Influence of Trajectory on Accuracy of Hazard Estimation During Lunar Landing

Eleanor S. Crane*† and Stephen M. Rock‡

*‡Aerospace Robotics Laboratory, Stanford University, Stanford, CA, 94305, USA
*Moon Express, Inc., Building 20-1 S Akron Road, Moffet Field, CA, 94035, USA

A vision-based hazard estimation system for lunar landers is presented and performance improvements are demonstrated by modifying the lander trajectory over the landing zone. A previously developed technique for inferring the size of rocks based on the measured size of their shadows is the primary hazard information source which feeds an Extended Kalman Filter (EKF) for recursive estimation of discrete hazard objects through the course of the descent. A dense, probabilistic hazard map of the landing zone is created by using the output of the EKF. An overview is given of the algorithmic steps required to extract hazard information from images, estimate the hazard and spacecraft state in the EKF, and create the hazard probability map. Results from a comparison of two trajectories over simulated terrain is given to show that a change in trajectory can produce an improvement in hazard map uncertainty. As a consequence of lower map uncertainty, a greater fraction of the landing zone can be identified as clear of hazards and so potentially suitable for touchdown.

I. Introduction

Hazard Detection and Avoidance (HDA) systems will be an enabling technology for the next generation of autonomous lunar landers, as the addition of this capability will open up far more of the Moon’s surface for exploration than has previously been available. Areas of the Moon with high geological interest and diversity, such as boulder fields, which could be considered too risky for a lander with a standard sensor suite, could be landed in safely by using terrain-relative sensing during descent in order to map these hazards and select the final touchdown site relative to them. This work is motivated more specifically by the challenges of small spacecraft, which have not only a low tolerance to rocks and other hazards but also limited power and mass resources with which to perform the HDA task. To this end, a vision-based system is proposed for such landers. A passive imaging system has the advantage of being a low power solution, but its utility is limited to the lunar day. However, the lunar surface, where it is consistently illuminated, can be a very effective environment for vision-based techniques due to high-contrast lighting and the lack of atmospheric distortion.

The view of the hazard objects, and hence the hazard information available in a given image is dependent on the pose of the camera, and hence the spacecraft. Therefore, by carefully selecting the sequence of spacecraft poses during decent, that is, the trajectory, it is possible to improve the quality and quantity of information collected during imaging and thereby improve the hazard map created for landing site retargeting. A new estimation framework for vision-based HDA has been implemented which employs previously developed image processing techniques for the detection phase and leverages the complete measurement history in order to create a probabilistic hazard map. By changing the trajectory, the information stream available to the recursive filter is modified in order to produce a map with higher confidence. This paper will show that for a relatively small change in the descent trajectory, it is possible to achieve a substantial decrease in the uncertainty of the hazard map created during descent.

*PhD Candidate, Aeronautics & Astronautics, Stanford University, AIAA Student Member
†GNC/Software Engineer, Moon Express, Inc.
‡Professor, Aeronautics & Astronautics, Stanford University, AIAA Fellow
An overview of the image processing techniques used to extract hazard data, the Extended Kalman Filter framework used to blend the data, and the probabilistic representation of hazard field are detailed. This paper also compares two different trajectories over a number of potential landing areas to show that, as a result of decreased uncertainty, the proportion of unobstructed sites identified in flight can be increased by modifying the trajectory.

This paper focuses on showing trajectory-dependent improvement of HDA; future work will extend this idea to include information-seeking guidance inputs in order to actively drive down hazard map uncertainty. Hazard maps are the starting point for all landing site selection algorithms and an improvement in map confidence will flow through any cost map constructed for retargeting.

II. Background

The estimation part of the HDA task involves collecting and organizing hazard data, then selecting a new site relative to those hazards. Methods for hazard data collection are largely dependent on the chosen sensing method. Once the hazard data has been obtained, the problem of constructing a cost map for landing site selection in flight has been studied at length, and can be largely decoupled from the choice of sensor. This work will form the basis of future investigations into retargeting, but is not directly applicable for creating the hazard maps.

Work in the area of small hazard data collection has been largely driven by active range-sensing methods such as LIDAR. Several LIDAR-based systems have been proposed and developed for use on the Moon and Mars to varying levels of technology readiness. LIDAR-based work has focused more on sensor modeling and digital elevation map generation, the results of which are not directly applicable to a vision system, although the link between measurement quality and pose has been investigated for these systems. While LIDAR is well-suited to the task of hazard detection, the high power required to operate such a system make it, at present, challenging to house on a small lander with limited resources.

