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THE TREND IN RETIREMENT

by

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The Trend in Retirement
Preliminary Version
Comments Welcome!

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Abstract

A model with leisure production and endogenous retirement is used to explain the declining labor-force participation rates of elderly males. The model is calibrated using the health and retirement study. The model is able to predict both the increase in retirement since 1850 and the observed drop in market consumption at the moment of retirement. The increase in retirement is driven by rising real wages and a falling price of leisure goods over time.

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Keywords: retirement, leisure, home production, consumption-drop, technological progress
Subject Area: Macroeconomics

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1 Introduction

Today’s senior citizens spend their time gardening, travelling, and enjoying a wide range of entertainment goods. Less than 20 percent are in the workforce, instead they allocate their time among various home and leisure activities. The world was quite a different place in 1880, when more than 75 percent of men over the age of 65 were participating in the labor market. Labor-force participation rates of men aged 65 and over continually declined during the period 1880 to 1990. Throughout this same period, life expectancy rates rose implying that the portion of a man’s life spent in retirement has increased.

Figure 2 shows the labor-force participation rates of men aged 65 and over for the period 1850 to 1990 in the United States, France, Great Britain, and Germany, and the participation rates of men aged 55 to 64 in the United States. The decline in the labor-force participation rates occurred in all four countries. This decline cannot be accounted for by increasing life expectancies. Participation rates fell for all ages above 65. In addition, participation rates have fallen among men aged 55 to 64. In 1880, 96 percent of men aged 60 were in the labor-force, by 1990 only 39 percent were. For men in their late fifties, participation rates have been declining since 1900 but started to decline at a faster rate around 1960. Figure 3 shows the retirement rates of men aged 50 and over by five-year-age groups for the period 1850 to 2000. The retirement rate is the ratio of the number of men who are retired to the total of all men in the labor-force and retired.

The combination of rising life expectancies and declining labor-force participation rates of the elderly have led to an increase in the expected duration of retirement. In fact, a twenty-year-old male in 1850 expected to spend approximately 6 percent of his adult life retired, while a twenty-year-old male in 1990 is expected to spend 30 percent of adult life retired. Figure 3 also shows how the percentage of his adult life a 20 year-old man can expect to spend retired has changed over this period.

1.1 Retirement

Retirement is defined as a planned withdrawal from the labor-force by older workers. For the majority of workers this withdrawal involves switching from full-time work to being fully retired, despite the fact that both a standard life-cycle model in which agents can choose

1 See Costa (1998), Chapter 2 for a in-depth discussion of trends in labor-force participation. The source for Figure 2 is Costa (1998), p. 29, Tables 2A.1 and 2A.2.
2 The data for the retirement rates in Figure 3 is from: Ruggles, Steven, et al. 2004. Integrated Public Use Microdata Series: Version 3.0. (IPUMS) Minneapolis, MN: Minnesota Population Center. It can be found at http://www.ipums.org. The retirement rates for each age group were computed by observing that: \% retired = (\% not in the labor-force − \% never participating)/(1 − \% never participating).
3 Adult life excludes the first twenty years. The data for the expected portion of life in retirement in Figure 3 is taken from Lee (2001), Table 1, p. 645. It is based on the same IPUMS data as used to compute the retirement rates. The expected length of retirement is computed assuming 20 year-olds have perfect information about future mortality rates.
hours and empirical evidence suggest that workers would prefer to gradually reduce hours.\textsuperscript{4} A variety of theories have been proposed to explain why a majority of older workers withdraw once and completely from the labor market. They include the inability of older workers, in demanding jobs, to handle physical and/or mental stress, minimum hours constraints and schedule inflexibility, and employer incentives and pensions. Evidence from the Health and Retirement Survey and the Health and Retirement Study has pointed to minimum hours constraints and schedule inflexibility as the largest factors influencing retirement decisions. For example, Hurd and McGarry (1993) find, using the Health and Retirement Survey, that the ability to change hours of work, pensions, and health insurance have an important effect on retirement decisions. While Gustman (2004) concludes, based on the Health and Retirement Study, that relaxing minimum hours constraints would significantly increasing the percentage of older people who continue working.

Table 1: Hours per Week Spent in Various Activities for Men by Age Group in 1985

<table>
<thead>
<tr>
<th>Activity</th>
<th>25-54</th>
<th>55-64</th>
<th>65+</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sleeping</td>
<td>54.9</td>
<td>57.5</td>
<td>58.7</td>
</tr>
<tr>
<td>Working or commuting</td>
<td>40.1</td>
<td>23.7</td>
<td>8.0</td>
</tr>
<tr>
<td>Recreation</td>
<td>35.8</td>
<td>42.7</td>
<td>51.1</td>
</tr>
<tr>
<td>Travelling</td>
<td>11</td>
<td>9.2</td>
<td>7.2</td>
</tr>
<tr>
<td>Grooming and child care</td>
<td>10.9</td>
<td>10.2</td>
<td>12.3</td>
</tr>
<tr>
<td>Eating and preparing meals</td>
<td>9.5</td>
<td>12</td>
<td>12.6</td>
</tr>
<tr>
<td>House and Yard Work</td>
<td>9.2</td>
<td>13.5</td>
<td>16.7</td>
</tr>
<tr>
<td>Shopping</td>
<td>4.7</td>
<td>5.4</td>
<td>5.6</td>
</tr>
<tr>
<td>Other</td>
<td>2.1</td>
<td>2</td>
<td>2.4</td>
</tr>
</tbody>
</table>

Upon retiring one must reallocate his time. How do retired people spend their time? Table 1 gives a breakdown of men’s time use for different age groups in 1985.\textsuperscript{5} Older men allocate more of their time to home and leisure activities such as preparing food, home improvements, gardening, shopping, and recreation, and less of their time to market work. For example, men age 55 to 65 spend approximately 19 percent more time on recreation than men aged 25 to 54, while men age 65 and over spend nearly 43 percent more time. By definition, retirement is a period in life in which a person has a large amount of time to allocate to non-market activities.

One important characteristic of the retirement period is what has come to be known as the retirement-consumption puzzle. First documented by Banks, Blundell, and Tanner (1998), the retirement-consumption puzzle is that while standard life-cycle models and the permanent income hypothesis imply that people smooth their consumption over their life-

\textsuperscript{4} To some extent older workers have been successful at remaining in the labor-force and reducing hours, since the percentage of older workers working part-time has been rising since, at least, 1940. See Chapter 5 of Costa (1998) for details.

\textsuperscript{5} Source for Table 1 is Godbey (1997), p. 207, Table 19.
time, a significant drop in consumption at the moment of retirement is observed in the data. Note that the drop in consumption observed in the data is actually a significant drop in expenditures on non-durables. It is assumed in these works that expenditure is equivalent to consumption. Researchers have had difficulty generating the drop in consumption at retirement with a standard life-cycle model, despite adding features such as uncertainty about the moment of retirement, mortality risk, and work-related expenses. While some researchers have argued that these results shed doubt on the rational agent model and support models in which agents are either time inconsistent or non-forward looking, a growing body of papers have pointed to the importance of the relationship between the amount of non-market time and the marginal utility of consumption as a possible remedy. Agents desire to smooth utility over their lifetimes. When the marginal utility of consumption is independent of the fraction of time an agent spends engaging in non-market activities, the agent smooths consumption in order to smooth utility. On the contrary, when the marginal utility of consumption depends on non-market time a discrete increase in non-market time at retirement results in a discrete jump in market consumption. Whether the change in consumption is positive or negative depends upon the elasticity of substitution between non-market time and consumption and the concavity of the utility function.

One class of models in which the marginal utility of market consumption often depends on the amount of non-market time are those in which agents engage in home production. There is an extensive literature demonstrating the importance of home production in explaining a variety of phenomena. In addition, recent empirical works have found increasing evidence that equating expenditure on market goods with consumption is misleading since people who have more free-time spend more time shopping and engaging in home production. Thus, not only is actual consumption a combination of consumption of market goods and consumption of goods produced at home using non-market time, but people with more time pay less for market goods. For example, evidence from time-use data shows that retired people spend more time on home activities and shopping then non-retired people. Table 2 shows the average number of hours per week spent in various home activities by retired men versus non-retired men aged 60 to 64 and 65 to 69 in 2001. Retired men aged 65 to 69 spend twice as many hours a week as non-retired men in the same age group shopping and nearly 50 percent more time preparing meals.

Another class of models in which the marginal utility of market consumption depends on the amount of non-market time are those that specify a utility function which is non-separable in consumption and leisure. Laitner and Silverman (2005) show that a model of this form when estimated to match data on consumption growth over the life-cycle and retirement provides estimates of structural parameters, such as the intertemporal elasticity of substitution, which are similar to others found in the literature. In addition, they demonstrate that the estimated model does a fairly good job of reproducing wealth holdings at

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7 For the time inconsistency etc. arguments see Bernheim, Skinner, and Weinberg (2001).
8 For examples see Reid, Margaret G. (1934); Becker (1965); Benhabib, Rogerson, and Wright (1991); Greenwood, Seshadri, and Vandenbroucke (2005); and Rios-Rull (1993) and the references therein.
9 For example see Aguiar and Hurst (June 2004) and Aguiar and Hurst (November 2004).
10 Source for Table 2 is Hurd and Rohwedder (2003), Tables 11 and 12. Based on data from the Consumption and Activities Survey, a supplement of the Health and Retirement Survey.
Table 2: Average Number of Hours per Week Spent on Home-Related Activities by Older Men in 2001

<table>
<thead>
<tr>
<th>Activity</th>
<th>Aged 60-64</th>
<th></th>
<th>Aged 65-69</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Not retired</td>
<td>Retired</td>
<td>Not retired</td>
<td>Retired</td>
</tr>
<tr>
<td>House cleaning</td>
<td>2.88</td>
<td>3.16</td>
<td>1.78</td>
<td>3.21</td>
</tr>
<tr>
<td>Washing/ironing</td>
<td>1.03</td>
<td>1.14</td>
<td>1.05</td>
<td>0.92</td>
</tr>
<tr>
<td>Yard work/gardening</td>
<td>2.10</td>
<td>4.07</td>
<td>2.05</td>
<td>4.97</td>
</tr>
<tr>
<td>Shopping</td>
<td>3.16</td>
<td>3.41</td>
<td>1.97</td>
<td>3.96</td>
</tr>
<tr>
<td>Meal preparation</td>
<td>3.46</td>
<td>4.51</td>
<td>3.11</td>
<td>4.57</td>
</tr>
<tr>
<td>Money management</td>
<td>0.78</td>
<td>0.84</td>
<td>0.80</td>
<td>1.09</td>
</tr>
<tr>
<td>Home improvements</td>
<td>0.88</td>
<td>2.32</td>
<td>0.67</td>
<td>1.91</td>
</tr>
</tbody>
</table>

retirement similar to those from HRS data.

