Communication, Correlation of Beliefs and Asset Price Fluctuations∗

Hiroyuki Nakata†

Essex Finance Centre
University of Essex

June 16, 2004

Abstract

This paper studies how communication amongst agents influences the equilibrium of a financial economy. We set up a standard overlapping generations (OLG) model with assets, while allowing for heterogeneous beliefs. The paper explicitly describes how communication generates correlation of beliefs, and show that communication can be embedded in the models of rational beliefs that do not model communication explicitly a priori. We confine our attention to a Markovian economy, in which the beliefs of the agents are all Markovian. Simulation results are provided to examine the effects of communication, in which beliefs of the agents are classified in accord to the reactions to communication.

1 Introduction

This paper studies how communication amongst agents influences the equilibrium of a financial economy while allowing for heterogeneous beliefs. More specifically, we introduce communication in a standard overlapping generations (OLG) model with heterogeneous beliefs.

We postulate that communication amongst different economic agents is a mechanism that generates correlation of beliefs. Note that it is essential that heterogeneity of beliefs is present in order for communication to have an impact on the equilibrium (through correlation of beliefs), because the only role communication can play is to possibly remove (the impacts of initial) asymmetric information when rational expectations or a common prior

∗Preliminary. Comments are welcome. The author appreciates comments from the participants of the workshops at Essex and Yokohama National University.

†Address for correspondence: Department of Accounting, Finance and Management, University of Essex, Wivenhoe Park, Colchester, CO4 3SQ, United Kingdom. E-mail: hnakata@stanfordalumni.org or hnakata@essex.ac.uk.
is assumed. Once correlation of beliefs is present in a general equilibrium model of a financial economy, it is obvious that the equilibrium will be influenced by such correlation. Amongst the studies on heterogeneous beliefs, some studies of rational beliefs examine the effects of correlation of beliefs numerically, e.g. Kurz and Schneider (1996), Kurz and Beltratti (1997), and Kurz and Motolese (2001). In fact, their results indicate that correlation of beliefs amplifies the volatility of the economy. They, however, do not describe the mechanism that generates correlation of beliefs, while we explicitly describe such a mechanism by introducing communication. In other words, correlation of beliefs is endogenous in our model.

However, it is not clear in what ways communication generates correlation of beliefs, and thus, how it influences the equilibrium. It is therefore of interest to study the effects of communication on the dynamical (equilibrium) behaviour of the economy. To be more specific, we shall address the following questions:

- Does communication have a tendency to stabilize or destabilize the market?
- Under what conditions will the market be destabilized, versus being stabilized?
- How do the results depend on the nature of the market (with public information)?
- What are the implications of these results for public policy in financial markets?

We are interested in situations in which there is abundant commonly shared information, because such situations are supposed to be more common in major financial markets today. For example, there are numerous newsletters that are available to the public. In particular, it is a common practice for leading (financial) analysts to publicly announce their forecasts or opinions about various financial instruments and/or the (macro-)economic climate. In addition, because of the strict regulations on insider dealing, at least in theory, there should not be any immense problems that arise from asymmetric information in major financial markets today. Hence, it is natural to think that people frequently communicate and share information in financial markets as our model describes below.

---

1. Blackwell and Dubins (1962) show that the conditional probabilities merge in the limit as agents accumulate the same information when mutual absolute continuity of measures is assumed. Also, Geanakoplos and Polemarchakis (1982) show that a merging of opinions occurs within finite steps when two agents communicate back and forth when the information structure is characterized by information partitions under the common prior assumption. However, a merging of opinions is a degenerate case when the mutual absolute continuity of measures and/or the common prior assumption is absent. See Freedman (1965) and Nakata (2003).

2. We admit that asymmetric information even in the first order sense is very important, not to mention the recent dramatic incidents at Enron and/or Worldcom.
With this in mind, we construct a model that incorporates communication under heterogeneous beliefs. In doing so, we confine our attention to the case in which announcements alone do not expand the state space of the economy.\footnote{Nakata (2001) studies the case in which announcements expand the state space of the economy.} For the class of announcements/communication specified, we show that a simulation model that is found in Kurz and Beltratti (1997), which does not model communication explicitly a priori, is compatible with our model. Moreover, we introduce ambiguity concerning the choice of effective beliefs, and represent it in terms of a probability, which is not understood by the agents. However, our treatment of ambiguity does not capture the concept of ambiguity found in the Ellsberg Paradox, i.e. Knightian uncertainty. Hence, we do not follow the line of research that focuses on the issue, e.g. Gilboa and Schmeidler (1989), Epstein and Wang (1994, 1996), although we introduce a notion of ‘set of probability beliefs’.

To provide answers to the above questions, we construct a simulation model, which is identical to the one found in Kurz and Beltratti (1997). However, the simulation model does incorporate communication in the light of our construction. This implies that we can reinterpret the results of the rational belief equilibrium models that do not model communication a priori by asserting that there is one. In fact, one advantage of such a reinterpretation is that we can classify the effective beliefs of the agents in a more intuitive way to see under what conditions communication destabilizes or stabilizes the economy.

From a more theoretical perspective, our classification of effective beliefs suggests a way to restrict the class of rational beliefs, which is fairly large. Namely, rather than taking an axiomatic approach, we suggest an empirical approach. To be more specific, we classify the beliefs in accord with reaction to communication (e.g. conformist, contrarian, etc.), which is observable and also characterises the correlation of beliefs, while correlation of beliefs/measure itself is not observable. We discuss advantages of an empirical approach over the axiomatic one briefly.

The rest of the paper proceeds as follows. In Section 2, the structure of the economy and the beliefs of the agents are explained first, and then the equilibrium of the economy is defined. Then, the definition of rational beliefs is given so as to define the rational belief equilibrium, which is central to our analysis. Section 3 examines a simulation model, which is the same as the one found in Kurz and Beltratti (1997). The simulation results are interpreted in the light of our construction of beliefs and communication to answer the above questions. Finally, Section 4 concludes the paper.

2 The Model

In this section, we first introduce a standard OLG model with financial assets albeit with communication. Then, we describe the structure of beliefs of the agents. After describing the young agent’s problem and its optimality condi-
tions, we define the competitive equilibrium of the economy, while restricting our attention to Markovian economies. Finally we define the Markov rational belief equilibrium of the economy.

2.1 The Structure of the Model

2.1.1 The Structure of the Economy

The structure of the economy is essentially the same as that of Kurz and Beltratti (1997), except that we model communication explicitly. Consider a standard OLG economy with \( H \) young agents in each generation which we denote by \( h = 1, 2, ..., H \) (\( H \) is some finite positive integer). Also, there are \( H \) old agents in each period. There is a single perishable consumption good, whose price is normalized to unity in every period \( t \). We assume that only young agents receive an endowment \( W^h_t \) (\( t = 1, 2, ... \)) of this consumption good, except that in the initial period (\( t = 0 \)) old agents (in period 1; born in period 0) receive endowments of the stock specified below (\( \theta^h_0 \) with \( \sum_{h=1}^{H} \theta^h_0 = 1 \)). Furthermore, each young agent is a replica of the old agent who preceded him, where a replica refers to the preferences and the set of beliefs. This makes us interpret the streams of agents as ‘dynasties’ or ‘types’. Also, there is a single infinitely lived firm owned by the agents. Let \( P_t \) denote the stock price of the firm in period \( t \) and \( \theta^h_t \) the shareholding of young agent \( h \) purchased in period \( t \). We assume without loss of generality that the aggregate supply of shares is fixed to unity in every period. The firm’s technology generates an exogenous random stream of returns \( \{D_t\}_{t=0}^{\infty} \), and we call it the dividend stream. We assume that \( D_t > 0 \) for all \( t \). For the agents, shareholding yields income from the dividend as well as a capital gain or loss. In addition, there is a market for a zero net supply, short term riskless debt instrument which we call a ‘bill’. To summarize, the economy has three markets: (a) a market for the consumption good with an aggregate supply equaling the total endowment and the total dividends, (b) a stock market with a total supply of unity, and (c) a market for a zero net supply, short term riskless debt instrument which we call a ‘bill’. We list the notation as follows: for each agent \( h \),

\[
\begin{align*}
C_{1h}^t &: \text{consumption of agent } h \text{ when young in period } t; \\
C_{2h}^{t+1} &: \text{consumption of } h \text{ when old in } t+1 \text{ (the agent was born in } t); \\
d_{t+1} &: D_{t+1}/D_t: \text{the random growth rate of dividends}; \\
\theta^h_t &: \text{amount of stock purchases by young agent } h \text{ in period } t; \\
B^h_t &: \text{amount of one-period bill purchased by young agent } h \text{ in period } t; \\
W^h_t &: \text{endowment of young agent } h \text{ in period } t; \\
P_t &: \text{the price of the stock in period } t; \\
p_t &: P_t/D_t: \text{the price/dividend ratio in period } t; \\
q_t &: \text{the price of the one-period bill in period } t. \text{ This is a discount price.}
\end{align*}
\]