Vision-based systems can alleviate the strain on the power resources of small landers as it is a low-power sensing solution. As cameras are a common sensor on landers, they could be used for multiple purposes during the mission, including hazard detection during descent. Previous research into vision-based landing aids has been, in general, more focused on terrain correlation for navigation purposes, rather than hazard detection. The extension of the cruise missile system DSMAC to lunar landers has been been considered and modifications for the lunar environment have been proposed. These crater identifications can provide a position correction relative to a map of the planet which can be used in targeting. The VisiNAV system goes further and, when no mapped features are visible, tracks features of opportunity between successive images in order to provide a motion constraint and bound IMU integration error. These systems are better suited to the goal of precision landing, which bounds the size of the landing ellipse relative to a site on the planet’s surface, rather than the hazard avoidance task.

Stereo vision has also been proposed as a solution for hazard detection during descent for some reference missions. Range discrimination based on stereo is limited by the camera baseline, which was the lander diameter in the proposed system. As such, the capacity for growing the system to increase the range sensitivity for increasingly smaller hazards is limited on a small lander. These same techniques and other homography methods exist in the vision literature to estimate hazardous slopes in concert with hazard objects. These vision-based slope techniques could also be combined with the hazard object estimation framework described in this paper to create a more robust HDA system.

More closely related to the HDA task, shadow-based vision techniques for hazard detection have been developed and tested by the robotics division at JPL. This work has been tested on orbital, aerial, and laboratory imagery, with site selection demonstrated. The published work has focused on the image processing necessary to do the rock detection, but not on an estimation framework to leverage these measurements throughout the descent. This paper will build on the hazard detection algorithms developed and proved at JPL, and incorporate them into a larger estimation and guidance architecture for performance optimization. The framework presented in this paper has been tailored for use with this shadow detection technique, but the larger idea of leveraging the trajectory design to improve a vision-based hazard detection system is not dependent on this particular image processing technique and could be adapted to a different detection algorithm.
III. Approach

The construction of hazard field probability maps requires several algorithmic steps. The hazard information is first obtained via the image processing step; this information is then exploited in an EKF estimation framework. The filtered spacecraft states and hazard objects are finally transformed into a probabilistic representation of the terrain. This section presents an overview of these algorithms.

A. Image Processing

The primary sensing mechanism to be used for detecting rocks in orbital and descent imagery is based on an image processing technique developed for terrain relative guidance at JPL. This technique infers the size and centroid of rocks by identifying their shadows in the image. The method first gamma-corrects the image in order to increase contrast. The image is then segmented into regions of shadow and light; individual shadow contours are then constructed from the edges of the shadowed regions, with some noise rejection before creating discrete hazard objects. Using the known angle of the sun with respect to the ground plane, the shadow direction and length can be determined. A hemispherical rock model is used to locate the highest point, and center, of the rock as the point on the shadow contour closest to the sun direction. By using the known sun elevation angle from the ground plane, the rock height can be estimated. More complicated rock models have been developed, but the hemispherical model is taken as a simple but conservative model based on the assumption that most rocks are wider than they are tall.

The rock detection process is illustrated below in Figure 1 using a test image taken in a laboratory setup. The original image is shown on the left, with the shadow outlines, extracted using a Canny edge detector on the gamma-corrected image. The individual rock locations and radii, in pixels, inferred from these shadow outlines are shown on the image on the right. Those measured rocks which correspond to actual rocks are shown in green. Those measurements shown in red have been labeled as false positives. However, while the red crosses may be false positive sightings of rocks, they are generally located in shadowed regions of the surface. The ridges which produced these shadows are generally not safe landing areas, and so are not considered an issue with the method. Not all radii were measured accurately from this single image, but successive images from multiple view-points enables more accurate estimate when combined in a filter.

