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# Differential Carrier Phase GPS Techniques for Space Vehicle Rendezvous

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## Biography

Kurt Zimmerman is a Ph.D. candidate in the Electrical Engineering Department at Stanford University. He received his M.S.E.E. from Stanford in 1990, and his B.S.E.E. from Carnegie Mellon University in 1989. He has been affiliated with the Stanford Aerospace Robotics Laboratory since 1990, investigating control of autonomous robot systems. Mr. Zimmerman also worked on planetary robot vehicles while at Carnegie Mellon. He is a member if Eta Kappa Nu, Tau Beta Pi, and IEEE.

Dr. Robert H. Cannon is the Charles Lee Powell Professor of Aeronautics and Astronautics at Stanford University. He received his Sc.D. degree in Mechanical Engineering from MIT, and his B.S.M.E degree from the University of Rochester. Dr. Cannon is currently the Director of the Aerospace Robotics Laboratory at Stanford, researching aerospace, underwater, and flexible-manipulator robot systems.

## Abstract

The objective of this research effort is to demonstrate that Differential Carrier Phase GPS techniques can be employed as the primary means of sensing both the relative position and the relative attitude of two space vehicles for precise, autonomous rendezvous maneuvers in Low Earth Orbit. In pursuit of this goal, an experimental system that can be tested in a well-controlled indoor laboratory environment has been built. Ideally, this system will be transferable to a real space system with little or no modification. Since the experiments take place indoors where GPS satellite signals cannot be received, several GPS pseudolite transmitters have been built and installed around the laboratory to provide the necessary GPS signals. The indoor GPS environment created by the close-range pseudolite transmitters poses some additional constraints on the algorithms used to extract relative position and relative attitude from the carrier phase measurements. Therefore, a secondary objective of this research is to clarify the differences between an indoor GPS system and the orbiting GPS satellite constellation, and to extend Differential Carrier Phase techniques such that they can be applied to near-field (indoor) systems as well as far-field (outdoor and space) systems. This paper presents the theoretical formulation and results of a two-dimensional position control experiment, an intermediate step toward the full rendezvous experiment.

## 1 Introduction

This work is motivated by the need to increase the efficiency and safety of assembly, maintenance, inspection, and repair tasks in the high-risk environment of Low Earth Orbit. Examples of such tasks include satellite retrieval, Orbital Replacement Unit (ORU) change-outs on satellites and the proposed space station, and assembly of modules and truss structures for advanced space missions. Currently these tasks are performed by astronauts through hundreds of hours of Extra-Vehicular Activity (EVA). Highly autonomous robot systems can make these tasks more routine and lower risk.

Both communication bandwidth limitations and data delay between the robot and the human supervisor lead to the need for a highly autonomous robot system that can react to unpredictable situations. The degree of autonomy required for such a robotic system can only be achieved through reliable, high-bandwidth sensors on-board the robot that enable locally closed dynamic control loops. Specifically, in order to perform a rendezvous or intercept task, it is necessary to sense the relative position and orientation between the robot and the target. This research project will take advantage of Differential Carrier Phase GPS technology to perform a precise intercept and capture of a free-floating target by an autonomous free-flying space robot.

The prospect of equipping satellites and other space systems with a GPS receiver and a means of broadcasting their GPS measurements so that they may be serviced by a robotic space vehicle is not an overbearing constraint, since 1) these satellites will probably use the GPS receiver as a navigational instrument in itself and 2) most space systems are equipped with communication systems that would be capable of broadcasting GPS measurement information to the service vehicle.

This paper presents:

- The fundamental research issues involved in developing an indoor testbed for GPS-based rendezvous.
- A description of the hardware system that has been developed.
- The theoretical analysis for a two-dimensional GPS-based position control experiment.
- Results and conlcusions from this experiment.