Next, we specify the structure of the dividend process. Our specification follows that of Mehra and Prescott (1985), which is standard in the literature.

\[4\text{We explain what we mean by a set of beliefs when we specify the structure of beliefs.}\]
Namely,
\[ D_{t+1} = d_{t+1} D_t, \tag{1} \]
where the stochastic process \( \{d_t\}_{t=1}^{\infty} \) is a stationary and ergodic Markov process. Following Kurz and Beltratti (1997), the state space of the process is \( D := \{d^H, d^L\} \), and the stochastic process \( \{d_t\}_{t=1}^{\infty} \) is driven by a transition probability matrix
\[
\begin{bmatrix}
\phi & 1 - \phi \\
1 - \phi & \phi
\end{bmatrix}.
\tag{2}
\]
With this specification, the dividends may well tend to rise over time; thus it is more convenient to focus on the growth rates of the economic variables. To this end, we define the following variables:

- \( w_h^t := W_h^t / D_t \): the endowment/dividend ratio of young agent \( h \);
- \( b_h^t := B_h^t / D_t \): the bill/dividend ratio of young agent \( h \) in \( t \);
- \( c_{1h}^t := C_{1h}^t / D_t \): the ratio of consumption to aggregate capital income when young;
- \( c_{2h}^{t+1} := C_{2h}^{t+1} / D_t^{t+1} \): the ratio of consumption to aggregate capital income when old.

In order to elucidate the sources of randomness of the economy, we assume that \( w_h^t = w^h \) are constant for all \( h, t \). Hence, the aggregate endowment of the consumption \( \sum_{h=1}^{H} W_h^t \) is proportional to the total dividend \( D_t \) in each period \( t \).

Before transactions of the financial instruments and consumption take place, each young agent makes an announcement \( Y_h^t \) publicly. Hence, the announcements become public information immediately; thus, there is no asymmetric information at least in the first order sense.

We assume that each young agent makes an announcement about his own effective belief \( Q_h^t \), which is the theory that reflects his view about the economy. In other words, each young agent announces his ‘state of belief’. For example he announces if he is ‘optimistic’ or ‘pessimistic’. Hence, it is a function of the effective belief:
\[ Y_h^t = \varphi_h(Q_h^t), \quad \forall t, \]
with
\[ \varphi_h : Q^h \mapsto \Re, \]
where \( Q^h \) denotes the set of possible effective beliefs, while we assume that the map \( \varphi_h \) is a one-to-one relationship between \( Y_h^t \) and \( Q_h^t \). Thus, \( Y_h^t \) is a real valued variable that completely represents \( Q_h^t \).

Because the effective belief is really the theory that reflects how the agent views the economy, the announcement can be interpreted as the agent’s opinion about the economy. To simplify the analysis, we make the following assumption concerning announcements:

\[ ^5\text{However, we assume the young agent } h \text{ himself does not understand the mapping } \varphi_h. \text{ This is to reflect the ambiguity concerning the choice of effective beliefs as we discuss below.} \]
Assumption 1: Each young agent $h$ announces his opinion $Y^h_t$ truthfully.\footnote{Even without this assumption, i.e. even there are strategic concerns about announcements, the essential qualitative results of the paper do not change as long as $\phi^h$ is a one-to-one mapping, which can be viewed as a decision rule in a strategic environment. We shall see why Assumption 1 is not essential when we discuss the impacts of communication.}

This assumption will be maintained throughout the current paper. By Assumption 1, we put strategic concerns about the announcement to one side, avoiding complications that would involve game theoretic considerations, which are not essential to our current focus. Because we are examining a general equilibrium model, we are interested in situations in which each single agent believes that he cannot affect the whole system as well as other agents’ decisions, i.e. the competitive assumption. Hence, this assumption is compatible with the setting of a general equilibrium model.

Moreover, following the literature in the Bayesian theory called the Expert Problem,\footnote{See for example Bayarri and DeGroot (1991), Genest and Schervish (1985), West (1992) and West and Crosse (1992).} we treat the opinions as random variables. However, to be more specific, we describe the structure of probability beliefs of the agents in the following section.

2.1.2 The Structure of Beliefs

In what follows, we specify the structure of beliefs. Instead of fixing a belief over generations within each dynasty $h$, we assume that the effective belief $Q^h_t$ is random over time, and is governed by a probability measure $\mu^h$. More specifically, the sequence of beliefs of dynasty $h$, i.e. $\{Q^h_t\}_{t=1}^{\infty}$, is an i.i.d. sequence, and $\mu^h$ is a probability measure on $(Q^h, B(Q^h))$, where $Q^h$ is the set of possible effective beliefs, which is assumed to be (at most) countable.\footnote{We assume $Q^h$ to be finite to simplify the analysis later. However, our construction of beliefs is valid as long as $Q^h$ is countable, and so is the analytical model.}

Hence, probability $\mu^h(Q^h_t = k)$ is constant over time for all $k \in Q^h$ since $\{Q^h_t\}_{t=1}^{\infty}$ is an i.i.d. sequence. While we can introduce a more complex structure here, for example $\{Q^h_t\}_{t=1}^{\infty}$ may be an AR(1) process, we do not complicate the matter here so as to keep the analysis simple.

Although there is a probability distribution $\mu^h$ over $Q^h$ the set of possible effective beliefs, which also define probability distributions, we stress that $\mu^h$ is not part of young agent $h$’s belief. Namely, young agent $h$ does not form a belief such that $Q^h(\cdot) = \int_{k \in Q^h} k(\cdot)\mu^h(\text{dk})$.\footnote{The object $Q^h$ is called the barycenter of $\mu$, and there exists a unique barycenter if the underlying probability space is a standard space. See Gray (1988) for details.} Rather, $\{Q^h_t = k\}$ is a probability event with respect to measure $\mu^h$, which is irrelevant for young agent $h$ in period $t$, since his belief is completely represented by $Q^h_t$ itself.

We note that the effective belief $Q^h_t$ is really the theory with which young agent $h$ in period $t$ views the economy. Hence, when the set $Q^h$ is not a singleton, there are multiple theories that might be adopted by young agent $h$. In fact, the randomness of $Q^h_t$ means that the actual theory in use is chosen
randomly in each period. This is possible when an agent is ambiguous about the choice of theory.

Now, we explain why it is reasonable to assume such an ambiguity. To begin with, it is reasonable to say that no agent actually knows the truth, and that, every ‘intelligent’ agent knows that he does not know the truth. Hence, with the knowledge that he does not know the truth, each agent relies on a theory, which always employs some assumption(s) by definition. Because each agent knows that it is impossible for an assumption to be always correct (otherwise it is the truth itself, which is not an assumption by definition), he is uncertain or ambiguous about the choice of theory as long as there are multiple theories available. Namely, each agent is ambiguous in the sense that he is not very sure which theory is the most relevant amongst others, yet he ultimately relies on a theory at the time when he is making decisions. This observation motivates us to randomize $Q^h_t$ rather than to fix it as a particular measure over time, while assuming that the set of theories $Q^h$ is inherited over generations within the dynasty $h$.

In addition, it is common that institutional investors including financial institutions adopt some sort of quantitative/statistical model to determine their portfolio choices in practice, and has become increasingly so recently. This means that they adopt a particular probabilistic model to make a portfolio choice, although they do alter the models from time to time. Alterations of models may simply be changes in the parameters of the models, or they may even involve changes in the structure of the model itself. Although such changes are common, there hardly exists a fixed rule/model for model selections. This observation is therefore clearly consistent with our construction of beliefs because each institution uses a probabilistic model out of several possible models in every period, whilst there remains ambiguity in the model selection process.

Moreover, we assume that the ambiguity concerning the choice of theories does not lead the agents to ‘mix’ different theories, e.g. put a weight of .3 on theory A and a weight of .7 on theory B, unlike a mixed strategy in game theory, because we do not allow for an agent to form a belief such that $Q^h(\cdot) = \int_{k \in Q^h} k(\cdot) \mu^h(\text{dk})$ as we noted above. Of course, it is possible that an agent forms a belief about beliefs. However, as long as the belief about beliefs changes over time, our construction remains valid. This is so, even if we consider beliefs about beliefs about beliefs ad infinitum, i.e. the infinite regress of hierarchical beliefs, unless the hierarchical structure of beliefs is time-invariant, in which case there is no ambiguity concerning the choice of theory or beliefs ultimately, because the hierarchy of beliefs as a whole defines a probability belief.10

Our construction of beliefs is capable of describing a common situation in which the same investor sometimes becomes optimistic and sometimes becomes pessimistic even though the data at hand are the same. We stress that it is the belief of the agent that determines if he is optimistic or not, not the data. We do not adopt the view that a particular investor/institution always believes in a particular theory over the periods, and that a change in

---

10See Brandenburger and Dekel (1993) for the discussion on hierarchical beliefs.
behaviour only occurs when the data changes. Rather, we allow for an agent to change his view (or mind) even though there is no change in data.

