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Associative Memory—II

Potentiated synapses between neurons in a pattern enable full recall

Fraction recalled increases faster than fraction stimulated.
Overlapping patterns degrade recall (and memory capacity).
Active dendrites can mitigate pattern overlap.

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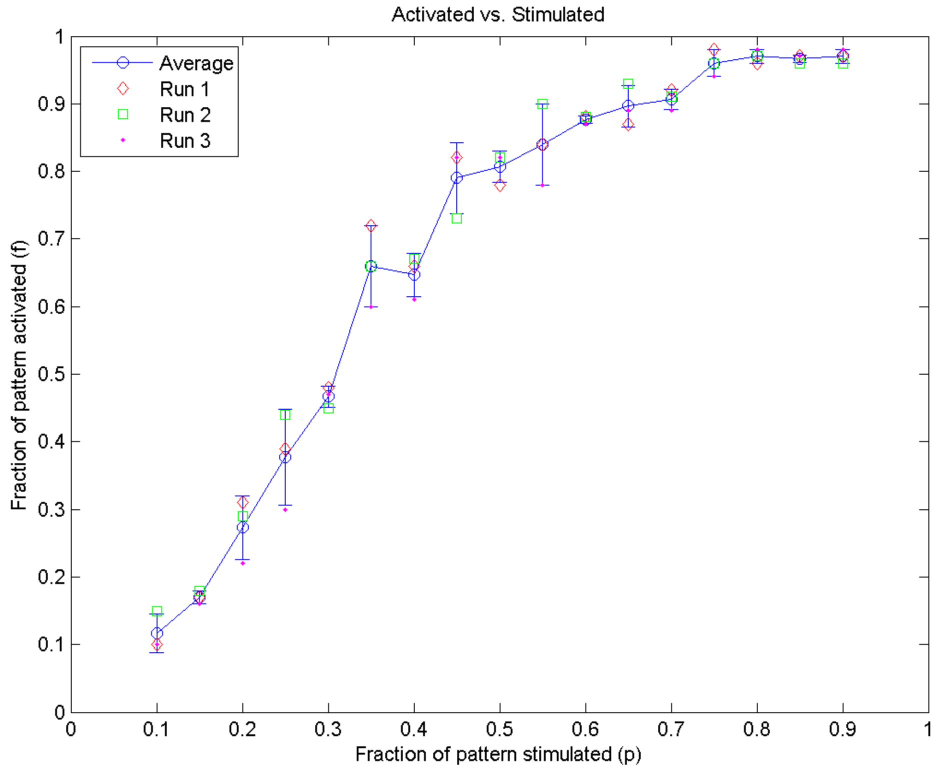
Recall performance



Over 90% of a pattern is recalled when 60% of its neurons are stimulated.

The stimulated group recruits neurons connected to it—if there are enough potentiated synapses.

These neurons may in turn recruit more neurons—if there is a high degree of connectivity.

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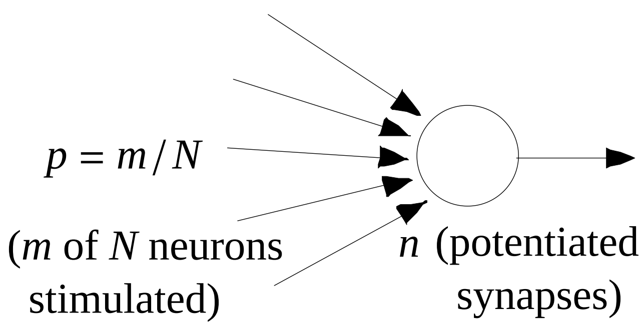
Simple model of recall



Fraction of potentiated synapses activated equals the fraction of pattern stimulated.

The number of a neuron's potentiated synapses that are activated, **k**, is modeled as a binomial distribution, **b**, with density:

$$b[k, n, p] = \binom{n}{k} p^k (1 - p)^{n-k}$$

where **p** is the fraction of the pattern stimulated and **n** is the total number of potentiated synapses.

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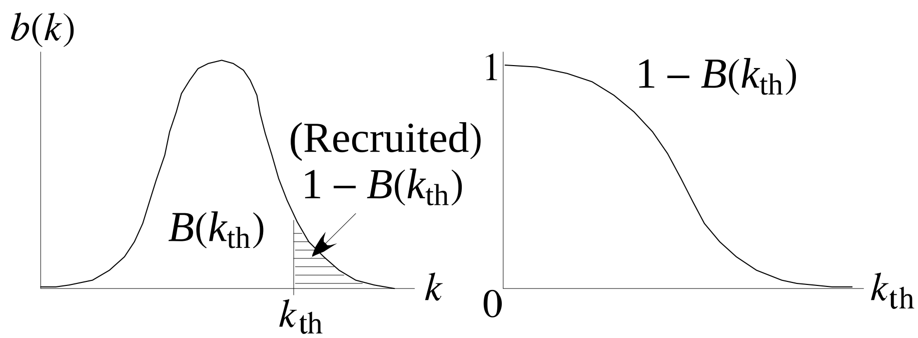
Probability a neuron is recruited



A neuron fires if k_{th} or more of its potentiated synapses are activated.