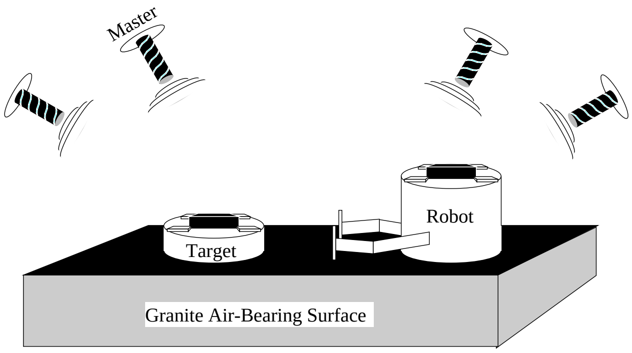

Figure 1: **Rendezvous Experiment Configuration**

## 2 Research Issues

There are several fundamental research issues that must be resolved prior to the successful completion of this project and prior to the use of GPS technology in a true space rendezvous mission. Some of these issues have already been resolved, as will be discussed, while others are still under investigation.

*The use of GPS as a sensor in real-time feedback control* – GPS sensing is susceptible to both predictable and unpredictable loss of sensor data due to occlusion, vehicle configuration, and multipath. Partial loss of signal information can be compensated for with measurements from several satellites in addition to incorporation of vehicle dynamics.

*The use of GPS as an indoor sensing technique* – The prospect of using GPS indoors presents new fundamental problems, such as described in [1]:

- *Handling non-planar wavefronts* – Since the transmitters are very close to the workspace, the wavefront will be spherical rather than planar. This leads to nonlinear phase measurement equations from which the position and attitudes must be derived.
- *Lack of pseudorange* – Pseudorange is essentially useless for indoor applications due to the relatively low accuracies available. The pseudolites are therefore not even designed to broadcast the data needed to perform pseudorange. This results in the need to reformulate already-existing attitude and integer-resolution algorithms to meet this constraint [2].
- *Higher risk of multipath* – Signal reflections off of walls can result in a much greater occurrence of multipath indoors than outdoors. This problem is being alleviated through the use of custom-designed helical antennas with conical-shaped beam patterns.
- *Near-far problem* – The close proximity of the transmitting sources to the receivers leads to very large variations in the power of the signal received as the vehicle traverses the workspace. The signal power of each of the pseudolites must be carefully adjusted so that the receiver does not cross-correlate signals (jam) at one extreme of the workspace and yet still be able to receive the signal at the other extreme of the workspace.
- *Calibration of indoor pseudolite constellation* – A method needs to be devised to accurately identify the location of the phase center of the pseudolite transmitters.

*Coordinated control of a multi-arm space robot* – The Stanford Aerospace Robotics Lab (ARL) has already demonstrated the coordinated control of a multi-arm free-flying space robot for rendezvous and capture [3], [4], [5]. These earlier experiments were successfully demonstrated through the use of an overhead vision sensing system which was employed to provide the relative vehicle/satellite position and orientation. A constraint of this vision system is that it operates in only two dimensions and requires an overhead, perpendicular view of illuminated target points on each object in the workspace. The GPS system will replace the overhead vision system as the source of relative position and attitude information.

There are other significant issues beyond the scope of this project that need to be resolved before GPS-based rendezvous can be attempted in space. These include:

*Effect of orbital dynamics on trajectory profiles* – Orbital trajectories need to be determined to bring the robot from several kilometers to within several meters of the target before high-precision carrier phase techniques can be employed. Carrier phase techniques cannot be applied at larger ranges due to the inability to efficiently resolve integers beyond several meters. Pseudorange will be used to navigate into close proximity.

*Selection of satellites* – Since the research presented here is being conducted in a controlled indoor setting, both vehicles are always tracking the same set of pseudolites. In space, however, the set of GPS satellites that are being tracked will constantly change, and a method for choosing which GPS satellites to track needs to be devised.

[6] and [7] have studied the issues of vehicle trajectory profiles and satellite selection for two vehicles rendezvousing from several hundred kilometers to within several meters.

*Antenna switching* – Both the robot and the target are equipped with GPS receivers that can multiplex between four antennas. This research presumes that all antennas are aligned and are dedicated to the same set of satellites. Although all six degrees of freedom will be sensed, the range of motion will be limited due to this constraint on antenna configuration. In space the vehicles will have full range of motion, and will therefore require switching between satellites as the vehicle rotates, in addition to the need to change satellite selection due to orbital motion, as mentioned above.