The approach taken here is slightly different from both home production and direct non-separability between consumption and leisure. Instead of producing home goods, agents produce enhanced-leisure, and agents derive utility from a non-separable function of market consumption and enhanced-leisure. Leisure production can be differentiated from home production by observing that consumer durables used in home production are time-saving goods. That is the more efficiency units of the good one has the less time one chooses to spend on housework. This implies that consumer durables and time spent on housework are Edgeworth-Pareto substitutes, since an increase in consumer durables should result in a decline in the marginal utility of housework and a reallocation of time to other activities. On the other hand, recreation goods and leisure time should be Edgeworth-Pareto complements since leisure goods by construction are time-using – the more leisure goods one has the more time one wants to spend engaged in leisure. To summarize, home production consists of combining time (housework) with time-saving consumer durables (washing machines) to produce a consumption good (clean clothes), and leisure production consists in combining time (leisure time) with time-using goods (TV’s) to produce a consumption good (entertainment). Other leisure-enhancing goods include vacation packages, golf passes, and concert tickets.

Why did retirement in the US rise? In addition to explaining the observed drop in expenditures on market non-durables at the time of retirement, adding leisure production to a model of retirement has implications for the timing of retirement. Namely, the timing of retirement should be influenced by the price of recreation goods used as inputs into the leisure production function. Technological progress in leisure goods since 1850 has lead to more or less continually falling prices of these goods over the last one and half centuries. The expansion in the variety of leisure goods available has been substantial, as well, increasing the options available to people on how to spend their non-market time. In addition real wages have been rising since the industrial revolution. Hence the time cost of leisure inputs has been decreasing while the value of time has been increasing. Leisure production’s impact on the timing of retirement will depend on which of these two effects dominates. Can the
declining cost of leisure goods help to explain the trend in retirement observed over this period?

What about social security? The first federal paid Social Security benefit was in 1940. By that time, the labor force participation rates of elderly males had already substantially declined. Of course, there were other forms of old-age insurance in existence before 1940, such as state run programs and union army pensions plans but the percentage of elderly receiving benefits was small and, for most, benefits were small. While benefits having been rising throughout the century. An increase in benefits doesn’t necessarily imply an increase in retirement. Benefits have increased because wages have increased. If people have no uncertainty about their future social security benefits they will adjust their savings appropriately. When changes in benefits are unanticipated, social security may have an impact on retirement decisions but whether this impact is significant or not is subject to controversy. For example Lumsdaine, Stock, and Wise (1994) find that while changes in pension plans have a significant effect on retirement, Social Security has only a modest effect. Krueger and Pischke (1992) use data from the Current Population Survey to estimate the effect of Social Security wealth on the labor supply of older US males. They find that growth in Social Security benefits can explain less than one sixth of the decline in the male labor force participation rate during the 1970’s. They conclude that other factors driving retirement must exist. Anderson, Gustman, and Steinmeier (1997) simulate a structural model of retirement and find that increases in pensions and Social Security can account for about a quarter of the total trend towards earlier retirement observed from 1960 to 1980. They state that this effect may be in fact overstated and in addition that pensions and Social Security had no effect on retirement by those above age 65. They conclude, similarly to Kreger and Pischke (1992), that other factors driving retirement must exist. See Costa (1998) for a discussion of the impact of Social Security on retirement and the references therein.

The rest of the paper is organized as follows. In Section 2, a standard consumer problem with an endogenous retirement decision is presented. The model is analyzed and its implications discussed. In Section 3, the baseline model with leisure production and endogenous retirement is presented. Section 4 provides a theoretical analysis of the model. Section 5 provides a discussion of changes in leisure and leisure goods since 1850. Section 7 discusses the model’s ability to explain the trend in retirement. Section 8 provides some insight into how some extensions and modifications of the baseline model would impact the results. Finally, section 9 concludes.

2 Implications of the Standard Model

First, to get some intuition, consider the implications of adding endogenous retirement to a standard utility maximization problem. The agent lives for $T$ years and his utility is defined over streams of market consumption, $c(t)$ and leisure, $l(t)$, where $t \in [0, T]$. He has one unit of time at each moment $t$. Non-retired agents spend the fraction $\bar{h}$ of their time on the market. Thus they spend the fraction $1 - \bar{h}$ of their time on leisure. Retired agents allocate all their time to leisure. Agents choose their consumption, $c(t)$ and the moment of
their retirement, $A \leq T$, where choosing $A = T$ implies that the agent never retires. Once retired, an agent cannot return to work. An agent solves

$$\max_{c(t), A} \left\{ U(c(t), A) = \int_0^T e^{-\theta t} u[c(t)] dt + \int_0^A e^{-\theta t} v[l(t)] dt + \int_A^T e^{-\theta t} v[l(t)] dt \right\}, \quad (P1)$$

where $\theta$ is the rate of time preference and it is assumed that $\theta > 0$. The functions, $u(\cdot)$ and $v(\cdot)$ are continuous and twice-differentiable with $u'(\cdot), v'(\cdot) > 0$ and $u''(\cdot), v''(\cdot) < 0$. The function $l(t)$ denotes the agents leisure and is defined by

$$l(t) = \begin{cases} 1 - \bar{h}, & t \leq A, \\ 1, & t > A. \end{cases} \quad (2.1)$$

The agent receives a stream of wages, $w(t)$ over his lifetime. His budget constraint is

$$\int_0^T e^{-rt} c(t) dt = \bar{h} \int_0^A e^{-rt} w(t) dt,$$

where $r$ is the fixed real interest rate and $r > 0$. The agent’s consumption stream and retirement age must satisfy the following constraints,

$$c(t) \geq 0, \quad \forall t,$$

and

$$0 \leq A \leq T.$$

The first-order condition for $c(t)$ is

$$e^{-\theta t} u'[c(t)] = e^{-rt} \lambda, \quad \forall t, \quad (2.2)$$

where $\lambda$ is the multiplier on the budget constraint in the Lagrangian. The first-order condition for the retirement date, $A$, is

$$e^{-\theta A}[v(1) - v(1 - \bar{h})] \leq e^{-rA} \lambda w(A) \bar{h}. \quad (2.3)$$

Which can be rewritten as

$$\underbrace{v(1) - v(1 - \bar{h})}_{\text{marginal cost}} \leq \underbrace{u'[c(A)] w(A) \bar{h}}_{\text{marginal benefit}}, \quad (2.4)$$

by combining with equation (2.2). The left-hand-side of equation (2.4) is the marginal cost of delaying the moment of retirement since it is the instantaneous gain in utility the agent receives at the moment he retires. The right-hand-side is the marginal benefit since it is the utility value of the additional earnings the agent gets by working in moment $A$. At an interior solution for the optimal retirement date, $A$, equation (2.4) will hold with equality. Differentiating (2.2) with respect to $t$ yields

$$\dot{c}(t) = (\theta - r) \frac{u'[c(t)]}{u''[c(t)]}. \quad (2.5)$$
2.1 Theoretical Analysis

Suppose $u(\cdot)$ is the constant-relative-risk-aversion utility function

$$u[c(t)] = \frac{c(t)^{1-\rho}}{1-\rho}, \quad (2.6)$$

where $\rho > 0$ and $\rho = 1$ implies log-utility. Then equation (2.4) becomes

$$v(1) - v(1 - \bar{h}) \leq c(A)^{-\rho}w(A)\bar{h}, \quad (2.7)$$

and

$$c(t) = c_0(A)e^{\frac{r-\theta}{\rho}t}. \quad (2.8)$$

Plugging (2.8) into the budget constraint and solving for $c_0$ gives

$$c_0(A) = \frac{\bar{h} \int_0^A e^{-rt}w(t)dt}{\int_0^T e^{\left(\frac{r-\theta}{\rho}-r\right)t}dt}. \quad (2.9)$$

In addition, suppose wages are growing at a constant rate, $\kappa$, so that

$$w(t) = w_0e^{\kappa t}.$$ 

Notice that at an interior solution for the optimal retirement date, $A^*$, equation (2.7) will hold with equality while a solution in which the agent never retires requires that the marginal benefit of working at the last moment of life be at least equal to the marginal cost. Since the objective function, in general, is not strictly concave in the retirement date, an $A^* \leq T$ which satisfies the first-order conditions for optimality is not guaranteed to be the solution to P1. Hence additional conditions must be checked. Proposition 1 gives a condition under which there exists exactly one $A^*$ which satisfies the first-order conditions and states that this $A^*$ is a maximizer.

**Proposition 1** If

$$v(1) - v(1 - \bar{h}) > c(T)^{-\rho}w(T)\bar{h}, \quad (2.10)$$

where

$$c(T) = c_0(T)e^{\frac{r-\theta}{\rho}T},$$

then there exists a unique $A^* < T$ satisfying the first-order conditions of problem P1 and it is the global maximizer.

