversion lower total firm value (the market value of the firm’s total earnings including the manager’s stake) by over 15% relative to the “first best” benchmark scenario in which the manager is perfectly diversified and has no discretion. The risk-averse manager’s precautionary motives cause her to hold surplus cash in contrast with the first best scenario in which the firm has a significant level of short-term debt (negative cash). As a result, the firm’s total leverage (long-term debt net of cash) is over 50% lower in the actual scenario than in the benchmark scenario.

Consistent with the evidence in recent studies (Graham and Harvey, 2001, Bertrand and Schoar, 2003, Lewellen, 2006), the firm’s capital structure varies widely with managerial characteristics such as ability, risk aversion and disutility of effort. As managerial characteristics vary, the firm can move from being a net issuer of short-term debt to one holding surplus cash. The long-term debt ratio declines with the manager’s ability, but increases with her risk aversion as well as her disutility of effort, while the short-term debt ratio declines with all three variables. The model predicts that the manager’s inside equity stake and the firm’s short-term (long-term) debt are positively (negatively) related, which is consistent with recently reported empirical evidence that managers with higher ownership stakes are more likely to choose short-term debt than long-term debt (Datta et al, 2005).

The intuition for the above results is as follows. The manager’s choice of long-term debt reflects its effect on her initial payoff from leveraging the firm, which is positively affected by ex post debt tax shields, and its effect on her valuation of her stream of future payoffs, which is negatively affected by the possibility of bankruptcy. In equilibrium, the manager captures the additional output or surplus she generates in each period due to her ability and effort. An increase in the manager’s ability (and/or a decrease in her risk aversion or disutility of effort) increases the surplus the manager generates in each period and, therefore, the marginal effect of her continuation value on the choice of long-term debt. Hence, long-term debt decreases with the manager’s ability and increases with her risk aversion or effort disutility. An increase in the manager’s ability, risk aversion, or disutility of effort increases the “cash” component of the manager’s compensation relative to the “equity”
component, thereby *increasing* the firm’s cash as a proportion of its value (alternately, *lowering* its short-term debt ratio; see footnote 1). The fact that short-term debt or cash plays an important role in facilitating optimal risk-sharing between diversified shareholders and the undiversified, risk-averse manager also explains why managerial discretion in short-term debt financing has a much greater impact on firm value than discretion in long-term debt financing.

Consistent with empirical evidence (see Rajan and Zingales, 1995), the relation between total leverage (long-term debt net of cash) and lagged profitability is negative in the baseline model. The relation could become positive as managerial characteristics deviate from their baseline values, in particular, when the manager’s ability and/or her risk aversion are below respective thresholds. In these scenarios, the firm has nonzero short-term debt (negative cash) in its capital structure. Our study, therefore, highlights the fact that it is important to appropriately control for manager-specific characteristics in empirical analyses of leverage dynamics.

We also investigate the effects of project characteristics on capital structure. Consistent with prior evidence, the long-term debt ratio and net leverage ratio decline with the average expected growth rate of the firm’s projects or its “growth opportunities” (for example, Rajan and Zingales, 1995). The firm’s short-term debt ratio also declines with growth opportunities. The firm’s *earnings risk* (standard deviation of earnings in each period) and its *project risk* (volatility of the *evolution* of earnings) have *differing* effects on the firm’s long-term and short-term debt ratios. The long-term debt ratio declines with the firm’s average project risk, but the short-term debt ratio varies non-monotonically. The long-term debt ratio, however, increases with the firm’s average earnings risk, while the short-term debt ratio declines. Project risk and earnings risk have differing effects because the earnings risk affects the manager’s incentive compensation in each period, while the project risk has long-term effects by influencing the manager’s valuation of her future payoff stream.

In summary, we develop a dynamic, structural model to obtain quantitative guidance on the effects of managerial discretion and lack of diversification on firms’ capital structures and security values. The model incorporates three important dimensions on which managers affect firms—financing, effort and project selection. Our analysis leads to predictions that are consistent with prior empirical evidence and also suggests new, testable implications for the effects of manager-specific characteristics on capital structure that could be examined in future empirical research.
2 Related Literature

A substantial body of literature examines the effects of bondholder-shareholder conflicts in dynamic “contingent claims” models (for example, Fischer et al, 1989, Leland and Toft, 1996, Goldstein et al, 2001, Ju et al, 2004). We adopt the philosophical perspective of these papers, which argue that it is important to analyze dynamic frameworks to obtain quantitative guidance on the significance of agency conflicts among firms’ stakeholders. In contrast, we incorporate managerial discretion in our framework so that manager-specific characteristics affect firms’ values and capital structures.

A number of studies incorporate managerial discretion in two-period frameworks with risk-neutral agents and either adverse selection (for example, John and John, 1993, Goswami et al, 1996), or moral hazard (Berkovitch et al, 2000). Chang (1993) analyzes the interaction between compensation structure, payout policy and capital structure in a two-period model in which risk-averse managers value control. Zwiebel (1996) develops a dynamic model with risk-neutral agents in which contracts are not feasible, and capital structure reflects the tradeoff between exogenous private control benefits and the need to ensure sufficient efficiency to prevent control challenges. Morellec (2004) analyzes the effects of the interactions between non-contractible private control benefits and taxes, and shows that they can explain observed leverage ratios. Parrino et al (2005) study the effects of different types of financing on the investment distortions created by a risk-averse manager with a given compensation structure.

We complement the above studies by examining the effects of a risk-averse manager’s discretion in (long-term and short-term) debt and equity financing, effort and project selection in a dynamic framework in which the manager’s incentive structure is endogenous. The surplus generated by the manager’s human capital (ability and effort), which she appropriates in equilibrium, are her endogenous control benefits. Berk et al (2006) develop a dynamic model to analyze the effects of managerial risk aversion on capital structure. The manager’s compensation structure and her degree of entrenchment are endogenously derived. As in our study, they show that managerial characteristics are key determinants of firms’ capital structures. We differ from their study by developing a framework with moral hazard (effort and project selection) and incentive compensation for managers. Further, in contrast with their study, long-term debt is risky in our framework so that credit spreads are nonzero.
Another strand of the literature analyzes the design of financial securities in frameworks in which all agents are risk-neutral (for example, DeMarzo and Fishman, 2006, DeMarzo and Sannikov, 2006, Biais et al, 2007). Because these studies focus on security design, they abstract from imperfections such as taxes. As our objective is to obtain quantitative assessments of the effects of managerial discretion, we incorporate taxes in our framework, which have a first order effect on firms’ capital structures as shown by recent studies (for example, Hennessy and Whited, 2005, Strebulaev, 2007). This study, therefore, takes the perspective of “tradeoff” models that a firm’s capital structure reflects the effects of external imperfections such as taxes and bankruptcy costs as well as internal agency conflicts among its various stakeholders. We also differ from the above studies in that the manager is risk-averse and can dynamically alter the firm’s projects.

3 The Model

At date zero, the manager of an all-equity firm approaches the capital markets to obtain financing for a capital investment \( I > 0 \). The manager raises the required investment through a combination of equity and debt. In particular, the manager could choose pure equity or pure debt financing. The amount raised through external financing could exceed \( I \). The manager’s ownership stake in the initial all-equity firm is \( g_{\text{initial}} \in (0, 1) \). The firm’s claims are publicly traded after date zero. The total earnings before interest and taxes (EBIT) are distributed among all the firm’s claimants: the manager, shareholders, debtholders, and the government (through taxes). The manager affects earnings through her ability as well as her dynamic effort and project choices. For simplicity, we ignore personal taxes and directly model the effects of taxes through a constant effective corporate tax rate \( \tau \in (0, 1) \). There are no losses of tax shields in financial distress. Security issuance costs are negligible and the risk-free interest rate is the same for all market participants.

All agents are fully rational and all structural parameters of the model are common knowledge. The firm’s outside investors are well-diversified value-maximizers. We carry out our analysis in the risk-neutral probability under which outside investors are risk-neutral (see Ch. 12 of Duffie, 2001).
3.1 The Firm’s Total EBIT Flow

The model is in continuous time with a time horizon $[0, \infty)$. The continuous time infinite horizon formulation is an analytically convenient abstraction of a more realistic discrete date, finite horizon framework in which the time interval between successive dates (for example, one quarter or one year) is “small” relative to the time horizon of the problem (for example, 10 or 20 years). The infinitesimal time interval $[t, t + dt]$ in the continuous time model should, therefore, be interpreted as an interval such as one quarter or one year in the real world.

In any period $[t, t + dt]$ ; $t \in [0, \infty)$, the firm’s existing assets generate a total EBIT flow $P(t)dt$ without any actions by the manager. The EBIT rate $P(\cdot)$ from existing assets is the key state variable in the model. The manager affects the total EBIT flow over time through her ability as well as her effort and project choices.

Effect of Manager’s Actions on the EBIT Flow in the Current Period

In each period $[t, t + dt]$, the manager chooses exactly one of two possible projects, labeled 1 and 2. If the manager exerts effort $e(t) > 0$ and chooses project $i \in \{1, 2\}$, the EBIT flow is

$$
\begin{align*}
\text{EBIT flow from existing assets} & \quad \text{Incremental EBIT flow generated by manager} \\
& \quad \frac{\text{d}Q_{i,e(t)}(t)}{P(t)dt} + P(t)[(\ell + e(t))dt + s_i dW(t)],
\end{align*}
$$

(1)

where $W$ is a Brownian motion. In (1), $\ell > 0$ is the manager’s ability, which is constant through time and is common knowledge.\(^2\) The parameters $s_1, s_2$ could be viewed as the manager’s human capital risks since they are associated with her human capital inputs (see also footnote 2). Alternately, they could be viewed as the projects’ earnings risks since they determine the standard deviation of earnings in each period under the projects. For concreteness, we adopt the latter terminology. We assume that $s_1 \geq s_2$, that is, project 1’s earnings risk is greater than or equal to that of project 2.

Effect of Manager’s Project Choices on EBIT Flows in Future Periods

The manager’s project choices also affect earnings in future periods by influencing the evolution of the firm’s assets or, alternately, the “scale” of the firm. If the manager chooses project $i \in \{1, 2\}$ in period $[t, t + dt]$, the process $P(\cdot)$, which determines the level of the firm’s EBIT flow in each period by (1), evolves as

$$
\text{d}P(t) = P(t)[\mu_i dt + \sigma_i dB(t)],
$$

(2)

\(^2\) Alternately, we could allow for the manager’s ability to be stochastic, but with a constant mean $\ell$, by interpreting the expression $\ell + s_i dW(t)$ as the manager’s ability (see also Berk et al, 2006).
where $B$ is a Brownian motion that is independent of $W$. The projects’ volatilities $\sigma_1, \sigma_2$ in (2) are positive constants with $\sigma_1 > \sigma_2$. The evolution of the firm’s assets due to the manager’s project choices is risky with project 1 being riskier than project 2. We refer to $\sigma_1, \sigma_2$ as the firm’s project risks to distinguish them from the firm’s earnings risks $s_1, s_2$. As we show later, the firm’s project risk and earnings risk have differing effects on its capital structure.

Note that, because our analysis is in the risk-neutral probability, the project drifts $\mu_1, \mu_2$ are actually the risk-neutral or risk-adjusted expected growth rates of $P(\cdot)$ under the two projects. We allow for $\mu_1$ to be greater than, equal to, or less than $\mu_2$. The project parameters $s_1, s_2, \mu_1, \mu_2, \sigma_1, \sigma_2$, which determine the EBIT flows over time, are common knowledge. The information generated by the EBIT process and the process $P(\cdot)$ is $\{\mathcal{F}_t\}$.

**Assumption 1**

*The manager’s effort choices are unobservable and, therefore, non-contractible. Her project choices, the total EBIT flow process $Q_{i,e}(\cdot)$, and the process $P(\cdot)$ are contractible.*

**EBIT Flow in the Absence of a Manager**

If the current manager leaves the firm (voluntarily or not) at any date $t$, and is not replaced by a new manager, the EBIT flow to the firm over the period $[t, t + dt]$ is the EBIT flow from existing assets $P(t)dt$. In the absence of a manager, however,

$$P(t + dt) = \vartheta P(t) \text{ where } \vartheta << 1. \tag{3}$$

The existence of a manager is, therefore, critical to the firm’s operations. The firm cannot generate an EBIT flow above the level $P(t)dt$ without a manager in place and, moreover, its assets depreciate so that its earnings in future periods are also negatively affected.

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3Since the manager chooses exactly one of the two projects in each period, we assume, without loss of generality, that the EBIT flows in each period under either project are driven by the same Brownian motion $W$ to simplify the notation. Similarly, the evolutions of the state variable $P(\cdot)$ under the two projects are driven by the same Brownian motion $B$. We could allow for the Brownian motions $W$ and $B$ to be correlated with each other. This generalization, however, introduces additional parameters that are difficult to identify when we calibrate the model to the data.

4We could extend the model to explicitly incorporate physical capital investments in each period. In this extension, earnings are affected by physical capital and the manager’s human capital. The analysis of this extension leads to implications for how managerial discretion affects the firm’s physical capital investments, but does not significantly alter the main results of this study regarding the effects of managerial flexibility in financing, effort and project selection. To simplify the analysis, therefore, we do not explicitly incorporate physical capital investment. We can also extend the framework to incorporate costs of switching between projects as well as allow for the manager to choose between multiple (more than two) projects over time. The details of these extensions are available upon request.
3.2 The Objectives of Outside Investors and the Manager

The firm’s outside investors are well-diversified price-takers who unanimously seek to maximize the market values of their claims. As mentioned earlier, our analysis is carried out under the risk-neutral probability in which investors are risk-neutral. In contrast, the manager has significant inalienable and non-diversifiable human capital invested in the firm, and is, therefore, exposed to firm-specific risk. She requires a premium for bearing firm-specific risk.

As in DeMarzo and Fishman (2006), the manager has linear inter-temporal preferences with a subjective discount rate. In contrast with DeMarzo and Fishman (2006), and similar to Alvarez and Jermann (2000), however, the manager’s subjective discount rate is \( \text{stochastic} \), which reflects her costs of bearing firm-specific risk. More precisely, the manager values a stream of future payoffs as their expectation weighted by her \textit{subjective valuation kernel} \( \Lambda^M(\cdot) \) that is given by

\[
\Lambda^M(t) = \exp \left[ -rt - \frac{1}{2}(\gamma_1)^2 t - \frac{1}{2}(\gamma_2)^2 t - \gamma_1 W(t) - \gamma_2 B(t) \right].
\] (4)

In the above, \( r \) is the constant risk-free rate, and \( \gamma_1 > 0, \gamma_2 > 0 \) are the manager’s constant subjective costs of bearing the risks associated with the Brownian motions \( W \) and \( B \), respectively, which determine the earnings and project risks (see 1 and 2).

If the manager’s compensation in period \([t, t+dt]\) is \( dc(t) \) and her effort level is \( e(t) \), her valuation of her future \textit{total} payoffs (including her disutility of effort) is

\[
U(c, e) = E \left[ \int_0^\infty \Lambda^M(t)[dc(t) - \frac{1}{2} \kappa e(t)^2 P(t) dt] \right],
\] (5)

where \( \frac{1}{2} \kappa e(t)^2 P(t) dt \) (\( \kappa > 0 \) is a constant that is common knowledge) is the manager’s disutility of effort in period \([t, t+dt]\) that is expressible in monetary terms. The manager’s disutility of effort is proportional to the state variable \( P(\cdot) \). Because \( P(\cdot) \) effectively represents the “scale” of the firm, the manager’s disutility of effort increases with the scale of the firm.

In the formulation (4), we could, in general, allow for the manager to have differing costs of risk, \( \gamma_1, \gamma_2 \), associated with the two Brownian motions \( W \) and \( B \). It is, however, difficult to separately identify \( \gamma_1 \) and \( \gamma_2 \) when we calibrate the model to the data. We, therefore, assume henceforth that \( \gamma_1 = \gamma_2 = \gamma > 0 \). From (4) and (5), when \( \gamma = 0 \), the manager is perfectly diversified and incurs no additional costs of bearing firm-specific risk. The parameter \( \gamma \), therefore, directly captures the manager’s lack of diversification and determines her costs of bearing firm-specific risk. We
indirectly infer the baseline value of \( \gamma \), which we henceforth refer to as the manager’s cost of risk, by calibrating the model to the data in Section 7.

Analogous to the theory of risk-neutral valuation in traditional asset pricing theory (Duffie, 2001), the form of the manager’s valuation kernel \( \Lambda^M(\cdot) \) permits a convenient representation of the manager’s valuation of her future total payoffs as their discounted expectation, where the discounting is at the risk-free rate and the expectation is under an equivalent probability measure that we term the manager’s subjective valuation probability. More precisely, we can use Girsanov’s theorem (see Chapter 6 of Duffie, 2001) to show that the manager’s objective function (5) can also be expressed as

\[
U(c,e) = E^M\left[ \int_0^\infty \exp(-rt)\left[ dc(t) - \frac{1}{2} ke(t)^2 P(t) dt \right] \right],
\]

where the expectation is under the manager’s subjective valuation probability. The processes \( Q_{i,e}(\cdot) \) and \( P(\cdot) \) evolve as follows in the manager’s subjective valuation probability.

\[
dQ_{i,e}(t) = P(t) dt + P(t) \left[ (\ell + e(t) - \gamma s_i) dt + s_i dW^M(t) \right],
\]

\[
dP(t) = P(t) \left[ (\mu_i - \gamma \sigma_i) dt + \sigma_i dB^M(t) \right],
\]

where

\[
W^M(t) = W(t) + \gamma t, \quad B^M(t) = B(t) + \gamma t
\]

are Brownian motions in the manager’s valuation probability. In (8),

\[
\mu_1^M = \mu_1 - \gamma \sigma_1, \quad \mu_2^M = \mu_2 - \gamma \sigma_2
\]

are the drifts of the projects in the manager’s subjective valuation probability. Alternately, these are the risk-adjusted drifts of the projects from the manager’s perspective.\(^5\) The following assumption ensures that all value functions are finite.

**Assumption 2**

\( r > \mu_i \), which implies that \( r > \mu_i^M \), \( i \in \{1, 2\} \).

### 3.3 The Long-Term Debt Structure

For tractability, we assume that any long-term debt issued at date zero has infinite maturity, is non-callable, and is completely amortized so that debtholders are entitled to a coupon payment

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\(^5\)The manager’s objective (6) implies that the manager values a deterministic stream of payoffs by discounting them at the risk-free rate \( r \).
θdt in each period \([t, t + dt]\). The coupon payment per unit time, θ, which determines the firm’s long-term debt structure, is later determined endogenously. Hereafter, we refer to the coupon payment rate as simply the *coupon*. For the time being, the firm’s capital structure consists of equity and long-term debt. In Section 6, we implement the manager’s optimal contract through financial securities. In this implementation, the firm’s capital structure is *dynamic*. It consists of inside equity, outside equity, long-term debt issued at date zero, and dynamic risk-free borrowing or lending (short-term debt or cash).  

### 3.4 Contracting and Bankruptcy

The manager receives dynamic incentives through contracts that could be explicitly contingent on the contractible EBIT flow process. As in studies such as Aghion and Bolton (1992), we consider an “incomplete contracting” framework in which only single-period contracts are enforceable. As argued by studies in the incomplete contracting literature, long-term contracts could be impossible to enforce for several reasons including costs of writing and enforcing long-term agreements, managers’ inalienable rights to withdraw their human capital, the fact that the firm’s pool of shareholders changes over time, changes in the “outside options” of both parties and the possibility of hold up, etcetera. Further, as in Fudenberg et al (1990), Aghion and Bolton (1992), and studies in the “signalling” literature (see Chapter 6 of Tirole, 2006), the manager has the bargaining power and offers a contract to shareholders in each period. The existence of significant bargaining power and real authority for managers is consistent with empirical and anecdotal evidence. Murphy (1999) reports that CEOs often also serve as chairmen of the board of directors, influence the selection of board members, and effectively determine their own compensation structures. More generally, the “manager” in our framework represents the firm’s “insiders” who have real authority over the firm’s decisions (financing, compensation, project selection) and possess bargaining power with “outsiders”. The portion of total output that the manager appropriates due to her bargaining power are her *endogenous* control benefits.