# 3 System Description

## 3.1 Experiment Configuration

The final experiment configuration is as shown in Figure 1. GPS signals are generated by six pseudolites distributed around the laboratory above the workspace of the vehicle. The robot is commanded at a high, task level through a graphical user interface to rendezvous and capture the moving target. The robot combines its phase measurements with the target's phase measurements to estimate the relative position and attitude and plan an intercept trajectory in real-time. Once it is within capture range, the robot will grasp the target using its manipulators.

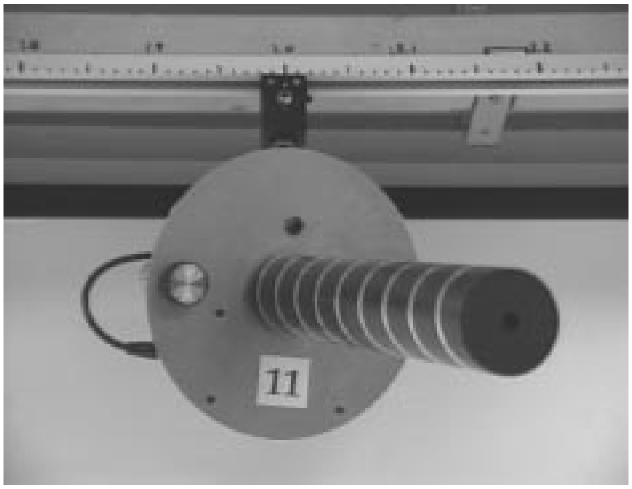

Figure 2: **Mounted Pseudolite**

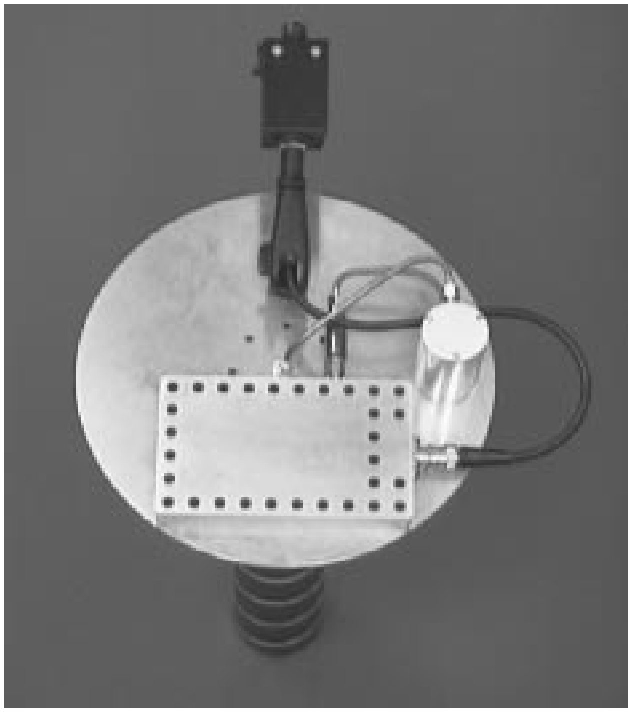

Figure 3: **Pseudolite Transmitter**

### 3.2 Pseudolite Constellation

Each pseudolite produces its own L1 carrier phase signal modulated by its own unique C/A code. In order to perform differential carrier phase measurements between the robot and the target vehicle, the receivers on both vehicles must be synchronized to within one millisecond, so that the carrier phase measurements from each are tagged to the same millisecond epoch. Synchronization is achieved through a "Master Pseudolite" which broadcasts a 50bps GPS data signal modulated on top of the C/A code. This 50bps data signal contains valid Time Of Week (TOW) data in its handover word (HOW) [8]. This enables the two receivers to automatically synchronize to the same millisecond epoch, hence enabling differential carrier phase measurements between the two receivers. Figure 2 shows a mounted pseudolite (broadcasting as PRN 11). The pseudolite is completely self-contained, and can be mounted anywhere around the room on a standard track-lighting fixture which supplies power at 12V.