**Proof.** See Appendix. 

The intuition behind the proof of Proposition 1 is based on the shape of the marginal cost (left-hand-side of equation (2.4)) and marginal benefit (right-hand-side of equation (2.4)) curves. The marginal cost curve is constant while the marginal benefit curve is strictly convex. In addition, the marginal benefit of working goes to infinity as the retirement date goes to zero. The reasoning is that as the date of retirement goes to zero, the agent’s wealth
Figure 1: Schematic of marginal cost and marginal benefit curves.

and total consumption go to zero as well, driving the marginal utility of consumption to infinity. Thus the curves can cross zero, one, or two times. Figure 1 illustrates the case where the curves intersect twice, at $A_1$ and $A_2$. Notice that if the length of the agent’s life is such that at the last moment of life, $T$, the marginal benefit of working is less the marginal cost then the marginal benefit curve must cross the marginal cost curve only once. In addition, the value of $A$ at this crossing must be the optimum. In the example illustrated in Figure 1 this situation occurs when $T$ falls between $A_1$ and $A_2$. In that case, $A_1$ would be the global maximizer of $P_1$.

Note that condition (2.10) in Proposition 1 is a sufficient but not a necessary condition for existence of an interior solution to $P_1$. To see this consider a situation in which the marginal benefit and marginal cost curves are as in Figure 1 but the length of the agent’s life is such that $T > A_2$. In this situation, $A_1$ may be the global maximizer of $P_1$ but condition (2.10) does not hold. In order to determine whether retiring at $A_1$ or $T$ (never retiring) is optimal, the value of the objective function at these two points must be compared.

How does an increase in the agent’s wage level affect his retirement decision? When the agent’s income goes up he faces two opposing effects. On one hand, he is now wealthier and can afford to retire earlier in life (income effect), but on the other hand, the ‘price of retirement’ has gone up making retirement more costly (substitution effect). Hence when the income effect dominates, an increase in the agent’s income level will cause him to retire earlier in life, and when the substitution effect dominates, an increase in his income level will cause him to retire later. The following lemma states the relationship formally.

**Proposition 2** The optimal retirement date, $A^*$ is:

(i.) increasing in the the initial wage rate, $w(0)$, if $\rho < 1$,
(ii.) independent of the initial wage rate, $w(0)$, if $\rho = 1$ (log utility),
(iii.) decreasing in the the initial wage rate, \( w(0) \), if \( \rho > 1 \).

**Proof.** See Appendix.

In Lemma 3 it is shown that the retirement date is increasing in the length of life, \( T \). The intuition is straightforward, if the agent lives longer he must work longer to pay for his extra years of consumption.

**Proposition 3** The optimal retirement date, \( A^* \), is increasing in the length of life, \( T \).

**Proof.** See Appendix.

Proposition 2 suggests that this simple model has the potential to reproduce at least some of the increase in retirement observed during the twentieth century. Both wages and life expectancy rose during this period. Even though Proposition 3 predicts that as the length of life rises the age of retirement increases, if \( \rho > 1 \) than the effect of rising real wages could lead to a falling age of retirement. But due to the separability of utility in leisure time and consumption the model is unable to match the data on the drop in consumption at retirement. In addition, the model cannot be used to assess the importance of the falling prices and increasing variety, quality, and availability of leisure goods. These facts motivate the introduction of the baseline model presented in the next section.

## 3 The Baseline Model

Consider a model in which agents value both market consumption and enhanced-leisure, which is produced by combining leisure time with leisure goods. Take note of two points. First, let leisure time and leisure goods be Edgeworth-Pareto complements so that an increase in one factor increases the agent’s marginal utility of the other factor. This feature of the model captures the notion that leisure goods are time-using by definition. Second, the data described earlier suggests that leisure time and leisure goods are Hicksian complements, i.e., a fall in the price of one results in an increase in the demand for both. Under this notion of complementarity, the rise in real income and fall in the price of leisure goods will lead to agents purchasing more of the leisure goods and retiring earlier to spend more time enjoying them. The connection between the two notions of complementarity is a matter of calibration, and will be discussed later.

Non-separability between market consumption and enhanced-leisure, combined with the restriction that agents must supply labor indivisibly while working, will result in entry into retirement triggering a drop in market consumption. Lost utility from consumption goods will be offset by increased utility from leisure. Of course, everyone does not retire at the same time. Agents will differ along two dimensions: their market ability and their ability to produce leisure or leisure productivity. Agents with a high market ability will have the luxury of retiring earlier since they do not need to work as long to save enough income to fund their retirement. While low ability agents will have to work longer, as they want to
have enough savings to be able to afford leisure goods later in life.\textsuperscript{11} Agents with a high productivity in leisure production will also work longer and retire later. These agents work longer so that they can purchase more of the leisure goods, taking advantage of their high leisure productivity. Agents with low leisure productivity have less of an incentive to work late into life since they their low leisure productivity reduces their benefit from leisure goods.

Time is continuous and indexed by \( t \). The economy consists of overlapping generations. Each generation has a maximum length of life \( T \). Agents face a constant probability of dying per a unit time, \( \eta \), at all moments of their life except at age \( T \) where the probability of dying is 1. The assumption of a constant probability of dying and a maximum length of life is made for tractability not realism. Agents are characterized by their type \( s \in S \equiv \{ (\tau, x, z) \in \mathbb{R}_+ \times \mathbb{R}_+ \times \mathbb{R}_+ \} \), where \( \tau \) denotes the date of the agent’s birth. An agent born at moment \( \tau \) will be age \( a = t - \tau \) at time \( t \). The variable \( x \) is an agent’s ability to do market work and \( z \) is his ability to produce leisure. The ability \( x \) is governed by the distribution \( F(x) \) while \( z \) is governed by the distribution \( G(z) \). There is no correlation between an agent’s market ability and leisure ability.

### 3.1 Agents’ Maximization Problem

An agent born at date \( \tau \) with market ability \( x \) and leisure productivity \( z \) chooses paths for market consumption and the purchase of leisure goods over his lifetime, \( c_s(a) \) and \( g_s(a) \), respectively, and the age of his retirement, \( A_s \), to maximize his expected lifetime utility given by

\[
\int_0^{A_s} e^{-(\theta + \eta)a} U[c_s(a), n_s(a)] da + \int_{A_s}^T e^{-(\theta + \eta)a} U[c_s(a), n_s(a)] da.
\] (3.11)

The parameter \( \theta \) is the rate of time preference and \( e^{-\eta a} \) is the probability of being alive at age \( a \). The momentary utility function is of the constant relative risk aversion form so

\[
U[c_s(a), n_s(a)] = \frac{\{c_s(a)^\alpha n_s(a)^{1-\alpha}\}^{1-\sigma}}{1-\sigma},
\] (3.12)

and it is assumed that \( \sigma > 0 \) and \( \sigma = 1 \) implies log-utility.

Agents have one unit of time at each moment of their lives. A non-retired agent, with ability \( x \), inelastically supplies a fraction \( \bar{h} \) of his time to the market and receives labor income \( w(t)\bar{h}x \), where \( w(t) \) is the wage per an efficiency unit of labor at time \( t \). He dedicates the rest of his time to the production of leisure. Retired agents spend all their time on leisure production. Thus time spent on leisure production is defined as

\[
l_s(a) = \begin{cases} 
1 - \bar{h}, & a \leq A_s, \\
1, & a > A_s.
\end{cases}
\] (3.13)

The economy also contains goods which aid in leisure production, here called leisure goods or leisure inputs. The price of the leisure good at time \( t \) is denoted as \( p_g(t) \). Time and leisure

\textsuperscript{11} See section 8 for a discussion on how the model could be modified to remove the monotonic relationship between income and retirement age.
goods are combined to produce enhanced-leisure using a constant elasticity of substitution production function, or

\[ n_s(a) = \left\{ \zeta g_s(a)^{\chi} + (1 - \zeta)[z l_s(a)]^{\chi} \right\}^{\frac{1}{\chi}}, \]

where \( \chi \leq 1 \) and \( \chi = 0 \) implies a Cobb-Douglas production function. The parameter \( \chi \) controls the degree of substitutability between leisure time and leisure goods. If \( \chi > 0 \) then the elasticity of substitution between leisure time and leisure goods will be greater than one and the two factors will be Hicksian substitutes. While \( \chi < 0 \) implies an elasticity of substitution that is less than one and Hicksian complimentarily between leisure time and leisure goods. The parameter \( \zeta \) is the weight on leisure inputs relative to leisure time in the production function.

The following restriction is imposed on the model,

\[ 1 - \sigma - \chi > 0, \]

which ensures that leisure time and leisure goods are Edgeworth-Pareto complements in utility. Notice that when \( \sigma \geq 1 \), this restriction guarantees that \( \chi < 0 \) and then a decrease in the price of leisure goods will lead to an increase in the demand for leisure time.

Notice that the allocations of consumption goods and leisure goods over time may be discontinuous at the retirement date, \( A_s \), since the fraction of time the agent allocates to leisure changes from \( 1 - \bar{h} \) to \( 1 \) at this date. Hence the agent’s maximization problem is written in such a way as to incorporate this discontinuity.