![Image of rock detection process](image)

Figure 1. Intermediate steps in rock detection image processing

B. Filtering Framework

A recursive estimation system for estimating the spacecraft and hazard states, in the form of a sequential, Multiplicative Extended Kalman Filter, has been developed. The spacecraft motion is propagated using an inertial measurement unit and is updated using measurements from a radar altimeter and the camera. The filter estimates the spacecraft translational states and the location and radius of rock hazards in a local East-North-Up (ENUP) frame. The attitude state is tracked via a three element Gibbs vector and a reference quaternion. Accelerometer and gyro biases are tracked inside the filter for use in spacecraft motion updates. The internal state vector is given by

\[
\tilde{x} = \begin{bmatrix}
\tilde{\mathbf{r}}_{sc} & \tilde{\mathbf{v}}_{sc} & \tilde{\mathbf{b}}_{accel} & \tilde{\mathbf{a}}_g & \tilde{\mathbf{b}}_{gyro} & x_1 & y_1 & r_1 & \ldots & x_n & y_n & r_n
\end{bmatrix}
\]

(1)
where \( r_{sc} \) and \( v_{sc} \) are the spacecraft position and velocity in the ENUP frame, \( \tilde{b} \) are the IMU biases, and \( \tilde{a}_g \) is the Gibbs vector attitude in the body frame. The reference quaternion is given by \( \tilde{q}_{ref} \) and updated with the filter. The hazard objects are appended to the end of the state as they are discovered, with the East, \( x_j \), and North, \( y_j \), positions and estimated radius \( r_j \) for each hazard \( i \). Each hazard is modeled as a hemisphere, and thus its shape is described by only one parameter.

### 1. Time Update

The estimator is driven by a standard spacecraft sensor, an inertial measure unit (IMU). The equations of motion are propagated by the spacecraft acceleration in the ENUP frame and the spacecraft angular rate in the body frame. For the short duration of terminal descent, the estimation framework assumes that the Moon does not rotate or translate through space. This eases the computational burden on the spacecraft since the hazards on the ground remain stationary in the estimation frame. The equations of motion governing the translational motion are given as

\[
\dot{v}_{sc} = a_{sc} - 2\omega_M \times \dot{b} - \ddot{\omega}_M \times v_{sc} \approx \ddot{a}_{sc}
\]

\[
\dot{r}_{sc} = v_{sc} - \ddot{\omega}_M \times r \approx \dddot{v}_{sc}
\]

where \( \ddot{\omega}_M \) is the rotation rate of the Moon, which is assumed to be negligible. The acceleration of the spacecraft in the ENUP frame is constructed from the IMU accelerometer measurement and the estimate of the spacecraft attitude with respect to the ENUP frame.

\[
a_{sc} = B \cdot R_{ENUP} \cdot a_{IMU} + \tilde{g}_M
\]

where \( \tilde{g}_M \) is an estimate of the local gravity. The orientation of the spacecraft, as kept by the reference quaternion \( \tilde{q}_{ref} \) and driven by the rate measured by the IMU, evolves as

\[
\tilde{q}_{ref}(t + \Delta t) = \left(I \cos(\frac{1}{2} \Delta t) + \frac{1}{||\omega_{sc}||} \Omega(\omega_{sc}) \sin(\frac{1}{2} ||\omega_{sc}|| \Delta t)\right) \tilde{q}_{ref}(t)
\]

\[
\tilde{\omega}_{sc} = \tilde{\omega}_{IMU} - \tilde{b}_{gyro}
\]

The reference quaternion is used for navigation purposes, but the internal attitude state is kept via a three-element Gibbs vector, \( \tilde{a}_g \), which tracks the small deviations in attitude away from the reference. The Gibbs vector is reset to null vector at the beginning of every time update; it is updated after the reference quaternion is propagated forward in time according to

\[
\delta q = \tilde{q}(t + \Delta t) \otimes \tilde{q}(t)^{-1} \quad \tilde{a}_g = \frac{\delta q}{\delta q}
\]

At the completion of filter operation, which updates \( \tilde{a}_g \), the internal attitude state is transformed back into a quaternion with unit norm.

\[
\tilde{q} = \left[\begin{array}{c}
\frac{1}{2} \tilde{a}_g
\end{array}\right] \otimes \tilde{q}
\]

### 2. Measurement Update

The state is corrected by measurements from a radar altimeter, which provides range to the surface along the radar foresight direction. In terminal descent, while the hazard filter is active, the underlying surface is assumed to be planar. At higher altitudes, a spherical central-body model can be used instead. The flat-surface measurement equation is given by

\[
\hat{R} = \frac{\hat{z}}{\cos \theta_{elevation}} = \frac{\hat{z}}{\tilde{q}_1 - \tilde{q}_2 - \tilde{q}_3 + \tilde{q}_4}
\]
The camera provides image-derived measurements, which include the pixel locations of the rock center in the current image, \( u_j \) and \( v_j \), and the shadow-derived measurement of rock radius, \( r_j \), in meters. The pixel measurements are estimated using a projective camera model