Figure 3 shows the shielded GPS pseudolite transmitter box as it is mounted on the back of the helical antenna. For size reference, the ground plate of the antenna is 8.5" in diameter. The transmitter electronics board was designed by the Stanford GPS Laboratory for use on their automated landing system [9]. The antenna is designed to broadcast L1 in the normal mode, as described in [10]. The beam pattern is conical in shape, and the beamwidth can easily be controlled by adjusting the length of the antenna.

### 3.3 Robot and Target Vehicles

Each of the robot and target vehicles are equipped with a six channel GPS receiver that is capable of multiplexing between four antennas, i.e. up to 24 carrier phase measurements on each vehicle prior to taking differences. The receivers are off-the-shelf TANS Quadrex receivers from Trimble Navigation, with customized internal software. Measurements from the target vehicle are time-tagged and broadcast out by its communications computer through a radio-link Ethernet transceiver. The robot receives the measurement and combines them with its own measurements to estimate its position and orientation relative to the target vehicle. All communications are handled through the Network Data Delivery Service, a TCP-based[1] communications package designed for distributed control systems [11].

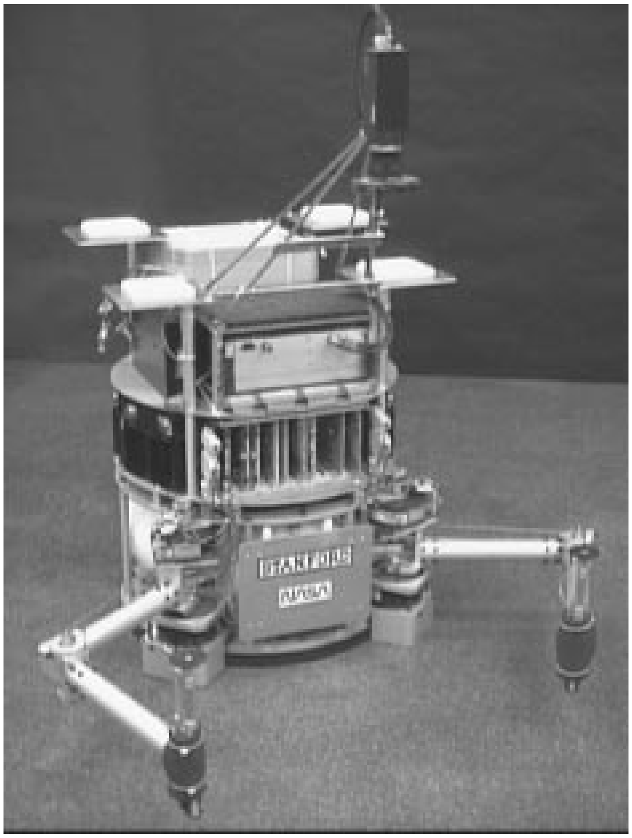

Figure 4: **ARL Free-Flying Space Robot**

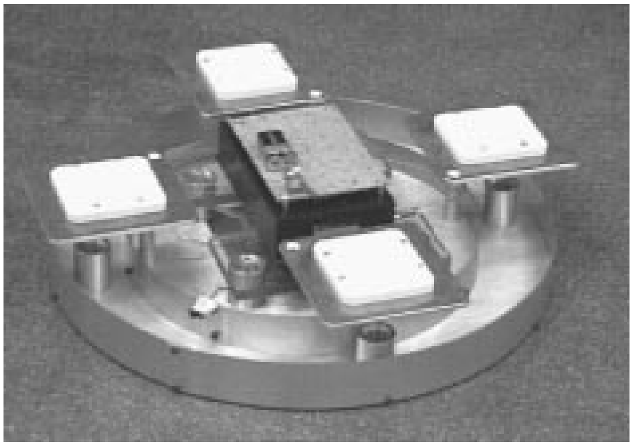

Figure 5: **Target Vehicle**

The robot is depicted in Figure 4. The robot uses an air-cushion support system to achieve the drag-free, zero-g characteristics of space in two dimensions. It is a self-contained autonomous vehicle, complete with on-board VME-bus computers, radio-link Ethernet transceiver, batteries, cold-gas propulsion sys-

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[1] Internet Transmission Control Protocol

tem with eight on-off thrusters, and dual cooperating manipulators. The control software is written in "C" and is being developed using $ControlShell^{TM}$ [12] and the $VxWorks^{TM}$ Operating System. The target vehicle shown in Figure 5 is also equipped with its own communication and flotation devices.