At the beginning of each period, the manager offers a contract to shareholders, which specifies her end-of-period payoff as well as her project and effort choices or *actions* over the period. The

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6We can extend the model to allow for finite maturity long-term debt. Specifically, we can adopt the approach of Leland (1998) wherein long-term debt has infinite maturity, but is continuously retired at par and replaced with identical new debt, which leads to a finite “average” debt maturity. This extension, the details of which are available upon request, does not alter the main implications of this study.
contracts must be incentive compatible for the manager with respect to her unobservable effort choices and satisfy shareholders’ dynamic participation constraints (we describe these in Section 3.7). In each period, the manager receives her contractually specified payoff from the total earnings net of corporate taxes. The remaining earnings are distributed among debtholders and shareholders.

As in Leland and Toft (1996) and Leland (1998), debt payments are serviced entirely as long as the firm is solvent. In financial distress, debt payments are serviced through the dilution of equity. Since shareholders are protected by limited liability, bankruptcy occurs when it is no longer optimal for shareholders to continue servicing debt. The absolute priority of debt is enforced at bankruptcy and the firm is subsequently controlled by debtholders as an all-equity firm. The manager continues to operate the firm after bankruptcy and contracts with the new shareholders of the firm: the debtholders. The firm bears deadweight costs as a result of bankruptcy that are reflected in a proportional reduction in future earnings. More precisely, if $T_b$ is the bankruptcy (stopping) time, the state variable $P(\cdot)$ falls by a proportion $\varsigma \in (0, 1)$ at bankruptcy so that

$$P(T_b) = (1 - \varsigma)P(T_b-) .$$

The post-bankruptcy period is otherwise identical to the period during which the firm is solvent. The manager dynamically chooses between projects 1 and 2 in each period and exerts effort. The effects of the manager’s actions on total earnings are as described in (1) and (2). The bankruptcy costs modeled above comprise of direct costs as well as indirect costs that arise from imperfections in the firm’s product market such as its relationships with customers and suppliers, which directly affect its asset base or output-generating capacity. In particular, these costs are due to sources external to the manager-firm relationship.

We simultaneously describe the contracting before and after bankruptcy because, in equilibrium, post-bankruptcy actions and earnings, which are rationally anticipated by all agents, affect pre-bankruptcy actions and earnings. Without loss of generality, the sequence of single-period contracts between the manager and shareholders before bankruptcy could be viewed as a single long-term contract that is implemented by this sequence (see Fudenberg et al, 1990). Similarly, the sequence of single-period contracts between the manager and debtholders after bankruptcy are viewed as a single long-term contract. Without loss of generality, we further simplify the notation by

\footnote{We could generalize the model to allow for debtholders to re-lever the firm after bankruptcy, but this complicates the notation without altering the main implications of the study.}
concatenating the pre and post-bankruptcy contracts and directly referring to the single combined contract for the manager. The pre-bankruptcy portion of the contract is between the manager and shareholders and the post-bankruptcy portion is between the manager and debtholders.

Formally, a contract $\Gamma \equiv [dc_m(\cdot), \xi(\cdot), e(\cdot)]$ for the manager is a stochastic process describing the manager’s compensation payments $dc_m(\cdot)$, project choices $\xi(\cdot)$, and effort choices $e(\cdot)$, before and after bankruptcy. The process $dc_m(\cdot)$ is $\mathcal{F}_t$-adapted while $\xi(\cdot)$ and $e(\cdot)$ are $\mathcal{F}_t$-predictable. Shareholders’ choice of bankruptcy time is an $\mathcal{F}_t$-stopping time $T_b$.

### 3.5 Payoffs to Shareholders and Debtholders

For a contract $\Gamma \equiv [dc_m(\cdot), \xi(\cdot), e(\cdot)]$ and bankruptcy time $T_b$, it follows from (1) that the firm’s total after-tax earnings in any period $[t, t + dt]$ before and after bankruptcy is

$$dc_f(t) = \begin{cases} (1 - \tau)dQ_{\xi(t),e(t)}(t) + \tau \theta dt, & t < T_b \\ (1 - \tau)dQ_{\xi(t),e(t)}(t) & t \geq T_b \end{cases}$$

(12)

The above reflects the shielding of the coupon payment $\theta dt$ from corporate taxes. The payoff to debtholders during the period is

$$dc_d(t) = \begin{cases} \theta dt, & t < T_b \\ dc_f(t) - dc_m(t) & t \geq T_b \end{cases}$$

(13)

After bankruptcy, debtholders receive the residual payout flow after payments to the manager as described by the second equation in (13). From (12) and (13), the payoff to shareholders, which is the total after-tax earnings net of payments to the manager and debtholders is

$$dc_s(t) = \begin{cases} [dc_f(t) - dc_m(t) - dc_d(t)], & t < T_b \\ 0 & t \geq T_b \end{cases}$$

(14)

### 3.6 The Dynamic Bankruptcy Game

As the manager’s contract is affected by the possibility of bankruptcy and also influences the timing of bankruptcy, the contract and the bankruptcy time must be simultaneously determined in equilibrium of a dynamic game, the bankruptcy game, between the manager and shareholders.
Given that shareholders will declare bankruptcy at the stopping time $T_b$, the manager’s “best response” or optimal choice of contract solves

$$\hat{\Gamma}(T_b) = \arg \max_{\Gamma \equiv (d_{cm}(\cdot, \xi)(\cdot), e(\cdot)) \in \Omega(T_b)} E^M_t \left[ \int_{t=0}^{T_b} \exp(-r t) \left[ d_{cm}(t) - \frac{1}{2} \kappa e(t)^2 P(t) \right] dt \right]$$

$$+ \int_{t=T_b}^{\infty} \exp(-r t) \left[ d_{cm}(t) - \frac{1}{2} \kappa e(t)^2 P(t) \right] dt.$$  

In the above, the expectation is under the manager’s subjective valuation probability defined in (7) and (8). For clarity, we explicitly indicate the pre- and post-bankruptcy periods in the manager’s objective function above. The manager chooses her optimal contract from the set $\Omega(T_b)$ of feasible contracts which are implemented by sequences of single-period contracts and satisfy participation and incentive compatibility constraints that we describe in the next sub-section. The notation indicates that this set depends on the bankruptcy time $T_b$.

Given that the manager chooses a contract $\Gamma$, shareholders’ “best response” or optimal choice of bankruptcy time solves

$$\hat{T}_b(\Gamma) \equiv \arg \max_{T \in \mathcal{F}_t}\mathbb{E}_{\xi, e} \left[ \int_{t=0}^{T} \exp(-r t) d_{cm}(t) \right],$$

where the maximization is over all $\mathcal{F}_t$-stopping times.

In equilibrium, the manager’s choice of contract and shareholders’ choice of bankruptcy time must simultaneously solve (16) and (17), that is, they represent a fixed point of the dynamic game described by (16) and (17). Note that the manager’s actions influence the timing of bankruptcy, and are also affected by the possibility of bankruptcy. Since the equilibrium contract and bankruptcy time solve (16) and (17) for a given long-term debt coupon $\theta$, we denote these as

$$[\Gamma^*(\theta); T^*_b(\theta)].$$

For simplicity, whenever there is no danger of confusion, we occasionally suppress the argument indicating the dependence of the contract and bankruptcy time on the long-term debt structure $\theta$.

### 3.7 Participation and Incentive Compatibility

In problem (16), the manager optimizes over the set of feasible contracts $\Omega(T_b)$, which are implemented by sequences of single-period contracts and satisfy participation and incentive compatibility constraints that we now describe. To simplify the discussion in this section, we use the generic term
“shareholders” to also refer to the debtholders after bankruptcy because they are the new shareholders of the all-equity firm post-bankruptcy.

By the discussion at the end of Section 3.1, \( P(t)dt \) is the EBIT flow in period \([t, t + dt]\) from the firm’s existing assets, that is, in the hypothetical absence of a manager. Hence, the after-tax earnings that shareholders and debtholders can guarantee themselves for the period is \((1 - \tau)P(t)dt + \tau \theta dt\) before bankruptcy and \((1 - \tau)P(t)dt\) after bankruptcy. Since debtholders are contractually guaranteed a payoff of \( dc_d(t) \) given by (13) before bankruptcy, shareholders can guarantee themselves a payoff of

\[
(1 - \tau)P(t)dt + \tau \theta dt - dc_d(t) = (1 - \tau)(P(t) - \theta) dt
\]

Similarly, shareholders can guarantee themselves a payoff \((1 - \tau)P(t)dt\) after bankruptcy.

The existence of a manager is critical to the firm. We assume the depreciation proportion \( \vartheta \) in (3) is sufficiently low that the value of equity in the absence of a manager (in “autarky”) is substantially lower than its value if a manager is in place and shareholders receive their reservation payoff in each period. Further, all potential managers are identical in all respects to the incumbent. Hence, shareholders accept a contract if and only if the market value of equity at each date under the contract is at least as great as the market value of equity at that date if they receive their reservation payoff \((1 - \tau)(P(t) - \theta) dt\) in each subsequent period until bankruptcy. Hence, a feasible contract \( \Gamma \) must satisfy the following dynamic participation constraints for shareholders:

\[
E_{\xi, e} \left[ \int_{u=t}^{T_b} \exp(-ru) dc_d(u) | F_t \right] \geq E_{\xi} \left[ \int_{u=t}^{T_b} \exp(-ru)((1 - \tau)(P(u) - \theta) du) | F_t \right], \quad t < T_b, \\
E_{\xi, e} \left[ \int_{u=t}^{\infty} \exp(-ru) dc_d(u) | F_t \right] \geq E_{\xi} \left[ \int_{u=t}^{\infty} \exp(-ru)((1 - \tau)(P(u) du) | F_t \right], \quad t \geq T_b
\]

The subscripts on the expectations on the left-hand sides of the inequalities above indicate its dependence on the manager’s actions \( \xi(\cdot), e(\cdot) \). The expectations on the right hand sides depend on the project choices \( \xi(\cdot) \) specified by the contract because they affect the evolution of the state variable \( P(\cdot) \) (see 2) (recall that the contract is implemented by a sequence of single-period contracts).

A contract \( \Gamma \equiv (dc_m(\cdot), \xi(\cdot), e(\cdot)) \) is incentive compatible with respect to the manager’s effort choices if and only if it is optimal for the manager to exert effort \( e(\cdot) \) specified by the contract given the payoff stream \( dc_m(\cdot) \), project choices \( \xi(\cdot) \), and bankruptcy time \( T_b \).
3.8 The Optimal Long-Term Debt Structure

The manager chooses the firm’s long-term debt structure to maximize her subjective valuation of her payoff at date zero and her future total payoffs from operating the firm (including her disutility of effort). As the manager’s ownership stake in the initial all-equity firm is \( g_{\text{initial}} \), her payoff at date zero equals the proportion \( g_{\text{initial}} \) of the total proceeds from financing net of the required investment outlay \( I \). With rational expectations, the proceeds from debt and equity issuance at date zero are equal to their respective market values. For a given coupon, \( \theta \), the market values of long-term debt, \( D_\theta(0) \), and equity, \( S_\theta(0) \), are given by

\[
D_\theta(0) = E_{\xi^*(\theta),e^*(\theta)} \left[ \int_{t=0}^{T_b^*} \exp(-rt)dc_d^*(t) + \int_{t=T_b^*}^\infty \exp(-rt)dc_d^*(t) \right], \tag{21}
\]

\[
S_\theta(0) = E_{\xi^*(\theta),e^*(\theta)} \left[ \int_{t=0}^{T_b^*} \exp(-rt)dc_s^*(t) \right]. \tag{22}
\]

In (21) and (22), \( \xi^*(\theta), e^*(\theta) \) are the manager’s project and effort choices, and \( T_b^*(\theta) \) is the bankruptcy time in equilibrium of the bankruptcy game for the given coupon \( \theta \) defined in (18) (the arguments indicate that these depend on \( \theta \)). The payoffs to debtholders and shareholders before and after bankruptcy, \( dc_d^*(\cdot), dc_s^*(\cdot) \), are described in (13) and (14). The manager’s payoff at date zero is, therefore, \( g_{\text{initial}}[D_\theta(0) + S_\theta(0) - I] \). The manager’s valuation of her future total payoffs or continuation value at date zero is

\[
M_\theta(0) = E_{\xi^*(\theta),e^*(\theta)} \left[ \int_{t=0}^{T_b^*} \exp(-rt)[dc_m^*(t) - \frac{1}{2}\kappa e_m^*(t)^2P(t)]dt + \int_{t=T_b^*}^\infty \exp(-rt)[dc_m^*(t) - \frac{1}{2}\kappa e_m^*(t)^2P(t)]dt \right]. \tag{23}
\]

where the expectation above is under the manager’s subjective valuation probability. The manager’s optimal choice of long-term debt structure at date zero solves

\[
\theta^{\text{opt}} = \arg \max_\theta g_{\text{initial}}[D_\theta(0) + S_\theta(0) - I] + M_\theta(0). \tag{24}
\]

In equilibrium of the entire model in which the initial debt structure, the manager’s contract, and the bankruptcy time are all determined endogenously, the coupon on long-term debt is \( \theta^{\text{opt}} \), and the \textit{ex post} contract of the manager and the bankruptcy time are \( \Gamma^*(\theta^{\text{opt}}), T_b^*(\theta^{\text{opt}}) \), respectively.
4 The Equilibrium

We analyze the equilibrium in three steps. In step one, we derive the manager’s optimal contract for a given bankruptcy time $T_b$ and long-term debt structure $\theta$ as described in (16). In step two, we prove the existence of equilibrium of the bankruptcy game for a given long-term debt structure $\theta$ in which the contract and bankruptcy time simultaneously solve (16) and (17). In step three, we characterize the manager’s optimal choice of long-term debt structure, which solves (24). We are able to analytically determine the properties of the manager’s optimal contract for a given bankruptcy time and coupon (step one). The contract and bankruptcy time in equilibrium of the bankruptcy game (step two), and the manager’s optimal long-term debt structure choice (step three) are computed numerically in Section 7.

We consider stationary, Markov perfect equilibria of the dynamic bankruptcy game in which the manager’s and shareholders’ decisions at any date only depend on the current value of the state variable $P(\cdot)$. In other words, the manager’s contract in any period $[t, t + dt]$ only depends on $P(t)$ and there exists a constant bankruptcy level such that shareholders declare bankruptcy at date $t$ if and only if $P(t)$ is lower than the bankruptcy level. The bankruptcy time is the time at which the process $P(\cdot)$ hits the bankruptcy level, which is an $\mathcal{F}_t$-stopping time.

The stationary, Markov perfect equilibrium of the dynamic bankruptcy game for a given long-term debt structure $\theta$ is characterized by the pair $(\Gamma^*(\theta), p^*_b(\theta))$. $\Gamma^*(\theta)$ is the manager’s optimal contract given that bankruptcy is declared when the state variable falls to the level $p^*_b(\theta)$. $p^*_b(\theta)$ is shareholders’ optimal choice of bankruptcy level given the manager’s contract $\Gamma^*(\theta)$.

4.1 The Optimal Contract for a Given Bankruptcy Level and Debt Structure

4.1.1 The Manager’s Optimal Compensation and Effort Choices

The following theorem characterizes the manager’s compensation and effort in each period under the optimal contract for a given bankruptcy level $p_b$.

Theorem 1 (The Manager’s Optimal Compensation and Effort)

For a given bankruptcy level $p_b$, the contract $\Gamma \equiv (d_{cm}(\cdot), \xi(\cdot), e(\cdot))$ is optimal for the manager only if the following hold at each date $t$ before or after bankruptcy:

(a) The dynamic participation constraints (20) are satisfied with equality at each date.
(b) If $\xi(t) = i \in \{1, 2\}$ (the manager chooses project $i \in \{1, 2\}$ in period $[t, t + dt]$ under the contract), her end-of-period payoff is

$$dc_m(t) = a(t)P(t)dt + b(t)dQ_{i,e(t)}(t), \text{ where}$$

$$b(t) = 1 - \tau - \kappa \gamma s_i; \quad a(t) = (1 - b(t))(\ell + b(t)/\kappa) - b(t); \quad e(t) = \frac{b(t)}{\kappa}$$

(c) The manager’s conditional valuation of her end-of-period total payoff (including her disutility of effort) is

$$E^M\left[ \exp(-rt)\left[ dc_m(t) - \frac{1}{2}\kappa(e(t))^2P(t) \right] | \mathcal{F}_t \right] = g(t)P(t)dt, \text{ where}$$

$$g(t) = g_i = \ell + b(t)\left( \frac{1}{\kappa} - \gamma s_i \right) - \frac{(b(t))^2}{2\kappa}$$

Proof. See Appendix A.

As the bankruptcy level $p_b$ is fixed, and only single-period contracts are enforceable, it is suboptimal for the manager to cede any surplus to the firm’s current shareholders (above their reservation payoff) in any period. Hence, the dynamic participation constraints (20) are satisfied with equality under the optimal contract. By (25), the manager’s optimal compensation is affine in the EBIT flow. The payout flow and the manager’s disutility of effort are proportional to the state variable $P(\cdot)$. Therefore, the parameter $a(t)$, which determines the fixed portion of her compensation, her “pay performance sensitivity” $b(t)$, and effort $e(t)$ in any period are constants conditional on her project choice $i \in \{1, 2\}$ for the period. By (26), the manager’s incentive structure is locally determined by the firm’s earnings risks $s_1, s_2$. As we discuss in the next sub-section, because the manager’s optimal dynamic project choices are affected by the project risks $\sigma_1, \sigma_2$, her global payoff structure—the variation of her payoffs over the entire region of firm performance—depends on the earnings and project risks $s_1, s_2, \sigma_1, \sigma_2$.

4.1.2 The Manager’s Optimal Dynamic Project Choices

By Theorem 1, it suffices to restrict consideration to the sub-class of contracts $(dc_m(\cdot), \xi(\cdot), e(\cdot))$ for which $dc_m(t)$ and $e(t)$ are given by (25) and (26). Hence, a contract is optimal for the manager if and only if her dynamic project choices solve

$$\sup_{\xi} E^M_{\xi} \int_{t=0}^{\infty} \exp(-rt)\left[ dc_m(t) - \frac{1}{2}\kappa e(t)^2P(t) \right] dt,$$
where the expectation now only depends on the manager’s dynamic project choice policy \( \xi(\cdot) \). To analyze the stochastic control problem \((29)\), we begin by using the law of iterated expectations and \((27)\) to rewrite \((29)\) as

\[
\sup_{\xi} E_{\xi}^{M} \int_{t=0}^{\infty} \exp(-rt) E_{\xi}^{M}[dc_{m}(t) - \frac{1}{2}KE(t)^{2}P(t)dt|F_{t}] \\
= \sup_{\xi} E_{\xi}^{M} \int_{t=0}^{\infty} \exp(-rt)g(t)P(t)dt,
\]

where \( g(t) \) is defined in \((27)\). Let \( M(p) \) denote the optimal value function for the manager at any date \( t \) when the current value of the state variable \( P(t) = p \). This is the manager’s expected continuation value from following her optimal project choice policy. By the dynamic programming principle of optimality, we can heuristically write

\[
M(p) = \max_{i \in \{1, 2\}} \left[ \text{Current Period Flow} \int g_{i}pd\tau + \exp(-rd\tau) E_{\tau}^{M}[M(P_{i}(t+\tau))]|F_{\tau} \right],
\]

where \( P_{i}(t+\tau) \) is the end-of-period value of the state variable \( P \) when the manager chooses project \( i \in \{1, 2\} \) for the period. Under the manager’s subjective valuation probability, it follows from \((8)\) and \((10)\) that

\[
dP_{i}(t) = P_{i}(t)[\mu_{i}^{M} dt + \sigma_{i}dB^{M}(t)].
\]

Applying Ito’s lemma in \((31)\), \( M(p) \) satisfies the following formal Hamilton-Jacobi-Bellman equation (see Fleming and Soner, 1992):

\[
\sup_{i \in \{1, 2\}} \left[ L_{i}^{M}(M) + g_{i}p \right] = 0,
\]

where \( L_{1}^{M}, L_{2}^{M} \) are differential operators defined as

\[
L_{i}^{M}(M) = \frac{1}{2} \sigma_{i}^{2} p \frac{d^{2}M}{dp^{2}} + \mu_{i}^{M} p \frac{dM}{dp} - rM.
\]

The manager’s optimal policy is determined by the sign of her project tradeoff

\[
\Omega_{1} - \Omega_{2} := \frac{g_{1}}{r - \mu_{1}^{M}} - \frac{g_{2}}{r - \mu_{2}^{M}}.
\]

**Theorem 2 (The Manager’s Optimal Dynamic Project Choices)**

(a) Suppose that the project tradeoff \( \Omega_{1} - \Omega_{2} > 0 \). The manager always chooses the high-risk project 1 after bankruptcy. Prior to bankruptcy, there exists \( \hat{p} \geq p_{b} \) such that the manager chooses project 1 whenever \( P(\cdot) > \hat{p} \), and chooses project 2 whenever \( p_{b} < P(\cdot) < \hat{p} \).
Suppose that the project tradeoff $\Omega_1 - \Omega_2 \leq 0$. The manager always chooses the low-risk project 2 after bankruptcy. Prior to bankruptcy, there exists $\tilde{p} \geq p_a$ such that the manager chooses project 2 whenever $P(\cdot) > \tilde{p}$, and chooses project 1 whenever $p_a < P(\cdot) < \tilde{p}$.