\[
\begin{bmatrix}
\hat{u}' \\
\hat{v}' \\
\hat{w}'
\end{bmatrix} = C_{\text{cam}} C_{\hat{x}} \begin{bmatrix}
\hat{x}_{\text{feature}} \\
1
\end{bmatrix} = \begin{bmatrix}
\hat{u}'/\hat{w}' \\
\hat{v}'/\hat{w}'
\end{bmatrix}
\]

where the projective model is given by the intrinsic camera properties in \( C_{\text{cam}} \) and the extrinsic properties of the camera, \( C_{\hat{x}} \), encapsulated by the estimated spacecraft position and the camera position with respect to the spacecraft frame. These matrices are given by

\[
C_{\text{cam}} = \begin{bmatrix}
f_x & 0 & c_x \\
0 & f_y & c_y \\
0 & 0 & 1
\end{bmatrix} \quad C_{\hat{x}} = \begin{bmatrix}
R(\hat{q}) & -R(\hat{q})\hat{T}_{\text{cam}}
\end{bmatrix}
\]

where \( R(\hat{q}) \) is the direction cosine matrix equivalent of the estimated quaternion \( \hat{q} \) between the ENUP frame and the camera frame, and \( \hat{T}_{\text{cam}} \) is the estimated position of the camera with respect to the origin of the ENUP frame, expressed in the camera frame.

The measurement of rock radius is used directly in the filter, and so no linearization is required to incorporate it into the EKF. The filter runs at a high rate, 100 Hz, in order to feed the attitude loop, with radar and image measurements incorporated into the filter at their native, lower rate, using the sequential framework.\(^{16}\)

### C. Probability Maps

In order to perform landing site evaluations, the estimated state is transformed from discrete hazard objects into a dense map of the terrain which shows the probability of a particular map location containing a hazardous object. Conceptually, this probability is comprised of three elements. First, for sites which appear clear, there is some probability that a hazardous object smaller than the current camera resolution is present, \( P_{\text{haz, unseen}} \), and the probability that a rock larger than the camera resolution has been missed, \( P_{\text{haz, missed}} \). Map sites also have some probability of being occupied by a hazard object being tracked in the filter, \( P_{\text{haz, seen}} \). The EKF framework tracks each object using a Gaussian probability distribution, which is exploited to determine the probability that a particular map location is covered by that object. The total probability is given as a sum of the elements outlined above; these elements are explained in detail in this section.

\[
P_{\text{haz}}(x_i, y_i) = P_{\text{haz, unseen}}(x_i, y_i) + P_{\text{haz, missed}}(x_i, y_i) + P_{\text{haz, seen}}(x_i, y_i)
\]

(12)

The probability map is initialized, before any images are taken, with a uniform probability equal to the fraction of the total surface area covered by rocks large enough to damage the lander, with diameter \( D_{\text{min}} \), as given by the Golombek model.\(^{17}\) The Golombek model is given by a cumulative distribution function (CDF), which gives the fractional area covered by rocks of diameter \( D \) or larger, given the total coverage fraction by rocks of any diameter, \( k \). The prior probability is given by

\[
P_{\text{haz, unseen,prior}}(x_i, y_i) = F_G(D_{\text{min}}, k) = k e^{-D_{\text{min}}(1.79+0.152/k)}
\]

(13)

As images are taken during the descent, rocks large enough to be detected at the current image resolution, given by \( D_{\text{res}} \), can be added to the filter. However, if no rock is detected at a certain location in an image, this information implies a reduction in the probability of that location being hazardous. The probability of a hazardous object being present in that location is given by

\[
P_{\text{haz, unseen}}(x_i, y_i) = F_G(D_{\text{min}}, k) - F_G(D_{\text{res}}, k)
\]

(14)

The false negative rate of the detection technique must be taken into account in order to allow for the possibility that a rock larger than the camera resolution was missed. The success rate of the detection method is dependent on the number of pixels which comprise the shadow length.\(^{12}\) Shadows which are comprised of only a few pixels can be rejected as noise; a shadow composed of many pixels is less ambiguous.
and so the true detection rate increases with shadow length. The false negative \((FN)\) detection rate is modeled as a linear function of the number of pixels, called \(q\). The total probability that a larger hazard exists in a location is then given as a sum of the probability that a rock of certain size exists, multiplied by the probability that a rock of such a size was missed:

\[
P_{\text{haz,missed}}(x_i, y_i) = \sum_q \left( F_{\text{C}}(q+1) \tan(\beta_{\text{sun}}) \ast \text{resolution, } k - F_{\text{C}}(q) \tan(\beta_{\text{sun}}) \ast \text{resolution, } k \right) FN(q) \tag{15}
\]

Once the spacecraft is low enough that the image resolution is smaller than the threshold for hazardous rocks, the only contribution to the probability of hazards at a seemingly clear location is the false negative probability given above.