### 3.4 Additional System Components

As noted in section 2, earlier experiments with the robot took advantage of an overhead vision system for tracking objects. This same system is being used for comparative purposes with the GPS sensing system. [1] documents that the vision system can provide repeatability on the *sub-millimeter* level and absolute accuracies of a few centimeters across the entire workspace.

## 4 Theoretical Analysis

The final goal of this project requires the estimation of both the relative attitude and position of the robot and the target vehicle. An intermediate step toward this goal has been the formulation of the equations for a two-dimensional GPS-based position control experiment[2] This section provides the derivation of the estimation equations needed to perform the two-dimensional control experiment. The following assumptions and constraints were made:

1. Only the relative position vector between the robot and the target will be estimated.
2. The position of the target vehicle is known and fixed. Only the position of the robot will change.
3. The initial relative position is known. This means that as long as the pseudolites stay in lock after initialization, the integer ambiguity problem can be ignored.
4. Pseudorange cannot be used in the formulation. As described in section 2, the pseudolites do not provide the information necessary for performing pseudorange measurements.
5. The receivers provide synchronous measurements.
6. Only one antenna on each of the vehicles is used, and it is considered to be the origin of the frame of reference for each vehicle.

Note that all vectors in the formulation are in a common world frame, which in the case of the experiment is the center of the granite air-bearing surface. The variables are:
Constants:

| | |
|---|---|
| $\lambda$ | L1 carrier wavelength |
| $c$ | Speed of light |

Known variables:

| | |
|---|---|
| $P_j$ | Vector from origin to pseudolite j |
| $T$ | Vector from origin to target vehicle |

Unknown variables:

| | |
|---|---|
| $R$ | Vector from origin to robot |
| $\tau_{Pj}$ | Clock error on pseudolite j |
| $\tau_R$ | Clock error on robot receiver |
| $\tau_T$ | Clock error on target receiver |
| $k_j$ | Integer ambiguity along carrier from pseudolite j. |

The initial values of $k_j$ are assumed known as noted in assumption (3).

The measurement equations are as follows:

$$\phi_{Rj} = |R - P_j| + c\tau_{Pj} + c\tau_R + \lambda k_{Rj} \qquad (1)$$

$$\phi_{Tj} = |T - P_j| + c\tau_{Pj} + c\tau_T + \lambda k_{Tj} \qquad (2)$$

Where $|X|$ is the magnitude of vector $X$. The clock errors of the receivers and the pseudolites can be removed by performing a double difference on these measurement equations, as described in [13]. The first difference equations are obtained by subtracting equations ( 2) from ( 1) for each pseudolite j:

$$\begin{aligned} \Delta\phi_j &= \phi_{Rj} - \phi_{Tj} \\ &= |R - P_j| - |T - P_j| + c\tau_{RT} + \lambda k_{RTj} \qquad (3) \end{aligned}$$

The second difference equations are obtained by subtracting equation $j$ from equation $k$ where $k = (j+1)$ mod $N$, and $N$ is the number of pseudolites for which valid signals are being received by both receivers[3]. The second difference will also eliminate satellite ephemeris errors, selective availability errors, and ionospheric and tropospheric distortion errors, in the case of using the real GPS satellites rather than indoor pseudolites.

$$\begin{aligned} \nabla\Delta\phi_{jk} &= \Delta\phi_j - \Delta\phi_k \\ &= |R - P_j| - |R - P_k| - \\ &\quad |T - P_j| + |T - P_k| + k_{RTjk} \qquad (4) \end{aligned}$$

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[2] Actually, all three positions $x$, $y$, and $z$ are estimated, but only $x$ and $y$ can be controlled.