Proof. In Appendix B.

When the project tradeoff is positive, the manager’s valuation of her future payoffs from choosing project 1 exceeds her valuation from choosing project 2 in the hypothetical absence of bankruptcy so that project 1 “dominates” project 2. Hence, the manager always chooses project 1 after bankruptcy when the firm is all-equity. Prior to bankruptcy, in “good” states where the state variable $P$ exceeds a trigger, the low probability of future bankruptcy implies that the manager prefers project 1. In “bad” states where $P$ is below the trigger, the high probability of bankruptcy and the associated personal costs for the manager could cause the manager to lower risk by choosing project 2. The manager switches projects or shifts risk only when her personal costs significantly outweigh the gains from choosing project 1. The intuition above is reversed when the project tradeoff is negative so that project 2 “dominates” project 1. Hence, depending on her risk aversion, and the projects’ drifts and volatilities, the manager could either increase or decrease risk in financial distress.

The proof of the theorem also describes the (necessary and sufficient) conditions under which the manager shifts risk (switches projects) prior to bankruptcy. The characterization of the structure manager’s optimal policy (in particular, the fact that it is characterized by a single switching trigger), and the precise conditions for risk-shifting, also significantly simplify the numerical algorithm we use to analyze the impact of managerial discretion in Section 7.

Theorem 2 implies that, in general, the manager’s optimal dynamic project choices vary with the firm’s performance as measured by the state variable $P$. By Theorem 1, the manager’s pay-performance sensitivity and expected payoff in any period depends on her choice of project. These results taken together suggest that the manager’s payoff structure over the entire range of firm performance is globally nonlinear, which is consistent with payoff structures observed in reality. If the manager had no discretion in project selection, however, her pay-performance sensitivity would be constant over time by Theorem 1 and her payoff structure would be affine in firm performance. Therefore, the incorporation of managerial discretion in project selection in the model plays a key role in generating the observed nonlinearity in payoff structures.
4.2 The Endogenous Bankruptcy Level for a Given Long-Term Debt Structure

For a given bankruptcy level \( p_b \), the manager’s optimal contract is described by Propositions 1 and 2, where her optimal project choice policy is characterized by a switching trigger \( \hat{p} \). Since the manager’s optimal policy depends on the bankruptcy trigger \( p_b \), the optimal switching trigger is a function of the bankruptcy trigger. However, given that the manager follows a policy of switching projects at the trigger level \( \hat{p} \), shareholders, who are protected by limited liability, optimally choose to declare bankruptcy at a level \( p_b' \) that might differ from \( p_b \). The shareholders’ optimal choice of bankruptcy trigger depends on the switching trigger \( \hat{p} \) that characterizes the manager’s project choice policy. Hence, we have the sequence of mappings \( p_b \to \hat{p} \to p_b' \), which leads to the mapping

\[ \Psi : p_b \to p_b'. \]

In the stationary Markov perfect equilibrium of the dynamic bankruptcy game described by (16) and (17), the endogenous bankruptcy level \( p_b^* \) is a fixed point of the mapping \( \Psi \)

\[ p_b^* = \Psi(p_b^*). \quad (36) \]

The manager’s contract \( \Gamma^* \) in equilibrium of the bankruptcy game is described by Propositions 1 and 2 with the bankruptcy level equal to \( p_b^* \).

We now make the above arguments precise. For brevity, we only describe the analysis for the scenario in which the manager’s project tradeoff is positive. Given that the manager follows a policy of switching projects at \( \hat{p} \) described by part (i) of Theorem 2, we now describe the determination of the values of debt and equity and the level at which shareholders optimally declare bankruptcy.

**Theorem 3 (The Values of Debt and Equity for a Given Project Choice Policy)**

Suppose the manager follows the policy of switching projects at the trigger \( \hat{p} \) when the firm is solvent. The market values of total outstanding debt \( D(p) \), and equity, \( S(p) \), when the value of the state variable \( P \) is \( p \), and the level \( p_b' \) at which shareholders optimally declare bankruptcy solve:

\[
L_1 D + \theta = 0 \text{ for } p > \hat{p} \\
L_2 D + \theta = 0 \text{ for } p_b' < p < \hat{p},
\]

\[
\lim_{p \to \infty} D(p) < \infty, \quad D(p_b') = \frac{(1 - \varsigma)(1 - \tau)p_b'}{r - \mu_1}. \quad (37)
\]
\[ L_1 S + (1 - \tau)(p - \theta) = 0 \text{ for } p > \hat{p} \]
\[ L_2 S + (1 - \tau)(p - \theta) = 0 \text{ for } p_b' < p < \hat{p}, \]
\[ \lim_{p \to \infty} \frac{S(p)}{p} < \infty, \quad S(p_b') = S(p_b) = 0. \]  
(38)

In the above, \( L_1, L_2 \) are operators defined as
\[ L_i(H) = \frac{1}{2} \sigma_i^2 p^2 \frac{d^2 H}{dp^2} + \mu_i p \frac{dH}{dp} - rH; \quad i \in \{1, 2\}. \]  
(39)

**Proof.** In Appendix C.

In contrast with (34), the (risk-neutral) project drifts \( \mu_1, \mu_2 \) appear in these equations because the market values of debt and equity are the risk-neutral expectations of payoffs to debtholders and shareholders, respectively, discounted at the risk-free rate. The last equations in (38) are the value matching and smooth pasting conditions for the value of equity at the endogenous bankruptcy trigger \( p_b' \) (see Leland, 1998). The bankruptcy trigger \( p_b' \) is the solution to a nonlinear equation that cannot be solved analytically, but can be easily solved numerically. We can, however, obtain analytical expressions for the values of debt and equity as functions of \( p_b' \) (see 108 and 110 in Appendix C). The set of equations (37) and (38) define the mapping \( \Psi \) from the given bankruptcy trigger \( p_b \) to the trigger \( p_b' \) at which shareholders would optimally declare bankruptcy. The following result shows the existence of a fixed point of this mapping.

**Theorem 4 (Existence of Equilibrium of the Bankruptcy Game)**

There exists a solution \( p_b^* \) to the fixed point problem described by (36), (37) and (38). Hence, there exists a stationary, Markov perfect equilibrium of the bankruptcy game.

**Proof.** In Appendix D.

### 4.3 The Optimal Long-Term Debt Structure

The third and final step in the characterization of the equilibrium is the determination of the manager’s optimal choice of coupon (which determines the firm’s long-term debt structure) as discussed in Section 3.8. The manager’s contract in equilibrium of the bankruptcy game for a given coupon is described by Propositions 1 and 2 with the bankruptcy level equal to \( p_b^* \). For expositional convenience, we now explicitly indicate the dependence of the contract and bankruptcy level on
the coupon by denoting them as $\Gamma^*(\theta)$ and $p^*_b(\theta)$, respectively. We similarly denote the switching trigger, which describes the manager’s dynamic project choices by Theorem 2, as $p^*(\theta)$.

By (24), the manager’s optimal choice of the coupon on long-term debt solves

$$
\theta^{opt} \equiv \arg \max_\theta \left[ g_{initial}[D_\theta(0) + S_\theta(0) - I] + M_\theta(0) \right],
$$

where the values of debt, $D_\theta(0)$, and equity, $S_\theta(0)$, for a given coupon $\theta$ solve (37) and (38) with the switching trigger and the bankruptcy level taking the values $p^*(\theta)$ and $p^*_b(\theta)$, respectively. By Theorem 2, the manager’s optimal value function $M_\theta(0)$ for a coupon $\theta$ (see 24) solves (87) with the switching trigger and bankruptcy level set to the values $p^*(\theta)$ and $p^*_b(\theta)$.

5 Benchmark Scenarios

To analyze the various dimensions of managerial discretion—financing, effort and project selection—we investigate three benchmark scenarios in which they are successively “shut down”.

5.1 Benchmark One: Long-Term Debt Maximizes Total Economic Value

In (24), the manager chooses the long-term debt structure to maximize the sum of her payoff at date zero and her continuation value (the valuation of her future payoffs after date zero). To investigate the effect of the manager’s discretion in long-term debt financing, we consider the benchmark scenario in which the manager chooses the long-term debt structure to maximize the total economic value, which is the discounted (at the risk-free rate) risk-neutral expectation of the firm’s total after-tax earnings less the manager’s disutility of effort. More precisely, the manager chooses the long-term debt structure to maximize

$$
\theta^{opt}_1 \equiv \arg \max_\theta D_\theta(0) + S_\theta(0) + M^1_\theta(0),
$$

where

$$
M^1_\theta(0) = E_{\xi^*_m(\theta),\xi^*_{m}(\theta)} \left[ \int_{t=0}^{\infty} \exp(-rt) \left[ dc^*_m(t) - \frac{1}{2} \kappa e^*_m(t)^2 P(t)dt \right] \right]
$$

In contrast with (23), $M^1_\theta(0)$ (the superscript indicates that this is the manager’s continuation value in benchmark scenario one) is the risk-neutral expectation of the manager’s future total payoffs rather than the expectation under her subjective valuation probability. The manager’s effort and project choices continue to be described by Theorems 1, 2, 3, and 4. However, the
coupon on long-term debt is now chosen to solve (41) instead of (40). The manager, therefore, has
discretion in effort and project choices, but chooses the long-term debt structure to maximize the
total economic value of the firm.

5.2 Benchmark Two: Long-Term Debt and Project Choices Maximize Total
Economic Value

We now “turn off” the manager’s discretion in financing and project selection. In this benchmark
scenario, the manager chooses the long-term debt structure and the firm’s projects to maximize the
firm’s total economic value. As in Section 4, we first derive the project choices and the bankruptcy
time for a given long-term debt structure, and then analyze the optimal choice of the long-term
debt structure.

The manager continues to have discretion in effort so that her compensation \(dc_m(t)\) and effort
\(e_m(t)\) in each period \([t, t + dt]\) are described by Theorem 1. For a given long-term debt structure
and bankruptcy time \(T_b\), the manager’s project choices maximize the total economic value
\[
\sup_{\xi} E_{\xi} \int_{t=0}^{\infty} \exp(-rt) \left[ dc_f(t) - \frac{1}{2} \kappa e_m(t)^2 P(t) dt \right].
\] (43)

In contrast with (29), the expectation above is in the risk-neutral probability. The term inside
the integral above is the firm’s total after-tax earnings net of the manager’s disutility of effort.
The following theorem describes the economic value-maximizing project choice policy. We omit its
proof for brevity because it follows along similar to lines to that of Theorem 2.

Theorem 5 (The Economic Value Maximizing Dynamic Project Choice Policy)

Analogous to (27), define \(\tilde{g}_1, \tilde{g}_2\) as follows:
\[
E \left[ \exp(-rdt) \left( dc_m(t) - \frac{1}{2} \kappa e(t)^2 P(t) \right) | \mathcal{F}_t \right] = \tilde{g}_i P(t) dt,
\] (44)

where the expression on the left hand side is the risk-neutral expectation of the manager’s end-of-
period total payoff.

(a) Suppose that \(\tilde{\Omega}_1 - \tilde{\Omega}_2 = \frac{1-\tau+\tilde{g}_1}{r-\mu_1} - \frac{1-\tau+\tilde{g}_2}{r-\mu_2} > 0\). There exists an endogenous trigger value \(\tilde{p} \geq p_b\)
of the state variable \(P(\cdot)\) such that the manager chooses the high risk project 1 whenever \(P(\cdot) > \tilde{p}\)
and the low risk project 2 whenever \(p_b < P(\cdot) < \tilde{p}\) when the firm is solvent. After bankruptcy, the
firm always chooses the high-risk project 1.
(b) If $\tilde{\Omega}_1 - \tilde{\Omega}_2 \leq 0$, there exists an endogenous trigger value of the state variable such that the firm chooses the low risk project 2 above the trigger and the high risk project 1 below the trigger before bankruptcy. After bankruptcy, the firm always chooses the low-risk project 2.

A direct comparison of Theorems 2 and 5 reveals that the manager’s project choice policy differs significantly from the economic value-maximizing project choice policy when the respective project tradeoffs $\Omega_1 - \Omega_2$ and $\tilde{\Omega}_1 - \tilde{\Omega}_2$ have opposite signs. For example, if $\Omega_1 - \Omega_2 > 0$, but $\tilde{\Omega}_1 - \tilde{\Omega}_2 < 0$, the manager chooses project 1 in “good” states and project 2 in “bad” states, while the firm value-maximizing policy chooses project 2 in “good” states and project 1 in “bad” states. If the project tradeoffs have the same sign, however, the project choices in the two scenarios have the same structure. Hence, the impact of the manager’s discretion in project selection on firm value is likely to be substantial when the project tradeoffs in the two scenarios have opposite signs and relatively small when they have the same sign. We explore the effects of these conflicts on the firm’s value and capital structure in Section 7.

We can proceed as in Section 4.2, to characterize the equilibrium of the bankruptcy game where the project choices and the bankruptcy level are endogenously determined. As in Section 5.1, the manager chooses the long-term debt structure to maximize the total economic value.

5.3 Benchmark Three: Long-Term Debt, Project Choices and Effort Maximize Total Economic Value

In this benchmark scenario, the manager chooses the long-term debt structure, effort, and projects to maximize the total economic value of the firm. In particular, this scenario corresponds to the case where complete contracts for the manager are enforceable. To focus attention on the impact of managerial discretion, we continue to assume that the firm incurs (direct and indirect) bankruptcy costs. As discussed earlier, these costs arise from sources external to the manager-firm relationship such as imperfections in the firm’s product market (The magnitude of distortions due to managerial discretion relative to this benchmark scenario would be further amplified if bankruptcy costs in this scenario are assumed to be absent.).

In this benchmark scenario, the manager appropriates the after-tax surplus she generates in each period and chooses her effort and projects to maximize total economic value. Therefore, the
manager’s compensation in each period is given by

\[ dc_m(t) = (1 - \tau)(dQ(t) + e(t)) - P(t)dt, \quad \xi(t) \in \{1, 2\}. \] (45)

Recall that the state variable \( P(\cdot) \) falls as in (11) at the bankruptcy time. We can characterize the manager’s optimal project choices for a given bankruptcy level through results that parallel Theorem 5 and also show the existence of equilibrium of the bankruptcy game. We do not state these results explicitly for brevity. The long-term debt structure solves

\[ \theta_{opt}^3 = \arg \max_{\theta} [D_3^3(0) + S_3^3(0) + M_3^3(0) - I], \] (46)

where \( D_3^3(0), S_3^3(0), M_3^3(0) \) are the market values of debt, equity, and the manager’s stake (the superscripts indicates that this is benchmark scenario three) when the debt structure is \( \theta \).

6 Dynamic Capital Structure

As in recent studies such as DeMarzo and Fishman (2006), DeMarzo and Sannikov (2006), and Biais et al (2007), the manager’s equilibrium contract can be implemented through dynamic holdings in financial securities, specifically “inside” equity and short-term (single-period) risk-less cash borrowing or lending. By (25), the manager’s compensation is affine in the total earnings. By (12) and (13), we can rewrite the manager’s payoff (25) in period \([t, t + dt] \) before bankruptcy as

\[ dc_m(t) = \bar{b}(t)|dc_f(t) - dc_d(t) + \bar{a}(t)dt|, \text{ where } \] (47)

\[ \bar{b}(t) = \frac{b_{opt}(t)}{1 - \tau}; \quad \bar{a}(t) = (1 - \tau)[\theta_{opt} + \frac{a_{opt}(t)}{b_{opt}(t)} P(t)] \] (48)

In the above, \( \theta_{opt}dt \) is the coupon payment on long-term debt that is optimally chosen at date zero as described in Section 4.3. \( b_{opt}(t) \) is the manager’s pay-performance sensitivity in period \([t, t + dt] \) and \( a_{opt}(t) \) determines the “fixed” portion of her compensation (given by Theorems 1 and 2) in the equilibrium of the entire model in which the long-term debt structure, the manager’s contract and the bankruptcy time are all determined endogenously.

By (47) and (48), the manager’s optimal compensation structure can be implemented through an inside equity stake \( \bar{b}(t) \) and a cash flow \( \bar{a}(t)dt \) to all equity holders (inside and outside) in each
period. Depending on their sign, the cash flows $\bar{u}(t)dt$ correspond to dynamic, risk-free, single-period (short-term) borrowing or lending. In this implementation of the manager’s contract, the firm’s capital structure is dynamic and consists of inside and outside equity, long-term debt issued at date zero and dynamic, short-term risk-free borrowing or lending. The long-term debt issued at date zero determines the firm’s “base” leverage with the short-term borrowing or lending representing dynamic and stochastic fluctuations about the base leverage level. The values of long-term debt, outside equity, and the value of the sequence of short-term cash borrowing or lending (the cash reserve) at any date $t$ when the firm is solvent are

\[
\text{Long-Term Debt} = E^{-\Gamma_{\text{opt}}} \left[ \int_t^{T_b^{\text{opt}}} e^{-r(s-t)} \theta^{\text{opt}} ds + \int_{T_b^{\text{opt}}}^{\infty} e^{-r(s-t)} (1 - \tau) P(s) ds \right],
\]

\[
\text{Outside Equity} = E^{-\Gamma_{\text{opt}}} \int_t^{T_b^{\text{opt}}} e^{-r(s-t)} (1 - \bar{b}(s)) [dc_f(s) - \theta^{\text{opt}} ds + \pi(s) ds],
\]

\[
\text{Cash Reserve} = E^{-\Gamma_{\text{opt}}} \int_t^{\infty} e^{-r(s-t)} \bar{\pi}(s) ds,
\]

where $\Gamma_{\text{opt}}$ is the manager’s contract, $T_b^{\text{opt}}$ is the bankruptcy time, $\theta^{\text{opt}}$ is the coupon on long-term debt and $dc_f(s)$ is the total after-tax earnings in period $[s, s + ds]$ in equilibrium of the entire game.

If the cash reserve described above is negative, the firm has positive short-term risk-free debt. In the baseline model obtained by calibrating the model to the data in the following section, the firm has a positive cash reserve at date zero. We show, however, that the firm can move from having positive cash to nonzero short-term debt as underlying parameter values, especially manager-specific characteristics, vary. In this implementation of the manager’s contract, which we use to calibrate the model to the data, the cash reserve can be used to make long-term debt interest payments in financial distress. Bankruptcy occurs when the firm has no cash to make interest payments and no additional funds can be raised by issuing equity.

7 Numerical Analysis

To obtain quantitative implications for the impact of managerial discretion on firms, we first calibrate the model to the data to determine the baseline values for the model parameters. We then explore the “comparative statics” of the model by varying parameters about their baseline values.

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8 Short-term debt or cash is subject to the same risk-free (borrowing or lending) interest rate $r$. Note that, in our continuous-time framework, the “interest” portion of short-term debt is of $O(dt^2)$. Hence, tax shields on these interest payments are also of $O(dt^2)$ and can be neglected.


7.1 Model Calibration

**Risk-Free Rate, Tax Rate, Bankruptcy Costs:** We set the risk-free rate \( r \) to 0.06, which is consistent with the average risk-free rate estimated by Cochrane (2005). We set the effective corporate tax rate \( \tau \) to 0.15, which is consistent with the estimates of Graham (2000).\(^9\) We set the proportional bankruptcy cost parameter \( \varsigma \) to 0.15, which is the midpoint of the range \([0.10, 0.20]\) of proportional financial distress costs reported in Andrade and Kaplan (1998).