If no hazards were measured at a given landing site area, the probability of safety would evolve uniformly, in the manner given above, over the whole area. However, as rocks are measured and filtered, positive hazard information can also be added to the probability map. All rock objects are modeled with Gaussian distributions, as part of the EKF framework, for their location and radius. The position uncertainty is most easily encapsulated by creating an uncertainty radius around the mean of the filtered hazard location. In the current filter framework, the North and East position estimates are represented with identical Gaussian distributions, as part of the EKF framework, for their location and radius. The position uncertainty is most easily encapsulated by creating an uncertainty radius around the mean of the filtered hazard location. In the current filter framework, the North and East position estimates are represented with identical Gaussian distributions, since no preferential measurement direction is used in the filter measurement noise. The position estimates are independent from one another, meaning that the uncertainty is given by two independent, identically distributed normal random variables. The norm of these variables, the radial representation, is represented by a Rayleigh distribution, given by

\[
f(p; \sigma_{\text{loc}}) = \frac{p}{\sigma_{\text{loc}}^2} e^{\frac{-p^2}{2\sigma_{\text{loc}}^2}} \tag{16}
\]

For computational ease, the Rayleigh distribution is modeled as a Gaussian, which is generated via parameter fitting for each rock. A closed form solution is available for the true probability and will be implemented for model validation, but this approximation does not alter the fundamental shape of the probability map since the approximation is a member of same natural exponential family as the true distribution. Using the fit parameters, \(\hat{\mu}_{\text{locj}}\) and \(\hat{\sigma}_{\text{locj}}\), a new, normal random variable can be created by adding the radial position error, \(P\) to the rock radius, \(r\)

\[
R_{\text{effj}} = P_j + r_j \tag{17}
\]

\[
R_{\text{effj}} \sim \mathcal{N}(\mu_{r_j} + \hat{\mu}_{\text{locj}}, \sigma_{r_j}^2 + \hat{\sigma}_{\text{locj}}^2) \tag{18}
\]

which represents the effective radial error associated with each rock. This random variable is normally distributed, but is physically limited to positive values of \(R_{\text{eff}}\). A truncated normal distribution\(^{18}\) is then created in order to be a valid probability in the realizable set. The truncated normal distribution for a variable \(R \sim \mathcal{N}(\mu, \sigma^2)\) where \(R \in [0, \infty)\) is given by

\[
f_{R_{\text{eff}}}(r; \mu, \sigma) = \frac{f_{R_{\text{eff}}}(r)}{\sigma(1 - \Phi(\frac{-\mu}{\sigma}))} \tag{19}
\]

where \(f_{R_{\text{eff}}}(r; \mu, \sigma)\) is the pdf of the complete distribution and \(\Phi\) is the standard normal cumulative distribution function. The probability that a particular map location is occupied by rock \(j\) is equivalent to the probability that the effective radius of that rock is larger than the distance from the map location. For this, the cumulative distribution is required.

\[
F_{R_{\text{eff}}}(r; \mu, \sigma) = \frac{\Phi(\frac{r - \mu}{\sigma}) - \Phi(\frac{-\mu}{\sigma})}{1 - \Phi(\frac{-\mu}{\sigma})} \tag{20}
\]

For any map location \([x_i, y_i]\) that is a distance \(d_i\) away from a particular rock \(j\), the probability that that site is hazardous is given by the probability that the rock occupies that particular site, which is given by

\[
P_{\text{haz,seen}}(x_i, y_i) = 1 - F_{R_{\text{effj}}}(d_i) \tag{21}
\]
which is the probability that the effective radius of the rock \( j \) is greater than or equal to the distance between the site and the center point of rock \( j \). This probability is discounted by the probability that the detection was a false positive (FP), which is, again, a linear function of the number of pixels in the detected shadow.

\[
P_{\text{haz,seen}}(x_i, y_i) = \sum_j FP(q_j)(1 - F_{Reff_j}(d_i))
\] (22)

As above, the total probability at each point on the map is given by the sum of the probability that a hazard below the camera resolution is present, a hazard above the camera resolution was missed, and the probability that each measured hazard extends to a particular location.