As in Kalemli-Ozcan, Ryder, and Weil (2000), the economy contains a life-insurance company which offers actuarially-fair annuities to the agents. Annuities allow agents to share mortality risk and, as was first shown by Yaari (1965), since the agents have no bequest motive they will use annuities as their sole instrument of investment. If \( r \) is the constant bond market interest rate, agents with probability of dying \( \eta \) face a rate of return on annuities of \( r + \eta \). Thus the agent’s life-time budget constraint is

\[
\int_{0}^{A_s} e^{-(r+\eta)a}c_s(a)da + \int_{A_s}^{T} e^{-(r+\eta)a}c_s(a)da + \int_{0}^{A_s} e^{-(r+\eta)a}p_g(a + \tau)g_s(a)da + \int_{A_s}^{T} e^{-(r+\eta)a}p_g(a + \tau)g_s(a)da = \int_{0}^{A_s} e^{-(r+\eta)a}\bar{h}w(a + \tau)da. \tag{3.14}
\]

Hereafter the \( s \)-subscript is dropped for ease of notations.\(^{12}\)

The first-order condition for market consumption is

\[
\alpha e^{-(\theta+\eta)a}c(a)^{(1-\sigma)(1-\alpha)} = \lambda e^{-(r+\eta)a}, \quad \forall a \in [0, T], \tag{3.15}
\]

where \( \lambda \) is the multiplier on (3.14) in the Lagrangian. The first-order condition for purchase of the leisure good is

\[
(1 - \alpha)\zeta e^{-(\theta+\eta)a}c(a)^{(1-\sigma)(1-\alpha)} = \lambda e^{-(r+\eta)a}p_g(a + \tau), \quad \forall a \in [0, T]. \tag{3.16}
\]

\(^{12}\) See Appendix Part A for additional notes on the agent’s maximization problem.
The first-order condition for the retirement age is

$$\lambda e^{(\theta-r)A} \left[ xhw(A+\tau) - (\bar{c}_A - \xi_A) - p_g(A+\tau)(\bar{g}_A - g_A) \right] \leq \left( \frac{(c_A^{1-\alpha})}{1-\sigma} - \frac{(\pi_A^{1-\alpha})}{1-\sigma} \right) \left( \frac{\xi_A}{1-\sigma} - \frac{\pi_A}{1-\sigma} \right) \left( \frac{\xi_A}{1-\sigma} - \frac{\pi_A}{1-\sigma} - \rho \right) \left( \frac{\xi_A}{1-\sigma} - \frac{\pi_A}{1-\sigma} - \rho \right) \frac{\xi_A}{1-\sigma} - \frac{\pi_A}{1-\sigma}, \right. \tag{3.17}$$

where $\bar{c}_A$ is consumption of the market good at the moment of retirement, given that the agent is still working, or

$$\bar{c}_A = c(A), \tag{3.18}$$

$\xi_A$ is defined as

$$\xi_A = \lim_{a \to A^+} c(a), \tag{3.19}$$

and $\bar{g}_A$, $\pi_A$, and $\pi_A$ are defined similarly. To understand equation (3.17), consider the problem of an agent who is deciding whether or not he should retire at age $A$. If he retires, his instantaneous utility changes. His net gain in instantaneous utility is on the left-hand-side of equation (3.17). This the marginal cost of postponing retirement. The right-hand-side is the marginal benefit. It is the utility value of the savings of the agent at age $A$ if he is working net of his savings at $A$ if he is retired. As long as the marginal benefit of working exceeds the marginal cost, the agent will not retire. Thus an agent could die having never retired. At an interior solution for the optimal retirement date, $A$, equation (3.17) will hold with equality.

Solving (3.15) for $c(a)$ and differentiating with respect to $a$ gives

$$\frac{\dot{c}(a)}{c(a)} = \frac{1}{\Phi} \left[ \theta - r - \Psi \frac{\tilde{n}(a)}{n(a)} \right], \tag{3.20}$$

where

$$\Phi = (1-\sigma)\alpha - 1,$$

and

$$\Psi = (1-\sigma)(1-\alpha).$$

Totally differentiating (3.16) with respect to $g$ and $a$, plugging in (10.23), and solving for $\dot{g}(a)$ yields

$$\frac{\dot{g}(a)}{g(a)} = \left( \frac{\hat{p}_g(a+\tau)}{p_g(a+\tau)} - \frac{\theta - r}{\Phi} - \Psi \frac{\tilde{n}(a)}{n(a)} \right) \left( \chi - 1 - \zeta \left( \frac{\Psi}{\Phi} + \chi \right) g(a)^{\chi n(a)^{-\chi}} \right)^{-1}. \tag{3.21}$$

Equations (10.23) and (3.21) are a system of differential equations in $c(\cdot)$ and $g(\cdot)$.

### 3.2 Optimum

It is now possible to define an optimum for this economy. Given distributions of market ability and leisure TFP, $F(x)$ and $G(z)$, and prices, $\{w(t) : t \geq 0\}$, $r$, and $\{p_g(t) : t \geq 0\}$, an optimum of this economy consists of a set of allocations $\{c_s(a), g_s(a) : a \in [0,T]\}$ for all $s \in S$, that solve the agents’ maximization problems.
4 Theoretical Analysis

This section looks at relationship between key parameters in the model and an agents’s optimal retirement age, $A_s$. Hereafter, it is assumed that wages are growing at a constant rate, $\kappa$, or

$$w(t) = w(0)e^{\kappa t},$$

and the price of the leisure good is falling at rate $\gamma$ or

$$p_g(t) = p_g(0)e^{-\gamma t}.$$  

Since the models purpose is to generate the long-run trend in retirement, not capture small movements, these assumptions seem reasonable and greatly simplify the analysis and computation.

First consider how the age of retirement is related to the agent’s market ability, $x$. The following proposition defines the relationship for the case of Cobb-Douglas leisure production.

**Proposition 4** The agents optimal retirement age, $A_s$, is independent of his market ability, $x$, when the leisure production function is Cobb-Douglas.

**Proof.** See Appendix.

Proposition 4 suggests that the relationship between $A_s$ and $x$ changes when $\chi$ switches signs. In fact, $A_s$ is increasing in $x$ for $\chi > 0$ and decreasing in $x$ for $\chi < 0$ as can be seen in numerical simulations. The relationship is intuitive. First consider the case of $\chi < 0$. In this case, leisure goods and leisure time are Hicksian complements. An increase in the agent’s market productivity generates an increase in his income, allowing him to purchase more of the leisure good. The increase in his quantity of the leisure goods increases his demand for leisure time. Since the only way to increase leisure time is by retiring, the agent retires earlier. When $\chi > 0$ however, the demand for leisure time and hence retirement is decreasing in the leisure good. Thus, facing an increase in his market productivity, an agent chooses to retire later.

The relationship between the agent’s retirement age and his leisure productivity, $z$, is exactly the opposite. When leisure goods and leisure time are Hicksian complements ($\chi < 0$), an agent’s retirement age is increasing in $z$. While the retirement age is decreasing in $z$ for $\chi > 0$. When $z$ increases, if leisure time and leisure goods are Hicksian complements, the agent wants to increase his amount of the leisure good. To afford more of the leisure good he must work for a longer portion of his life. When leisure time and leisure goods are substitutes, an increase in $z$ leads to a decrease in leisure good purchases and an earlier age of retirement. Proposition 5 documents the relationship between the retirement date and $z$ for the Cobb-Douglas case.

**Proposition 5** The agents optimal retirement age, $A_s$, is independent of his productivity in leisure production, $z$, when the leisure production function is Cobb-Douglas.
Proof. See Appendix.

Just as in the simple model, the retirement date is increasing in the length of life, $T$. Proposition 6, below, states this result formally and provides a proof for the special case of log utility and Cobb-Douglas leisure production. The proposition is supported numerically.

**Proposition 6** The agents optimal retirement age, $A_s$, is increasing in his length of life, $T$.

Proof. See Appendix. Proof provided only for the special case of log utility and Cobb-Douglas leisure production.

## 5 Leisure

Americans today spend much more time and money on leisure than they did a hundred years ago. In addition to increases in retirement since the nineteenth century, hours spent working per week have declined. In 1890 manufacturing workers, on average, spent approximately 60 hours working a week. By 1940, the work week consisted of 40 hours. Since 1940, total hours worked per year by men aged 18 to 64 have continued to decline due to increases in paid vacation time, and sick and personal days.\(^{13}\) While hours spent working for pay by women have risen, due to increased participation, the amount of time women spend doing housework has declined.\(^{14}\) Time use data suggests that, overall, hours of free time have increased for both men and women. For example, Godbey and Robinson (1997) find that from 1965 to 1985 hours per week spent in free time increased by 5 hours for both sexes.\(^{15}\) This free time has been used to enjoy a variety of leisure activities.

The increase in time spent on leisure activities has been concurrent with an increase in expenditure on leisure goods. While the average American in 1900 spent approximately 2 percent of his earnings on recreation goods, in 2001 Americans allocated more than 8 percent of their expenditure to the goods (see Lebergott (1996)). Expenditure on recreation goods does not include expenditure on transportation. Yet approximately 30 percent of the average total miles driven with a car each year are driven for social and recreational trips.\(^{16}\) When 30 percent of expenditure on transportation is included in recreational expenditure, recreation’s expenditure share rises from about 4 percent in 1900 to nearly 12 percent in 2001, as can be seen in Figure 4.

Increased time spent on leisure activities and increased expenditure on leisure goods can be attributed to a combination of rising real wages and falling prices of leisure goods.

---

\(^{13}\) The weekly hours data is from the Historical Statistics Series D 803 and D 847. Table 2, p. 95, in Godbey and Robinson (1997) shows that, based on time use diaries, average hours spent at work decreased for employed men from 1965 to 1985. Series D 116-118 show an increase in vacations, and sick/personal days.

\(^{14}\) See Greenwood, Seshadri, Yorukoglu (2005) for a discussion.

\(^{15}\) Table 6, p. 126.

which together have made leisure activities more affordable. Figure 5 shows the fall in the time cost of a particular selection of leisure goods throughout the twentieth century.\textsuperscript{17} The cost decreases at a rate of approximately 2 percent per year. The three largest groups of recreation goods included in the series are newspapers, magazines, and books; radios, televisions, and phonographs; and admissions to movies and concerts. The price series for newspapers, magazines, and books is based largely on retail prices. Thus it does not capture changes in quality nor does it account for the increase in the variety of reading material available. The price series for radios, televisions, and phonographs was generated by combining Bureau of Labor Statistics (BLS) data with retail price data from Sears’ catalogs. Technological advancement in the production of electronics led to enormous declines in the price of radios and televisions. For example, the retail price of radios fell by 98 percent between 1926 and 1970 at an average rate of 5.2 percent per year. A price index for televisions based on the Sears catalog suggests that the price of televisions decreased by 50 percent at an annual rate of 2.2 percent during the period 1952 to 1983.