**Earnings Risks, Project Risks, Project Drifts, Initial EBIT Rate, and Investment:** In the baseline model, we assume the manager only chooses one project over time with a drift \( \mu_1 \) and volatility \( \sigma_1 \). We later explore the effects of managerial discretion in project selection. To set the drift and volatility, we first determine the total value of the firm (*including* the manager’s stake) if it were hypothetically un-levered. The value of the un-levered firm is the value of total after-tax earnings. By Theorem 1, the manager’s optimal effort is a constant \( e \) (because she only chooses project 1 in this baseline scenario). Hence, the un-levered firm value \( G(t) \) at any date \( t \) is

\[
G(t) = E_t \int_t^{\infty} (1 - \tau) \exp(-ru)P(u)[(1 + \ell + e)du + s_1dW(u)] = \frac{(1 - \tau)P(t)(1 + \ell + e)}{r - \mu_1}. \tag{50}
\]

By (50), the ratio of the *average* after-tax earnings to firm value over any period \([t, t + dt]\) is

\[
\frac{(1 - \tau)P(t)(1 + \ell + e)dt}{G(t)} = (r - \mu_1)dt. \tag{51}
\]

Using data from COMPUSTAT, we estimate the average ratio of the annual after-tax earnings to firm value (market value of equity + book value of debt) for non-financial firms over the period 1993-2004 to be approximately 0.08. We use (51) as our proxy for the ratio of after-tax earnings to firm value. Setting the time interval ‘dt’ to be one year in (51), we have \( r - \mu_1 = 0.08 \). Since \( r = 0.06 \), we set \( \mu_1 = -0.02 \) to match the average after-tax earnings to firm value ratio.

We proxy the *asset value* or *book value* as the value of the un-levered firm *net* of the manager’s stake. Since the manager appropriates the surplus she generates through her human capital, it follows from (50) that the asset value \( A(t) \) at any date \( t \) is

\[
A(t) = \frac{(1 - \tau)P(t)}{r - \mu_1}. \tag{52}
\]

\(^9\)Recall that taxation is symmetric and the effective corporate tax rate also incorporates the effects of personal taxes in the model. An effective corporate tax rate of 0.15 is consistent with the estimates of the tax advantage of debt using Graham’s (1996) data on corporate tax rates as well as personal tax rates on interest and dividend income.
We set the initial investment outlay $I$ equal to the asset value at date zero. We normalize the initial EBIT rate $P(0)$ so that the asset value is 100. By (2) and (52), the asset value evolves as follows:

$$dA(t) = \mu_1 A(t) dt + \sigma_1 A(t) dB_1(t).$$

From (53), $\sigma_1$ is the volatility of the asset value. We set the baseline value of $\sigma_1$ to 0.35; the mean volatility of asset value for U.S. non-financial firms over the period 1993-2004. Table II of Hennessy and Whited (2005) reports that the average ratio of the standard deviation of the shock to income over assets for U.S. non-financial firms is 0.117. From (1) and (52), the ratio of the standard deviation of the annual after-tax earnings to asset value is $(r - \mu_1)s_1$. We, therefore, set $s_1 = 0.117/(r - \mu_1) = 1.46$.

**Managerial Characteristics:** We indirectly determine the baseline values of the manager’s ability $\ell$, cost of risk $\gamma$ and disutility of effort $\kappa$. We determine these parameters by matching the model’s predictions for the manager’s inside equity stake, the firm’s average cash and average long-term debt (defined in 49) to their corresponding values in the data.

Holderness et al (1999) report that the mean equity ownership of insiders (officers and directors) of U.S. exchange-listed firms in 1995 is approximately 21%. Because the manager is a proxy for the firm’s insiders in the model, we choose the manager’s average inside equity stake to be 0.21. Using COMPUSTAT annual data, we estimate the average ratio of cash holdings to firm value (market value of equity + book value of debt) for U.S. non-financial firms over the period 1993-2004 to be approximately 9.7% and the average ratio of long-term debt to firm value to be approximately 17.7%. For each candidate set of values of the manager’s ability, disutility of effort, and cost of risk, we compute the average ratios of long-term debt to firm value, and cash to firm value by simulating the model over a period of 12 years. The baseline values of the managerial characteristics are those that best fit (in the least squares sense) the predicted to actual statistics.$^{10}$

**Baseline Parameter Values:** Table 1 lists the baseline values of all the parameters in the model.

We estimate the manager’s ability $\ell$ to be 0.15, the manager’s cost of risk $\gamma$ to be 0.27, and her disutility of effort $\kappa$ to be 1.7. The estimated values of the manager-specific parameters almost exactly match the average inside equity stake, cash holdings as a proportion of firm value, and long-term debt as a proportion of firm value in the data.$^{11}$

$^{10}$The implicit assumption we make is that the average inside equity stake, long term debt ratio, and cash ratio for
Table 1: Baseline Parameter Values

<table>
<thead>
<tr>
<th>Economy Parameters</th>
<th>Firm Parameters</th>
<th>Manager Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r$</td>
<td>$\tau$</td>
<td>$\zeta$</td>
</tr>
<tr>
<td>0.06</td>
<td>0.15</td>
<td>0.15</td>
</tr>
</tbody>
</table>

7.2 Numerical Results

7.2.1 Baseline Outputs

Table 2 describes the output economic variables we calculate in each set of simulations, and their values in the baseline model. Note that the firm value is the market value of the firm’s stream of total after-tax earnings and, therefore, includes the manager’s (or the insiders’) stake. To illustrate the effects of managerial discretion, we also calculate these variables in the benchmark scenarios one and three described in Section 5. Because the manager only chooses one project over time in the baseline case, benchmark scenarios one and two in Section 5 coincide. In our simulations, we normalize the initial value $P(0)$ of the state variable so that the asset value (the value of the un-levered firm net of the manager’s stake) defined in (52) is 100. The market value of the firm’s short-term risk-free debt at date zero, which is the negative of its cash reserve defined in (49), is

$$\text{Short-Term Debt} = E_{T^{\text{opt}}} \int_0^\infty e^{-rt}(-\pi(t))dt. \tag{54}$$

If the short-term debt ratio is negative, the firm has positive cash reserves and vice versa.

Table 2: Baseline Outputs

<table>
<thead>
<tr>
<th>Variable</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Firm Value</td>
<td>Market Value of Total After-Tax Cash Flows to the Firm at Date Zero</td>
</tr>
<tr>
<td>Long-Term Debt Ratio</td>
<td>Market Value of Long-Term Debt at Date Zero/Firm Value</td>
</tr>
<tr>
<td>Short-Term Debt Ratio</td>
<td>Market Value of Short-Term Debt at Date Zero/Firm Value</td>
</tr>
<tr>
<td>Leverage Ratio</td>
<td>Long-Term Debt Ratio + Short-Term Debt Ratio</td>
</tr>
<tr>
<td>Credit Spread</td>
<td>Yield on Long-Term Debt - Risk-Free Rate</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Variable</th>
<th>Actual</th>
<th>Benchmark One</th>
<th>Benchmark Three</th>
</tr>
</thead>
<tbody>
<tr>
<td>Firm Value</td>
<td>128.17</td>
<td>128.42</td>
<td>158.25</td>
</tr>
<tr>
<td>Long-Term Debt Ratio</td>
<td>18.47%</td>
<td>38.13%</td>
<td>27.38%</td>
</tr>
<tr>
<td>Short-Term Debt Ratio</td>
<td>-11.09%</td>
<td>-23.38%</td>
<td>34.56%</td>
</tr>
<tr>
<td>Leverage Ratio</td>
<td>7.38%</td>
<td>14.76%</td>
<td>61.93%</td>
</tr>
<tr>
<td>Credit Spread</td>
<td>1.60%</td>
<td>3.12%</td>
<td>2.90%</td>
</tr>
</tbody>
</table>

the baseline firm are equal to the corresponding average values across the firms in the sample.

\textsuperscript{11}Our numerical experiments suggest that the parameters are identified by the statistics.
Comparing the actual scenario with benchmark scenario one (in which the long-term debt structure maximizes economic value), we see that the manager’s discretion in long-term debt financing has a relatively small impact on firm value. The long-term debt ratio and the cash ratio (the negative of the short-term debt ratio) in the actual scenario are both significantly lower than in benchmark scenario one. The total leverage ratio of the firm (long-term debt net of cash) is lower in the actual scenario by about 7.37%. Overall, the results indicate that managerial discretion in long-term debt financing has a relatively minor impact on the firm’s value, but a substantial effect on its capital structure and credit spread.

A comparison of the firm values in the actual scenario and the “first best” benchmark scenario three (in which the manager’s effort and the long-term debt structure maximize total economic value) shows that the agency conflicts arising from the manager’s lack of diversification and risk aversion lower firm value by over 15.0% relative to the first best scenario. Managerial discretion has a very large effect on short-term debt. While the risk-averse manager’s precautionary motives cause the firm to have positive cash (negative short-term debt) in the actual scenario, the firm has a substantially positive level of short-term debt in the first best scenario in which the manager is perfectly diversified and has no discretion. Comparing the overall leverage ratios, the risk-averse manager under-levers the firm by over 50% relative to the first best scenario.

Recall from Section 5.3 that the benchmark scenario three corresponds to the case where the manager is perfectly diversified and complete contracts for the manager are enforceable. The results of Table 2 show the substantial quantitative impact of incomplete contracting and managerial lack of diversification on the firm’s value and capital structure. The table also shows that managerial discretion in effort and short-term debt financing has a much greater impact on firm value than discretion in long-term debt financing. We postpone the discussion of the intuition for these results to Section 7.2.3 where we analyze the effects of managerial discretion in project selection.

7.2.2 The Effects of Manager Characteristics

We now examine the effects of manager characteristics—ability $\ell$, disutility of effort $\kappa$, and cost of risk $\gamma$—on the firm’s value and capital structure. To sharpen the intuition for the results, we abstract away from managerial discretion in project selection in this analysis, that is, the manager only chooses project 1 over time. All our numerical results, and the intuition underlying them,
hold when managerial discretion in project selection is also incorporated. The following result that analytically describes the effects of manager characteristics on the firm’s long-term debt.

**Theorem 6 (The Effects of Manager Characteristics on Long-Term Debt)**

*The long-term debt coupon and the long-term debt value decline with the manager’s ability, increase with her cost of risk, and increase with her disutility of effort.*

**Proof.** See Appendix E.

The intuition for the above results is as follows. By (40), the long-term debt structure is chosen to maximize the sum of the manager’s initial payoff and her continuation value. For a given long-term debt coupon $\theta$, the manager’s initial payoff is $g_{\text{initial}}[F_{\theta}(0) - I]$, where $F_{\theta}(0)$ is the market value of the firm at date zero net of the manager’s stake. Manager characteristics directly affect the surplus the manager generates in each period, which is captured by the manager in equilibrium. As a result, for a given coupon, manager characteristics affect the manager’s continuation value, but do not affect her initial payoff. The manager’s optimal choice of long-term debt balances the beneficial effects of *ex post* debt tax shields on her initial payoff with the detrimental effects of debt on the manager’s continuation value through the increased likelihood of bankruptcy.

The surplus the manager generates in each period increases with her ability, which increases the marginal effect of the manager’s continuation value relative to her initial payoff. In other words, as her ability increases, she cares less about her initial payoff from leveraging the firm compared with her continuation value. She, therefore, chooses lower long-term debt, which increases her continuation value at the cost of her lowering her initial payoff. On the other hand, an increase in the manager’s cost of risk or disutility of effort increases the costs of providing incentives to the manager, thereby lowering the surplus she generates in each period. As a result, the marginal effect of her continuation value is lowered relative to her initial payoff, that is, the manager “effectively” becomes more myopic. Hence, she chooses greater long-term debt.

The effects of manager characteristics on short-term debt is ambiguous for general parameter values. By (48) and (54), short-term debt decreases with the long-term debt coupon $\theta^{opt}$ and with the ratio $a^{opt}(t)/b^{opt}(t)$ of the parameters that determine the “risk-free” and “risky” components of the manager’s compensation (see 26). By (26) and (48), an increase in the manager’s ability does not affect her inside equity stake but increases the cash component of the manager’s compensation, which has a negative effect on the firm’s short-term debt. Recall, however, that the long-term
debt coupon also decreases with the manager’s ability, which has a positive effect on the firm’s short-term debt by (48) and (54). For general parameter values, therefore, the effect of managerial ability on the firm’s short-term debt is ambiguous. An increase in $\gamma$ or $\kappa$ increases the cost of providing incentives to the manager and, therefore, increases the “cash” portion of the manager’s compensation relative to the “equity” portion. Hence, $a^{opt}(t)/b^{opt}(t)$ increases, which has a positive (negative) effect on the firm’s cash (short-term debt). However, an increase in $\gamma$ or $\kappa$ also increases the long-term debt coupon by Theorem 6, which, in turn, increases the bankruptcy level. Hence, the effects of $\gamma$ and $\kappa$ on the short-term debt value are also ambiguous.

As we see in our subsequent numerical results, when the other model parameters are assigned their baseline values, the positive (negative) effects of an increase in the manager’s ability, cost of risk, or disutility of effort on the firm’s cash (short-term debt) dominate the negative (positive) effects. The firm’s cash (short-term debt), therefore, increases (decreases) with the manager’s ability, cost of risk and disutility of effort.

**The Effects of the Manager’s Ability**

Table 3 displays the variation of the output variables with the manager’s ability $\ell$. The effect of managerial ability on total firm value (which includes the manager’s stake) is potentially ambiguous. As discussed above, ability has a positive effect on the surplus generated by the manager. By Theorem 6, however, the manager chooses lower long-term debt as her ability increases so that the value of debt tax shields is lowered. The surplus generated by the manager’s human capital, however, far exceeds the value of debt tax shields so that firm value (which includes the manager’s stake) increases with the manager’s ability. As the long-term debt value declines with the manager’s ability, the long-term debt ratio declines. The credit spread also declines with the manager’s ability since the level of long-term debt declines. For the baseline values of the other model parameters, the short-term debt value declines with ability so that the short-term debt ratio also declines. As the manager’s ability increases, the firm moves from holding positive short-term debt to one holding surplus cash. Because the long-term and short-term debt ratios both decline with managerial ability, the total leverage ratio also declines. The results of the table are summarized as follows:

**Implications 1**

Firm value increases with the manager’s ability, while the long-term debt ratio, short-term debt ratio, total leverage ratio, and the credit spread decline.
Table 3: The Effects of the Manager’s Ability

The table displays the variation of the output variables defined in Table 2 with the manager’s ability $\ell$. All other parameters are set to their baseline values in Table 1.

<table>
<thead>
<tr>
<th>$\ell$</th>
<th>Firm Value</th>
<th>Long-Term Debt</th>
<th>Short-Term Debt</th>
<th>Leverage</th>
<th>Credit Spread</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>114.60</td>
<td>43.73%</td>
<td>16.81%</td>
<td>60.54%</td>
<td>3.18%</td>
</tr>
<tr>
<td>0.05</td>
<td>119.28</td>
<td>33.48%</td>
<td>8.35%</td>
<td>41.83%</td>
<td>2.51%</td>
</tr>
<tr>
<td>0.10</td>
<td>123.76</td>
<td>25.22%</td>
<td>-1.15%</td>
<td>24.06%</td>
<td>2.01%</td>
</tr>
<tr>
<td>0.15</td>
<td>128.17</td>
<td>18.47%</td>
<td>-11.09%</td>
<td>7.38%</td>
<td>1.60%</td>
</tr>
<tr>
<td>0.20</td>
<td>132.74</td>
<td>14.33%</td>
<td>-22.11%</td>
<td>-7.77%</td>
<td>1.36%</td>
</tr>
<tr>
<td>0.25</td>
<td>137.23</td>
<td>10.27%</td>
<td>-32.37%</td>
<td>-22.10%</td>
<td>1.10%</td>
</tr>
<tr>
<td>0.30</td>
<td>141.95</td>
<td>8.10%</td>
<td>-43.15%</td>
<td>-35.04%</td>
<td>0.96%</td>
</tr>
</tbody>
</table>

The Effects of the Manager’s Cost of Risk

Table 4 shows the effects of the manager’s cost of risk $\gamma$ on the output variables in Table 2. Firm value and the short-term debt ratio decline with $\gamma$, while the long-term debt ratio and credit spread increase. Because the surplus the manager generates declines with $\gamma$, firm value declines with $\gamma$. As the long-term debt value increases by Theorem 6, the long-term debt ratio increases. As in the case of the manager’s ability, the firm’s cash (short-term debt) increases (decreases) with $\gamma$ when the other model parameters are assigned their baseline value. The decline in short-term debt dominates the decline in firm value so that the short-term debt ratio also declines. The increase in the long-term debt ratio is more than offset by the decrease in the short-term debt ratio so that the firm’s total leverage ratio declines with $\gamma$. As $\gamma$ increases, the increase in the costs of risk-sharing with the manager causes her inside equity stake to decline (see 25 and 48) and the firm to move from having positive short-term debt to one holding surplus cash. The results of the table are summarized as follows:

Implications 2

The long-term debt ratio and the credit spread increase with the manager’s cost of risk. Firm value, the manager’s inside equity stake and the short-term debt ratio decrease with the manager’s cost of risk or, alternately, the cash ratio increases with the manager’s cost of risk.

The Effects of the Manager’s Disutility of Effort

Table 5 shows the effects of the manager’s disutility of effort $\kappa$ on the output variables defined in Table 2. Comparing the table with Table 7, we see that the effects of $\kappa$ are similar to those of $\gamma$. The negative effects of $\kappa$ on the surplus generated by the manager dominate the positive effects of $\kappa$ on
Table 4: The Effects of the Manager’s Cost of Risk
The table displays the variation of the output variables defined in Table 2 with the manager’s cost of risk $\gamma$. All other parameters are set to their baseline values in Table 1.

<table>
<thead>
<tr>
<th>$\gamma$</th>
<th>Firm Value</th>
<th>Long-Term Debt</th>
<th>Short-Term Debt</th>
<th>Leverage</th>
<th>Credit Spread</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>165.80</td>
<td>3.64%</td>
<td>57.18%</td>
<td>60.82%</td>
<td>0.63%</td>
</tr>
<tr>
<td>0.09</td>
<td>153.15</td>
<td>6.64%</td>
<td>47.54%</td>
<td>54.18%</td>
<td>0.88%</td>
</tr>
<tr>
<td>0.18</td>
<td>140.83</td>
<td>12.65%</td>
<td>29.97%</td>
<td>42.62%</td>
<td>1.30%</td>
</tr>
<tr>
<td>0.27</td>
<td>128.17</td>
<td>18.47%</td>
<td>-11.09%</td>
<td>7.38%</td>
<td>1.60%</td>
</tr>
<tr>
<td>0.36</td>
<td>115.45</td>
<td>23.38%</td>
<td>-45.60%</td>
<td>-22.22%</td>
<td>1.78%</td>
</tr>
<tr>
<td>0.45</td>
<td>108.29</td>
<td>25.32%</td>
<td>-72.09%</td>
<td>-46.77%</td>
<td>1.82%</td>
</tr>
<tr>
<td>0.54</td>
<td>102.77</td>
<td>24.08%</td>
<td>-89.31%</td>
<td>-65.23%</td>
<td>1.88%</td>
</tr>
</tbody>
</table>

The value of debt tax shields through the higher long-term debt coupon. Hence, firm value declines with $\kappa$. Because the long-term debt value increases with $\kappa$, the long-term debt ratio and credit spread increase with $\kappa$. When other parameters are set to their baseline values, the short-term debt value declines with $\kappa$ and, moreover, dominates the decline in firm value. Hence, the short-term debt ratio declines with $\kappa$. The manager’s inside equity stake also declines with $\kappa$ because of the increase in the cost of providing incentives to the manager. The results are summarized as follows:

Implications 3

*The long-term debt ratio and the credit spread increase with the manager’s disutility of effort. Firm value, the manager’s inside equity stake and the short-term debt ratio decrease with the manager’s disutility of effort or, alternately, the cash ratio increases with the manager’s disutility of effort.*

Table 5: The Effects of the Manager’s Disutility of Effort
The table displays the variation of the output variables defined in Table 2 with the manager’s disutility of effort $\kappa$. All other parameters are set to their baseline values in Table 1.