Note however that our construction of beliefs does not capture the concept of ambiguity found in the Ellsberg Paradox, because at any point of time, each agent forms a particular probability belief anyway, although we introduce a notion of ‘set of probability beliefs’. Hence, we do not follow the literature that focuses on modelling this sort of ambiguity (or Knightian uncertainty), e.g. Gilboa and Schmeidler (1989), Epstein and Wang (1994, 1996).

Although as long as the set \( Q^h \) is countable, the analytical model remains the same, we assume for simplicity that for every agent \( h \),

\[
Q^h := \{ Q^h_H, Q^h_L \},
\]

and for every \( h, t \),

\[
\mu^h\{ Q^h_t = Q^h_H \} = \alpha^h.
\] (3)

Namely, the effective belief of young agent \( h \) in period \( t \) is \( Q^h_H \) with a frequency of \( \alpha^h \) and \( Q^h_L \) with a frequency of \( 1 - \alpha^h \). Moreover, we may say optimistic when \( Q^h_t = Q^h_H \), and pessimistic when \( Q^h_t = Q^h_L \). Furthermore, we assume that \( Q^h_H \) and \( Q^h_L \) are stationary measures.

Because we assume that \( Q^h := \{ Q^h_H, Q^h_L \} \), we can specify the announcements as follows:

\[
Y^h_t = \begin{cases} 
  y^h_H & \text{if } Q^h_t = Q^h_H, \\
  y^h_L & \text{if } Q^h_t = Q^h_L,
\end{cases}
\]

with \( y^h_H \neq y^h_L \) if and only if \( Q^h_H \neq Q^h_L \).

Next, we assume that the actual effective belief of young agent \( h \) in period \( t \) has the following structure:

\[
\hat{Q}^h_t(\cdot) = \begin{cases} 
  Q^h_H(\cdot|Y^h_t = y^h_H) & \text{if } Q^h_t = Q^h_H, \\
  Q^h_L(\cdot|Y^h_t = y^h_L) & \text{if } Q^h_t = Q^h_L,
\end{cases}
\] (4)

Note that we define \( \hat{Q}^h_t \) as a conditional measure of \( Q^h_S \) given \( \{ Y^h_t = y^h_S \} \) for \( S = H, L \). However, we call both the unconditional measure \( Q^h_t \) and the conditional measure \( \hat{Q}^h_t \) the effective belief since the difference is explicitly or implicitly understood.

Observe that \( Y^h_t \) is treated as a random variable as we stated earlier. By following the literature of the Expert Problem, it is clear that we can treat \( Y^h_t \) as a random variable as long as the probability measure \( Q^h_t \) is not known. On the other hand, it is not obvious at all for the case of the young agent \( h \) in period \( t \), who knows \( Q^h_t \). Nevertheless, we postulate that the same applies to the young agent \( h \) in period \( t \) as well, i.e. \( Y^h_t \) is a random variable even from his perspective. It may appear that this treatment is not reasonable. However, we argue that this treatment really reflects the agent’s ambiguity concerning the choice of theory or beliefs. Although the agent does not form a probability belief on the possible effective beliefs, he understands that there are multiple possible effective beliefs. Hence, he also understands that there are multiple possible opinions. This implies that \( Y^h_t \) is not a constant for the agent as well.
However, there is a one-to-one relation between $Q^h_t$ and $Y^h_t$. Hence, it is impossible to have $Q^h_t = Q^h_H$ and $Y^h_t = y^h_t$ simultaneously. Nevertheless, the one-to-one relationship is not understood by the agent $h$ himself. To summarize, young agent $h$’s belief possesses the following features concerning his own opinion $Y^h_t$:

- Young agent $h$ in period $t$ knows what his opinion $Y^h_t$ is.
- Young agent $h$ in period $t$ understands that his opinion could have been different potentially.
- There is a one-to-one relationship between $Q^h_t$ and $Y^h_t$, which is not understood by the young agent $h$ in period $t$ himself.

This observation motivates our construction of actual effective beliefs (4). In fact, it is easy to check that (4) satisfies all of the above three features.

### 2.1.3 Young Agent’s Problem

We now turn our attention to each individual agent’s optimization problem. We assume that each (young) agent in period $t$ forms an effective belief $Q^h_t$ (or $\hat{Q}^h_t$) as described above when he makes his decisions.\(^{11}\) In particular, instead of assuming rational expectations or a common prior, we allow for heterogeneous beliefs.

Before describing the optimization problem of young agents, let us summarize the timing of the model in each period.

**The Timing in Each Period**

1. Each young agent $h$ forms a probability belief (effective belief) $Q^h_t$ (or $\hat{Q}^h_t$).
2. Each young agent $h$ makes an announcement $Y^h_t$ publicly.
3. $d_t$ realizes and transactions take place.

The optimization problem of a young agent $h$ in period $t$ after observing the announcements (of others) is the following:

$$\max_{(C^1_t, \theta^h_t, C^2_{t+1})} E_{Q^h_t}\{u^h(C^1_t, C^2_{t+1}) \mid G^h_t}\}$$  \hspace{1cm} (5)

s.t.  \hspace{1cm} 

$$C^1_t + P_t \theta^h_t + q_t B^h_t = W^h_t$$  \hspace{1cm} (6)

$$C^2_{t+1} = \theta^h_t \cdot (P_{t+1} + D_{t+1}) + B^h_t,$$  \hspace{1cm} (7)

where $C^1_t$ denotes the consumption of $h$ when young in period $t$, $C^2_{t+1}$ the consumption of $h$ when old in period $t + 1$ (bearing in mind $h$ was born in

\(^{11}\)We assume that each agent is an expected utility maximizer, whose preference is represented by the subjective belief $Q^h_t$ and a von Neumann-Morgenstern utility function, although he is ambiguous about the choice of $Q^h_t$ potentially.
period $t$), and $\mathcal{G}_t^h$ the information set of young agent $h$ in period $t$ at the time of portfolio choice.

To enable us to compute equilibria, we assume agent $h$’s utility function to be of the CES form

$$ u^h(C_{t+1}^{1h}, C_{t+1}^{2h}) = \frac{1}{1 - \nu^h} (C_{t+1}^{1h})^{1-\nu^h} + \frac{\beta^h}{1 - \nu^h} (C_{t+1}^{2h})^{1-\nu^h}, \quad \nu^h > 0, $$

where $\beta^h \in (0, 1)$ is the discount factor and $\nu^h$ is the parameter that indicates the degree of relative risk aversion of agent $h$. Then, the first-order conditions (the Euler equations) for the optimization problem of a young agent $h$ in period $t$ will be

$$ -P_t \cdot (C_{t}^{1h})^{-\nu^h} + \beta^h E_{Q_t^h} \{(C_{t+1}^{2h})^{-\nu^h} \cdot (P_{t+1} + D_{t+1}) | \mathcal{G}_t^h \} = 0,$$

$$ -q_t \cdot (C_{t}^{1h})^{-\nu^h} + \beta^h E_{Q_t^h} \{(C_{t+1}^{2h})^{-\nu^h} | \mathcal{G}_t^h \} = 0.$$

We can describe these conditions by using ratios $(p_t, q_t, d_t, c_{t}^{1h}, c_{t+1}^{2h}, b_{t}^{h})$ instead of absolute values $(P_t, D_t, C_{t+1}^{1h}, C_{t+1}^{2h}, B_{t}^{h})$ as follows:

$$ p_t \cdot (c_{t}^{1h})^{-\nu^h} = \beta^h E_{Q_t^h} \{(c_{t+1}^{2h}d_{t+1})^{-\nu^h} \cdot (p_{t+1} + 1)d_{t+1} | \mathcal{G}_t^h \}, $$

$$ q_t \cdot (c_{t}^{1h})^{-\nu^h} = \beta^h E_{Q_t^h} \{(c_{t+1}^{2h}d_{t+1})^{-\nu^h} | \mathcal{G}_t^h \}, $$

$$ c_{t}^{1h} = -p_t \theta_t^h - q_t b_t^h + w_t^h, $$

$$ c_{t+1}^{2h} = \theta_t^h \cdot (p_{t+1} + 1) + \frac{b_t^h}{d_{t+1}}. $$

Now, we make the following assumption to make the simulations of this model tractable:

**Assumption 2:** Each young agent $h$ in period $t$ believes that the economy is Markovian.

By Assumption 2, each agent believes that the joint process $\{p_t, q_t, d_t, Y_t\}_t$ is Markov, where $Y_t = (Y_t^1, Y_t^2, ..., Y_t^{\mathcal{H}})$. It follows that the conditions above will be rewritten as

$$ p_t \cdot (c_{t}^{1h})^{-\nu^h} = \beta^h E_{Q_t^h} \{(c_{t+1}^{2h}d_{t+1})^{-\nu^h} \cdot (p_{t+1} + 1)d_{t+1} | p_t, q_t, d_t, Y_t \}, $$

$$ q_t \cdot (c_{t}^{1h})^{-\nu^h} = \beta^h E_{Q_t^h} \{(c_{t+1}^{2h}d_{t+1})^{-\nu^h} | p_t, q_t, d_t, Y_t \}, $$

$$ c_{t}^{1h} = -p_t \theta_t^h - q_t b_t^h + w_t^h, $$

$$ c_{t+1}^{2h} = \theta_t^h \cdot (p_{t+1} + 1) + \frac{b_t^h}{d_{t+1}}. $$

It follows that the demand correspondences of the young will be time-invariant: for every $h, t$,

$$ \theta_t^h = \theta^h(p_t, q_t, d_t, Y_t; Q_t^h), \quad (8) $$

$$ b_t^h = b^h(p_t, q_t, d_t, Y_t; Q_t^h). \quad (9) $$

Observe that the demand is influenced by $Y_t^h$ and $Q_t^h$. This suggests that communication may have impacts on the equilibrium of the economy by generating correlation of beliefs.
2.2 The Equilibrium

We have so far defined the optimization problem of a young agent and derived its solution, i.e. the demand correspondences. In what follows, we define the equilibrium of the economy by introducing the market clearing conditions in addition to the optimality conditions of the young agents’ problems.