\[
P_{\text{haz}}(x_i, y_i) = P_{\text{haz,unseen}}(x_i, y_i) + P_{\text{haz,missed}}(x_i, y_i) + P_{\text{haz,seen}}(x_i, y_i)
\] (23)

IV. Experiment

A. Sim Environment Overview

In order to demonstrate the influence of trajectory on hazard map quality, two realistic descent trajectories are simulated and the resulting probability maps are compared. This experiment was implemented in a high-fidelity lunar landing simulation environment which has been developed in MATLAB™ with the capability of generating and processing synthetic images of terrain in order to incorporate image measurements into a dynamic, closed-loop simulation of a landing scenario. The simulation environment employs a six degree-of-freedom dynamics model, driven by lunar environment and actuator models. In addition to the image measurements, an IMU and radar altimeter are modeled. All these sensor measurements are incorporated an implementation of the online estimation system described above.

1. Hazard Field Generation

The simulation environment can generate a random rock field of varying densities based on the Golombek model\(^{17}\) for the fractional area of a region which is covered by rocks. While the model was originally developed for Mars, studies of rock density on the Moon have shown that the model remains valid and that rock density varies between 3 and 18\%.\(^{19}\) For convenience in generating images with shadow fields, simulated rocks are modeled as cubes. This assumption leads to greater ambiguity in finding the center point of the rock in the processing step, but as the technique remains effective on rocks of this type, the model is taken as conservative at the current level of development.

![Figure 2. Simulated lander performing synthetic imaging during descent](image)
2. Synthetic Imaging

Synthetic images are created within the simulation environment using the OpenGL renderer in MATLAB\textsuperscript{TM} by tuning the rendering properties to match those of the simulated descent camera. All images are rendered in greyscale at a constant size of 640 × 480 pixels. An example image is shown below in Figure 3, with blur and noise added to the image. This image was taken over a randomly generated rock field with 7\% coverage at an altitude of 30m.

![Example Synthetic Image](image)

Figure 3. Example Synthetic Image

3. Trajectory Generation

In order to generate trajectories for use in these investigations, a Modified Apollo Guidance\textsuperscript{20} scheme has been employed. This scheme uses a two-point boundary value problem formulation or the position and velocity states, which yields acceleration commands to be implemented via the lander orientation and main engine thrust. The details of the attitude control system were not considered vital to this experiment and so, for convenience, the lander orientation is set directly and no attitude control system is modeled. As a simple model, a SLERP method\textsuperscript{21} is used to slew the spacecraft between its current and desired attitudes, which effectively limits the slew rate and angular acceleration to realistic levels.

B. Experimental Setup

For this experiment, two trajectories are considered: a ‘nominal’ and a ‘modified’ path. The initial positions, at 600m altitude and 1000m of both crossrange and downrange, are identical. The velocity states are different, however, which accounts for the resulting differences in shape. The ‘nominal’ trajectory is more fuel efficient, whereas the ‘modified’ trajectory requires approximately 1kg more fuel. The more fuel efficient trajectory is called ‘nominal’ since, if there were no sensing constraints to be met, it would be preferable. The modified trajectory is characterized by two main changes from the nominal: it has greater loft and overflies the landing zone by approximately 60m before coming in to land. Both these trajectories transition into a constant velocity touchdown landing phase at 10m altitude and −2m/s vertical velocity. These two trajectories are compared in Figure 4 below.

These two trajectories were generated using the Modified Apollo Guidance scheme described above, but the initial conditions for this scheme were changed in order to produce a modified trajectory with specific properties, which were hypothesized to improve hazard estimation performance. The lander approaches the target site from the opposite direction as the sun, meaning that, in the descent images, the shadows from the rocks are occluded by the rocks themselves. The modified trajectory overflies the target site and is able to image the rock field shadows from an angle which is free from occlusion, thus providing a better measurement of the shadow size, and hence rock size, to the filter for mapping. The modified trajectory also spends more time imaging the target site: the modified trajectory has as two second duration increase over the nominal trajectory, at the cost of fuel. Some improvement in performance can be expected due to the noise filtering properties of the filter when working with more measurements of the same objects.
These two trajectories were flown over randomly generated rock fields with 5% rock density. The rock field was estimated and mapped during the descent and probability maps were created, but no retargeting of the lander was performed. Consequently, the lander flew exactly the same paths over each of the hazard fields. All algorithms were implemented and exercised identically in all flights. The only free variable between a pair of flights over a given terrain was the trajectory.