<table>
<thead>
<tr>
<th>$\kappa$</th>
<th>Firm Value</th>
<th>Long-Term Debt</th>
<th>Short-Term Debt</th>
<th>Leverage</th>
<th>Credit Spread</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.25</td>
<td>145.70</td>
<td>13.06%</td>
<td>16.57%</td>
<td>29.62%</td>
<td>1.36%</td>
</tr>
<tr>
<td>1.40</td>
<td>138.65</td>
<td>15.42%</td>
<td>11.03%</td>
<td>26.45%</td>
<td>1.48%</td>
</tr>
<tr>
<td>1.55</td>
<td>132.89</td>
<td>16.96%</td>
<td>2.98%</td>
<td>19.94%</td>
<td>1.54%</td>
</tr>
<tr>
<td>1.70</td>
<td>128.17</td>
<td>18.47%</td>
<td>-11.09%</td>
<td>7.38%</td>
<td>1.60%</td>
</tr>
<tr>
<td>1.85</td>
<td>124.23</td>
<td>19.96%</td>
<td>-39.85%</td>
<td>-19.89%</td>
<td>1.66%</td>
</tr>
<tr>
<td>2.00</td>
<td>120.80</td>
<td>20.52%</td>
<td>-66.09%</td>
<td>-45.57%</td>
<td>1.66%</td>
</tr>
<tr>
<td>2.15</td>
<td>117.84</td>
<td>21.04%</td>
<td>-82.68%</td>
<td>-61.64%</td>
<td>1.66%</td>
</tr>
</tbody>
</table>

Implications 2 and 3 suggest that the manager’s inside equity stake and the firm’s long-term (short-term) debt are negatively (positively) related, which is consistent with the evidence shown by Datta et al (2005) who find that managers with greater stock ownership are more likely to choose
short-term debt over long-term debt. Overall, consistent with the empirical findings of studies such as Graham and Harvey (2001) and Bertrand and Schoar (2003), Tables 3, 4 and 5 show the significant impact of manager-specific characteristics on the firm’s value and financing choices.

**Manager Characteristics and the Relation between Leverage and Lagged Profitability**

In the baseline model, the parameter $a^{opt}(t)$ (see 26 and 48), which determines the “cash” component of the manager’s compensation in each period is positive. It follows from (48) that the parameter $\bar{a}(t)$ increases with the value of the state variable $P(t)$ that, in turn, implies that the value of the firm’s cash reserve at any date increases with $P(t)$. Hence, the firm’s total leverage (long-term debt net of cash) declines with its lagged profitability (as represented by the value of $P(t)$). The negative relation between leverage and lagged profitability is consistent with extensive prior empirical evidence (see, for example, Rajan and Zingales, 1995).

As manager-specific parameters deviate from their baseline values, however, the relation between leverage and lagged profitability could become positive. By (26), $a^{opt}(t)$ decreases as the manager’s ability $\ell$ and/or her cost of risk $\gamma$ decrease. It follows that, if $\ell$ or $\gamma$ is sufficiently low, $a^{opt}(t)$ could become negative so that the relation between leverage and lagged profitability becomes positive. These implications, as well as those of Tables 4, 5, and 3 suggest that it is important to appropriately control for manager-specific characteristics in empirical analyses of leverage dynamics.

**7.2.3 The Impact of Managerial Discretion in Project Selection**

We now examine the impact of managerial discretion in project selection. A project $i \in \{1, 2\}$ is characterized by its drift $\mu_i$, project risk $\sigma_i$, and earnings risk $s_i$. We set $s_1 = s_2$ to focus attention on the effects of the firm’s project risks $\sigma_1, \sigma_2$. Over the period 1993-2004, the standard deviation of the distribution of asset volatilities of US non-financial firms is 0.22 and the mean is 0.35. We choose $\sigma_1 = 0.4, \sigma_2 = 0.3$, which are approximately a quarter standard deviation above/below the mean. We choose the drifts of the two projects to lie in the range $[-0.025, -0.015]$, which is equally spaced about the baseline value $-0.02$. Note that the risk-neutral drift of the high-risk project 1 could be higher or lower than that of project 2, which (as we shall see) significantly influences the magnitude of the impact of managerial discretion. Accordingly, we consider three possible cases in

---

12The long-term debt value also increases with $P(t)$, but its increase is much smaller than that of the cash reserve. The long-term debt value is relatively flat when the firm is financially healthy and only changes significantly in financial distress.
our analysis: (i) $\mu_1 = \mu_2 = -0.02$, (ii) $\mu_1 = -0.015, \mu_2 = -0.025$, (iii) $\mu_1 = -0.025, \mu_2 = -0.015$.

We set all other parameters of the model to their baseline values in Table 1. As in our earlier simulations, we set the initial value $P(0)$ of the state variable so that the asset value (the un-levered firm value net of the manager’s stake) is 100. Table 6 displays the output variables described in Table 2 in the actual scenario, and the three benchmark scenarios described in Section 5.

Table 6: The Impact of Managerial Discretion in Project Selection
The table displays the output variables described in Table 2 in the actual scenario, and the three benchmark scenarios described in Section 5. In each scenario, results are displayed for three possible cases: $\mu_1 = \mu_2 = -0.02$, $\mu_1 = -0.015, \mu_2 = -0.025$, and $\mu_1 = -0.025, \mu_2 = -0.015$. The project volatilities are $\sigma_1 = 0.4, \sigma_2 = 0.3$. All other parameters are set to their baseline values in Table 1.

<table>
<thead>
<tr>
<th></th>
<th>Actual</th>
<th>B1</th>
<th>B2</th>
<th>B3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Firm Value</td>
<td>$\mu_1 = -0.02, \mu_2 = -0.02$</td>
<td>128.46</td>
<td>129.85</td>
<td>130.80</td>
</tr>
<tr>
<td></td>
<td>$\mu_1 = -0.015, \mu_2 = -0.025$</td>
<td>128.11</td>
<td>129.58</td>
<td>156.15</td>
</tr>
<tr>
<td></td>
<td>$\mu_1 = -0.025, \mu_2 = -0.015$</td>
<td>128.81</td>
<td>130.09</td>
<td>130.09</td>
</tr>
<tr>
<td>Long-Term Debt Ratio</td>
<td>$\mu_1 = -0.02, \mu_2 = -0.02$</td>
<td>19.47%</td>
<td>40.51%</td>
<td>40.18%</td>
</tr>
<tr>
<td></td>
<td>$\mu_1 = -0.015, \mu_2 = -0.025$</td>
<td>17.23%</td>
<td>38.88%</td>
<td>37.66%</td>
</tr>
<tr>
<td></td>
<td>$\mu_1 = -0.025, \mu_2 = -0.015$</td>
<td>21.75%</td>
<td>41.51%</td>
<td>41.51%</td>
</tr>
<tr>
<td>Short-Term Debt Ratio</td>
<td>$\mu_1 = -0.02, \mu_2 = -0.02$</td>
<td>-12.11%</td>
<td>-25.46%</td>
<td>-23.56%</td>
</tr>
<tr>
<td></td>
<td>$\mu_1 = -0.015, \mu_2 = -0.025$</td>
<td>-10.34%</td>
<td>-24.17%</td>
<td>-22.94%</td>
</tr>
<tr>
<td></td>
<td>$\mu_1 = -0.025, \mu_2 = -0.015$</td>
<td>-13.89%</td>
<td>-26.38%</td>
<td>-26.38%</td>
</tr>
<tr>
<td>Leverage Ratio</td>
<td>$\mu_1 = -0.02, \mu_2 = -0.02$</td>
<td>7.37%</td>
<td>15.05%</td>
<td>16.62%</td>
</tr>
<tr>
<td></td>
<td>$\mu_1 = -0.015, \mu_2 = -0.025$</td>
<td>6.89%</td>
<td>14.71%</td>
<td>14.71%</td>
</tr>
<tr>
<td></td>
<td>$\mu_1 = -0.03, \mu_2 = -0.01$</td>
<td>7.86%</td>
<td>15.13%</td>
<td>15.13%</td>
</tr>
<tr>
<td>Credit Spread</td>
<td>$\mu_1 = -0.02, \mu_2 = -0.02$</td>
<td>1.20%</td>
<td>2.56%</td>
<td>3.28%</td>
</tr>
<tr>
<td></td>
<td>$\mu_1 = -0.015, \mu_2 = -0.025$</td>
<td>1.25%</td>
<td>2.73%</td>
<td>3.59%</td>
</tr>
<tr>
<td></td>
<td>$\mu_1 = -0.025, \mu_2 = -0.015$</td>
<td>1.14%</td>
<td>2.33%</td>
<td>2.33%</td>
</tr>
</tbody>
</table>

Firm values in the benchmark scenario B1 (in which the long-term debt structure maximizes economic value) are only slightly larger than in the actual scenario. Scenario B1 differs from the actual scenario in that the manager’s discretion in long-term debt financing is “turned off”. Firm value or the value of the “total pie” is potentially affected by the value of debt tax shields and the distortions in the manager’s project choices (see Theorem 2) due to the debt structure. As shown by Theorem 2, however, the long-term debt structure only distorts the manager’s project choices when the firm’s earnings are below a threshold, that is, in financial distress. When the long-term debt structure is chosen at date zero either by the manager to maximize her value or to maximize the total economic value, the probability of financial distress is relatively small. Hence, the difference in firm values between the actual scenario and scenario B1 largely stems from the difference in the values of debt tax shields, which is small compared with the size of the total pie.
A comparison of scenario B1 with benchmark scenario B2 (in which the manager’s discretion in long-term debt financing and project selection is “turned off”) permits the quantification of the effects of the manager’s discretion in project selection. Managerial discretion in project selection has a minor effect on firm value when $\mu_1 \leq \mu_2$ (the high-risk project 1 has lower risk-neutral drift), but a substantial effect when $\mu_1 > \mu_2$ (the high-risk project 1 has higher risk-neutral drift).

The intuition for these results hinges on the discussion following Theorem 5. First note that, because $s_1 = s_2$, it follows from Theorem 1 that $\tilde{g}_1 = \tilde{g}_2$, where $\tilde{g}_1, \tilde{g}_2$ are defined in (44). By Theorem 5, the economic-value maximizing policy chooses project 1 in “good” states and project 2 in “bad” states when $\mu_1 > \mu_2$. On the other hand, the economic value-maximizing policy chooses project 2 in “good” states and project 1 in “bad” states when $\mu_1 < \mu_2$.

When the manager has discretion in project selection, it follows from Theorem 2 that the manager’s optimal policy depends on her project tradeoff $\Omega_1 - \Omega_2$. $\Omega_1$ and $\Omega_2$ (defined in 35) depend on the projects’ drifts $\mu^M_i = \mu_i - \gamma \sigma_i, i \in \{1, 2\}$ in the manager’s subjective valuation probability. For the baseline value of the manager’s cost of risk $\gamma$ in Table 1, $\Omega_1 < \Omega_2$ when $\mu_1, \mu_2 \in \{-0.025, -0.015\}$. It follows from Theorem 2(b), therefore, that the manager chooses project 1 in “bad” states and project 2 in “good” states.

By the discussion above, the structure of the manager’s policy is similar to that of the economic value-maximizing policy when $\mu_1 \leq \mu_2$, but dramatically differs from it when $\mu_1 > \mu_2$. In other words, when the high-risk project 1 also has higher NPV, the manager’s risk aversion causes her to under-invest in project 1, thereby significantly lowering firm value. On the other hand, the manager’s risk aversion has little effect on firm value when the high-risk project 1 has lower NPV because such a project also has a lower subjective value to the manager.

A comparison of the benchmark scenarios B2 and B3 (in which the manager’s discretion in financing, project selection and effort are “shut off”) permits the quantification of the impact of managerial discretion in effort. The firm values in scenario B2 are significantly lower than in scenario B3 illustrating the effect of the manager’s risk aversion on her effort choices. The table shows that managerial discretion in effort has the most significant impact on the firm’s value and leverage. Discretion in effort, however, largely influences the firm’s leverage through its short-term debt (or cash) rather than long-term debt. The risk-averse manager’s precautionary motives cause the firm to hold surplus cash in the actual scenario as well as benchmark scenarios 1 and 2, while it
has positive short-term debt in the first best scenario B3. As discussed earlier, the firm’s long-term debt structure primarily reflects the traditional tradeoff between debt tax shields and bankruptcy costs. On the other hand, as described in Section 6, the firm’s short-term debt plays a key role in providing optimal explicit incentives to the manager to exert effort. Hence, the firm’s short-term debt ratio is significantly influenced by managerial discretion in effort.

### 7.2.4 The Effects of Average Project Risk

Table 7 shows the variation of the economic variables with the firm’s average project risk. As in Table 6, we consider three possible cases: (i) $\mu_1 = \mu_2 = -0.02$, (ii) $\mu_1 = -0.015, \mu_2 = -0.025$, (iii) $\mu_1 = -0.025, \mu_2 = -0.015$.

<table>
<thead>
<tr>
<th>$\sigma_1$</th>
<th>$\sigma_2$</th>
<th>Firm Value</th>
<th>Long-Term Debt Ratio</th>
<th>Short-Term Debt Ratio</th>
<th>Leverage Ratio</th>
<th>Credit Spread</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_1 = -0.02$</td>
<td>0.25</td>
<td>0.15</td>
<td>129.52</td>
<td>23.17%</td>
<td>-15.81%</td>
<td>7.36%</td>
</tr>
<tr>
<td>0.3</td>
<td>0.2</td>
<td>129.05</td>
<td>21.32%</td>
<td>-14.06%</td>
<td>7.25%</td>
<td>0.54%</td>
</tr>
<tr>
<td>0.35</td>
<td>0.25</td>
<td>128.92</td>
<td>20.44%</td>
<td>-13.90%</td>
<td>6.54%</td>
<td>0.88%</td>
</tr>
<tr>
<td>0.4</td>
<td>0.3</td>
<td>128.46</td>
<td>19.47%</td>
<td>-12.11%</td>
<td>7.37%</td>
<td>1.20%</td>
</tr>
<tr>
<td>0.45</td>
<td>0.35</td>
<td>128.17</td>
<td>18.47%</td>
<td>-11.09%</td>
<td>7.37%</td>
<td>1.60%</td>
</tr>
<tr>
<td>$\mu_2 = -0.02$</td>
<td>0.25</td>
<td>0.15</td>
<td>129.46</td>
<td>22.97%</td>
<td>-15.59%</td>
<td>7.37%</td>
</tr>
<tr>
<td>0.3</td>
<td>0.2</td>
<td>129.15</td>
<td>22.15%</td>
<td>-14.67%</td>
<td>7.48%</td>
<td>0.64%</td>
</tr>
<tr>
<td>0.35</td>
<td>0.25</td>
<td>128.57</td>
<td>21.23%</td>
<td>-12.11%</td>
<td>9.13%</td>
<td>0.88%</td>
</tr>
<tr>
<td>0.4</td>
<td>0.3</td>
<td>128.53</td>
<td>20.21%</td>
<td>-12.67%</td>
<td>7.54%</td>
<td>1.31%</td>
</tr>
<tr>
<td>0.45</td>
<td>0.35</td>
<td>128.28</td>
<td>19.16%</td>
<td>-11.64%</td>
<td>7.52%</td>
<td>1.73%</td>
</tr>
<tr>
<td>$\mu_1 = -0.015$</td>
<td>0.25</td>
<td>0.15</td>
<td>129.46</td>
<td>22.97%</td>
<td>-15.59%</td>
<td>7.37%</td>
</tr>
<tr>
<td>0.3</td>
<td>0.2</td>
<td>129.15</td>
<td>22.15%</td>
<td>-14.67%</td>
<td>7.48%</td>
<td>0.64%</td>
</tr>
<tr>
<td>0.35</td>
<td>0.25</td>
<td>128.57</td>
<td>21.23%</td>
<td>-12.11%</td>
<td>9.13%</td>
<td>0.88%</td>
</tr>
<tr>
<td>0.4</td>
<td>0.3</td>
<td>128.53</td>
<td>20.21%</td>
<td>-12.67%</td>
<td>7.54%</td>
<td>1.31%</td>
</tr>
<tr>
<td>0.45</td>
<td>0.35</td>
<td>128.28</td>
<td>19.16%</td>
<td>-11.64%</td>
<td>7.52%</td>
<td>1.73%</td>
</tr>
<tr>
<td>$\mu_2 = -0.025$</td>
<td>0.25</td>
<td>0.15</td>
<td>129.46</td>
<td>22.97%</td>
<td>-15.59%</td>
<td>7.37%</td>
</tr>
<tr>
<td>0.3</td>
<td>0.2</td>
<td>129.15</td>
<td>22.15%</td>
<td>-14.67%</td>
<td>7.48%</td>
<td>0.64%</td>
</tr>
<tr>
<td>0.35</td>
<td>0.25</td>
<td>128.57</td>
<td>21.23%</td>
<td>-12.11%</td>
<td>9.43%</td>
<td>0.88%</td>
</tr>
<tr>
<td>0.4</td>
<td>0.3</td>
<td>128.47</td>
<td>20.21%</td>
<td>-12.67%</td>
<td>7.54%</td>
<td>1.31%</td>
</tr>
<tr>
<td>0.45</td>
<td>0.35</td>
<td>128.28</td>
<td>19.16%</td>
<td>-11.64%</td>
<td>7.52%</td>
<td>1.73%</td>
</tr>
<tr>
<td>$\mu_1 = -0.025$</td>
<td>0.25</td>
<td>0.15</td>
<td>129.46</td>
<td>22.97%</td>
<td>-15.59%</td>
<td>7.37%</td>
</tr>
<tr>
<td>0.3</td>
<td>0.2</td>
<td>129.15</td>
<td>22.15%</td>
<td>-14.67%</td>
<td>7.48%</td>
<td>0.64%</td>
</tr>
<tr>
<td>0.35</td>
<td>0.25</td>
<td>128.57</td>
<td>21.23%</td>
<td>-12.11%</td>
<td>9.43%</td>
<td>0.88%</td>
</tr>
<tr>
<td>0.4</td>
<td>0.3</td>
<td>128.47</td>
<td>20.21%</td>
<td>-12.67%</td>
<td>7.54%</td>
<td>1.31%</td>
</tr>
<tr>
<td>0.45</td>
<td>0.35</td>
<td>128.28</td>
<td>19.16%</td>
<td>-11.64%</td>
<td>7.52%</td>
<td>1.73%</td>
</tr>
</tbody>
</table>

The average risk of the firm’s projects has differing effects on the long-term debt ratio, the short-term debt ratio, and the net leverage ratio. The long-term debt ratio declines with average risk, which is consistent with considerable empirical evidence (for example, Bradley, et al, 1984, Rajan and Zingales, 1995). Similar to the discussion in Section 7.2.2, the manager’s choice of long-term debt reflects its effects on her initial payoff and her continuation value by (40).

By (26), the project volatilities $\sigma_1, \sigma_2$ have no effect on the manager’s compensation structure.
and, therefore, on the surplus she generates in each period. The manager’s continuation value, however, declines with the average risk of the firm’s projects because the manager’s risk aversion lowers the “effective” drifts of the projects from the manager’s standpoint (see 7 and 8), which increases the manager’s expected personal bankruptcy costs. Higher average project risk also lowers the market value of long-term debt. The long-term debt ratio, therefore, declines with average project risk. From the table, we see that the credit spread on long-term debt increases with average risk reflecting the higher likelihood of bankruptcy.

The decline in the long-term debt coupon with average risk has a positive effect on the firm’s short-term debt (see (48) and (54). The increased likelihood of bankruptcy, however, has a negative effect on the value of the firm’s short-term debt. The interplay between these effects, as well as the fact that firm value declines with average project risk, causes the short-term debt ratio to vary non-monotonically with average risk in general. The net leverage ratio, being the sum of the long-term and short-term debt ratios, also varies non-monotonically with the firm’s average risk.

Implications 4

The long-term debt ratio declines with the average project risk, while the credit spread increases. The short-term debt ratio and the leverage ratio vary non-monotonically.