2.2.1 The Definition of the Competitive Equilibrium

In addition to the optimality conditions for young agents’ problems, the equilibria of the economy are characterized by the market clearing conditions: for every period \( t \), the markets clear if

\[
\sum_{h=1}^{H} \theta^h(p_t, q_t, d_t, Y_t; Q^h_t) = 1, \\
\sum_{h=1}^{H} b^h(p_t, q_t, d_t, Y_t; Q^h_t) = 0.
\]

We therefore define a stable Markov competitive equilibrium of our economy as follows:

**Definition:** Sequences of probability measures \( \{Q^1_t, Q^2_t, ..., Q^H_t\}_{t=1}^{\infty} \) and a joint stochastic process \( \{p_t, q_t, (\theta^h_t, b^h_t), d_t, Y_t; h = 1, 2, ..., H\}_{t=1}^{\infty} \) with initial portfolios \((\theta^0_h, b^0_h), h = 1, 2, ..., H\) associated with the true probability measure \( \Pi \) constitute a stable Markov competitive equilibrium if

1. \((p_t, q_t, \theta^h_t, b^h_t, d_t, Y_t; h = 1, 2, ..., H)\) satisfy conditions (8), (9), (10) and (11) for all \( t \).

2. \( \Pi \) is a stable measure, and each sequence \( \{Q^h_t\}_{t=1}^{\infty} \) constitutes a stable measure for all \( h \).

By construction, the equilibrium prices will be a sequence generated by a time-invariant map as follows:

\[
\begin{bmatrix}
    p_t \\
    q_t
\end{bmatrix} = \Phi(d_t, Y_t; Q^1_t, Q^2_t, ..., Q^H_t)
\]

\[
\Phi(d_t, Y_t), \quad \forall t.
\]

The last line follows from the fact that there is a one-to-one relation between \( Y^h_t \) and \( Q^h_t \) for all \( h, t \).

It is clear from the equilibrium map (13) that the primitives of the economy are the dividend growth rate \( d_t \) and the announcements \( Y_t \) given the preferences. It follows that although the economy appears to be a joint stochastic process of \((p_t, q_t, d_t, Y_t)\), which is assumed to be Markov, it is sufficient to describe the joint stochastic process of \((d_t, Y_t)\) since they determine

---

\(^{12}\)See Gray (1988) for details concerning a stable measure.

\(^{13}\)Effective beliefs \((Q^1_t, Q^2_t, ..., Q^H_t)\) can be viewed as part of the primitives of the economy although they are redundant with the announcements \( Y^h_t \).
the prices \((p_t, q_t)\). In other words, the states of the prices are partitioned by the states of \(d_t\) and \(Y_t\), and consequently, at most \(2 \times 2^H = 2^{1+H}\) prices will be observed in this economy.

### 2.2.2 The True Probability Measure and the Stationary Measure

In this paper we focus on stable Markov competitive equilibria, which are defined above. Because of the stochastic stability, there exists a stationary measure \(m\) that is induced from the true probability measure that governs the equilibrium prices (and allocations) over time. However, we stress that this does not mean that the true measure is stationary. In fact, there are (infinitely) many stable measures from which the same stationary measure \(m\) can be induced, and the true measure is only one of them.

From the equilibrium map (13), we know that there are at most \(2^{1+H}\) prices in the economy, which are determined by \((d_t, Y_t)\). Hence, the equilibrium process of the economy is fully specified if we specify the true measure \(\Pi\) that governs \((d_t, Y_t)\), which is a stable Markov process. However, we are only interested in the long term averages/frequencies of the economic variables; thus, we instead specify the stationary transition probability matrix \(\Gamma\) for the joint stochastic process \((d_t, Y_t)\), which characterizes the stationary measure \(m\) that is induced by the true probability measure \(\Pi\).

In doing so, we require the stationary transition probability matrix the following:

- the marginal distribution for \(d_t\) is specified by the stationary dividend (growth rate) process (2),
- the marginal distributions for \(Y^h_t\) specify \(Y^h_t\) to be i.i.d. with \(m_{Y^h}\{Y^h_t = y^h_t\} = \alpha^h\) for all \(h\).

There are many matrices that satisfy these conditions. However, we specify the one that is found in Kurz and Beltratti (1997) so that we can reinterpret their simulation results in the light of our construction of the beliefs and the economy.

### 2.2.3 Rational Expectations Equilibrium

To see that heterogeneity of beliefs is crucial for communication to have an impact on the equilibrium, we shall examine the special case of rational expectations equilibrium (REE). In a REE, \(Q^h_t = \Pi\) for all \(h, t\), where the true probability measure \(\Pi\) is induced by (2) and by the equilibrium map (13). Furthermore, when \(Q^h_t = \Pi\) is a stationary measure\(^{14}\), the following holds:

\[ Y^h_t = \bar{y}, \quad \forall h, t, \]

\(^{14}\)In the context of our particular economy where there is no source of non-stationarity except for the beliefs of the agents, rational expectations do not really make sense unless \(\Pi\) itself is a stationary measure. The basic principle behind rational expectations is compatibility between the empirical data and the probability law, which is the same as that of rational beliefs. However, rational expectations do not allow for heterogeneous beliefs, and that it is assumed that every agent knows the true probability measure.
where $\bar{y}$ is a constant.

Hence, for every $h$,

\[
\theta^h_t = \theta^h(p_t, q_t, d_t, Y_t; Q^h_t) = \hat{\theta}^h(p_t, q_t, d_t, Y_t; \Pi) = \hat{\theta}^h(p_t, q_t, d_t, \bar{y}; \Pi) = \tilde{\theta}^h(p_t, q_t, d_t), \quad \forall t,
\]

where $\bar{y} := (\bar{y}, \bar{y}, \ldots, \bar{y}) \in \mathbb{R}^H$. The last line follows, because $\bar{y}$ is determined uniquely by $\Pi$, while $\Pi$ is implicit in (the determination of) the map $\hat{\theta}^h(\cdot)$. Similarly for $b^h_t$, the following holds:

\[
b^h_t = \tilde{b}^h(p_t, q_t, d_t), \quad \forall t.
\]

Hence, the equilibrium map will be reduced to

\[
\begin{bmatrix}
p_t \\
q_t
\end{bmatrix} = \hat{\Phi}(d_t).
\] (14)

This leads us to conclude the following:

**Proposition 1:** Communication (or exchange of opinions) has no impact on a REE.

As we can see from the above explanations, communication has no impact on a REE, because each agent can pinpoint the announcement of the other agent correctly with probability one with respect to the true probability measure $\Pi$. Therefore, announcements are not really intrinsic random variables on the subjective probability spaces of the agents in this case. In other words, announcements do not expand the state space. Note however that this particular result rests crucially on the assumption that $Q^h_t = \Pi$ for all $h, t$.

Note that the presence of asymmetric information is essential for communication to make sense under the assumption of rational expectations. In other words, communication is not really communication under rational expectations unless there is some sort of information asymmetry, because each agent knows what others know otherwise. In contrast, when we allow for heterogeneous beliefs, and in particular, when we assume that the probability measure itself is not known to others, this is no longer the case.

### 2.2.4 Rational Belief Equilibrium

Now, we allow for heterogeneous beliefs unlike the REE. Nevertheless, we require every sequence of effective beliefs $\{Q^h_t\}_{t=1}^\infty$ to constitute a rational belief. The generic condition/definition is the following:

**Definition:** A sequence of effective beliefs $\{Q^h_t\}_{t=1}^\infty$ constitutes a rational belief if it induces a stationary measure that is equivalent to the one induced
In what follows, we show that there exists a sequence of effective beliefs \( \{Q^h_t\}_{t=1}^\infty \) that constitutes a rational belief. Let \( X \) denote the state space of \((p_t, q_t, d_t, Y_t)\) for all \( t \), and \( X^\infty \) the state space for the entire sequence. Let \( \mathcal{B}(X^\infty) \) denote the Borel \( \sigma \)-field generated by \( X^\infty \). Then, the true stochastic process of the economy is described by a stochastic dynamical system \( (X^\infty, \mathcal{B}(X^\infty), T, \Pi) \), where \( T \) denotes the shift transformation. However, we can expand the probability space to incorporate the sequences of effective beliefs \( \{Q^h_t\}_{t=1}^\infty \).