While the BLS does incorporate some quality-adjustment in their price series, evidence suggests that BLS price indices for consumer durables have been biased-upward due to lack of quality adjustment.\textsuperscript{18} An alternative price index for televisions, generated by Gordon (1990), suggests that the quality-adjusted price of televisions, including costs for repair and energy, fell by 80 percent over the period 1952 to 1983 at an annual rate of 4.34 percent, nearly double the rate suggested by the Sears’ index or more than four times the 1 percent rate of decline in the BLS’s Consumer Price Index. Not only did the quality-adjusted prices of radios and televisions fall dramatically but the variety of stations and programs increased. Figure 6 shows the rise in the number of radio and television stations in the United States from 1921 to 1998.\textsuperscript{19} This increase in variety can be thought of as another dimension of quality-improvement. The price series for leisure goods would demonstrate an even faster rate of decline had these quality-adjustments been included.

The leisure goods price series in Figure 5 not only underestimates the effect of improvement in quality but, in addition, is imperfect because many important leisure goods are not included. One example is automobiles which, as mentioned above, are an important good for leisure activities. The price of automobiles decreased by 85 percent since 1906, with the fastest rate of decline occurring from 1906 to 1940, when it declined at an annual average rate of 5.5 percent. The prices of other transportation goods, as well as goods which add in communication and travel, should also be considered.

\textsuperscript{17} The time cost index of leisure goods is obtained by dividing a price index of leisure goods by a real wage index. Thus the time cost falls due to a combination of the falling relative price of leisure goods and rising real wages. The price of leisure goods falls at an average annual rate of 1 percent. Sources for the price index: For the period 1901 to 1934, data from Owen (1969), Table 4-B, p. 85 is used; for the period 1935 to 1968, data is from the Historical Statistics Series E 165; and for 1969 - 2001, the data is taken from the Bureau of Labor Statistics’ Handbooks of U.S. Labor Statistics, 2nd. Ed. (1998), p. 263 and 6th Ed. (2003), p. 308. The US real wage index is from Williamson (1995) and the Bureau of Labor Statistics.

\textsuperscript{18} For instance, Bils and Klenow (2001) conclude that price indices for consumer durables are biased upward by at least 0.80% per year.

\textsuperscript{19} The source for Figure 6 is the Historical Statistics: Series R 93, 94, and 96 for the period 1920 to 1970. It is extended to 2000 using data from the Statistical Abstract of the United States.
The notion that the price of leisure goods has a significant impact on the demand for leisure time was first pointed out by Owen (1971). Owen argues that a significant amount, about 25 percent, of the decline in weekly hours of U.S. males during the period 1901 to 1961 is due to the falling relative price of recreation goods. He argues that the other 75 percent of the decline is due to rises in the real hourly wage. More recently, Vandenbroucke (2005) calibrates a model in which agents produce and derive utility from enhanced-leisure. Vandenbroucke finds that the decline in the price of leisure goods during the first half of the twentieth century can explain a significant part of the decline in weekly hours per worker.

6 Retirement Lifestyle

Falling prices and the increased availability and variety of leisure goods have led to the development of a retirement lifestyle. Most people today expect to spend their senior years enjoying a leisurely lifestyle with a recreation-intensive schedule. Evidence from time-use studies suggests that as men age they spend more time using leisure goods, such as televisions, radios, stereos, books, magazines, and newspapers. For example, according to Godbey (1997), in 1985, men aged 55 to 64 spent 13 percent more time watching television than men aged 25 to 54, while men over the age of 64 spent 81 percent more time. Men over the age of 65 also spent nearly double the amount of time men aged 25 to 54 spent reading and listening to music. Table 3 gives a breakdown of time spent in various leisure activities by age groups for men in 1985.20

Table 3: Hours per Week Spent in Various Leisure Activities for Men by Age Group in 1985

<table>
<thead>
<tr>
<th>Activity</th>
<th>Age 25-54</th>
<th>Age 55-64</th>
<th>Age 65+</th>
</tr>
</thead>
<tbody>
<tr>
<td>Participating in organizations</td>
<td>0.9</td>
<td>2.2</td>
<td>1.1</td>
</tr>
<tr>
<td>Attending events</td>
<td>0.9</td>
<td>0.6</td>
<td>0.1</td>
</tr>
<tr>
<td>Visiting</td>
<td>6.6</td>
<td>6.7</td>
<td>6.0</td>
</tr>
<tr>
<td>Playing or watching sports</td>
<td>2.9</td>
<td>2.8</td>
<td>3.0</td>
</tr>
<tr>
<td>Hobbies</td>
<td>2.5</td>
<td>3.5</td>
<td>3.5</td>
</tr>
<tr>
<td>Talking or socializing</td>
<td>2.8</td>
<td>3.1</td>
<td>4.5</td>
</tr>
<tr>
<td>Watching TV</td>
<td>16.1</td>
<td>18.2</td>
<td>24.9</td>
</tr>
<tr>
<td>Reading</td>
<td>2.6</td>
<td>4.7</td>
<td>6.7</td>
</tr>
<tr>
<td>Listening to music</td>
<td>0.5</td>
<td>0.9</td>
<td>1.3</td>
</tr>
</tbody>
</table>

An important component of the retirement lifestyle, which goes largely unobserved when looking at time-use data based on daily diaries, is travelling. Evidence from data on consumer expenditures suggests that Americans aged 55 and older spend larger fractions of their income on vacationing, touring, and sight-seeing than those of younger ages. For example, for the years 1972 and 1973, Americans aged 55 to 64 spent 2.5 percent of their income on

20 The source for Table 3 is the same as that for Table 1. See footnote 5.
vacations and pleasure trips, and those aged 65 and over spent 3.3 percent, while those aged 25 to 54 spent only 2.0 percent. Vacations’ share of total expenditure increased throughout the 1970s. By the years 1980 and 1981, the expenditure share of 55 to 64 year-olds was 3.7 percent, and the share of those age 65 and over was 4.3 percent, compared with a share of 3.0 percent for those aged 25 to 54. In fact, an article published in Nation’s Business in 1981 points out the growing numbers of older Americans vacationing and touring:

Persons 55 and older are the major customers for round-the-world cruises where fares may range between 15,000 dollars and 150,000 dollars, and, at the same time, are the bread-and-butter market for low-cost motor coach excursions – accounting for nearly one third of all bus charter trips in the country.

The increased attractiveness of travelling for older Americans has mostly likely been due, not only to the declining costs, but also to technologies which have made travelling more comfortable and less exhausting. Aron (1999) talks about trips to Yellowstone National Park in 1888. She points out that not only where such trips expensive – a 25 day tour cost 275 dollars and a one-way stage-coach ride from San Francisco to Yosemite cost 80 dollars – but “could often be a taxing endeavor,” since tourists had no way of avoiding “the heat and dust of the desert and the inevitable discomfit that accompanied getting around Yellowstone or down in to the Yosemite valley.” A round-trip motorcoach tour from San Francisco to Yellowstone today cost 115 dollars: that’s more than 14 times cheaper than the one-way stage coach ride and it’s air-conditioned.21

The evolution of the retirement lifestyle has been concurrent with a growing independence of elderly people. Instead of moving in with and relying on adult children or other relatives, increased wealth along with technological progress has allowed more and more seniors to maintain their own households. Figure 7 shows how the percentage of retired men who are heads of households has risen since 1880.22 While less than 50 percent of men 65 and older were household heads in 1880, nearly 90 percent of them were in 1990. As independent heads of households seniors are able to choose locations which suit their lifestyle as retirees. Thus there has been a movement of elderly people to warmer climates where milder temperatures make life more enjoyable. One popular location for retirees is Florida. Figure 8 shows how the percentage of the United States’ and Florida’s population which is over the age of sixty-four has changed since 1870.23 In 1870, only 1.2 percent of Florida’s population was over the age of 64 compared to 2 percent of the United States’. The share of senior citizens in Florida’s population grew steadily surpassing that of the United States’ in approximately 1940. By 1990 more than 18 percent of Floridians were senior citizens compared to 12.5 percent of Americans. Many retirees spend part of the year in Florida and part of the year travelling. A man who retired in 1972 reported:

We live in a 31-foot Airstream trailer – spend seven months in winter in a park in Melbourne, Fla., where we have ever kind of activity. We dance and square

22 The source for Figure 7 is Costa (1998), Table 6A.1, p. 130.
23 Data used for Figure 8 is from the Historical Statistics, Series A 119, 133, 195 and 209 for the period 1870 to 1970. Extended to 2000 with data from the Statistical Abstract of the United States.
dance and party all winter. Then in summer we travel about – stop and spend some days with children and grandchildren and rest of time traveling to rallies in caravans and sightseeing from Canada through 48 states and Mexico.  

An important cost associated with moving to a new area and traveling is the cost of maintaining communication with friends and family. Technological progress in communication since the nineteenth century has increased the speed and quality of communication and greatly reduced the price. The most important breakthrough in communication was the invention of the telephone. In 1883 the first telephone system was setup connecting New York City to Boston. Soon people could talk live with friends and family members in cities across America but costs where high and increased quickly with distance. In 1915, a three minute daytime call to Philadelphia from New York City cost more than 10 dollars while a call to San Francisco cost nearly 250 dollars. Throughout the twentieth century costs not only fell but converged. In 1970 the same calls cost 2.22 dollars and approximately 6 dollars, respectively. Figure 9 shows the declining cost of long-distance calls from New York City to four other cities in the United States during the period from 1902 to 1970. Since 1970 the cost of long-distance calls has continued to decline and the price has become less dependent on the distance. Most phone companies charge less than 10 cents per minute to call from anywhere to anywhere in the United States and phone calls from computers can be made for free.