7.2.5 The Effects of Earnings Risk

Table 8 shows the variation of the economic variables with the firm’s average earnings risk $s_{\text{average}} = \frac{s_1 + s_2}{2}$. To clarify the intuition for the results, we assume that $s_1 = s_2$. Our results and the intuition underlying them are unaltered if we consider the case where $s_1 \neq s_2$.

Firm value declines with earnings risk. By (26), an increase in the earnings risk (alternately, human capital risk; see footnote 2) increases the costs of risk-sharing with the manager, thereby lowering the surplus she generates in each period. More interestingly, earnings risk has opposing effects on the firm’s long-term and short-term debt ratios. While the long-term debt ratio increases with earnings risk, the short-term debt ratio decreases. The decline in the short-term debt ratio more than offsets the increase in the long-term debt ratio so that the overall leverage ratio decreases.

The intuition for the effects of earnings risk on the debt ratios is as follows. Earnings risk has a negative effect on the surplus the manager generates in each period and, therefore, her continuation value. By Theorem 3 and (40), however, it does not directly affect the manager’s initial payoff at
Table 8: The Effects of Earnings Risk

The table displays the output variables described in Table 2 as the earnings risk $s = s_1 = s_2$ varies. The results are displayed for three possible cases: $\mu_1 = \mu_2 = -0.02$; $\mu_1 = -0.015$, $\mu_2 = -0.025$; and $\mu_1 = -0.025$, $\mu_2 = -0.015$. All other parameters are set to their baseline values in Table 1.

<table>
<thead>
<tr>
<th>$s$</th>
<th>Firm Value</th>
<th>Long-Term Debt Ratio</th>
<th>Short-Term Debt Ratio</th>
<th>Leverage Ratio</th>
<th>Credit Spread</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_1 = -0.02$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.35</td>
<td>131.42</td>
<td>19.03%</td>
<td>2.10%</td>
<td>21.13%</td>
<td>1.20%</td>
</tr>
<tr>
<td>1.4</td>
<td>130.08</td>
<td>19.23%</td>
<td>-3.56%</td>
<td>15.67%</td>
<td>1.20%</td>
</tr>
<tr>
<td>1.45</td>
<td>128.73</td>
<td>19.43%</td>
<td>-10.51%</td>
<td>8.92%</td>
<td>1.20%</td>
</tr>
<tr>
<td>1.5</td>
<td>127.49</td>
<td>20.57%</td>
<td>-19.98%</td>
<td>0.59%</td>
<td>1.25%</td>
</tr>
<tr>
<td>1.55</td>
<td>126.14</td>
<td>20.79%</td>
<td>-31.56%</td>
<td>-10.77%</td>
<td>1.25%</td>
</tr>
<tr>
<td>$\mu_2 = -0.02$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.35</td>
<td>131.42</td>
<td>18.86%</td>
<td>2.21%</td>
<td>21.07%</td>
<td>1.26%</td>
</tr>
<tr>
<td>1.4</td>
<td>130.18</td>
<td>19.96%</td>
<td>-4.13%</td>
<td>15.82%</td>
<td>1.31%</td>
</tr>
<tr>
<td>1.45</td>
<td>128.84</td>
<td>20.17%</td>
<td>-11.08%</td>
<td>9.08%</td>
<td>1.31%</td>
</tr>
<tr>
<td>1.5</td>
<td>127.49</td>
<td>20.38%</td>
<td>-19.89%</td>
<td>0.49%</td>
<td>1.31%</td>
</tr>
<tr>
<td>1.55</td>
<td>126.25</td>
<td>21.51%</td>
<td>-32.08%</td>
<td>-10.57%</td>
<td>1.37%</td>
</tr>
<tr>
<td>$\mu_1 = -0.015$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.35</td>
<td>131.31</td>
<td>18.28%</td>
<td>2.71%</td>
<td>20.99%</td>
<td>1.08%</td>
</tr>
<tr>
<td>1.4</td>
<td>129.96</td>
<td>18.47%</td>
<td>-2.95%</td>
<td>15.52%</td>
<td>1.08%</td>
</tr>
<tr>
<td>1.45</td>
<td>128.61</td>
<td>18.67%</td>
<td>-9.92%</td>
<td>8.75%</td>
<td>1.08%</td>
</tr>
<tr>
<td>1.5</td>
<td>127.38</td>
<td>19.82%</td>
<td>-19.41%</td>
<td>0.41%</td>
<td>1.13%</td>
</tr>
<tr>
<td>1.55</td>
<td>126.03</td>
<td>20.03%</td>
<td>-31.00%</td>
<td>-10.98%</td>
<td>1.13%</td>
</tr>
</tbody>
</table>

An increase in the earnings risk, therefore, increases the marginal effect of the manager’s initial payoff relative to her continuation value on her optimal long-term debt structure choice. She, therefore, chooses greater long-term debt. The increase in long-term debt coupled with the decline in firm value with earnings risk causes the long-term debt ratio to increase with earnings risk. By Theorem 1 and (26), an increase in the earnings risk lowers the power of incentives to the manager, and increases the “cash” portion of her compensation. By (49), this has a positive effect on the firm’s cash reserves and a negative effect on its short-term debt.

Comparing Tables 7 and 8, we conclude that distinct components of the firm’s risk have differing effects on its capital structure. While the long-term debt ratio declines with project risk, it increases with earnings risk. The short-term debt ratio varies non-monotonically with project risk, but decreases with earnings risk. By the intuition for these findings discussed earlier, the differing effects arise due to the fact that earnings risk directly affects the manager’s incentive compensation in each period. The project risks, on the other hand, have longer-term effects by influencing the manager’s valuation of her stream of future payoffs.

Implications 5

The long-term debt ratio and credit spread increase with the firm’s earnings risk. The short-term
debt ratio and the net leverage ratio decline with the firm’s earnings risk.

7.2.6 The Effects of Growth Opportunities

Table 9 shows the variation of the economic variables with the average drift of the firm’s projects. The drifts $\mu_1, \mu_2$ both lie in the interval $[-0.03, -0.01]$. We consider three possible cases: (i) $\mu_1 = \mu_2$, (ii) $\mu_1 > \mu_2$, (iii) $\mu_1 < \mu_2$. In these simulations, we set the initial value of the EBIT rate $P(0)$ so that the un-levered firm value in the scenario in which $\mu_1 = \mu_2 = -0.03$ is 100.

Table 9: The Effects of Growth Opportunities
The table displays the output variables described in Table 2 as the average drift $\frac{\mu_1 + \mu_2}{2}$ of the firm’s projects varies. The results are displayed for three possible cases: $\mu_1 = \mu_2$, $\mu_1 > \mu_2$, and $\mu_1 < \mu_2$. The initial value of the EBIT rate $P(0)$ is chosen so that the un-levered firm value in the scenario where $\mu_1 = \mu_2 = -0.03$ is 100. All other parameters are set to their baseline values in Table 1.

<table>
<thead>
<tr>
<th>Case</th>
<th>$\mu_1$</th>
<th>$\mu_2$</th>
<th>Firm Value</th>
<th>Long-Term Debt Ratio</th>
<th>Short-Term Debt Ratio</th>
<th>Leverage Ratio</th>
<th>Credit Spread</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_1 = \mu_2$</td>
<td>-0.03</td>
<td>-0.03</td>
<td>128.11</td>
<td>21.23%</td>
<td>-10.34%</td>
<td>10.89%</td>
<td>1.45%</td>
</tr>
<tr>
<td></td>
<td>-0.025</td>
<td>-0.025</td>
<td>136.12</td>
<td>20.11%</td>
<td>-12.60%</td>
<td>7.51%</td>
<td>1.31%</td>
</tr>
<tr>
<td></td>
<td>-0.02</td>
<td>-0.02</td>
<td>144.60</td>
<td>20.09%</td>
<td>-12.75%</td>
<td>7.34%</td>
<td>1.23%</td>
</tr>
<tr>
<td></td>
<td>-0.015</td>
<td>-0.015</td>
<td>154.08</td>
<td>19.18%</td>
<td>-12.85%</td>
<td>6.33%</td>
<td>1.11%</td>
</tr>
<tr>
<td></td>
<td>-0.01</td>
<td>-0.01</td>
<td>165.64</td>
<td>17.96%</td>
<td>-14.04%</td>
<td>3.92%</td>
<td>1.05%</td>
</tr>
<tr>
<td>$\mu_1 &gt; \mu_2$</td>
<td>-0.025</td>
<td>-0.035</td>
<td>124.26</td>
<td>22.50%</td>
<td>-9.86%</td>
<td>12.64%</td>
<td>1.52%</td>
</tr>
<tr>
<td></td>
<td>-0.02</td>
<td>-0.03</td>
<td>128.11</td>
<td>21.23%</td>
<td>-10.34%</td>
<td>10.89%</td>
<td>1.45%</td>
</tr>
<tr>
<td></td>
<td>-0.015</td>
<td>-0.025</td>
<td>136.12</td>
<td>20.11%</td>
<td>-12.60%</td>
<td>7.51%</td>
<td>1.31%</td>
</tr>
<tr>
<td></td>
<td>-0.01</td>
<td>-0.02</td>
<td>144.60</td>
<td>20.09%</td>
<td>-12.75%</td>
<td>7.34%</td>
<td>1.23%</td>
</tr>
<tr>
<td></td>
<td>-0.005</td>
<td>-0.015</td>
<td>154.08</td>
<td>19.18%</td>
<td>-12.85%</td>
<td>6.33%</td>
<td>1.11%</td>
</tr>
<tr>
<td>$\mu_1 &lt; \mu_2$</td>
<td>-0.035</td>
<td>-0.025</td>
<td>136.12</td>
<td>20.11%</td>
<td>-12.60%</td>
<td>7.51%</td>
<td>1.31%</td>
</tr>
<tr>
<td></td>
<td>-0.03</td>
<td>-0.02</td>
<td>144.60</td>
<td>20.09%</td>
<td>-12.75%</td>
<td>7.34%</td>
<td>1.23%</td>
</tr>
<tr>
<td></td>
<td>-0.025</td>
<td>-0.015</td>
<td>154.08</td>
<td>19.18%</td>
<td>-12.85%</td>
<td>6.33%</td>
<td>1.11%</td>
</tr>
<tr>
<td></td>
<td>-0.02</td>
<td>-0.01</td>
<td>165.64</td>
<td>17.96%</td>
<td>-14.04%</td>
<td>3.92%</td>
<td>1.05%</td>
</tr>
<tr>
<td></td>
<td>-0.015</td>
<td>-0.005</td>
<td>178.25</td>
<td>16.47%</td>
<td>-14.62%</td>
<td>1.85%</td>
<td>1.02%</td>
</tr>
</tbody>
</table>

Not surprisingly, firm value increases with the average drift of the firm’s projects because its expected total earnings increases. The long-term debt ratio, the short-term debt ratio, the net leverage ratio and the credit spread all decline with the average expected growth rate. The decline of the long-term debt ratio and leverage ratio with the average expected growth rates is consistent with empirical evidence of a negative relation between a firm leverage and its “growth opportunities” (for example, Rajan and Zingales, 1995). The intuition for the decline of the long-term debt ratio is as follows. As the average expected growth rate increases, the likelihood of bankruptcy declines, ceteris paribus. This has a positive effect on the manager’s continuation value as well as her
incentive to issue greater long-term debt at date zero to exploit ex post debt tax shields. However, firm value also increases with the average expected growth rate, which has a negative effect on the long-term debt ratio. The negative effect of the increase in firm value on the long-term debt ratio dominates so that the long-term debt ratio decreases.

The explanation for the decline in the short-term debt ratio with the average drift is as follows. By (26), the manager’s optimal pay-performance sensitivity \( b^{opt} (t) \) and the parameter \( a^{opt} (t) \), which determines the cash portion of her compensation, do not depend on the project drifts \( \mu_1, \mu_2 \). Further, as discussed earlier, the parameter \( a^{opt} (t) \) is positive in the baseline model. By (48), therefore, the parameter \( \bar{a} (t) \) declines with the state variable \( P(t) \). An increase in the average drift increases the likelihood of “high” realizations of the \( P(t) \). It follows from (49) that the firm’s cash increases with the average drift or, alternately, short-term debt declines. The cash reserve increases with average drift to a greater extent than firm value so that the cash ratio increases, that is, the short-term debt ratio declines.

**Implications 6**

The long-term debt ratio, short-term debt ratio, net leverage ratio, and credit spread decline with the firm’s growth opportunities.

**8 Conclusions**

We develop a dynamic structural model to obtain quantitative guidance on the impact of managerial discretion on firms. The model incorporates taxes, bankruptcy costs, managerial risk aversion, and three important dimensions on which managers affect firms, namely, financing, effort and investment/project selection. We indirectly infer the key manager-specific parameters of the model—ability, risk aversion, and disutility of effort—by matching its predictions to aggregate data. Our analysis of the calibrated model shows that the agency conflicts between undiversified, risk-averse managers and well-diversified value-maximizing outside investor have a major impact on firms’ values, capital structures, and the values of their risky debt and equity. Managerial discretion and risk aversion could substantially reduce firm value and lower leverage ratios relative to the benchmark scenario in which the manager is perfectly diversified and has no discretion.

Managerial discretion in effort, project selection, and short-term debt financing have a much
greater impact on the firm’s value and capital structure than discretion in long-term debt financing. Capital structure varies widely with managerial characteristics. The firm’s long-term debt ratio and credit spread decline with the manager’s ability, and increase with her risk aversion as well as her disutility of effort, while the firm’s short-term debt ratio declines with all three variables. The different components of firm risk—earnings risk and project risk—have opposing effects on long-term and short-term debt. Consistent with empirical evidence, the relation between leverage and lagged profitability is negative in the baseline model, and long-term debt and net leverage ratios decline with firms’ average risk and growth opportunities.
Appendix A: Proof of Theorem 1

The proof requires a precise interpretation of equations (1) and (2) describing the firm’s EBIT process and the state variable \( P \). For the project choice policy \( \xi(\cdot) \) specified by the contract, we consider the EBIT process \( Q_\xi \) and the state variable \( P_\xi \) (the subscripts indicate their dependence on the project choice policy \( \xi \)) to be given random processes on a probability space with the manager’s human capital (ability and effort) altering the probability distributions of these processes (see Holmstrom and Milgrom, 1987). The uncertainty is represented by a probability space \((\Omega, \mathcal{F}, \mathcal{R})\) on which is defined two independent standard Brownian motions \( \hat{W} \) and \( \hat{B} \). Let \( \{\mathcal{F}_t\} \) denote the complete and augmented filtration generated by \( \hat{W} \) and \( \hat{B} \). Define the processes \( P_\xi, Q_\xi \) by

\[
dP_\xi(t) = \sigma_{\xi(t)}P_\xi(t)d\hat{B}(t), \quad dQ_\xi(t) = s_{\xi(t)}P_\xi(t)d\hat{W}(t);
\]

where \( \xi(t) \in \{1, 2\} \). We use the Girsanov transformation to obtain a new probability measure under which the processes \( Q_\xi, P_\xi \) evolve as in (1) and (2). Define the processes

\[
\zeta_{\xi,e}(t) := \exp \left[ \int_0^t \left( 1 + \ell + e(u) \right) \left( s_{\xi(u)} \right)^{-1} d\hat{W}(u) + \mu_{\xi(u)} \left( \sigma_{\xi(u)} \right)^{-1} d\hat{B}(u) - \frac{1}{2} \int_0^t \left( 1 + \ell + e(u) \right)^2 \left( s_{\xi(u)} \right)^{-2} + \mu_{\xi(u)}^2 \left( \sigma_{\xi(u)} \right)^{-2} |du| \right], \quad (56)
\]

\[
W_{\xi,e}(t) = \hat{W}(t) - \int_0^t \left( 1 + \ell + e(u) \right) \left( s_{\xi(u)} \right)^{-1} du
\]

\[
B_{\xi,e}(t) = \hat{B}(t) - \int_0^t \mu_{\xi(u)} \left( \sigma_{\xi(u)} \right)^{-1} du \quad (57)
\]

The process \( \zeta_{\xi,e}(\cdot) \) is a positive square integrable martingale.\(^{13}\) For each finite date \( T \), define the new probability measure \( \Pi_{\xi,e}^T \) on the \( \sigma \)-field \( \mathcal{F}_T \subset \mathcal{F} \) by

\[
\frac{d\Pi_{\xi,e}^T}{d\mathcal{R}} = \zeta_{\xi,e}(T). \quad (58)
\]

Let \( \Pi_{\xi,e} \) be a probability measure on \( \mathcal{F} \) such that \( \Pi_{\xi,e} = \Pi_{\xi,e}^T \) on \( \mathcal{F}_T \). By Girsanov’s theorem (see Oksendal, 2003), the processes \( W_{\xi,e}(\cdot), B_{\xi,e}(\cdot) \) are Brownian motions under the measure \( \Pi_{\xi,e} \).\(^{14}\) Further, under this measure, the processes \( Q_{\xi}(\cdot), P_{\xi}(\cdot) \) evolve as

\[
dQ_{\xi}(t) = \left[ 1 + \ell + e(t) \right] P_{\xi}(t)dt + s_{\xi(t)}P_{\xi}(t)dW_{\xi,e}(t)
\]

\[
dP_{\xi}(t) = \mu_{\xi(t)}P_{\xi}(t)dt + \sigma_{\xi(t)}P_{\xi}(t)dB_{\xi,e}(t). \quad (59)
\]

Equation (59) describes the evolution of the EBIT process and the process \( P_{\xi} \). It is identical to equations (1) and (2), but the Brownian motion and the probability measure depend on the effort and project choice processes.

\(^{13}\)The effort process is assumed to satisfy the Novikov condition (see Oksendal, 2003). The Novikov condition is satisfied by the equilibrium effort process because it is deterministic.

\(^{14}\)This somewhat involved construction of the probability measure \( \Pi_{\xi,e} \) is necessitated by the fact that Girsanov’s theorem is only valid on a finite time horizon \([0, T]\) (see Section 6N of Duffie, 2001)
We can similarly use Girsanov’s theorem to construct a new measure $\Pi^M_{\xi,e}$ representing the manager’s subjective valuation probability and Brownian motions

$$W^M_{\xi,e}(t) = W_{\xi,e}(t) + \gamma t, \quad B^M_{\xi,e}(t) = B_{\xi,e}(t) + \gamma t,$$

with respect to which the processes $Q_\xi$ and $P_\xi$ evolve as in (7) and (8), respectively.

The processes below are required in the sequel. For any contract $\Gamma \equiv (dc_m(\cdot), \xi(\cdot), e(\cdot))$ and bankruptcy time $T_b$, we define the cumulative value process of the manager as

$$\mathcal{U}_\Gamma(t) := E^M_{\xi,e} \left[ \int_0^\infty \exp(-r(u - t))(dc_m(u) - \frac{1}{2}\kappa e(u)^2 P_\xi(u)du) \mid \mathcal{F}_t \right],$$

which is the manager’s conditional valuation of her future total payoffs (including disutilities of effort) at any date including her past total payoffs from the contract carried forward at the risk-free rate. Here, $E^M_{\xi,e} \mid \mathcal{F}_t$ denotes conditional expectation at date $t$ under the probability measure $\Pi^M_{\xi,e}$ defined in (58).

**Remark 1**

The discounted cumulative value process of the manager, $e^{-rt}\mathcal{U}_\Gamma(t)$, is a square-integrable $\{\mathcal{F}_t\}$-martingale under the measure $\Pi^M_{\xi,e}$.