To do so, we denote the true probability measure on the space \((X^\infty \times Q^h, \mathcal{B}(X^\infty \times Q^h))\), whose ‘marginal measure’ for \( X^\infty \) is \( \Pi \) and that for \( (Q^h)^\infty \) is \( \bar{\mu}^h \), while \( \mu^h \) is the marginal measure for for \( Q^h \).\(^{15}\) Then, the expanded true stochastic process is described as a dynamical system such that \( (\Omega^h, \mathcal{B}^h, T, \hat{\Pi}^h) \), where \( \Omega^h := (X^\infty \times Q^h)^\infty \) and \( \mathcal{B}^h := \mathcal{B}((X^\infty \times Q^h)^\infty) \). This construction is similar to the rational belief structure in Nielsen (1996), although the set of beliefs consist of measures on \((X, \mathcal{B}(X))\) rather than on \((X^\infty, \mathcal{B}(X^\infty))\) there. Also, any interdependence between those measures and \( X \) is not allowed there, while it is allowed here. Hence, we follow Kurz and Schneider (1996) with respect to such an interdependence, whilst the structure that incorporates measures on measures follow Nielsen (1996).

With this in mind, we prove a theorem that is analogous to the conditional stability theorem (Theorem 2) in Kurz and Schneider (1996), although they introduce random variables that represent the effective beliefs as conditional measures, which they call the generating variables. Namely, while they study the stability properties of the joint system of \( X \) and the generating variables, we study the stability properties of the joint system of \( X \) and \( Q^h_t \).

Before stating the theorem, we introduce some notation to be more precise concerning the construction of the probability space(s). Let \( \hat{\Pi}^h_k \) denote the conditional probability of \( \hat{\Pi}^h \) given a particular sequence of effective beliefs \( k \in (Q^h)^\infty \):

\[
\hat{\Pi}^h_k(\cdot) : (Q^h)^\infty \times \mathcal{B}(X^\infty) \mapsto [0, 1].
\]

For each \( A \in \mathcal{B}(X^\infty) \), \( \hat{\Pi}^h_k \) is a measurable function of \( k \) and for each \( k \), \( \hat{\Pi}^h_k(\cdot) \) is a probability on \((X^\infty, \mathcal{B}(X^\infty))\). For \( A \in \mathcal{B}(X^\infty) \) and \( B \in \mathcal{B}((Q^h)^\infty) \), we have

\[
\hat{\Pi}^h(A \times B) = \int_{k \in B} \hat{\Pi}^h_k(A) \mu^h(dk),
\]

where \( \mu^h \) is a probability measure on \((Q^h)^\infty, \mathcal{B}((Q^h)^\infty)) \). Also, as we noted above,

\[
\Pi(A) = \hat{\Pi}^h(A \times (Q^h)^\infty), \quad \forall A \in \mathcal{B}(X^\infty),
\]

\[
\mu^h(B) = \hat{\Pi}^h(X^\infty \times B), \quad \forall B \in \mathcal{B}((Q^h)^\infty).
\]

When \( (\Omega^h, \mathcal{B}^h, T, \hat{\Pi}^h) \) is a stable dynamical system with a stationary mea-
sure \( m^{\hat{\Pi}^h} \), we define the two marginal measures of \( m^{\hat{\Pi}^h} \) as follows:

\[
m(A) := m^{\hat{\Pi}^h} (A \times (Q^h)^\infty), \quad \forall A \in \mathcal{B}(X^\infty),
\]

\[
m_{Q^h}(B) := m^{\hat{\Pi}^h} (X^\infty \times B), \quad \forall B \in \mathcal{B}((Q^h)^\infty).
\]

Also let \( \hat{m}_k \) denote the stationary measure of \( \hat{\Pi}^h_k \), which is a measure on \( (X^\infty, \mathcal{B}(X^\infty)) \).

When the dynamical system \( (\Omega^h, \mathcal{B}^h, T, \hat{\Pi}^h) \) has the above construction, we have the following theorem:

**Theorem 1:** Let \( (\Omega^h, \mathcal{B}^h, T, \hat{\Pi}^h) \) be a stable and ergodic dynamical system. Then,

(a) \( (X^\infty, \mathcal{B}(X^\infty), T, \hat{\Pi}^h_k) \) is stable and ergodic for \( \hat{\Pi}^h \) a.e. \( k \).

(b) \( \hat{m}^h_k \) is independent of \( k \), and \( \hat{m}^h_k = m \).

(c) If \( (X^\infty, \mathcal{B}(X^\infty), T, \hat{\Pi}^h_k) \) is stationary then the stationary measure of \( \hat{\Pi}^h_k \) is \( \Pi \). That is

\[
\hat{m}^h_k = m = \Pi.
\]

(Proof) The proof essentially follows that of Theorem 2 in Kurz and Schneider (1996). First \( X \) is clearly a Polish space, since it is a set of countable isolated points.\(^{16}\) Also, \( Q^h \) is a Polish space, because it is assumed that \( Q^h \) is at most a countable set, and thus, it is a set of countable isolated points. It follows that \( (X \times Q^h)^\infty \) is also a set of countable isolated points; thus, it is a Polish space, too. Because \( Q^h \), whose role is essentially the same as that of the generating variable in Kurz and Schneider, is countable at most, it is straightforward that the dynamical system in this paper possesses the same properties as the one in Kurz and Schneider; thus, the proof of Theorem 2 of Kurz and Schneider applies. Q.E.D.

Suppose \( Q^h \) is a probability measure on \( (\Omega^h, \mathcal{B}^h) \), and that, \( (\Omega^h, \mathcal{B}^h, T, Q^h) \) is a stable and ergodic dynamical system. Then Theorem 1 states that any stable measure \( Q^h \) on \( (\Omega^h, \mathcal{B}^h) \) implies a stationary measure \( m^h_k = m^h \) for all \( k \in (Q^h)^\infty \), where \( m^h \) is the marginal measure of \( m^{Q^h} \) on \( (X^\infty, \mathcal{B}(X^\infty)) \), which is the stationary measure induced by \( Q^h \).\(^{17}\)

Recall that our construction of effective beliefs allows us to specify \( Q^h \) as follows:

\[
Q^h_s (A) = Q^h_s (A|Y^h_t = Y^h_0) = Q^h_s (A | (Q^h)^\infty (Q^h_t \times k^{(t)})), \quad \forall A \in \mathcal{B}(X^\infty),\ k^{(t)},\ S = H, L(16)
\]

where \( k^{(t)} \) denotes the sequence of effective beliefs other than period \( t \). Clearly this is a special case of a conditional measure of \( Q^h \). Moreover, the definition of rational belief restricts the class of stable measures \( Q^h \) to

\(^{16}\)It is a finite set in our model. However, we keep the theorem as generic as possible.

\(^{17}\)We need the superscript \( h \) for \( m^h \) here while there is none in Theorem 1 (b), because the marginal measure of \( Q^h \) on \( (X^\infty, \mathcal{B}(X^\infty)) \) is subjective, while that of \( \hat{\Pi}^h \) is the true probability.
satisfy the property such that $m^h = m$. Hence, Theorem 1 ensures that every sequence of effective beliefs $\{Q^h_t\}_{t=1}^{\infty}$ constitutes a rational belief as long as the stable measure $Q^h$ satisfy the condition (on the stationary measure $m^h$).

In what follows, we explicitly show how to describe rational beliefs in our Markovian economy. Recall that the equilibrium map (13) is

$$\begin{bmatrix} p_t \\ q_t \end{bmatrix} = \Phi(d_t, Y_t), \quad \forall t.$$ 

Hence, the equilibrium is driven by a stable Markov process of $(d_t, Y_t)$ as we noted above. Namely, we need to define a dynamical system on $(V^\infty, \mathcal{B}(V^\infty))$, where $V$ is the state space of $(d_t, Y_t)$, as a stable Markov process.

For the computation of the long-term frequencies or long-term averages of the economic variables, it is sufficient to specify a stationary transition matrix $\Gamma$ that specifies the transition probabilities from $(d_t, Y_t)$ to $(d_{t+1}, Y_{t+1})$, i.e. $\Gamma$ is on $V \times V$, which induces a stationary measure, and that, the stationary measure is the one that is induced by the true probability measure. Note that, the true process may not be stationary, but it is enough for us to specify its induced stationary measure that is fully characterized by the transition probability matrix $\Gamma$ to compute the long-term frequencies.