The rise in the retirement lifestyle, driven by rising incomes and falling costs of leisure goods, has led to an increased withdrawal of older men from the labor-force. Can the model outlined above reproduce the trend of increasing retirement since 1870 when it is calibrated to the rising incomes and falling prices of leisure goods?

7 Numerical Analysis

The following experiment is devised to bring the model to the data: First the model is calibrated to the year 2000 by matching data on labor-force participation rates and the drop in market consumption at the moment of retirement from the Health and Retirement Study. Then the calibrated model is run to reproduce the retirement rates of the elderly population, aged 50 to 80, for the period 1850 to 2000.

7.1 Computation

In order to ease the computation of the statistics of interest the model and statistics are computed for a discrete set of types. Each agent is characterized by his birth year, $\tau$, his market productivity, $x$, and his leisure productivity, $z$. Agents are assumed to be born at age twenty and while agents do live continuously, it is assumed that agents are born at five year intervals. So, for example, the group of people between the ages of 75 and 79 in the year 2000 are represented in the model economy by the cohort of those born in 1943. Thus

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24 Morse and Gray (1980).
25 The source for Figure 9 is the Historical Statistics, Series R 13 to 16.
at any moment in time the population of people aged 50 to 80 is represented by six cohorts ages 52, 57, and so on up to 77.

There are 10 possible values for \( x \) and 10 for \( z \) defined by the sets \( \mathcal{X} = \{ x_i, i = 1 \ldots 10 \} \) and \( \mathcal{Z} = \{ z_i, i = 1 \ldots 10 \} \) where the logarithms of the \( x_i \)'s and \( z_i \)'s are evenly spaced. The distributions for \( x \) and \( z \), \( F(x) \) and \( G(z) \) are defined such that they approximate a truncated lognormal distribution. The truncation points are such that 0.05 percent of the area underlying the original distribution is removed from each side. Thus the sets of weights on the \( x_i \)'s and \( z_i \)'s are each characterized by two parameters: the mean, denoted by \( \mu_x \) for \( x \) and \( \mu_z \) for \( z \), and standard deviation, denoted \( \sigma_x \) and \( \sigma_z \), of the lognormal distribution they approximate.

Given an agent’s type \( s \) and the series for prices and wages the agent’s maximization problem is solved numerically by a combination of a grid search over the retirement date, \( A \), and a more efficient gradient-based root-finding algorithm. Care is taken to ensure that a potential solution is the global maximizer by checking the second-order condition and corners.

### 7.2 Calibration

Some of the parameters are set based on data or previous works. These parameters and their values are summarized in Table 7.2. The mean of the ability distribution, \( \mu_x \) and the wage in 1943 are normalized to one. The standard deviation of the ability distribution is constant over time and set such that the income distribution in the economy matches the U.S. income distribution in the year 1979.\(^{26}\) The rate of time preference, \( \theta \) is set to 0.03 which implies an annual discount factor of 0.97. The coefficient of risk aversion, \( \sigma \) is set to 1.3, implying an intertemporal elasticity of substitution (IES) for services of 0.77.

The annual growth rate of wages of 1.0 percent is for the period 1830 to 2000. It was determined using an extension of the real wage index of Williamson (1995).\(^{27}\) Similarly, the rate at which the price of the leisure good falls is estimated from the leisure price series used to compute the time cost of leisure goods presented in Figure 5. The interest rate was chosen to approximate the average return on one year Treasury bills over the time period 1962 to 2005. The fraction of time spent working is set to 46 percent. This is the average time spent working of males in the United States over the period 1830 to 2000.\(^{28}\)

Cohorts, entering the economy at age twenty, face a constant probability of dying, \( \eta \), at each moment of their adult life, except for the last moment, \( T \), in which they die with probably one. In order to account for increasing life expectancy, \( \eta \) and \( T \) are allowed to vary over time. Thus each cohort will face a different probability of dying and maximum length of life. The parameter values are determined as follows: If \( l \) is the random variable

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\(^{26}\) According to Gottschalk and Smeeding (1997), Table 3, the adjusted disposable personal income of a household at the 80th percentile is 2.7 times higher than one at the 20th percentile in 1979. This implies a standard deviation is 0.59.

\(^{27}\) The index was extended using wage data from the Bureau of Labor Statistics.

\(^{28}\) Data on weekly hours worked by U.S. males is from Whaples (1990) and the Statistical Abstracts of the United States.
Table 4: Parameter values used in baseline model that were set using data or other sources.

<table>
<thead>
<tr>
<th>Set</th>
<th>Std. dev. of market ability distribution</th>
<th>0.59</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta$</td>
<td>Rate of time preference</td>
<td>0.03</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Coefficient of risk aversion</td>
<td>1.3</td>
</tr>
<tr>
<td>$w_{1943}$</td>
<td>1943 wage</td>
<td>1</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>Growth rate of wages</td>
<td>0.01</td>
</tr>
<tr>
<td>$r$</td>
<td>Interest rate</td>
<td>0.063</td>
</tr>
<tr>
<td>$\bar{h}$</td>
<td>Fraction of time spent working</td>
<td>0.46</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Rate of price decline</td>
<td>0.0055</td>
</tr>
</tbody>
</table>

“length of adult life” than for an agent in the economy $l$ has the probability distribution function,

$$f(l) = \begin{cases} \eta e^{-\eta l}, & 0 < l < T, \\ e^{-\eta T}, & l = T, \end{cases}$$

and the expected length of adult life is

$$E(l) = \frac{1}{\eta}(1 - e^{-\eta T}). \tag{7.22}$$

For a cohort born in year $\tau$, $\eta$ and $T$ are chosen such that two conditions are satisfied. First, the cohorts expected length of adult life, computed using equation (7.22), must match the life expectancy of a twenty year-old U.S. male in year $\tau$ from data. Second, all cohorts must face the same probability of living to their maximum length of life, which is set by assuming that the youngest cohort in the economy, the cohort who is age twenty in 1968, has a maximum length of life of 150 years. The second condition ensures that the imposition of a maximum length of life has the same impact on the retirement decisions of all cohorts. The life expectancy of twenty-year-old males in the U.S. increased at a fairly constant rate from approximately 37 years in 1850 to approximately 53 years in 1970.\textsuperscript{29} Using life expectancies based on the linear trend, from the cohort that is twenty in 1793 to the cohort that is twenty in 1968, $\eta$ falls from 0.034 to 0.019.

The rest of the parameters are chosen such that the model matches the data along seven key moments. The empirical moments are computed using data from the 2000 Health and Retirement Study. The first six moments are the retirement rates of the six cohorts alive in the year 2000.\textsuperscript{30} The last moment is the size of the observed drop in market consumption. Empirical works, using a variety of different data sets consisting of both U.S. and British

\textsuperscript{29} Data on life expectancies is from Haines (1994) for the period 1850 to 1900 and from the Historical Statistics Series B 118 for the period 1900 to 1970.

\textsuperscript{30} The retirement rate is defined at the end of Section 6. The retirement rates for the six cohorts were computed using the formula in footnote 2.
households, have estimated the drop in market expenditure on non-durables to be anywhere in the range from 10 percent to 30 percent of before retirement expenditure.\textsuperscript{31} Using data from the 2000 HRS, Hurd and Rohwedder (2003) find that expenditure on market non-durables drops by 16.8 percent for singles and 11.6 percent for married couples at the moment of retirement of the household head, with an average drop in consumption of 13.6 percent. The average drop is used as the first moment that that model much match. The consumption drop statistics derived from the 2000 HRS are consistent with those found using other data sets. For example, Bernheim (2001) finds an average drop in expenditure on food of 14 percent using data from Panel Study of Income Dynamics and Aguiar (2004) finds an average drop in expenditures on food of 17 percent using data from the Continuing Survey of Food Intakes conducted by the U.S. Department of Agriculture.

Denote the six cohorts who are between the ages of 50 and 80 in the year 2000, from oldest to youngest, by $\tau_1$ through $\tau_6$. The minimization is done as follows. Define the following vector of unknown parameters:

$$\delta = (\alpha, \zeta, \chi, \mu_z, \sigma_z, p_{g, 1943}).$$

Given $\delta$, the model’s prediction for the labor-force participation rate of cohort $\tau_i$ is denoted by $L_i(\delta)$, and the model’s prediction for the average drop in market consumption is denoted by $D(\delta)$. The exercise, now, consists of two steps: First, $\delta$, is chosen to minimize the sum of the deviations between the model’s output and the empirical moments computed from the HRS. Formally:

$$\hat{\delta} = \arg \min_{\delta} \left\{ (d - D(\delta))^2 + \sum_{i=1}^{6} [l_i - L_i(\delta)]^2 \right\}.$$ 

Note that $\chi$ was restricted to be less than $1 - \sigma$ to ensure that leisure goods and leisure time are Edgeworth-Pareto complements. Second, the model’s predictions, $D(\hat{\delta})$ and $L_i(\hat{\delta})$, for $i = 1, \ldots, 6$, are computed using $\hat{\delta}$. The results of the minimization are shown in Table 5. Since there are seven moments and only six parameters, the model cannot match the moments perfectly but it does an excellent job. The model underestimates the consumption drop, predicting an average drop of 9.4 percent, but reproduces the distribution of retirement rates well. It slightly over estimates the retirement rates of the 55-59 aged cohort and the 60-64 aged cohort. This may be because the model does not account for the impact of pensions and Social Security on retirement. Most pensions as well as Social Security are only given to retirees of a certain age. Thus some people may delay retirement a few years until they reach the eligible age.\textsuperscript{32} The first age Social Security can be collected is age 62 with additional benefits for those who wait until age 65. If some HRS respondents delayed their retirement until age 62 or 65, then retirement rates from the model would overestimate those observed in the data for the 55 to 64 year-olds.

