SIAG/OPT Views-and-News

A Forum for the SIAM Activity Group on Optimization

Volume 16 Numbers 1-2 October 2005

Contents

Tributes to George Dantzig and Leonid Khachiyan

George Dantzig: A Personal Perspective

Walter Murray
Department of Management Science and Engineering, Terman Center, Stanford University, Stanford, CA 94305, USA (`walter@stanford.edu`).

My earliest memory of George is from 1969. He was being helped by Dick Cottle (a role Dick played the whole time I knew George) into a boat going to the Island of Bender, which is just off Bandol in the south of France. Even then he seemed a frail old man. It was only later I came to realize that George was a lot tougher both physically and mentally than his outward appearance would suggest.

It is not my intent here to catalog George's mathematical contribution. Anyone interested in that could not do better than to start with Dick's recent book "The Basic George B. Dantzig". I shall attempt to shine some light on one facet, the view from where I stood. I would not be writing this if George was simply a great mathematician, to me he was much more and it is this extra dimension that set him apart. Nonetheless, I shall make one mathematical comment. To me George had impeccable mathematical taste and instinct. This is not entirely divorced from his character. He was patient and always took the long view. Perhaps he knew he would live to be ninety. This contrasts sharply with the almost frenetic rush to publish quickly that now permeates much of science. George was driven by curiosity and had much in common with his namesake “Curious George”.

Even when George was alive the celebrations of his many birthday milestones gave occasion to reflect on George’s life. He has been repeatedly referred to as the “Father of Linear Programming”. For his 90th birthday, I commented that perhaps more correctly George should be referred to as the “Father of Linear Programmers”. In all the years I knew George I never heard him make one derogatory remark about anyone in the field (and there were many occasions when he would have had just cause). It was as if we were part of his family and George never spoke ill of his family. George was an intensely loyal man. It is not that George was incapable of being derogatory; indeed his invective on some topics such as lawyers or the higher administration of the University was, in typical George fashion, at an extreme point. Of course it was always delivered in a calm quiet voice. George never needed to raise his voice to get attention. He was in particularly good form when Condoleezza Rice was Provost and she forced the OR Department to merge with the EES Department. George never used his status to get his own way and was always happy to rely only on the force of his argument.

I now think that “Father” is not quite the correct term. Fathers are often strict and need to keep their children on the path they have determined. In my own life George reminds me much more of my Grandmother. She had lived though two world wars and outlived two husbands (my two Grandfathers). Nothing I ever did fazed her in the least. Whenever I was in trouble, which was quite frequent, I always went to her first. She would always be on my side no matter how wrong I had been. She had her idiosyncrasies, just like George, but that just made her all the more endearing.

George was fun. He always had a half smile on his face as if he was mentally recalling some amusing story. You could always tell he was about to say something outrageous when he pursed his lips to try and suppress the smile. Faculty meetings with George present were never boring. George liked to take a rise out of certain people. He only chose those he knew would take it in the spirit it was intended and could handle the public embarrassment it sometimes entailed. His favorite targets were Pete Veinott and Curtis Eaves. Both laughed louder than the rest of us when the comments were made. Of course neither could respond in kind. That would have been like punching a teddy bear.

George’s concern for people in the field seemed to have no limits. It was not just lowly people, but would be extended to everyone, including senior faculty. If he thought somebody’s work was suffering or they were not getting sufficient grant money, then George would find some way to help, often without them knowing. It was not just his willingness to help but the fact he must have been continually checking on everybody to see they were all right. When George asked how you were doing he really wanted to know. Had George not been Jewish he would have got my vote for Pope.

George wrote wonderful letters for people, even for people he did not know. If I ever wanted support for some cause, such as a student applying for a fellowship, I could always rely on George to put his weight behind it. Given that he was very busy he sometimes would ask me to write the letter and would sign anything I wrote without question. It was always easier and more fun to write a letter in George’s name than my own. I just had to lay it on with a trowel. Maybe George knew Disraeli.

His attitude to the efforts of people in the field was like that of a doting parent, whatever the quality of the work he always saw the positive side of it. Michael Saunders and I once pointed out to George that the work of a student was really very similar to some work that had been published. George’s response was to praise the student for rediscovering not any old rubbish but really good stuff. It may be that the discovery of the Simplex method was due to George’s positive outlook. George told me that Von Neumann had commented that had he discovered the Simplex algorithm he would have dismissed it as impractical.

It was not just students who George encouraged. Over the years George must have received a lot of mail and met many people from many walks of life who suggested improvements in the Simplex algorithm and other things. He listened to them patiently much as you would with a child. As recently as 1999 I got email from a professor at a foreign university who claimed to have discovered how to modify the Simplex algorithm to avoid artificial variables and that the new algorithm did not appear to need more iterations than the number of rows or variables. The evidence supporting this claim came from tests on problems with no more than ten rows or variables. The message concluded by mentioning he had spoken recently with Prof. Dantzig about the method and George had found it interesting and suggested he contact me. This was typical of George being a little mischievous.