In addition, the on-board filter is given the same initialization errors in all cases. For this experiment, the hazard map begins empty in all cases and there is no prior information available on any of the large rocks potentially visible from orbit. This capability exists within the filter, but was not exercised for these trials.

V. Results

The quality of the hazard maps resulting from different trajectories is examined. First, the evolution of the probability map over simplified terrain is presented to visualize the development of map uncertainty. Second, complete probability maps of realistic terrain are presented to show that the modified trajectory produces a probability map with lower uncertainty as evidenced by tighter contours of probability around the hazards. Finally, as a method for quantifying the improvement in uncertainty, thresholds are applied to the continuous probability map which will classify terrain as clear, occupied, or unknown. The thresholded maps are used to show the decrease in the size of uncertain regions and improvements in the accuracy of classification.

A. Probability Evolution over simplified terrain

In order to more closely examine this evolution of the probability map, one particular slice of the probability map which contains four rocks, of varying sizes, is shown in Figure 5. This demonstration terrain has been drastically simplified from the randomly generated terrain for purposes of clarity. The map resulting from
the nominal trajectory is shown on the left, with the map from the modified trajectory on the right. At high altitude, the probability of a rock is equal to the prior, that is the fractional area covered by rocks which are dangerous to the lander, which is only 10%. As the camera resolution increases, large hazards are added to the map. The initial covariance is large and so the area of uncertainty surrounding each measurement is also large. The initial position estimate can be seen to be biased, which is a result of the particular camera pose which made the observation. As more observations are made, the position, both laterally and out of plane, evolve and improve. During these observations, camera resolution continues to increase to the level where all hazardous rocks can be seen, and the probability of occupation can decrease to expose clear sites.

**Figure 5. Probability Contour Evolution with Altitude**

The improvements to map uncertainty can be observed by examining the differences between the map of the small hazard at 298m East. At 100m altitude, the modified map shows a lower uncertainty regarding the hazard, which can be seen in the steeper slope of the drop off in probability back to the low probability areas surrounding the hazard. This drop off in probability continues to steepen in the modified map, as can be seen in the map constructed at 50m altitude. In the nominal map, this object has moved out of plane with respect to this slice at 275m East, leaving only a region of uncertainty.

**B. Map uncertainty improvements**

The uncertainty decrease is examined in results from two sets of full probability maps created by flying same nominal and modified trajectories over randomly generated lunar terrain. The probability contours of the final map generated during the nominal trajectory is shown on the left and the modified trajectory on the right in Figure 6. The true rock field is shown in both maps using black squares to denote the location of
rocks. The basic shape of both maps is similar in that both maps are converging to the underlying rock distribution. However, the modified map contours show a much tighter convergence around the individual hazards due to better convergence in the filter of the position and size of the hazards. By decreasing uncertainty around the hazard, larger swaths of very low probability areas are exposed in the map.

![Figure 6. Final Probability Map Contour Comparison](image)

The best map to use as an in-flight decision aid will have minimal uncertainty. In order to quantify the uncertainty present in each map, thresholds are applied to the continuous distribution in order to classify a particular site as either unknown or identified. If a site has a high probability of being either clear of hazards or occupied, it can be identified. Those areas of intermediate probability are considered unknown, as they cannot be confidently identified as either clear or occupied. The quality of the map can be evaluated by calculating the total area fraction which is classified as unknown.

These same maps from Figure 6 from the two candidate trajectories can be evaluated more quantitatively by comparing the areas of uncertainty in each map. For visualization purposes, these classifications are shown in red for ‘occupied’ terrain, yellow for ‘unknown’, and green for ‘clear’ in Figure 7. Occupied sites are obviously unsuitable for landing, whereas unknown sites are not well-defined enough for a further classification to be made. Clear sites are of the greatest interest since these sites represent the set of potentially landable terrain. For this experiment, a site must have a probability of occupation less than 0.25% in order to be labeled clear; at all altitudes, a site must have a probability of occupation greater than 80% to be labeled as occupied. At 300m altitude, before any observations of the terrain have been made, both maps are equivalent with the prior probability being uniform throughout. All the terrain is classified as unknown due to the lack of information.