The values of the parameters that were chosen through the minimization procedure are given in Table 7.2. Note that the value for $\alpha$, implies an IES for consumption of 0.92. This

\textsuperscript{31} Often times expenditure on food is used as a proxy for non-durable expenditure.

\textsuperscript{32} This notion is developed in Gustman and Steinmeier (2004).
value is well within the range suggested in the literature.\textsuperscript{33} In addition, leisure goods share of total expenditure in the model is approximately 12 percent for the year 2000. This result is in line with the data on leisure shares presented in Figure 4.

Table 6: Parameter values used in baseline model that were chosen to match the model to moments based on data from the 2001 HRS.

<table>
<thead>
<tr>
<th>Calibrated in Minimization</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$ Market consumption’s share of total consumption</td>
<td>0.28</td>
</tr>
<tr>
<td>$\xi$ Weight on leisure goods in leisure production function</td>
<td>0.10</td>
</tr>
<tr>
<td>$\chi$ Controls elasticity of substitution between leisure goods and leisure time</td>
<td>–0.45</td>
</tr>
<tr>
<td>$\mu_z$ Mean of leisure productivity distribution</td>
<td>1.5</td>
</tr>
<tr>
<td>$\sigma_z$ Std. dev. of leisure productivity distribution</td>
<td>1.1</td>
</tr>
<tr>
<td>$p_{g,1943}$ 1943 price of leisure goods</td>
<td>6.5</td>
</tr>
</tbody>
</table>

7.3 Evolution of Retirement

The model’s prediction for the trend in retirement was obtained by running the calibrated model over the time period from 1830 to 2000. The results are presented in Figure 10. The model is able to match the trend in retirement observed in the data. Notice that

\textsuperscript{33} For example Attanasio and Weber (1993) find that the IES of consumption should be in the range from 0.3 to 0.8 based on micro data while values as high as 1 are common in real business cycle literature. See Guvenen (2005) for an interesting discussion.
the retirement rate predicted by the model for the eldest three cohorts underestimates the rate observed from the data in the later years. Starting around 1980 real wages fell and relative price of leisure goods leveled out. Consequently retirement rates slowed and, in the case of the eldest three cohorts, stopped rising. In the model, the real wage and relative price of leisure goods change at constant rates. Thus the model is only designed to capture the overall trend and cannot reproduce the leveling out of retirement rates occurring from 1980 to 2000. Since the model is calibrated to the year 2000, it is not surprising that it underestimates the retirement rates of the eldest cohorts in later years.

8 Extensions

Currently the model predicts a positive relationship between retirement and income which is not supported in the data. This relationship could easily be removed by allowing agents market productivity to decline over their life-time or after some age. Another possible extension would be to make the probability of dying a function of age. It would be interesting to see the impact on the results of having more realistic survival probabilities.

An interesting application of this work might be to extend the model, making it suitable to study the evolution of female labor force participation. A model of female labor force participation would have to include a third possible use of time – spending time on home production. A model which incorporates both a home production function where housework and home durables are Edgeworth-Pareto substitutes and leisure production could be used to reproduce the movement of females from housework to market work and the labor force participation rates of older women.

Another interesting extension would be to use the model to study the impact of Social Security and/or pensions on retirement and the wealth distribution of the elderly. While the incorporation of a Social Security system into the economy may be a non-trivial extension of the model, the resulting tool could be used to study a variety of questions relating to Social Security. With the incorporation of unanticipated increases in Social Security benefits, one could study the impact that Social Security has on the retirement decision and the drop in market consumption at the moment of retirement.

9 Conclusion

Labor force participation rates of elderly US males have been declining since the nineteenth century. This decline cannot be accounted for by increased life expectancies as participation rates have decreased for each age above 65 and, in later years, for men aged 55 to 64. The decrease has resulted in an increase in the fraction of one’s life a man spends retired. Retired men spend the majority of their time engaged in leisure activities. Thus the story of retirement is a story of leisure. Technological progress in the production of leisure goods has led to a decline in their relative price. In addition, the increased availability of new goods and a continual increase in quality has made leisure activities more attractive and enjoyable. Combined with the rise in real wages, the result has been the emergence of the concept of
a retirement lifestyle. Retired individuals spend their time and money on vacations, sports such as golf, gardening, reading, and watching television. They can enjoy their retirement in the privacy of their own homes, possibly located in a warmer part of the country such as Florida, or they may even spend summers north and winters south. Increased wealth, along with declining costs associated with travel and communication has increased retirees options.

In order to assess the ability of rising real wages and declining prices of leisure goods to drive a decrease in labor force participation rates of elderly US males a model economy in which agents choose the moment of their retirement is developed. Agents in the economy produce enhanced-leisure by combining leisure time with leisure goods. Leisure goods are defined as goods which are time-using in the sense that they are Edgeworth-Pareto complements in utility with leisure time. Under the baseline calibration, leisure time and leisure goods are, in addition to Edgeworth-Pareto complements, Hicksian complements and thus the falling relative price of leisure goods results in an increased demand for leisure time.

An important characteristic of the retirement period is that it corresponds with a drop in market consumption by US households. One interpretation of the drop in market consumption is that it reflects households, facing a zero-one work choice, attempt to smooth utility over their lifetimes. This interpretation is captured by the model by choice of a utility function which is intratemporally nonseparable in market consumption and leisure and a baseline calibration which implies that the intertemporal elasticity of substitution for consumption and leisure services is sufficiently low. The result is that agents discretely decrease consumption of market goods at retirement to offset the discrete gain in utility derived from the discrete jump in leisure time.

The model is calibrated using data from the Health and Retirement Study by minimizing the distance between the model and data along seven key moments: the labor-force participation rates of six age groups and the average drop in consumption. The calibrated model is then used to recreate the evolution of the labor-force participation rates of the six groups. The model is able to generate the trends of decreased labor force participation observed by the six age groups over the period 1850 to 2000.
10 Appendix

10.1 Standard Model

Proposition 1 If

\[ v(1) - v(1 - \bar{h}) > \left[ c_0(T)e^{\frac{-\rho T}{\kappa - r}} \right]^{-\rho} w(T)\bar{h}, \]

then there exists a unique \( A^* < T \) satisfying the first-order conditions of problem P1 and it is the global maximizer.

Proof. First note that setting \( A^* = T \) can not be a maximizer since, by equation (2.10), the first-order condition is not satisfied. To show that there exists an \( A^* < T \) such that equation (2.7) holds with equality at \( A^* \), first define the right-hand-side of (2.7) as

\[ MB(A) = c(A)^{-\rho} w(A)\bar{h}, \]

and the left-hand-side as

\[ MC = v(1) - v(1 - \bar{h}). \]

Equation (2.10) implies that \( MC > MB(T) \). In addition,

\[ \lim_{A \to 0} MB(A) = \infty > MC. \]

Thus since \( MB(A) \) is continuous in \( A \), by the Intermediate Value Theorem there exists at least one \( A^* \) such that \( MC = MB(A^*) \). To see that this \( A^* \) is a global maximizer of P1, first note that if there is only one \( A^* \) satisfying the first-order condition it must be a maximizer since \( MB(\cdot) \) decreases from above \( MC \) to \( MB(T) \). Second note that

\[ \frac{d^2MB}{dA^2} = MB(A) \left[ \frac{\rho(\kappa - r)^2e^{(\kappa-r)A}}{(1 - e^{(\kappa-r)A})^2} + \Omega(A)^2 \right] > 0, \quad \forall A, \]

where

\[ \Omega(A) = \frac{\kappa - r + \theta - [(1 - \rho)(\kappa - r) + \theta]e^{(\kappa-r)A}}{1 - e^{(\kappa-r)A}}. \]

Thus \( MB \) is strictly convex in \( A \) and can intersect \( MC \) at most two times. But since \( MB > MC \) for \( A \) small and \( MB(T) < MC \), strict convexity implies that \( MB(\cdot) \) crosses \( MC \) exactly once. □

Proposition 2 The optimal retirement date, \( A^* \) is:

(i.) increasing in the the initial wage rate, \( w(0) \), if \( \rho < 1 \),

(ii.) independent of the initial wage rate, \( w(0) \), if \( \rho = 1 \) (log utility),

(iii.) decreasing in the the initial wage rate, \( w(0) \), if \( \rho > 1 \).
Proof. Totally differentiating (2.7) with respect to \( A \) and \( w(0) \) gives
\[
\frac{dA}{dw(0)} = \frac{(\rho - 1)(1 - e^{(\kappa - r)A})}{(\kappa - r + \theta - [(1 - \rho)(\kappa - r) + \theta]e^{(\kappa - r)A})w(0)},
\]
where \( \Omega(A) \) is as defined in the proof of Proposition 1. Note that by the second-order condition for optimality \( \Omega(A^*) < 0 \).

Proposition 3  The optimal retirement date, \( A^* \), is increasing in the length of life, \( T \).

Proof. Totally differentiating (2.7) with respect to \( A \) and \( T \) gives
\[
\frac{dA}{dT} = \frac{[(1 - \rho)r - \theta]e^{(1 - \rho)(r - \theta)T}w(A)}{(1 - e^{(1 - \rho)(r - \theta)T})(\kappa - r + \theta - [(1 - \rho)(\kappa - r) + \theta]e^{(\kappa - r)A})}
\]
where \( \Omega(A) \) is as defined in the proof of Proposition 1. Note that by the second-order condition for optimality \( \Omega(A^*) < 0 \).