George slowed physically towards the end but his mind and wit remained as sharp as ever. He also refused to let his physical infirmity hinder him. The extensive celebrations for 90th birthday that he attended were a testimony to that. A few months earlier he had also attended the 50th birthday party of Mukund Thapa. Mukund's parties always have a loud rock band, which George hated, and Indian food, which was also something not to George's taste. Nonetheless George showed up. Each time he attended Mukund's parties he would pointedly tear off the corners of a paper serviette, form them into plugs and then place them in his ears. Remarkably he could still carry on a conversation.

In the last year or so George had trouble walking without assistance. When he needed help, someone would put their arm around him and he would put his arm on their shoulder. Then while hugging him you would shuffle along. Even if the distance was short the journey could take some time. He did not seem to resent the need to be helped and it was an opportunity to demonstrate affection for George. The last time I helped George was when we were at Dick's house to celebrate George's 90th birthday. We were in the garden and George needed to move from there through the kitchen, the hall and into the dining room. George would use these opportunities to chat. It always seemed very personal since being physically very close we talked in a whisper. Whatever the conversation, it always ended with the same phase, "Whatever you do Walter, do not grow old". I am not sure what alternatives George thought I had, but I did not like to disagree with him. If he had seen me skiing or driving, he would have seen I was doing my best to comply. I did wonder if he thought it was just me unsuitable for old age or that he thought it good advice for everyone. Maybe he was a fan of the film "Logan's Run" or subscribed to the idea it would be better if we were all born old and got progressively younger

The last time I saw George was when he was in the hospital just prior to his death. I was accompanied by Peter Glynn and Gerd Infanger. George was attached to a number of tubes and was obviously heavily medicated. He was drifting in and out of consciousness and was struggling to breath. His frail body barely made a ripple in the blanket that covered him. It was a hard sight to observe. After some time the Nurse said "Prof. Dantzig, Peter, Gerd and Walter are here to see you.". There was a brief pause and then in a clear voice George said, "I am overwhelmed.". Even in his distressed state George was trying to make us feel better. A few days later George died at home. His last words were to ask his caregiver if it was all right to leave now. Having been told that it was, he then added, "Will you miss me?". Courage was described by Hemingway as grace under pressure. There are few who have the grace of George.

I never heard George complain (and you do not get to ninety without having a lot to moan about), raise his voice, be in a bad temper, or not be pleased to see me. He was a fabulous human being and if I had to choose between inheriting his mathematical talent or his human qualities I would not hesitate in choosing the latter.

Leonid Khachiyan, 1952–2005: An Appreciation

Michael Todd

School of Operations Research, Cornell University, Ithaca, NY 14953, USA (miketodd@cs.cornell.edu).

Leonid Khachiyan died of a heart attack a few days before his 53rd birthday in South Brunswick, NJ. He is survived by his wife of 20 years, Olga Pischikova Reynberg, and teenage daughters Anna and Nina, student and student-to-be at Rutgers University, where Khachiyan had taught since 1990. Previously he was a researcher at the Computing Center of the USSR Academy of Sciences, an adjunct professor at the Moscow Institute of Physics and Technology, and a visiting scientist at Cornell University.

This article is a tribute to Leo Khachiyan as a friend and an optimizer. I'll also give references for some of his key papers.

Leo was famous in the optimization community for his use of the ellipsoid algorithm to demonstrate that linear programming, in the Turing machine model, had a polynomial-time algorithm; for this work, he received the Fulkerson Prize of the American Mathematical Society and the Mathematical Programming Society. This was an astonishing result, not only in settling a long-open problem in complexity, but also in introducing radically new viewpoints and techniques to linear programming. While the ellipsoid method had been developed by David Yudin and Arkadi Nemirovski and, independently, by Naum Shor in 1976–77, for convex optimization, Khachiyan used it in a tour-de-force to crack the complexity problem for linear programming. Since the algorithm was designed for the real-number model, and required an estimate of the distance to an optimal solution, Khachiyan had to establish a number of bounds on sizes of solutions, volumes of polyhedra, and the precision required to carry out the computations, to achieve his goal. The result was first published in a 4-page note without proofs in Soviet Mathematics Doklady in February 1979 [3]. It was brought to the attention of Western researchers in a presentation at the Montreal Mathematical Programming Symposium in August 1979 and in a later publication by Peter Gács and Laci Lovász. Their presentation was far clearer than the original to those not used to thinking in the varying coordinate systems viewpoint of the Soviet researchers. Khachiyan's later 1980 paper [4] in the journal USSR Computational Mathematics and Mathematical Physics provided the proofs for the results in his earlier work.