The manager’s continuation value process $M_\Gamma(t)$ is

$$M_\Gamma(t) = E^M_{\xi,e} \left[ \int_t^\infty \exp(-r(u - t))(dc_m(u) - \frac{1}{2}\kappa e(u)^2 P_\xi(u)du) \mid \mathcal{F}_t \right],$$

which is the manager’s subjective valuation of her future total payoffs at date $t$. The manager’s adjusted cumulative value process represents the cumulative value process of the manager when she exerts effort $e(s); s \leq t$ and effort $\hat{e}(s); s \geq t$. Formally, we define

$$Y_{dc_m,\xi}(e(\cdot); t; \hat{e}(\cdot)) = \int_0^t e^{-r(u-t)}(dc_m(u) - \frac{1}{2}\kappa e(u)^2 P_\xi(u)du) + E^M_{\xi,e} \left[ \int_t^\infty e^{-r(u-t)}(dc_m(u) - \frac{1}{2}\kappa \hat{e}(u)^2 P_\xi(u)du) \mid \mathcal{F}_t \right].$$

The manager’s maximum conditional valuation process represents the manager’s maximum conditional valuation of her future payoffs at date $t$ given that he has exerted effort $e(s); s \leq t$ and is defined as

$$Z_{dc_m,\xi}(e(\cdot); t) := \sup_{\hat{e}(\cdot)} Y_{dc_m,\xi}(e(\cdot); t; \hat{e}(\cdot)).$$

**Structure of Incentive Compatible Contract**

To simplify the subsequent notation, we occasionally drop the subscripts denoting the dependence of the processes defined in (61)-(64) on the contractual parameters whenever there is no danger of confusion. A contract $\Gamma \equiv (dc_m(\cdot), \xi(\cdot), e(\cdot))$ is incentive compatible for the manager only if, for the given project choice policy $\xi(\cdot)$ and payoff stream $dc_m(\cdot)$, the manager’s optimal effort choices are given by the process $e(\cdot)$. The following lemma provides necessary conditions for the incentive compatibility of a contract $\Gamma$. 

47
Lemma 1

A feasible contract \( \Gamma \equiv (d_c(m, \cdot), \xi(\cdot), e(\cdot)) \) is incentive compatible for the manager only if

\[
d_{cm}(t) = a(t)P_\xi(t)dt + b(t)dQ_\xi(t) + b'(t)dP_\xi(t), \quad \text{where} \tag{65}
\]

\[
b(t) = \kappa e(t), \tag{66}
\]

and \( a(t), b'(t) \) are \( \mathcal{F}_t \)-measurable.

**Proof.** When the manager’s compensation and continuation value processes evolve as in (65), we show below that her optimal effort choices after any given date \( t \) coincide with the process \( e(\cdot) \) regardless of her prior history of effort choices. It will then follow that the process \( e(\cdot) \) describes the manager’s optimal effort choices over time. By the principle of optimality of dynamic programming (Oksendal, 2003), the effort \( e(t) \) is optimal for the manager in period \( [t, t + dt] \) for any prior effort process \( e(\cdot) \) only if\(^\text{15}\)

\[
e(t) = \arg \max_{e(\cdot)} E_{\xi, e}^M \left[ e^{-r dt} Z(e'(\cdot); t + dt) - Z(e'(\cdot); t) \mid \mathcal{F}_t \right] = \arg \max_{e(\cdot)} E_{\xi, e}^M \left[ dZ(e'(\cdot); t) - r Z(e'(\cdot), t) dt \mid \mathcal{F}_t \right]. \tag{67}
\]

In what follows, we derive the infinitesimal change \( dZ(e'(\cdot);t) \) and then use (67) to establish the statements of the Lemma. It follows from the definition (64) that

\[
Z(e'(\cdot); t) = \sup_{e''(\cdot)} Y(e'(\cdot); t; e''(\cdot)) = \sup_{e''(\cdot)} Y(e(\cdot); t; e''(\cdot)) + \int_0^t \exp(-r(s-t)) \frac{1}{2} \kappa |e'(s)^2 - e(s)'^2| P_\xi(s) ds
\]

\[
= Z(e(\cdot); t) + X(e'(\cdot); t), \tag{68}
\]

where

\[
X(e'(\cdot); t) := \frac{1}{2} \kappa \int_0^t \exp(-r(s-t)) [e'(s)^2 - e(s)'^2] P_\xi(s) ds. \tag{69}
\]

The second equality in (68) follows because the contract only depends on past history, and because the manager’s effort in any period only affects earnings in that period and not the evolution of the process \( P_\xi(\cdot) \). Hence, her prior effort choices over the interval \( [0, t] \) do not affect her future optimal effort choices. It may be readily verified from (68) and (69) that

\[
dZ(e'(\cdot); t) = dZ(e(\cdot); t) + \frac{1}{2} \kappa (e'(t)^2 - e(t)'^2) P_\xi(t) dt + r X(e'(\cdot), t) dt. \tag{70}
\]

Since \( e(\cdot) \) represents the manager’s optimal effort choices by hypothesis, it follows that

\[
Z(e(\cdot), t) = \mathcal{U}(e(\cdot), t). \tag{71}
\]

\(^{15}\)Since the conditional expectation only depends on the process \( e'(\cdot) \) prior to date \( t \), which is an arbitrary process anyway, we avoid complicating the notation unnecessarily in (67) by using the same letter to denote a candidate (possibly sub-optimal) level of effort in the infinitesimal interval \( [t, t + dt] \).
By Remark 1, the process $\exp(-r(.))Z(e(\cdot), \cdot)$ is a square-integrable $\{F_t\}$-martingale under the measure $\Pi_{\xi,e}^M$. It follows from the martingale representation theorem (see Oksendal, 2003) that there exist square-integrable, $\{F_t\}$-adapted processes $\omega_1(\cdot), \omega_2(\cdot)$ such that

$$dZ(e(\cdot); t) = rZ(e; t)dt + \omega_1(t)dB_{\xi,e}^M(t) + \omega_2(t)dM_{\xi,e}^M(t),$$

where the second equality above follows from (57), (60), and Ito’s lemma. Since the expectation in the dynamic programming equation (67) is taken under the measure $\Pi_{\xi,e}^M$, it follows from (57), (60) and (72) that $Z(e(\cdot); t)$ evolves under this measure as

$$dZ(e(\cdot); t) = rZ(e(\cdot); t)dt + \omega_1(t)s_{\xi(t)}^{-1}(e'(t) - e(t))dt + \omega_1(t)dB_{\xi,e}^M(t) + \omega_2(t)dM_{\xi,e}^M(t).$$

Substituting (73) in (70) yields

$$dZ(e'(\cdot); t) = rZ(e(\cdot); t)dt + \omega_1(t)s_{\xi(t)}^{-1}(e'(t) - e(t))dt + \frac{1}{2}\kappa(e'(t)^2 - e(t)^2)P_\xi(t)dt + rX(e'(\cdot), t)dt + \omega_1(t)dM_{\xi,e}^M(t) + \omega_2(t)dB_{\xi,e}^M(t).$$

Having derived the requisite expression for $dZ(e'(\cdot); t)$, we substitute it in (67) to obtain

$$e(t) = \arg\max_{e'(t)}[\omega_1(t)s_{\xi(t)}^{-1}e'(t) + \frac{1}{2}\kappa e'(t)^2P_\xi(t)].$$

It then follows that the effort $e(t)$ is optimal over the interval $[t, t + dt]$ only if

$$\omega_1(t)/P_\xi(t) = -s_{\xi(t)}\kappa e(t).$$

From the definition of the continuation value process $M(t)$ in (62), and using (71), we have

$$M(t) = Z(e(\cdot); t) + \int_t^\infty \exp(-r(u - t))\left[\frac{1}{2}\kappa e(u)^2P_\xi(u)du - dc_m(u)\right].$$

Using (72), (73) and (76), we obtain

$$dM(t) + dc_m(t) = f(t)dt + b(t)dQ_\xi(t) + \omega_2(t)s_{\xi(t)}^{-1}d\ln P_\xi(t).$$

where $b(t)$ is given by (66), and $f(t)$ is $F_t$-measurable. Because the contract $\Gamma$ is the concatenation of single-period contracts (only single-period contracts are enforceable), the change in the manager’s continuation value $dM(t)$ over the period $[t, t + dt]$ cannot depend on the firm’s earnings $dQ_\xi(t)$ during the period. It then follows from (78) that the manager’s compensation $dc_m(t)$ must have the form (65).

**Structure of Optimal Contract**

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Identity (72) is an almost sure relation that holds under all equivalent probability measures.
Because only single-period contracts are enforceable, the contract \( \Gamma \) must be \textit{sequentially optimal}, that is, it must be optimal in every continuation game corresponding to all dates and histories. Suppose that there is nonzero slack in the constraint (20) at some date \( t \) for a \( \mathcal{F}_t \)-measurable set \( A \). We can construct a new contract \( \Gamma' \) that modifies \( \Gamma \) by increasing the manager’s payoff in period \([t, t + dt]\) by \( \epsilon > 0 \) conditional on the prior history at date \( t \) lying in the set \( A \). For a sufficiently small \( \epsilon \), shareholders’ participation constraints are satisfied at date \( t \). They are clearly satisfied at all subsequent dates because \( \Gamma' \) is identical to \( \Gamma \) at these dates. Since the bankruptcy level \( p_b \) (and, therefore, the bankruptcy time \( T_b \)) is fixed, the manager’s valuation of her future payoff stream under \( \Gamma' \) is strictly greater than the valuation under \( \Gamma \) at date \( t \) so that \( \Gamma \) is not sequentially optimal. Because the date \( t \) and the set \( A \) are arbitrary, shareholders’ dynamic participation constraints (20) must be satisfied with equality at each date and state. It follows that the following \textit{period by period} participation constraints must be satisfied by the optimal contract:

\[
E_{\xi,e} \left[ dc_s(t) | \mathcal{F}_t \right] \geq E_{\xi} \left[ ((1 - \tau)(P_{\xi}(t) - \theta) dt) | \mathcal{F}_t \right], \ t < T_b,
\]

\[
E_{\xi,e} \left[ dc_s(t) | \mathcal{F}_t \right] \geq E_{\xi} \left[ ((1 - \tau)P_{\xi}(t) dt) | \mathcal{F}_t \right], \ t \geq T_b.
\]

The manager’s effort only affects the earnings in any period and \textit{not} the evolution of the process \( P_{\xi}(\cdot) \). It follows that, under the optimal contract, \( b'(t) \) in (65) must be zero. By Lemma 1, (79), and some algebra, we can show that the manager’s optimal choice of her \textit{pay-performance sensitivity} \( b(t) \) in period \([t, t + dt]\) (in which \( \xi(t) = i \in \{1, 2\} \)) solves

\[
b(t) = \arg \max_b \frac{b}{\kappa} - \frac{1}{2} b^2 - \gamma bs_i.
\]

By (79), the parameter \( a(t) \) of the manager’s compensation is

\[
a(t) = (1 - b(t))(\ell + b(t)/\kappa) - b(t).
\]

Equation (26) follows directly from (80) and (81), while equations (27) and (28) follow by taking the expectation of the manager’s end-of-period payoff under her subjective valuation probability.

**Appendix B: Proof of Theorem 2**

We prove part a) of the theorem; the proof of part b) follows along similar lines. We use the following dynamic programming verification theorem. The proof follows using the arguments in Section IV of Fleming and Soner (1992) and is omitted for brevity (it is available upon request).

**Theorem 7 (Dynamic Programming Verification Theorem)**

a) Suppose that \( M_{\text{bket}} \) is the manager’s continuation value function from a stationary project choice policy \( \xi_{\text{bket}}^* \) after bankruptcy and it satisfies the following HJB equation and boundary conditions:

\[
\max_{i \in \{1, 2\}} \left[ L_i M_{\text{bket}} + g_i p \right] = 0, \tag{82}
\]

\[
\lim_{p \to \infty} \left| p^{-1} M_{\text{bket}}(p) \right| < \infty, \tag{83}
\]
where the operators $L^M_1$, $L^M_2$ are defined in (34). $M^{bkpt}$ is the manager’s optimal continuation value function and $\xi^*_{bkpt}$ is her optimal project choice policy after bankruptcy.

b) At any date $t > 0$, suppose $M$ is the manager’s continuation value function from a stationary project choice policy $\xi^*$ before bankruptcy and it satisfies the following conditions:

$$\max_{i \in \{1, 2\}} [L^M_i M + g_i p] = 0,$$

(84)

$$M(p_b) = M^{bkpt}((1 - \varsigma)p_b) \lim_{p \to \infty} |p^{-1}M(p)| < \infty,$$

(85)

$M$ is her optimal value function and $\xi^*$ is her optimal project choice policy prior to bankruptcy.

We use the following lemma frequently in our proof.

**Lemma 2**

Let $\eta^+_i, \eta^-_i; i \in \{1, 2\}$ be the positive and negative root, respectively, of the quadratic equation

$$\frac{1}{2} \sigma_i^2 x^2 + (\mu_i^M - \frac{1}{2} \sigma_i^2) x - r = 0$$

(86)

We have $\eta^-_i < 1 < \eta^+_i$.\footnote{It is easy to show that the quadratic equation (86) must have one positive and one negative root.}

**Proof.** By Assumption ?? and the fact that $\eta^+_i, \eta^-_i$ are the roots of (86), we have $\frac{1}{2} \sigma_i^2 + (\mu_i^M - \frac{1}{2} \sigma_i^2) - r = \frac{1}{2} \sigma_i^2 (1 - \eta^+_i)(1 - \eta^-_i) < 0$. Hence, $\eta^-_i < 1 < \eta^+_i$. Q.E.D.

Let $M^q$ and $M^{bkpt}$ denote the manager’s continuation value functions before and after bankruptcy from the policy of (i) always choosing project 1 after bankruptcy and (ii) choosing project 2 for $p_b < p < q$ and project 1 for $p > q$. The functions $M^q$ and $M^{bkpt}$ must solve the following system of ordinary differential equations (ODEs):

$$L^M_1 (M^q) + g_1 p = 0 \text{ for } p > q,$$

$$L^M_2 (M^q) + g_2 p = 0 \text{ for } p_b < p < q,$$

$$L^M_1 (M^{bkpt}) + g_1 p = 0 \text{ for all } p,$$

$$M^q(p_b) = (1 - \varsigma)M^{bkpt}(p_b),$$

(87)

where $L^M_1, L^M_2$ are defined in (34). The boundary condition at bankruptcy follows from the fact that the state variable $P$ falls as described by (11) at the bankruptcy time.

We can solve the system (87) to show that the functions $M^q$ and $M^{bkpt}$ are given by

$$M^{bkpt}(p) = \Omega_1 p \text{ for } p > 0,$$

(88)

$$M^q(p) = A^q p^{\eta^-_q} + \Omega_1 p \text{ for } p > q,$$

$$= B^q p^{\eta^+_q} + C^q p^{\eta^-_q} + \Omega_2 p \text{ for } p_b < p < q,$$

$$M^q(p_b) = (1 - \varsigma)M^{bkpt}(p_b),$$

(89)
where \( \eta_i^+, \eta_i^- \), \( i \in \{1, 2\} \) are the roots of (86) and \( \Omega_1, \Omega_2 \) are defined in (35). \( A^q, B^q, C^q \) are constants that depend on the switching trigger \( q \), which are determined by the conditions that the function \( M^q \) is continuously differentiable on \((p_b, \infty)\) and continuous at \( p_b \).

We now proceed with the proof of the main theorem. We prove the theorem by proving four intermediate propositions that represent the various cases we need to consider.

**Proposition 1**
If the manager’s project tradeoff \( \Omega_1 - \Omega_2 > 0 \), her optimal project choice policy after bankruptcy is to always choose project 1.

**Proof.** From (88),

\[
L_2^M M^{bkpt} + g_2 p = - (\Omega_1 - \Omega_2)(r - \mu_2^M)p < 0,
\]

(90)

where the last inequality follows from the fact that the manager’s project tradeoff \( \Omega_1 - \Omega_2 > 0 \) by hypothesis, and \( r - \mu_2^M > 0 \) by Assumption ???. The function \( M^{bkpt} \), therefore, satisfies the HJB equation (82) and is the optimal value function of the manager after bankruptcy by Theorem 7.

**Proposition 2**
Suppose that \( \Omega_1 - \Omega_2 > 0 \) and \( \eta_1^- < \eta_2^- \). The manager’s optimal project choice policy before bankruptcy is to always choose project 1.

**Proof.** If the manager follows her optimal project choice policy of always choosing project 1 after bankruptcy, then the value function \( M^{pb} \) of always choosing project 1 before bankruptcy must be given by (89) with the switching trigger \( q \) equal to \( p_b \). The value function \( M^{pb} \) must be given by

\[
M^{pb}(p) = A^{pb}p^{\eta_1^-} + \Omega_1 p.
\]

(91)

The second term on the right hand side above represents the value function of choosing project 1 in the hypothetical scenario where the manager faces no personal costs due to bankruptcy. Since the value in the presence of personal costs must be strictly lower,

\[
A^{pb} < 0.
\]

(92)

From (91),

\[
L_2^M M^{pb} + g_2 p = A^{pb}p^{\eta_1^-} \left( \frac{1}{2} \sigma_2^2(\eta_1^-)^2 + (\mu_2^M - \frac{1}{2} \sigma_2^2)(\eta_1^-) - r \right) - (\Omega_1 - \Omega_2)(r - \mu_2^M)p
= A^{pb}p^{\eta_1^-} \left( \frac{1}{2} \sigma_2^2(\eta_1^- - \eta_2^-)(\eta_1^- - \eta_2^+)(\eta_1^- - \eta_2^-) - (\Omega_1 - \Omega_2)(r - \mu_2^M)p. \right.
\]

(93)

The second equality above follows from the fact that \( \eta_1^+, \eta_2^- \) are the roots of (86) for \( i = 2 \).

From (92) and the fact that \( \eta_1^- < \eta_2^- \) by hypothesis, the first term in the last expression above is negative. Since the project tradeoff \( \Omega_1 - \Omega_2 > 0 \) by hypothesis, and \( r - \mu_2^M > 0 \) by Assumption ???, the second term is positive. Hence, the entire expression is negative and the function \( M^{pb} \) satisfies (82) and is, therefore, the optimal value function of the manager before bankruptcy by Theorem 7.
Proposition 3
Suppose that $\Omega_1 - \Omega_2 > 0$, $\eta_2^- < \eta_1^-$ and
\[
\lim_{p \to p_b^+} L_2^M M^p + g_2 p \leq 0. \tag{94}
\]
The manager’s optimal project choice policy before bankruptcy is to always choose project 1.

Proof. The continuation value function $M^p$ of always choosing project 1 before bankruptcy is given by (91) where (92) holds. Because $\eta_2^- < \eta_1^-$ by hypothesis and $\lambda p < 0$ by the same arguments used to arrive at (92), the first term on the right hand side of the second equality in (93) is positive. The second term is also positive because $\Omega_1 - \Omega_2 > 0$ by hypothesis and $r - \mu_2^M > 0$ by Assumption ??.

Let us denote the function $L_2^M M^p + g_2 p$ as $f(p)$. We note that $f$ is a decreasing function so that the hypothesis (94) implies that $f(p) \leq 0$ for $p > p_b$. Hence, the function $M^p$ satisfies the HJB equation (82) and is the optimal value function of the manager before bankruptcy.

We use the following lemma in the proof of the subsequent proposition.

Lemma 3 (Super Contact Condition)
If $M^q$ is defined as in (87) with $p_b < q < \infty$ then the following equivalent conditions are necessary and sufficient for $M^q$ to be twice differentiable at $p = q$ is
\[
L_2^M M^q + g_2 p|_{p=q-} = 0, \tag{95}
\]
\[
L_1^M M^q + g_1 p|_{p=q-} = 0. \tag{96}
\]

Proof. Since $M^q$ is the value function of the policy of choosing project 2 for $p_b < q < q$ and project 1 for $p > q$, we have $L_2^M M^q + g_2 p = 0$ for $p < q$. Hence, $L_2^M M^q + g_2 p|_{p=q-} = 0$. If (95) also holds, then subtracting the two we obtain $L_2^M M^q|_{p=q+} - L_2^M M^q|_{p=q-} = 0$. Since $M^q$ is continuous and differentiable at $p = q$, it must also be twice differentiable. We can show that condition (96) is equivalent to (95) using similar arguments. Q.E.D.

Proposition 4
Suppose that $\Omega_1 - \Omega_2 > 0$, $\eta_2^- < \eta_1^-$ and
\[
\lim_{p \to p_b^+} L_2^M M^p + g_2 p > 0. \tag{97}
\]
There exists $p^* > p_b$ such that the manager optimally chooses project 2 for $p_b < p < p^*$, and project 1 for $p > p^*$.

Proof. The idea of the proof of the proposition is to use Lemma 3 to show the existence of a trigger $p^*$ such that the value function $M^{p^*}$ given by (89) (setting $q = p^*$) is twice differentiable at $p^*$. We then proceed to show that the function $M^{p^*}$ is the optimal value function of the manager implying that $p^*$ is the ”optimal switching trigger”.