### 10.2 Period Budget Constraint

The agent’s period budget constraint is
\[
c(a) + p_g(a)g(a) + \frac{db(a)}{da} = xhw(a + \tau) + (r + \eta)b(a), \quad a \leq A,
\]
and
\[
c(a) + p_g(a + \tau)g(a) + \frac{db(a)}{dt} = (r + \eta)b(a), \quad a > A,
\]
where \( b(a) \) is the agent’s annuity holdings at age \( a \). If (10.24) and (10.25) hold for all \( a \in [0, T] \), agents have no initial asset holdings, and
\[
e^{-(r + \eta)T}b(T) \geq 0,
\]
holds, which rules out Ponzi schemes, then the lifetime budget constraint, (3.14), holds.

To see this, first, for simplification, assume \( \tau = 0 \). Now rearrange (10.24), multiply through by \( e^{-(r + \eta)t} \), integrate from \( t = 0 \) to \( t = A \) and then from \( t = A \) to \( t = T \), and combine the results to obtain
\[
\int_0^A e^{-(r + \eta)t}c(t)dt + \int_A^T e^{-(r + \eta)t}c(t)dt + e^{-(r + \eta)t}p_g(t)g(t)dt + \int_A^T e^{-(r + \eta)t}p_g(t)g(t)dt = \int_0^T e^{-(r + \eta)t}w(t)dt - \int_0^A e^{-(r + \eta)t}p_g(t)g(t)dt + \int_0^T e^{-(r + \eta)t}p_g(t)g(t)dt = \int_0^T e^{-(r + \eta)t}w(t)dt - \int_0^A e^{-(r + \eta)t}p_g(t)g(t)dt + \int_0^T e^{-(r + \eta)t}(\frac{db}{dt} - rb(t))dt - \int_0^A e^{-(r + \eta)t}(\frac{db}{dt} - (r + \eta)b(t))dt.
\]
Notice that if
\[
f(t) = e^{-(r + \eta)t}b(t),
\]
26
then
\[ \frac{df(t)}{dt} = e^{-(r+\eta)t} \left( \frac{db}{dt} - (r + \eta)b(t) \right). \]
Hence (10.24) reduces to
\[ \int_0^A e^{-(r+\eta)t}c(t)dt + \int_A^T e^{-(r+\eta)t}c(t)dt + \int_0^A e^{-(r+\eta)t}p_g(t)g(t)dt + \int_A^T e^{-(r+\eta)t}p_g(t)g(t)dt = \int_0^T e^{-(r+\eta)t}w(t)dt - e^{-(r+\eta)T}b(T) + b(0), \]
which is analogous to (3.14) under the restrictions defined above.

### 10.3 Cobb-Douglas Leisure Production

If the leisure production function is Cobb-Douglas \((\chi = 0)\) then (3.21) and (10.23) become
\[
\dot{g}(a) = \frac{\theta - r + \gamma \Phi}{\Phi + \zeta \Psi} = \Theta, \quad (10.27)
\]
and
\[
\dot{c}(a) = \frac{1}{\Phi} \left[ \theta - r - \zeta \Psi \frac{\dot{g}(a)}{g(a)} \right] = \Xi. \quad (10.28)
\]
Thus \(g(a)\) and \(c(a)\) can be defined explicitly as
\[
g(a) = \begin{cases} 
g(0)e^{\Theta a}, & 0 < a \leq A, 
g_A e^{\Theta(a-A)}, & A < a \leq T, \end{cases} \quad (10.29)
\]
and
\[
c(a) = \begin{cases} 
c(0)e^{\Xi a}, & 0 < a \leq A, 
c_A e^{\Xi(a-A)}, & A < a \leq T. \end{cases} \quad (10.30)
\]
Evaluating equation (3.15) at \(a = 0\) yields
\[
\lambda = \alpha c(0)^\Phi \left( zg(0)\zeta (1 - \tilde{h})^{1-\zeta} \right)^\Psi. \quad (10.31)
\]
Plugging (10.31) back into (3.15) and evaluating at \(a = A\) with \(l(a) = 1\) determines
\[
c_A = (1 - \tilde{h})^{(1-\zeta)\Psi} e^{\Xi A} c(0). \quad (10.32)
\]
Plugging (10.31) into (3.16) and evaluating at \(a = 0\) gives
\[
g(0) = \frac{(1 - \alpha)\zeta}{\alpha p_g(\tau)} c(0), \quad (10.33)
\]
while evaluating at \(a = A\) for \(l(a) = 1 - \tilde{h}\) and \(c_S\) given by (10.32) gives
\[
g_A = \frac{(1 - \alpha)\zeta(1 - \tilde{h})^{(1-\zeta)\Psi} e^{\Xi A}}{\alpha p_g(\tau + A)} c(0). \quad (10.34)
\]
Proposition 4 The agents optimal retirement age, $A$, is independent of his market ability, $x$, when the leisure production function is Cobb-Douglas ($\chi = 0$).

Proof. Since the proposition obviously holds for $A = T$, consider the case of an interior solution for $A$. Using the equations derived above, notice that, for a given $A$, $c(a)$ and $g(a)$ are linear in $c(0)$ for all $a \in [0, T]$. Also, notice that the limits $c_A$ and $g_A$, are linear in $c(0)$. Thus, one can see from (3.14) that, given $A$, $c(0)$ is linear in $x$. Hence, $c_A$, $\bar{c}_A$, $g_A$ and $\bar{g}_A$ are linear in $x$ and, from (10.31), $\lambda$ is linear in $x^{\psi \zeta + \Phi}$. Based on these relationships, it is clear that equation (3.17), which implicitly determines $A$, is independent of $x$. \[\blacksquare\]

Proposition 5 The agents optimal retirement age, $A$, is independent of his productivity in leisure production, $z$, when the leisure production function is Cobb-Douglas ($\chi = 0$).

Proof. The proposition holds for $A = T$. Consider the case of an interior solution for $A$. From the equations above, it is clear that for a given $A$, $c(a)$ and $g(a)$ are independent of $z$. As are the limits $c_A$ and $g_A$. From (10.31), one can see that $\lambda$ is linear in $z^{\Psi(1-\zeta)}$. The variable, \[n_A(a) = g_A(a)^{\zeta} z^{(1-\zeta)},\] is linear in $z^{1-\zeta}$, as is $n_A(a)$. Hence (3.17), which implicitly determines $A$ is independent of $z$. \[\blacksquare\]

10.3.1 Cobb-Douglas Leisure Production and Log Utility

In the case of log utility, or $\sigma = 1$, equation (10.27) becomes
\[
\dot{g}(a) = r - \theta + \gamma,
\]
and equation (10.28) becomes
\[
\dot{c}(a) = (r - \theta).
\]
Solving for $g(a)$ and $c(a)$ yields
\[
g(a) = g(0)e^{(r-\theta+\gamma)a}, \quad 0 \leq a \leq T;
\]
and
\[
c(a) = c(0)e^{(r-\theta)a}, \quad 0 \leq a \leq T.
\]
Notice that the paths for consumption and the leisure good are continuous even at the moment of retirement. Plugging $g(a)$ and $c(a)$ into the budget constraint and applying equation (10.31), it can be shown that $c(0)$ is defined by
\[
c(0) = \frac{\alpha w(\tau) \int_0^T e^{(\kappa-r)\alpha x h} da}{(\alpha + (1-\alpha)\zeta) \int_0^T e^{-\theta a} da}.
\]
The first-order condition for retirement simplifies to
\[
(1-\alpha)(1-\zeta) \ln(1-h) = -\lambda e^{(\theta-r)A} x \bar{h}w(A + \tau),
\]
where

\[ \lambda = \frac{\alpha}{c(0)}. \]

**Proposition 6** The agents optimal retirement age, \( A \), is increasing in his length of life, \( T \).

**Proof.** Clearly at a corner solution for the retirement date the proposition holds, so consider the case of an interior solution for \( A \). Denote by \( f(A, T) \) equation (10.40) where \( \lambda \) and \( c(0) \) are given by (10.3.1) and (10.40) so that

\[ f(A, T) = (1 - \alpha)(1 - \zeta) \ln(1 - h) + \frac{(\alpha + (1 - \alpha)\zeta)e^{(\theta-r)A}x\hat{h}w(A + \tau)e^{-\theta A} \int_0^T e^{-\theta a} da}{w(\tau) \int_0^A e^{(\kappa-r)a} x\hat{h} da}. \]

(10.41)

Since \( A \) must be a maximizer it is enough to show that \( df(A, T)/dT > 0 \). Differentiating (10.41) with respect to \( T \) yields

\[ \frac{df(A, T)}{dT} = \frac{(\alpha + (1 - \alpha)\zeta)e^{(\theta-r)A}x\hat{h}w(A + \tau)e^{-\theta T}}{w(\tau) \int_0^A e^{(\kappa-r)a} x\hat{h} da}. \]

(10.42)

Clearly \( df(A, T)/dT > 0 \). ■

**References**


Figure 2: Labor-Force Participation Rates of Men Aged 65 and over for the Period 1850 to 1990 in the United States, France, Great Britain, and Germany and Men Aged 55 to 64 in the United States.

Figure 3: Retirement Rates for Men Aged 50 and over by Age Group and the Expected Percentage of Life Spent in Retirement at the Age of 20 for the Period 1850 to 2000 in the United States.
Figure 4: The Share of Total Expenditure which is allocated to Recreation Goods including Transportation for Recreational and Social Activities for the Period 1900 to 2001.

Figure 5: Time Cost Index of Leisure Goods in the United States from 1901 to 2001.
Figure 6: Number of Radio and Television Stations in the United States from 1921 to 1998.

Figure 7: Percentage of Non-institutionalized Men 65 and Older Who Were Heads of Households.
Figure 8: Percentage of United States’ and Florida’s Total Population which is Over the Age of 64 from 1870 to 2000.

Figure 9: Telephone Toll Rates Between New York City and Selected Cities in the United States from 1902 to 1970. (Rate for a 3-minute daytime call.)
Figure 10: Models Prediction for Trend in Retirement versus the Trend in the Data.