After its development in 1947 by George Dantzig, the simplex method had sloughed off the challenge of a number of alternative algorithms, notably iterative methods based on fictitious play in 2-person games, in the '50s, and had found itself successfully applied to a wider range of vastly larger-scale problems through the '50s and '60s. Then, in the '70s, it ran into a theoretical no-man's-land with the newfound notion of polynomial-time algorithms and Victor Klee and George Minty's discovery of a class of problems for which Dantzig's pivot rule for the simplex method led to an exponential number of pivots. While more recent versions, such as the dual steepest-edge variant that appears to be the best at present, remain highly competitive with the more recent interior-point methods and an indispensable part of the arsenal of any optimizer, they still exhibit exponential behavior on some examples. (To some extent, their good behavior in practice has been explained via analyses of the expected behavior of the simplex method by a number of authors, and by the more recent smoothed analysis of Daniel Spielman and Shang-Hua Teng.)

Leo's result was a bombshell in this environment. The use of the ellipsoid method, with its approximation of the polyhedral feasible region by ellipsoids, seemed counter to all we held dear: vertices, edges, phase 1 – phase 2, and even finite convergence to an exact solution in exact arithmetic. Instead we had to start with gigantic spheres, and then generate a sequence of shrinking ellipsoids until one was found sufficiently small that its center could be rounded to give an exact solution — assuming that all the data was rational. This was a pretty wild way to approach a problem that we knew had a finite solution via pivoting, and in fact bore some resemblance to the iterative methods tried in the '50s, but with a twist: the changing shapes of the ellipsoids gave a sort of variable-metric slant to the earlier relaxation methods.

It was natural that such a result would get a huge amount of press. Linear programming was big business, and leading papers around the world tried to educate their readers to the significance of the result, with very spotty results. The ensuing brouhaha has been well documented in Gene Lawler's article [7]. The effect on the optimization community was more rational. Many people tried, and failed, to turn Leo's result into a practical method for the solution of large-scale linear programming problems. (Part of the problem lies in the fact that the algorithm seems to require in practice a number of iterations close to its worst case bound: it also leads to very ill-conditioned linear systems.) A lot of attention was turned to the amazing theory of Yudin and Nemirovski on the informational complexity of nonlinear programming. And a few people, notably Martin Grötschel, Laci Lovász, and Lex Schrijver, realized that the ellipsoid method could be used as a powerful tool in combinatorial optimization, thereby lending a (very) little credence to some of the outrageous claims that had been made in the popular press. (Just one example: the Guardian headlined its story: "Soviet Answer to the 'Traveling Salesmen.' " Of course, the ellipsoid method has not shed any light on the complexity of the Traveling Salesman Problem.) And the ellipsoid method was the first theoretically good algorithm for the burgeoning field of semidefinite programming.

So Khachiyan became famous: but what of his other research and its significance? Interestingly, his first work was concerned with the convergence rate of iterative processes for solving matrix games, and he obtained some negative results: the error decreased at best inversely with the iteration count. His fourth paper, at the age of 26, was the Doklady announcement that LP was in P. The ideas in that work, estimating the sizes of solutions, looking at rational or integer solutions, and using geometric ideas in combinatorics and optimization, appear in much of Leo's subsequent research. He extended the polynomial algorithm to convex quadratic programming with M. K. Kozlov and S. P. Tarasov, and then considered the size of solutions and the complexity of solving convex polynomial programming problems, in either continuous or integer variables. He wrote a lovely survey of results in this area for the Proceedings of the 1983 International Congress of Mathematicians [5]. Let me mention a couple of results from that work. He bounded the size of a solution to a system of convex polynomial inequalities by a 2-stage exponential function, and showed by a simple example ($x_1 \geq h$, $x_2 \geq x_1^2$, ..., $x_n \geq x_{n-1}^2$) that this was the best possible. Yet he showed that such a solution when the degree was fixed could be "compactly represented" in polynomial space using a solution of just polynomial size to a reduced subsystem, consistent if and only if the original system was; moreover, this characterization could be found by a polynomial algorithm. Finally, he extended Lenstra's well-known result in integer programming by showing that there was a polynomial algorithm for finding an exact solution to a convex polynomial programming problem in real and/or integer variables, if the degree and the number of variables was fixed.