[3] This is just *one* way of arranging the first differences to derive second differences.

From these nonlinear equations the optimal estimate of $R$ given the measurements $\nabla\Delta\phi_{jk}$ needs to be found. This can be done using a Newton-Raphson gradient descent algorithm as described in [14]. Provided below is a simplified version of the technique, ignoring measurement noise.

Given a system of equations in the form:

$$z = h(x) \tag{5}$$

where

| | |
|---|---|
| $h(x)$ | a known nonlinear function of $x$ |
| $z$ | current measurement |
| $\bar{z}$ | measurement at previous time sample |
| $x$ | the unknown vector to be estimated |
| $\hat{x}$ | the current estimate of $x$ based on $z$ |
| $\bar{x}$ | the previous estimate of $x$ based on $\bar{z}$ |

( 5) is linearized by differentiating with respect to $x$ and then approximating $dz = \Delta z$ and $dx = \Delta x$:

$$dz/dx = dh(x)/dx = H(x) \tag{6}$$

where $H(x)$ is the linearized observation matrix evaluated at $x$. Now approximating

$$\begin{aligned} dz &\cong \Delta z = z - \bar{z} = z - h(\bar{x}) && (7)\\ dx &\cong \Delta x = x - \bar{x} && (8) \end{aligned}$$

leads to the linearized system of equations:

$$z - h(\bar{x}) = H(x)(x - \bar{x}) \tag{9}$$

with the approximate solution:

$$\hat{x} = \bar{x} + (H(\bar{x})^T H(\bar{x}))^{-1} H(\bar{x})^T (z - h(\bar{x})) \tag{10}$$

and gradient:

$$gradient = -H(\bar{x})^T (z - h(\bar{x})) \tag{11}$$

( 10) can be iterated upon *between* measurement updates by replacing $\bar{x}$ with $\hat{x}$ each iteration until the magnitude of the gradient ( 11) is smaller than a designated tolerance. ( 10) is also used each time a new measurement arrives, in which case $\bar{x}$ is set to $\hat{x}$ from the previous measurement.

# 5 Results

In order to verify the accuracy and feasibility of using the GPS system for feedback control, the following experiment was performed. The robot was commanded to move in a 1 meter by 1 meter square in the X/Y plane using GPS and the equations derived in the previous section to obtain the feedback measurement. Data was collected by the GPS receivers and the vision system for post analysis. The experiment was configured as follows:

- Four pseudolites, including one master pseudolite, provided the GPS signals. This is the minimum number of measurements necessary to solve the nonlinear equations for the relative position of the robot.
- The target was placed at a fixed and known location.
- The integer ambiguities between the robot and the target were initialized using the vision system.
- The receivers were synchronized via the 50bps data signal broadcast by the master pseudolite.
- Both receivers provided carrier phase measurements at 5Hz. The phase measurements were then converted to positions using the equations from section ?? and provided the feedback to the robot's controller.

The results in Figure 6 show the step response of the robot as measured by the GPS system. These responses demonstrate that the robot can be accurately controlled to well within a few centimeters of the desired locations. For example, the position error in Y from time=0 to time=30 seconds is 0.5 centimeters RMS.