On the other hand, we specified that the effective beliefs are determined randomly, either $Q^h_H$ or $Q^h_L$. Hence, we define pairs of transition probability matrices that correspond to the pair of effective beliefs $(Q^h_H, Q^h_L)$ as follows: young agent $h$ in period $t$ adopts a transition matrix $F^h_t$ by the following rule

$$F^h_t = \begin{cases} F^h_H & \text{if } Q^h_t = Q^h_H; \\
F^h_L & \text{if } Q^h_t = Q^h_L. \end{cases} \quad (17)$$

Recall that $Q^h_H$ is a measure on $(X^\infty, \mathcal{B}(X^\infty))$, where $X$ is the state space of $(p_t, q_t, d_t, Y_t)$ for all $t$, which is larger than $V$, the state space of $(d_t, Y_t)$. However, we are only interested in the long-term frequencies of the economic variables, and thus, we can ignore the states that occur with probability 0. Hence, it is sufficient for the transition probability matrices $F^h_H$ and $F^h_L$ to be on $V \times V$ rather than on $X \times X$.

With this and (17) in mind, we require the sequence of effective beliefs to satisfy the rationality condition, which is analogous to the one found in papers on rational beliefs (e.g. Kurz and Schneider [1996], Kurz and Beltratti [1997], Kurz and Motoleso [2001], etc.):

**Rationality Condition:** The transition matrices $F^h_H$ and $F^h_L$ of each agent $h$ must satisfy the following condition for the sequence of effective beliefs $\{Q^h_t\}_{t=1}^{\infty}$ to constitute a rational belief:

$$\alpha^h \cdot F^h_H + (1 - \alpha^h) \cdot F^h_L = \Gamma, \quad \forall h. \quad (18)$$

Because the frequency of the event $\{Q^h_t = Q^h_H\}$ is $\alpha^h$ with respect to the true probability $\mu$, agent $h$ uses the transition probability matrix $F^h_H$ with frequency $\alpha^h$. Hence, the rationality condition (18) requires the sequence of beliefs $\{Q^h_t\}_{t=1}^{\infty}$ to be compatible with the data that is generated by the
stationary transition probability matrix $\Gamma$. In other words, there is no way for the agents to reject the set of theories $Q^h$ for being invalid by observing the data.

Now we define a Markov Rational Belief Equilibrium as follows:

**Definition:** A Markov Rational Belief Equilibrium (RBE) is a stable Markov Competitive Equilibrium in which the sequences of effective beliefs $\{Q^h_t\}_{t=1}^\infty$ ($h = 1, 2, ..., H$) satisfy (3), (4) and the rationality condition (18).

The definition of a Markov Rational Belief Equilibrium allows for heterogeneous beliefs. However, it requires the sequence of beliefs to constitute a rational belief. Hence, it is required that both the true equilibrium process of the economic variables and the subjective process of them must be stable, but not necessarily stationary. However, it is not obvious at all how communication and/or the non-stationarity of beliefs impact the equilibrium of the economy. In the subsequent section, we therefore develop a simulation model to examine the impacts of communication and/or the non-stationarity of the beliefs.

## 3 Simulations

In this section, we examine the effects of communication on the equilibrium of the financial economy. To do so, we develop a simulation model, which is the same as the one in Kurz and Beltratti (1997). Also, we classify the effective beliefs to help examine the effects of communication. Then, we exhibit simulation results of three different cases: (a) Rational Expectations Equilibrium, (b) Rational Belief Equilibrium reported in Kurz and Beltratti (1997) and (c) another Rational Belief Equilibrium. Finally, we discuss the implications of the results.

### 3.1 The Simulation Model

First, we assume that the number of dynasties to be $H = 2$. Then, the number of states in each period is $2 \times 2^2 = 8$. It follows that we can define a map $\Phi^*$ between the state space of the index of the prices and the state space of $(d_t, Y^1_t, Y^2_t)$ (which is indexed by numbers from 1 to 8 rather than by $t$):

$$\begin{bmatrix}
1 \\
2 \\
3 \\
4 \\
5 \\
6 \\
7 \\
8
\end{bmatrix}
= \Phi^*
\begin{bmatrix}
d_1 = d^H, & Y^1_1 = y^1_H, & Y^2_1 = y^2_H \\
d_2 = d^H, & Y^1_2 = y^1_H, & Y^2_2 = y^2_l \\
d_3 = d^H, & Y^1_3 = y^1_L, & Y^2_3 = y^2_H \\
d_4 = d^H, & Y^1_4 = y^1_L, & Y^2_4 = y^2_l \\
d_5 = d^L, & Y^1_5 = y^1_H, & Y^2_5 = y^2_H \\
d_6 = d^L, & Y^1_6 = y^1_H, & Y^2_6 = y^2_l \\
d_7 = d^L, & Y^1_7 = y^1_L, & Y^2_7 = y^2_H \\
d_8 = d^L, & Y^1_8 = y^1_L, & Y^2_8 = y^2_l
\end{bmatrix}.
$$

(19)

We assume that the $8 \times 8$ stationary transition probability matrix $\Gamma$ has
the following structure:

$$\Gamma = \begin{bmatrix} \phi A & (1 - \phi) A \\ (1 - \phi) B & \phi B \end{bmatrix},$$

where $A$ and $B$ are $4 \times 4$ matrices which are characterized by ten parameters $(\alpha^1, \alpha^2, a_1, a_2, a_3, a_4, b_1, b_2, b_3, b_4)$:

$$A = \begin{bmatrix} a_1, \alpha^1 - a_1, \alpha^2 - a_1, 1 + a_1 - \alpha^1 - \alpha^2 \\ a_2, \alpha^1 - a_2, \alpha^2 - a_2, 1 + a_2 - \alpha^1 - \alpha^2 \\ a_3, \alpha^1 - a_3, \alpha^2 - a_3, 1 + a_3 - \alpha^1 - \alpha^2 \\ a_4, \alpha^1 - a_4, \alpha^2 - a_4, 1 + a_4 - \alpha^1 - \alpha^2 \end{bmatrix}, \quad (20)$$

$$B = \begin{bmatrix} b_1, \alpha^1 - b_1, \alpha^2 - b_1, 1 + b_1 - \alpha^1 - \alpha^2 \\ b_2, \alpha^1 - b_2, \alpha^2 - b_2, 1 + b_2 - \alpha^1 - \alpha^2 \\ b_3, \alpha^1 - b_3, \alpha^2 - b_3, 1 + b_3 - \alpha^1 - \alpha^2 \\ b_4, \alpha^1 - b_4, \alpha^2 - b_4, 1 + b_4 - \alpha^1 - \alpha^2 \end{bmatrix}. \quad (21)$$

The proposed structure of the stationary transition probability matrix $\Gamma$ satisfies the following properties required for the marginal distributions to hold:

- Marginal measures specify $Y^t_h$ to be i.i.d. with probability $\alpha^h$.

- The marginal measure for $d_t$ is specified by (2).

Note that the joint distribution of $(Y^t_1, Y^t_2)$ may depend on $d_t$. This implies that the effective beliefs $(Q^t_1, Q^t_2)$ may be jointly correlated with $d_t$.

Next, we specify the transition probability matrices that represent the beliefs of the agents. As we noted above, young agent $h$ in period $t$ uses $F^h_H$ when his belief is $Q^t_H$, and $F^h_L$ when $Q^t_L$. Because the rationality condition (18) must be satisfied, $F^h_L$ is determined by $F^h_H$ and $\Gamma$. Hence, we only specify $F^h_H$ as follows:

$$F^h_H = \begin{bmatrix} \phi A_1(\lambda^h) & (1 - \phi) B_1(\lambda^h) \\ (1 - \phi) B_1(\lambda^h) & B_2(\lambda^h) \end{bmatrix}, \quad (22)$$

where

$$A_1(\lambda^h) = \begin{bmatrix} \lambda^h A^1 \\ \lambda^h A^2 \\ \lambda^h A^3 \\ \lambda^h A^4 \end{bmatrix}, \quad A_2(\lambda^h) = \begin{bmatrix} (1 - \phi \lambda^h) A^1 \\ (1 - \phi \lambda^h) A^2 \\ (1 - \phi \lambda^h) A^3 \\ (1 - \phi \lambda^h) A^4 \end{bmatrix}, \quad (23)$$

$$B_1(\lambda^h) = \begin{bmatrix} \lambda^h B^1 \\ \lambda^h B^2 \\ \lambda^h B^3 \\ \lambda^h B^4 \end{bmatrix}, \quad B_2(\lambda^h) = \begin{bmatrix} (1 - (1 - \phi) \lambda^h) B^1 \\ (1 - (1 - \phi) \lambda^h) B^2 \\ (1 - (1 - \phi) \lambda^h) B^3 \\ (1 - (1 - \phi) \lambda^h) B^4 \end{bmatrix}. \quad (24)$$

where $A^i$ is the $i$th row of matrix $A$ and similarly for $B^i$ ($i = 1, 2, 3, 4$):

$$A^i = \begin{bmatrix} a_i, \alpha^1 - a_i, \alpha^2 - a_i, 1 + a_i - (\alpha^1 + \alpha^2) \end{bmatrix}, \quad (25)$$

$$B^i = \begin{bmatrix} b_i, \alpha^1 - b_i, \alpha^2 - b_i, 1 + b_i - (\alpha^1 + \alpha^2) \end{bmatrix}. \quad (26)$$
There are eight parameters \((\lambda^1_h, \lambda^2_h, \ldots, \lambda^8_h)\) for every \(h\), and it is clear that they determine how \(F^h_H\) deviates from \(\Gamma\). In particular, when \(\lambda^h_h > 1\), this means that the conditional probability of \(\{d_{t+1} = d^H\}\) given state \(i\) with respect to \(Q^h_H\) is higher than the probability specified in \(\Gamma\):

\[
Q^h_H\{d_{t+1} = d^H | \text{state } i\} > m\{d_{t+1} = d^H | \text{state } i\}, \text{ if } \lambda^h_h > 1.
\]