Another beautiful result [9], with Tarasov and I. I. Erlikh, replaced the sequence of circumscribing ellipsoids in the usual ellipsoid method with a sequence of inscribed ellipsoids (each inscribed in the current localizing set). This method allowed a decrease in the complexity of approximately solving a convex minimization problem by the factor n, the dimension of the problem, and thereby obtained the optimal (worst-case) complexity. The cost was that each iteration required the finding of an (approximately) largest volume inscribed ellipsoid; the authors suggested doing this via the original ellipsoid method (but without further function oracle calls)! This paper also had a surprising geometric theorem concerning volumes of inscribed ellipsoids, which Leo later improved. This concern with volumes led to later work on the complexity of polytope volume computation and on the conductance of Markov chains (involving another neat geometric inequality) to bound the mixing time for randomized methods.

After coming to the States, Leo's work continued some of its old themes, like his work on the complexity of maximal volume ellipsoids inscribed in a polytope and his fascinating paper on rounding polytopes [6], and added some new ones. He wrote a series of papers with Bahman Kalantari on various matrix scaling and balancing problems, and a series of papers with Mike Grigoriadis on fast approximations of multicommodity flows, of matrix games in sublinear time, and of block-angular convex programming problems, establishing a link to the work of Dantzig and Philip Wolfe on the decomposition principle. Indeed, one of their papers is entitled "Coordination

Complexity of Parallel Price-Directive Decomposition" [2].

In [1], Michael Fredman and Khachiyan established the surprising result that there is a quasi-polynomial-time algorithm for testing the duality of monotone disjunctive normal forms. This work had many applications, and led to a number of papers with Endre Boros, Vladimir Gurvich, his student Khaled Elbassioni, and others on various topics in combinatorics: hypergraphs, polymatroids, matroids, and enumerating all minimal solutions of implicitly stated monotone systems, with applications in minimal hypergraph traversals, data mining, machine learning, reliability theory, and in integer and stochastic programming. Finally, I want to mention his work with his student Lorant Porkolab. Porkolab and Khachiyan extended Leo's earlier work on convex polynomial programming to consider much more general formulae in the first-order theory of the reals and obtain related results. One consequence of their work [8] is that testing the feasibility of an inequality-constrained semidefinite programming problem in real or integer matrices of fixed dimension can be performed in polynomial time.

Let me conclude by telling a couple of stories that illustrate Leo's humor and sharp wit. Leo was very modest and kind to his friends, but he was also extremely cynical about politics and intolerant of condescension and pomposity. Since he had received the Young Investigators Award in Science and Technology, the Party Secretary at the Computing Center at the USSR Academy of Sciences indicated that it might be good for him to join the Party. Leo explained that he replied, with all innocence, "What party?" and added that he thought that was the right response. Later, he was looking at houses to buy near Rutgers, and was being shown around by a real estate agent, who was obviously trying to empathize as much as possible. She indicated that one house she showed him was close to the local synagogue, since she knew many Russian immigrants were Jewish, but Leo said he wasn't Jewish. Somewhat flustered, she said there were many churches close by. Leo saw his opening, and replied, "Actually, all I really believe in is the Communist Party." This caused some consternation, until finally the realtor saw a way to form a bond: "Well, they had some good ideas at the beginning."

Leonid Khachiyan was a great scholar and a much-loved father, husband, and friend. He will be sorely missed.

REFERENCES

[1] M. L. Fredman and L. G. Khachiyan, *On the complexity of dualization of monotone disjunctive normal forms*, J. Algorithms, 21 (1996), pp. 618–628.

[2] M. D. Grigoriadis and L. G. Khachiyan, *Coordination complexity of parallel price-directive decomposition*, Math. Oper. Res., 21 (1996), pp. 321–340.

[3] L. G. Khachiyan, *A polynomial algorithm in linear programming*, Doklady Akademiia Nauk SSSR, 224 (1979), pp. 1093–1096. (English Translation: Soviet Mathematics Doklady, 20 (1979), pp. 191–194.)

[4] L. G. Khachiyan, *Polynomial algorithms in linear programming*, Zhurnal Vychislitel'noi Matematiki i Matematicheskoi Fiziki, 20 (1980), pp. 51–68. (English Translation: USSR Computational Mathematics and Mathematical Physics, 20 (1980), pp. 53–72.)

[5] L. G. Khachiyan, *Convexity and complexity in polynomial programming*, in *Proceedings of the International Congress of Mathematicians, Warsaw*, PWN, Warsaw, (1984), pp. 1569–1577.

[6] L. G. Khachiyan, *Rounding of polytopes in the real number model of computation*, Math. Oper. Res., 21 (1996), pp. 307–320.

[7] E. L. Lawler, *The great mathematical Sputnik of 1979*, The Sciences, 1980, pp. 12–15.

[8] L. Porkolab and L. G. Khachiyan, *On the complexity of semidefinite programs*, J. Global Optim., 10 (1997), pp. 351–365.

[9] S. P. Tarasov, L. G. Khachiyan, and I. I. Erlikh, *The method of inscribed ellipsoids*, Soviet Mathematics Doklady, 37 (1988), pp. 226–230.