Step 1: We first show the existence of $p^*$ such that
\[
L_1^M M^{p^*} + g_1 p|_{p=p^*-} = 0. \tag{98}
\]
We prove the existence of \( p^* \) by showing that among the restricted sub-class of policies indexed by the parameter \( q \) where the manager chooses project 2 for \( p_2 < p < q \) and project 1 for \( p > q \), there exists \( q^* \) such that the policy of switching projects at \( q^* \) is optimal within this restricted sub-class of policies. We then show that \( q^* \) and the corresponding value function \( M^{q^*} \) satisfy (98) thereby implying that \( q^* \) is the required trigger \( p^* \).

**Step 2:** The continuation value functions \( M^{\infty} \) and \( M^{bkpt} \) of the policy of always choosing project 2 before bankruptcy and project 1 after bankruptcy are given by (89) and (88) with the switching trigger \( q \) equal to \( \infty \). Because the function \( M^{\infty} \) must satisfy the boundary conditions (85), it follows that \( M^{\infty} \) has the following functional form:

\[
M^{\infty}(p) = C^{\infty} p^{\eta_2^*} + \Omega_2 p, \\
M^{\infty}(p_h) = (1 - \varsigma) \Omega_1 p_h.
\]  

(99)

We now note that

\[
L_1^M M^{\infty} + g_1 p = C^{\infty} \left[ \frac{1}{2} \sigma_1^2 (\eta_2^-)^2 + (\mu_1 - \frac{1}{2} \sigma_1^2) (\eta_2^-) - r \right] p^{\eta_2^-} + \left[ (\Omega_1 - \Omega_2) (r - \mu_1^M) p \right].
\]  

(100)

Since \( \eta_2 < 1 \) by Lemma 2, \( \Omega_1 - \Omega_2 > 0 \) by hypothesis, and \( r - \mu_1^M > 0 \) by Assumption ??, the right hand side above is positive for sufficiently large \( p \). It follows as a special case of Theorem 7 that the policy of always choosing project 2 when the firm is solvent cannot be optimal within the restricted sub-class of policies characterized by a single switching trigger. The condition (97) implies that the policy of always choosing project 1 is also sub-optimal. By the continuity of \( M^{q} \) as a function of \( q \), therefore, there exists \( q^* \) such that the policy of switching projects at \( q^* \) is optimal within the restricted sub-class of policies characterized by a single switching trigger.

**Step 3:** We now show that \( q^* \) and \( M^{q^*} \) satisfy (98). Suppose, to the contrary that \( L_1^M M^{q^*} + g_1 p|_{p=q^-} > 0 \). We can use Itô’s lemma to show that there exists \( q' < q^* \) such that the value function \( M^{q'} > M^{q^*} \), which contradicts the optimality of the value function \( M^{q^*} \) within the restricted sub-class of policies where the manager switches projects at a single trigger level. Similarly, if \( L_1^M M^{q^*} + g_1 p|_{p=q^+} < 0 \), we can use arguments similar to those used in the proof of Lemma 2 to show that \( L_2^M M^{q^*} + g_2 p|_{p=q^+} > 0 \). We can then use Itô’s lemma to show that there exists \( q' > q \) such that \( M^{q'} < M^{q^*} \), which is again a contradiction. It follows that (98) is satisfied so that \( q^* \) is the required optimal switching trigger by Lemma 2.

**Step 4:** We now establish the optimality of the switching trigger \( p^* \) among all possible policies, not just those characterized by a single switching trigger. For \( p > p^* \),

\[
L_2^M M^{p^*} + g_2 p = A^{p^*} \left[ \frac{1}{2} \sigma_2^2 (\eta_1^-)^2 + (\mu_2 - \frac{1}{2} \sigma_2^2) (\eta_1^-) - r \right] p^{\eta_1^-} - \left[ (\Omega_1 - \Omega_2) (r - \mu_2^M) p \right].
\]  

(101)

Since \( \Omega_1 - \Omega_2 > 0 \) and \( r - (\mu_2^M - \frac{1}{2} \sigma_2^2) - \frac{1}{2} \sigma_2^2 > 0 \) by Assumption ??, the second term on the right hand side above is positive so that (98) can only hold if the first term is also positive. Because \( \frac{1}{2} \sigma_2^2 (\eta_1^-)^2 + (\mu_2^M - \frac{1}{2} \sigma_2^2) (\eta_1^-) - r = \frac{1}{2} \sigma_2^2 (\eta_1^- - \eta_2^-) (\eta_1^- - \eta_2^-) < 0 \) as \( \eta_2^- < \eta_1^- < \eta_2^* \), we must have \( A^{p^*} < 0 \). By arguments similar to those used in the proof of Proposition 3, the fact that \( L_2^M M^{p^*} + g_2 p|_{p=p^*} = 0 \) implies that \( L_2^M M^{p^*} + g_2 p|_{p=p^*} < 0 \) for \( p > p^* \).
implies that it is negative for \( p_b < p < p^* \). For \( p_b < p < p^* \),
\[
M^{p^*} = B^{p^*}p^{\eta_2^*} + C^{p^*}p^{\eta_1^*} + \Omega_2p. \tag{102}
\]
It follows from the above that
\[
L_1^M M^{p^*} + g_1p = B^{p^*}p^{\eta_2^*} \left[ \frac{1}{2} \sigma_1^2 (\eta_2^* - \eta_1^-)(\eta_2^* - \eta_1^+) \right] + C^{p^*}p^{\eta_1^*} \left[ \frac{1}{2} \sigma_1^2 (\eta_2^- - \eta_1^-)(\eta_2^- - \eta_1^+) \right] + \left[ (\Omega_1 - \Omega_2)(r - \mu_1^M)p \right]. \tag{103}
\]

We need to consider the two cases \( \eta_1^- \leq \eta_2^- \) and \( \eta_1^- > \eta_2^- \) separately.

Suppose that \( \eta_1^- \leq \eta_2^- \). After some tedious algebra that we omit for brevity, we can show that the twice differentiability of \( M^{p^*} \) at \( p = p^* \) implies that \( B^{p^*} > 0, C^{p^*} < 0 \). Since \( \eta_2^- < \eta_1^- < \eta_1^+ \leq \eta_2^+ \), the first term on the right hand side of (103) is positive and increasing in \( p \). Denote the sum of the second and third terms by \( f(p) \). \( f \) is increasing for \( p < p^* \). Hence, \( L_1^M M^{p^*} + g_1p \) is increasing for \( p < p^* \) so that (98) implies that it is negative for \( p < p^* \).

Suppose that \( \eta_1^- > \eta_2^- \). We can show that \( B^{p^*} < 0, C^{p^*} < 0 \). We can use arguments analogous to those used in the earlier case to show that \( L_1^M M^{p^*} + g_1p \) is increasing for \( p < p^* \) so that (98) implies that it is negative for \( p < p^* \). Therefore, \( M^{p^*} \) satisfies the HJB equation (82) and is the optimal value function of the manager before bankruptcy. The policy of switching projects at \( p^* \) is globally optimal. Q.E.D.

### Appendix C: Proof of Theorem 3

Applying the continuous time approximation, the market value of debt \( D(t) \) at any date \( t \) is
\[
D(t) = E_{x,e} \left[ \int_t^{T_b} \exp(-rt)c_d(t)dt + \int_{T_b}^\infty \exp(-rt)c_d(t)dt \right]. \tag{104}
\]
By (13), the fact that the participation constraints (20) are satisfied with equality, Ito’s lemma, and part a) of Theorem 2, the value of debt must satisfy the following for \( t > T_b \):
\[
L_1D + (1 - \tau)p = 0, \tag{105}
\]
\[
\lim_{p \to 0} D(p) = \lim_{p \to \infty} D(p)/p < \infty. \tag{106}
\]
We have directly represented the value of debt as a function of the current value \( p \) of the state variable \( P \). The solution of the above ODE is given by \( D(p) = \frac{(1 - \tau)p}{\tau - \mu_1^p} \). For \( t < T_b \), it follows from (13) and part a) of Theorem 2 that debt value satisfies
\[
D(p) = E \left[ \theta dt + \exp(-rt)D(P^i(t + dt)) \right], \tag{107}
\]
where \( P^i(t + dt) \) is the end-of-period value of the state variable in a period in which the manager chooses project \( i \). Applying Ito’s lemma to the above, we can show that the value of debt satisfies
(37) with the boundary conditions arising from the fact that the debt value is finite for all values of \( p \) and continuous at the bankruptcy level. The solution to the system (37) is

\[
D(p) = \begin{cases} 
Ap^\zeta + \frac{\theta}{r} & \text{for } p > \hat{p}, \\
Bp^\zeta + Cp^{\zeta_2} + \frac{\theta}{r} & \text{for } p' < p < \hat{p}, \\
\frac{(1 - \zeta)(1 - \tau)p'_b}{r - \mu_1} & \text{for } p' < \hat{p},
\end{cases}
\]

where the constants \( A, B, \) and \( C \) are explicitly determined by the conditions that the debt value is continuously differentiable on \((p'_b, \infty)\) and continuous at \( p'_b \). In (108), \( \zeta^+_i, \zeta^-_i; i \in \{1, 2\} \) are the positive and negative roots of the quadratic equations

\[
\frac{1}{2}\sigma_i^2 x^2 + \frac{\mu_i - 1/2}{2}\sigma_i^2 x - r = 0.
\]

The market value of equity is zero after bankruptcy. By (14), the fact that the participation constraints (20) are satisfied with equality, and part a) of Theorem 2, the value of equity \( S(p) \) at any date \( t \) prior to bankruptcy must satisfy

\[
S(p) = E\left[\left(1 - \tau\right)(p - \theta)dt + \exp(-r dt)S(P^i(t + dt))\right].
\]

By (13) and (14), the first term inside the expectation above is the after-tax payout flow to equity over the period \([t, t+dt]\). It follows from (109), Ito’s lemma, and the fact that shareholders optimally choose the bankruptcy level \( p'_b \) that the value of equity satisfies (38) whose solution is

\[
S(p) = \begin{cases} 
Kp^\zeta + \frac{(1 - \tau)p}{r - \mu_1} - \frac{(1 - \tau)\theta}{r} & \text{for } p > \hat{p}, \\
Lp^\zeta + Mp^{\zeta_2} + \frac{(1 - \tau)p}{r - \mu_2} - \frac{(1 - \tau)\theta}{r} & \text{for } p < \hat{p},
\end{cases}
\]

where \( K, L, M \) and the bankruptcy level \( p'_b \) are determined by the conditions that the equity value is continuously differentiable. Q.E.D.

Appendix D: Proof of Theorem 4

**Step 1:** For a given bankruptcy level \( p_b \), the manager’s optimal project choice policy is characterized by the optimal switching trigger \( \hat{p} = F(p_b) \). Since the manager’s optimal project choice policy is unique, \( F \) is a well-defined function. It also follows from the proof of Theorem 2 that \( F \) is a continuous function (we omit the straightforward, but laborious, proof for brevity). Given a project choice policy described by the switching trigger \( \hat{p} \), shareholders optimally declare bankruptcy at the level \( p'_b = G(\hat{p}) \) given by solving (37) and (38). Moreover, \( G \) is well-defined and continuous. Therefore, \( \Psi = G \circ A \) is also continuous.

**Step 2:** We now show that the range of the function \( \Psi \) is a compact, convex set. The range of the function \( F \) defined in Step 1 is \([0, \infty]\). However, \( \hat{p} = \infty \) corresponds to the policy of always choosing project 2 (recall the discussion after the statement of Theorem 2). Given this policy, the level \( G(\infty) \) at which shareholders optimally declare bankruptcy is clearly finite. Similarly, \( \hat{p} = 0 \)
corresponds to the policy of always choosing project 1. Given such a policy, the optimal bankruptcy level \( G(0) \) is also finite. Since \( G \) is continuous, its range \( C \) is compact. Further, because the domain of the function \( G \) is \([0, \infty]\), its range \( C \) is connected.

**Step 3:** Consider the restriction of the function \( \Psi \) to the set \( C \). Because \( \Psi \) maps the compact, convex set \( C \) into itself by steps 1 and 2, it has a fixed point by the Brouwer fixed point theorem.

**Appendix E: Proof of Theorem 6**

**The Effects of Ability and Disutility of Effort:** We first analyze the effects of the manager’s ability and disutility of effort on long-term debt. Since the manager chooses project 1 over time, it follows from Theorem 1 and (27) that the manager’s continuation value for a given coupon \( \theta \) is

\[
M_\theta(0) = g_1 E^M \left[ \int_0^{T_b(\theta)} P(t) dt + \int_{T_b(\theta)}^\infty P(t) dt \right] = g_1 M_\theta(0),
\]

where we explicitly indicate the pre and post-bankruptcy portions of the manager’s payoff stream for clarity. By Theorem 3, the endogenous bankruptcy time \( T_b(\theta) \) does not depend on the manager’s characteristics. It follows that the manager’s ability and disutility of effort affect the constant \( g_1 \) in (111), but not \( \overline{M}_\theta(0) \). By (26) and (28), \( g_i \) increases with \( \ell \) and decreases with \( \kappa \) so that the manager’s continuation value for a given coupon \( \theta \) increases with \( \ell \) and declines with \( \kappa \). The manager’s optimal choice of long-term debt coupon solves

\[
\theta^{opt} = \arg \max_\theta \left[ g_{initial}[F_\theta(0) - I] + g_1 M_\theta(0) \right] = \arg \max_\theta G(\theta, \ell, \kappa),
\]

where \( F_\theta(0) \) is the value of the firm net of the manager’s stake. We explicitly indicate the dependence of \( G \) on \( \theta, \ell \) and \( \kappa \) for clarity. Because the manager captures the surplus she generates in each period, \( F_\theta(0) \) does not depend on the manager’s ability or disutility of effort. If \( \pi \in \{\ell, \kappa\} \), by the implicit function theorem,

\[
\frac{\partial \theta^{opt}}{\partial \pi} = -\frac{\partial^2 G/\partial \theta \partial \pi}{\partial^2 G/\partial \theta^2} \bigg|_{\theta = \theta^{opt}}.
\]

By (112),

\[
\frac{\partial^2 G/\partial \theta^2}{\theta = \theta^{opt}} < 0 \text{ by the second order condition for a maximum.}^{18}
\]

The manager’s optimal choice of long-term debt coupon solves

\[
\theta^{opt} = \arg \max_\theta \left[ g_{initial}[F_\theta(0) - I] + g_1 M_\theta(0) \right] = \arg \max_\theta G(\theta, \ell, \kappa),
\]

where \( F_\theta(0) \) is the value of the firm net of the manager’s stake. We explicitly indicate the dependence of \( G \) on \( \theta, \ell \) and \( \kappa \) for clarity. Because the manager captures the surplus she generates in each period, \( F_\theta(0) \) does not depend on the manager’s ability or disutility of effort. If \( \pi \in \{\ell, \kappa\} \), by the implicit function theorem,

\[
\frac{\partial \theta^{opt}}{\partial \pi} = -\frac{\partial^2 G/\partial \theta \partial \pi}{\partial^2 G/\partial \theta^2} \bigg|_{\theta = \theta^{opt}}.
\]

\[
\frac{\partial^2 G/\partial \theta^2}{\theta = \theta^{opt}} < 0 \text{ by the second order condition for a maximum.}^{18}
\]

By (112),

\[
\frac{\partial^2 G/\partial \theta^2}{\theta = \theta^{opt}} < 0 \text{ because the bankruptcy level increases with } \theta. \text{ Since } g_1 \text{ increases with } \ell \text{ and decreases with } \kappa, \text{ it follows from (113) that } \theta^{opt} \text{ and the long-term debt value decrease with } \ell \text{ and increase with } \kappa.
\]

**The Effects of the Manager’s Cost of Risk:** The analysis of the effects of the manager’s cost of risk is a little more involved because the cost of risk \( \gamma \) affects \( g_1 \) as well as \( \overline{M}_\theta(0) \) in (111). By

---

^{18}It can be shown from (112) that the second order condition holds \textit{generically} for the set of possible values of \( \pi \) and \( \kappa \), that is, it holds except perhaps for isolated points.
the arguments in the proof of Proposition 2 (see especially 91), the manager’s continuation value from the policy of choosing project 1 when the long-term debt coupon is $\theta$ is

$$M_\theta(p) = A p^{\eta_1} + \frac{g_1 p}{r - \mu_1^M},$$

(115)

where the constant $A$ is determined by the boundary condition

$$Ap_\theta(\theta)^{\eta_1} + \frac{g_1 p_\theta(\theta)}{r - \mu_1^M} = (1 - \zeta) \frac{g_1 p_\theta(\theta)}{r - \mu_1^M},$$

(116)

which expresses the fact that the manager’s value function must be continuous at the bankruptcy level $p_\theta(\theta)$. By (115) and (116),

$$M_\theta(p) = -\frac{\zeta g_1 p_\theta(\theta)(\frac{p}{p_\theta(\theta)})^{\eta_1}}{r - \mu_1^M} + \frac{g_1 p}{r - \mu_1^M},$$

(117)

The second term on the right hand side above does not depend on $\theta$. Hence,

$$\frac{\partial M_\theta(p)}{\partial \theta} = -\frac{\zeta g_1 (1 - \eta_1)(\frac{p}{p_\theta(\theta)})^{\eta_1}}{r - \mu_1^M} p_\theta(\theta).$$

(118)

Because the bankruptcy level increases with the long-term debt coupon $\theta$, $p_\theta'(\theta) > 0$. Note also that, by our earlier discussion, the bankruptcy level does not depend on manager characteristics and, in particular, her cost of risk $\gamma$.

We now show that $\frac{\partial M_\theta(p)}{\partial \theta}$ increases with $\gamma$. By (10), $\frac{1}{r - \mu_i^M}$ decreases with $\gamma$. By (26) and (28), $g_1$ decreases with $\gamma$. Since $\eta_1$ is the negative root of the equation (86) (with $i = 1$), it follows from (10) that

$$\eta_1 = \frac{1}{2} \sigma_1^2 - \mu_1^M - \sqrt{(\frac{1}{2} \sigma_1^2 - \mu_1^M)^2 + 2r \sigma_1^2}$$

$$= \frac{1}{2} \sigma_1^2 - \mu_1 + \gamma \sigma_1 - \sqrt{(\frac{1}{2} \sigma_1^2 - \mu_1 + \gamma \sigma_1)^2 + 2r \sigma_1^2}$$

(119)

Differentiating the above with respect to $\gamma$, we have

$$\frac{d\eta_1}{d\gamma} = \frac{1}{\sigma_1} \left[ 1 - \frac{\frac{1}{2} \sigma_1^2 - \mu_1 + \gamma \sigma_1}{\sqrt{(\frac{1}{2} \sigma_1^2 - \mu_1 + \gamma \sigma_1)^2 + 2r \sigma_1^2}} \right] > 0,$$

(120)

Hence, $\eta_1$ increases with $\gamma$. Hence, $1 - \eta_1$ decreases with $\gamma$ and $(\frac{p}{p_\theta(\theta)})^{\eta_1}$ decreases with $\gamma$ (because $p > p_\theta$). By the above arguments and (118), we see that $\frac{\partial M_\theta(p)}{\partial \theta}$ increases with $\gamma$, that is, $\frac{\partial^2 M_\theta(p)}{\partial \theta \partial \gamma} > 0$.

By our earlier arguments, the manager’s initial payoff for a given coupon does not depend on her cost of risk. Similar to (113), therefore, if $\theta_{opt}(\gamma)$ is the manager’s optimal choice of long-term debt coupon when her cost of risk is $\gamma$, we have

$$\frac{\partial \theta_{opt}(\gamma)}{\partial \gamma} = -\frac{\partial^2 G}{\partial \theta \partial \gamma} |_{\theta = \theta_{opt}(\gamma)},$$

(121)

The denominator in the fraction on the right hand side above is negative by the second order condition for a maximum, while the numerator is positive because $\frac{\partial^2 M_\theta(p)}{\partial \theta \partial \gamma} > 0$ by the earlier discussion. It follows that $\frac{\partial \theta_{opt}(\gamma)}{\partial \gamma} > 0$, that is, the long-term debt coupon increases with $\gamma$. Hence, the long-term debt coupon and the long-term debt value increase with $\gamma$. 

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References