Because the simulation model itself is exactly identical to the one found in Kurz and Beltratti (1997), we can follow their interpretations about the transition probability matrices. However, we incorporated communication in the model so that there are more observable variables, that is, the announcements \((Y^1_t, Y^2_t)\). This addition provides us with another interpretation of the transition probabilities, which we shall explain below.

### 3.2 Classification of Beliefs

In what follows, we exhibit one example of classification of beliefs. We classify the beliefs by examining the differences between the unconditional expectations of the price of the common stock in the next period and the conditional expectations of it given the announcements. More specifically, the unconditional expectations of \(p_t\) with respect to \(Q^h_S\) is the following:

\[
M^h_S := \begin{cases} 
E_{Q^h_H}p_{t+1} & \text{if } S = H, \\
E_{Q^h_L}p_{t+1} & \text{if } S = L.
\end{cases} \tag{27}
\]

On the other hand, the conditional expectations of \(p_{t+1}\) with respect to \(Q^h_S\) given \(\{Y^{(h)}_t = y^{(h)}_H\}\), where \((h)\) denotes agent(s) other than \(h\), is the following:

\[
Z^h_S := \begin{cases} 
E_{Q^h_H}\{p_{t+1}|Y^{(h)}_t = y^{(h)}_H\} & \text{if } S = H, \\
E_{Q^h_L}\{p_{t+1}|Y^{(h)}_t = y^{(h)}_L\} & \text{if } S = L.
\end{cases} \tag{28}
\]

Observe that the following holds:

\[
M^h_S = \sum_{S' \in \{H, L\}} Q^h_S\{Y^{(h)}_t = y^{(h)}_{S'}\} E_{Q^h_S}\{p_{t+1}|Y^{(h)}_t = y^{(h)}_{S'}\}, \text{ for } S = H, L.
\]

Hence, it is sufficient to compare \(M^h_S\) and \(Z^h_S\) to examine the effects of announcements on the expectations of \(p_{t+1}\).

Before examining the simulation results, we explain why it is important to classify the beliefs. Recall that we are interested in providing answers to the questions address in the introduction. Namely, we are interested in the conditions under which the market will be destabilized or stabilized, and/or the implications for public policy in financial markets. Hence, by classifying the beliefs, we are able to see with what kind of beliefs the economy is destabilized by communication. Furthermore, we may be able to analyse a policy that stabilizes the economy by offsetting the effects of communication. We discuss this point further when we examine the simulation results.
3.3 Simulation Results

In what follows, we show the simulation results of three different cases. Amongst the three, one is the case of a Rational Expectations Equilibrium, another is the one reported in Kurz and Beltratti (1997). The other one is another RBE, which is shown to provide a comparison with the second case. In all of the cases, following Kurz and Beltratti (1997), we set $\beta^h = 0.84$ and $\gamma^h = 3.75$ for all $h$, as well as $\phi = 0.43$, $d_H = 1.054$ and $d_L = 0.982$. Furthermore, we set $\alpha^h = 0.5$ for all $h$. We refer the simulation results to the historical data of the following key variables:

- $\bar{p}$: the average price/dividend ratio.
- $\sigma_p$: the standard deviation of the price/dividend ratio.
- $\bar{R}$: the average risky rate of return on equities.
- $\sigma_R$: the standard deviation of the risky rate.
- $\bar{r}^F$: the average risk free interest rate.
- $\sigma_{rF}$: the standard deviation of the risk free interest rate.
- $\bar{\rho}$: the average equity premium.

We refer to the historical data that are reported in Kurz and Motoleso (2001). They used the updated version of the database for 1889 – 1998 compiled by Shiller (1981), while Mehra and Prescott (1985) used the same database for 1889 – 1978.

**Case 1: Rational Expectations Equilibrium**

In this case, the parameters are set as follows: $a_i = b_i = 0.25$ for all $i$, and $\lambda_j^h = 1$ for all $h, j$. Hence $F_H^h = F_L^h = \Gamma$ holds for all $h$. Thus, the dividend stream $\{d_t\}_{t=1}^{\infty}$ is the only source of randomness in the economy, which is a stationary Markov process. In fact, this case is corresponding to the result of Mehra and Prescott (1985), where rational expectations are assumed. Hence, communication has no impact on the economy as Proposition 1 shows $(M^h_S = Z^h_S = 22.93 = \bar{p}$ for all $h, S)$. The simulation results and the historical data are reported in Table 1.

<table>
<thead>
<tr>
<th>Variables</th>
<th>Simulation</th>
<th>Historical Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{p}$</td>
<td>22.93</td>
<td>23</td>
</tr>
<tr>
<td>$\sigma_p$</td>
<td>0.08</td>
<td>6.48</td>
</tr>
<tr>
<td>$\bar{R}$</td>
<td>6.25%</td>
<td>8.00%</td>
</tr>
<tr>
<td>$\sigma_R$</td>
<td>4.16%</td>
<td>18.08%</td>
</tr>
<tr>
<td>$\bar{r}^F$</td>
<td>5.68%</td>
<td>1.00%</td>
</tr>
<tr>
<td>$\sigma_{rF}$</td>
<td>0.91%</td>
<td>5.67%</td>
</tr>
<tr>
<td>$\bar{\rho}$</td>
<td>0.58%</td>
<td>7%</td>
</tr>
</tbody>
</table>

It is clear from the table that the simulation results fail to replicate the historical data. In particular, it appears that there is an equity premium...
puzzle, while the simulation yields much smaller fluctuations of the risky rate (and thus, the price/dividend ratio) than the historical data.

**Case 2: Rational Belief Equilibrium 1 (Kurz-Beltratti)**

In this case, the parameters are set as follows:

\[
(a_1, a_2, a_3, a_4) = (0.12, 0.43, 0.43, 0.12),
\]

\[
(b_1, b_2, b_3, b_4) = (0.001, 0.01, 0.01, 0.001),
\]

\[
\lambda_j^1 = 1.75 \text{ for all } j \text{ odd},
\]

\[
\lambda_j^2 = 0.25 \text{ for all } j \text{ even},
\]

\[
\lambda_j^3 = 0.25 \text{ for all } j \text{ odd},
\]

\[
\lambda_j^4 = 1.75 \text{ for all } j \text{ even}.
\]

In this case, heterogeneous beliefs are allowed, and communication may well have impacts on the economy. In fact, this is the main case reported in Kurz and Beltratti (1997). The simulation results are reported in Table 2.

**Table 2: Rational Belief Equilibrium 1 (Kurz-Beltratti)**

<table>
<thead>
<tr>
<th>Variables</th>
<th>Simulation</th>
<th>Historical Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{p}$</td>
<td>23.00</td>
<td>23</td>
</tr>
<tr>
<td>$\sigma_p$</td>
<td>1.93</td>
<td>6.48</td>
</tr>
<tr>
<td>$\bar{R}$</td>
<td>7.64%</td>
<td>8.00%</td>
</tr>
<tr>
<td>$\sigma_R$</td>
<td>17.58%</td>
<td>18.08%</td>
</tr>
<tr>
<td>$r^F$</td>
<td>1.17%</td>
<td>1.00%</td>
</tr>
<tr>
<td>$\sigma_r$</td>
<td>16.19%</td>
<td>5.67%</td>
</tr>
<tr>
<td>$\bar{\rho}$</td>
<td>6.47%</td>
<td>7%</td>
</tr>
</tbody>
</table>

As can be seen from the table, the simulation results succeed in replicate the equity premium and the fluctuations of the risky rate of the historical data. Note that the volatility of the risk free rate is too high compared to the historical data. However, it can be understood that the volatility of the risk free rate is well controlled by the Government or the Central Bank (the Fed). Hence, the result is not too surprising because public policy including monetary policy is absent in the model.