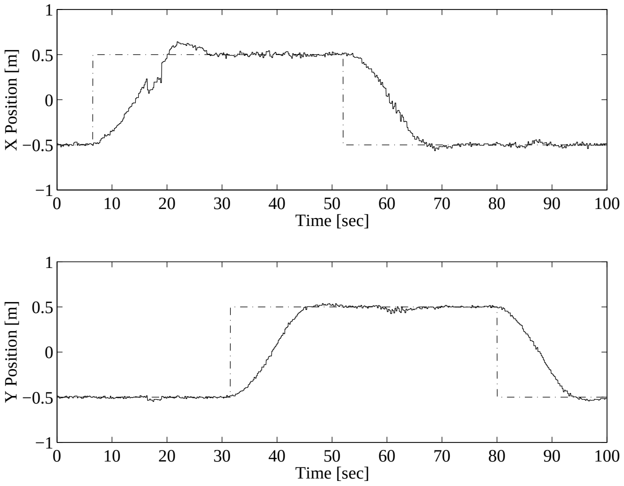

Figure 6: **Step Responses in X and Y**

Figure 7 is a top view of the robot's motion as it traversed the square path in a counter-clockwise manner. Note that the GPS signal overlays the desired path quite well, while the vision system, although providing a cleaner signal, is offset from the GPS and the desired path. This is due to the fact that the vision and GPS systems have not been calibrated to each other, and therefore the absolute positions deviate. Since the controller feedback came from the GPS measurements, the GPS signal overlays the commanded path. Also, some of the discrepancies toward the outer corners can be attributed to lens distortion in the vision system camera.

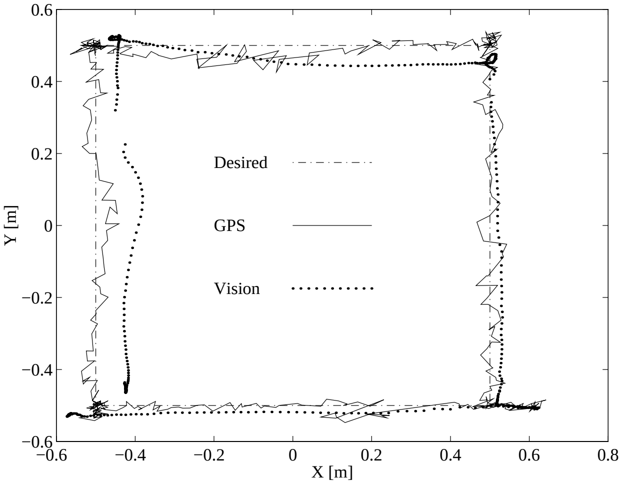

Figure 7: **X vs. Y Positions**

As a final check on the correctness of the system, the measurements from the vision system were converted to double differenced phases and then compared with the actual double differenced phases that were measured. The GPS phase and the phase derived from the vision system match closely, as Figure 8 indicates. The centimeter-level accuracy exhibited by the GPS phases is clearly shown here. The noise around $t = 17$ seconds is most likely due to multipath, and the effects of this noise on the estimated position in $x$ can be seen in Figures 6 and 7. This type of noise will be reduced through filtering and by taking measurements from additional pseudolites in future experiments.

# 6 Conclusions

The research presented in this paper has demonstrated a broad range of objectives: First, we have established a well-controlled environment in which to study and develop GPS systems for high-precision spacecraft rendezvous and capture. Second, we have experimentally demonstrated that differential carrier phase GPS can achieve the accuracies and bandwidth necessary to provide relative position measurements for closed-loop real-time control. Finally, we have presented the novel concept of applying GPS to control indoor systems. This has been achieved through modification of the fundamental theory and with only slight modifications to the software of off-the-shelf GPS receiver technology. The critical differences between indoor and outdoor GPS systems have been identified and several critical issues have been resolved.

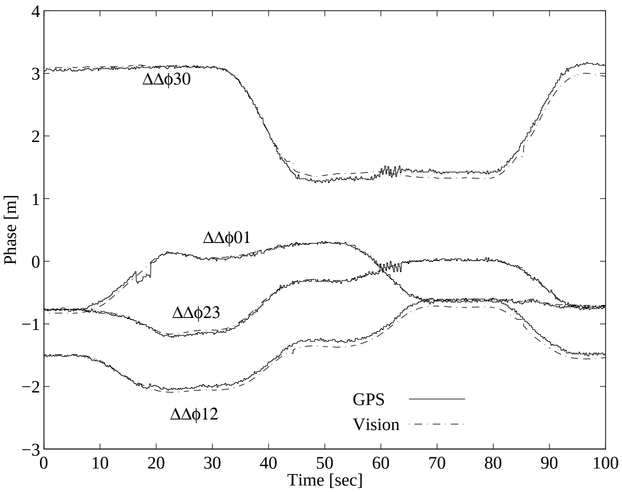

Figure 8: **Double Differenced Phases**

# 7 Acknowledgments

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