The modified trajectory observes part the landing area first, at an altitude of 250m. While there are areas labeled as occupied, at this high altitude, the map is still highly uncertain and these areas labeled as occupied at 200m continue to evolve during the course of the descent. At 100m altitude, much more of the hazard field is observable and more area has been labeled as occupied by both trajectories. The amorphous regions represent a collection of individual hazards that have not yet been resolved into discrete objects. High uncertainty can cause this situation, but it is also common for several small hazards to be identified as a single, larger obstacle at high altitude. Clear sites become identifiable as the spacecraft descends; as the uncertainty about each rock estimate decreases, sites can change from an unknown classification to clear. At the final altitude of 50m, the two maps appear similar: the layout of rocks is in agreement. The modified map shows, however, greater definition of individual hazards, where the nominal map shows more amorphous hazard zones in the same locations. The nominal map classifies much more of the area in the landing zone as either occupied or unknown; the modified map shows less uncertainty and identifies more areas as clear. At the final map altitude of 50m, the modified trajectory is able to remove uncertainty from an additional 9% of the map area.
C. Map classification improvements

In order to begin validating the probability assignment method, the success rate of classification is examined over the map area. There are almost 26000 units of map area which are classified by both the nominal
and the modified trajectories. Of all the terrain labeled clear with 99.75% probability, 100% of that terrain is actually clear. The terrain labeled as hazardous with 80% probability is actually occupied by a hazard approximately 85% of the time. While this initial comparison of classification shows strong correlation between the estimated and true probabilities, a more extensive Monte Carlo test of various terrains will be undertaken to fully validate the probability map method and improve the hazard classification accuracy.

The areas of terrain labeled in the two different maps is largely correct, but the modified trajectory labels a greater percentage of the terrain correctly. A comparison of the success rate in determining whether a particular point on the map is clear of any rocks is considered. For a site to be classified as clear, the probability that it is occupied must be less than the 0.25% threshold. In practice, this requires that no rocks be observed there and that any rocks near it must be well-known, with a low covariance, such that there is a low probability that they extend to the clear site. Since this is a binary classification problem, the standard $2 \times 2$ confusion matrix showing the rate of true and false classifications is illustrative of performance. This matrix can be constructed at any altitude, but will be examined here at 50m above the surface, which is taken as the last opportunity for site selection.

The confusion matrix for final set of clear sites designated during nominal trajectory is given below in Table 1. It is important to note that the algorithms are working well since there are very few false positives, although they are possible in this framework. However, for the nominal map, those sites which are labeled as clear represent only a tenth of the total area of clear sites. The set of sites available as potential landing sites is greatly reduced and may lead to a more dangerous site being selected.

<table>
<thead>
<tr>
<th>Actually Clear</th>
<th>Actually Occupied</th>
</tr>
</thead>
<tbody>
<tr>
<td>Classified Clear</td>
<td>0.11 0.0</td>
</tr>
<tr>
<td>Classified Not Clear</td>
<td>0.89 1.0</td>
</tr>
</tbody>
</table>

The corresponding confusion matrix for the modified trajectory is given in Table 2. As before, the terrain labeled clear is actually clear with very high probability. There is an increase, however, in the quantity of sites correctly identified as clear: 29% of the clear sites are available as potential landing sites. At the final decision altitude, the map from nominal trajectory is much more uncertain than the map from the modified trajectory. The result of this is that most of the area in the nominal map is labeled as unknown, as knowledge of the rocks has not converged to a level of confidence that allows clear sites to be seen near them. The filter state in the modified trajectory is better converged such that more area can be confidently labeled, correctly, as clear.

<table>
<thead>
<tr>
<th>Actually Clear</th>
<th>Actually Occupied</th>
</tr>
</thead>
<tbody>
<tr>
<td>Classified Clear</td>
<td>0.29 0.0</td>
</tr>
<tr>
<td>Classified Not Clear</td>
<td>0.71 1.0</td>
</tr>
</tbody>
</table>

The hazard mapping techniques are able to correctly identify terrain through both trajectories, and the benefits of HDA are available from any trajectory that satisfies some basic observability constraints, such as the trajectories shown here. But by considering the map improvements produced by the modified trajectory, it can be seen that the design of the trajectory can be an important contributor to the overall performance of the HDA system.

### VI. Conclusion

Changes in trajectory have been shown to have a significant effect on hazard map quality; by reducing map uncertainty, more potential landing sites can be exposed. The hazard mapping and probabilistic framework have been shown to converge to the true terrain shape in several examples. It then follows that guidance schemes which incorporate the need for mapping can improve HDA performance and landing success rates. The trajectories presented here were chosen heuristically but it would be possible to provide further mapping...
improvement, subject to fuel and safety constraints, by optimizing the trajectories using a dual control scheme.

Acknowledgments

This research was funded in part by a Stanford Graduate Fellowship.

References