Moreover, in this case, $M_S^h = 23.00 = \bar{p}$ for all $h, S$, while $Z_H^1 = 23.83$, $Z_L^1 = 22.17$, $Z_H^2 = 23.30$ and $Z_L^2 = 22.69$. Hence, the unconditional expectations are happened to be the same as the long term average of the price/dividend ratio, while the conditional expectations are not. Therefore, it is clear that communication does have impacts on the conditional expectations. Note in particular that the reactions to the announcements are in opposite directions depending on whether the effective belief is $Q_H^h$ or $Q_L^h$ for every $h$. 
Case 3: Rational Belief Equilibrium 2

In this case, the parameters are set as follows: \((a_i, b_i)\) are the same as in case 2 for all \(i\), and \(\lambda_j = 1.75\) for all \(h, j\). The simulation results are reported in Table 3:

<table>
<thead>
<tr>
<th>Variables</th>
<th>Simulation</th>
<th>Historical Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\bar{\rho})</td>
<td>22.26</td>
<td>23</td>
</tr>
<tr>
<td>(\sigma_\rho)</td>
<td>1.44</td>
<td>6.48</td>
</tr>
<tr>
<td>(\bar{R})</td>
<td>6.88%</td>
<td>8.00%</td>
</tr>
<tr>
<td>(\sigma_R)</td>
<td>10.77%</td>
<td>18.08%</td>
</tr>
<tr>
<td>(\bar{r}_F)</td>
<td>6.40%</td>
<td>1.00%</td>
</tr>
<tr>
<td>(\sigma_{r_F})</td>
<td>9.48%</td>
<td>5.67%</td>
</tr>
<tr>
<td>(\rho)</td>
<td>0.49%</td>
<td>7%</td>
</tr>
</tbody>
</table>

Heterogeneous beliefs are allowed in this case; thus, communication may have impacts on the economy. However, the simulation result is even worse than case 1 as far as the equity premium is concerned.

Moreover, in this case, \(M^h_H = Z^h_H = 22.89\) and \(M^h_L = Z^h_L = 21.06\) for all \(h\). Hence, communication has no impact on the conditional expectations. Observe that the difference in the unconditional expectations \(M^h_H\) and \(M^h_L\) is rather substantial, yet the fluctuations observed in this case are much smaller than the ones in case 2.

3.4 Discussion

We have seen simulation results of three different cases. The results are notably different amongst the three cases, and thus, it is worthwhile to examine the difference. However, to do so, we first need to clarify how to evaluate the results. In particular, we need to be clear about the way how we evaluate the effects of communication and/or the correlation of beliefs.

Consider the extended measurable space \(((X \times \mathcal{Q}^1 \times \mathcal{Q}^2)^\infty, \mathcal{B}((X \times \mathcal{Q}^1 \times \mathcal{Q}^2)^\infty))\), which is the true structure of the economy. Hence, if one knows this structure, he can define the true probability measure on this space. Consequently, with the true probability measure, one can define the correlation between \(Q^1_t\) and \(Q^2_t\) as well as the one between \(Y^1_t\) and \(Y^2_t\). Because it is assumed that there is a one-to-one relationship between \(Q^h_t\) and \(Y^h_t\), the correlations are equivalent.

However, an effective belief \(Q^h_t\) is only a probability measure on \((X^\infty, \mathcal{B}(X^\infty))\), not a measure on \(((X \times \mathcal{Q}^1 \times \mathcal{Q}^2)^\infty, \mathcal{B}((X \times \mathcal{Q}^1 \times \mathcal{Q}^2)^\infty))\). Hence, it is impossible to define the correlation between \(Q^1_t\) and \(Q^2_t\), while the correlation between \(Y^1_t\) and \(Y^2_t\) is still well defined with any effective belief \(Q^h_t\). Hence, from the perspective of agent \(h\), what really matters is only the observables \(Y_t\), since the effective beliefs are not observable.
Moreover, the reactions of the agents to the announcements characterize the relation between the effective belief $Q^h_t$ and the announcement of others $Y_t^{(h)}$. In effect, it characterizes the correlation between $Q^1_t$ and $Q^2_t$, i.e. correlation of beliefs. Hence, it is natural to classify the beliefs with respect to the reactions to the announcements. In addition, the explicit communication mechanism in our model enables us to interpret the correlation of beliefs in all simulation models of rational belief equilibria in a relatively simple way by asserting that there is one implicitly even if they do not model incorporate communication explicitly.

With this in mind, we provide answers to some of the questions addressed in the introduction. When we compare the results in case 2 and case 3, it is obvious that it is crucial for the agents to react differently to the announcements depending on the effective belief, i.e. whether $Q^h_H$ or $Q^h_L$, in order for the economy to experience large fluctuations. On the other hand, the discrepancy from the stationary measure or non-stationarity of beliefs alone does not generate large fluctuations, as we can see from case 2 and case 3 above.

Because we did not specify what the announcements really are, we are unable to provide a more concrete interpretations concerning the classification of beliefs. However, once we do so, and as long as the announcements represent the effective beliefs, this should not be a problem. For example, the announcements can be some sort of forecasts by the analysts or the financial institutions about the market. As we noted in the introduction, there are indeed abundant information of this sort in the financial markets.\(^{18}\) This opens up a possibility of concrete answers to the questions.

In doing so, we may use a result in the literature of the Expert Problem. In particular, the result of Genest and Schervish (1985) may well be very useful, which is summarised as follows. The reaction function of an agent to the announcements is linear in \(n\) moments of the announcements provided that the agent forms a belief up to the \(n\)th moment of the announcements, but does not specify the full joint distribution.\(^{19}\) For example, when agent \(h\) only specifies \(M^h_t\) and the expectation of the announcement of agent \(j\) (i.e. \(n = 1\)), then $Z^h_t$ is expressed as follows:

\[
Z^h_t = M^h_t + \nu^h_t \cdot (y^j_t - Z^h_t),
\]

where $Z^{hj}_t := E_{Q^h_t} Y^j_t$ and $\nu^{hj}$ is some constant that is determined subjectively by $Q^h_t$. Hence, we may classify the beliefs in terms of $\nu^{hj}$. More specifically, we may be able to classify the agents’ beliefs as (a) conformists, (b) contrarians, etc. Then, we can analyse the conditions of the markets by judging if conformists are prevailing, or contrarians are prevailing, and so forth.

For example, we know that $Z^h_H > M^h_H$ and $Z^h_L < M^h_L$ for all $h$ in case 2. Now, suppose we regard agent $h$’s announcement $Y^h_t$ as optimistic when

\(^{18}\)Unfortunately, the forecasts of the price/dividend ratio do not have a one-to-one relation with the effective beliefs in our model, because there are a continuum of possible combinations of transition probability matrices that induce the same stationary distribution (not measure!), which in turn result in the same expectations.

\(^{19}\)See Theorem 2.1 of Genest and Schervish (1985).
\( Y_t^h = 1 \) and pessimistic when \( Y_t^h = 0 \) for every \( h \). Then, it is clear that \( Z_{ht}^{hj} \in [0, 1] \) for all \( h, j, t \). Hence, if we apply the above formula, \( Z_{ht}^H > M_{ht}^H \) implies that \( \nu_{ht}^{hj} > 0 \). On the other hand, \( Z_{ht}^L < M_{ht}^L \) implies that \( \nu_{ht}^{hj} < 0 \). Hence, every agent \( h \) is a conformist when the effective belief is \( Q_{ht}^H \), while he is a contrarian when his effective belief is \( Q_{ht}^L \). To apply the above formula, we need to assume that the agents do not specify the full joint distribution, but only the first moments. Hence, we admit that this is an informal/casual interpretation. Nevertheless, we maintain that it gives some insights concerning the class of effective beliefs, and consequently, provides suggestions to the questions on the table.

4 Conclusion

We have shown that communication can be embedded in a model of rational belief equilibrium that does not incorporate communication explicitly a priori, e.g., Kurz and Beltratti (1997). The addition/inclusion of communication to the model enables us to evaluate correlation of beliefs in terms of conditional probabilities based on observable data, i.e., the announcements.

We stress that this is a substantial advantage because the entire knowledge of the true structure of the economy and the beliefs is needed to evaluate it when there is no explicit mechanism of communication. It is reasonable to presume that the lack of knowledge about the true structure of the economy applies to every one in reality. Hence, it is simply impossible to evaluate correlation of beliefs directly by focusing on the distributions of the measures \((Q_1^t, Q_2^t, ..., Q_H^t)\), since no one really knows the probability law that governs \((Q_1^t, Q_2^t, ..., Q_H^t)\). This is exactly why we would like to work with conditional probabilities based on the observables. Furthermore, this approach suggests that we may be able to restrict the class of rational beliefs empirically.

By classifying the beliefs of the agents in terms of conditional probabilities, the paper provides some suggestions to the questions address in the introduction. In particular, the simulation results indicate that the market is destabilized by communication when the agents react differently to the announcements with different effective beliefs. Because the simulation model does not incorporate public policy, it is impossible to evaluate what public policy is desirable. Yet the paper clearly suggests that some answers be given when a public policy is explicitly modelled.

References