The cumulative distribution, **B[k,n,p]**, yields the probability **r** that a neuron is recruited:

$$r = 1 - B[k_{th}, n, p]$$

where k_{th} is the minimum number of potentiated synapses that must be activated.

Note that this result applies to feedforward as well as recurrent networks. The only difference being that a recurrent network iterates the equation—inputing a new **p** that includes the recruited neurons.

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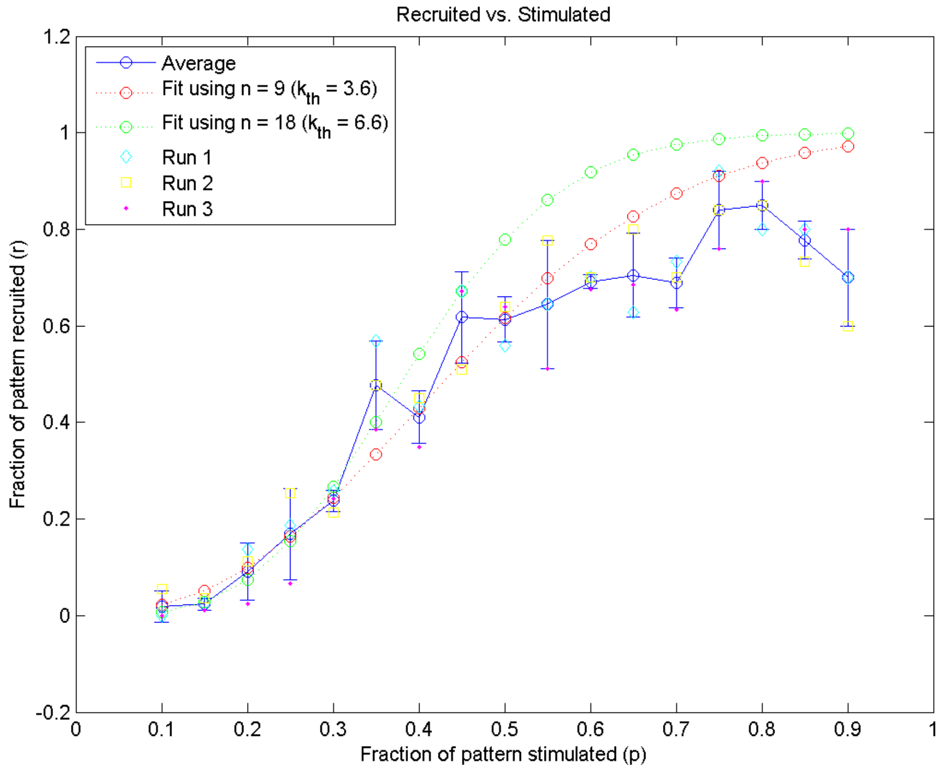
Recruited versus stimulated fractions



Binomial distribution yields a good fit, with $n = 9$ and $k_{th} = 3.6$.

Note that the stimulated (**p**) and recruited (**r**) fractions sum to give the fraction of the pattern recalled:

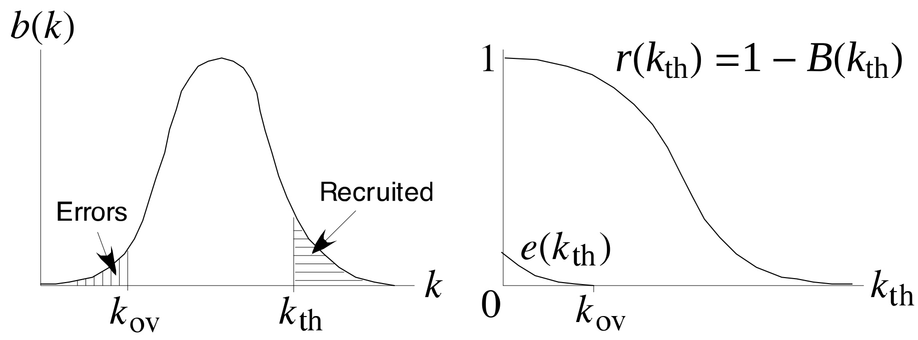
```
f = p + (1 – p) r
```

This relation was used to obtained the points plotted above (**r**) from the measured data (**f**):

```
r = (f – p) / (1 – p)
```

The good fit suggest that iteration is insignficant—behaves like a feedforward network—most likely because connectivity is sparse (21 or less out of a 100).

Overlapping patterns



Neurons in overlapping patterns are recruited when $k_{th} < k_{ov}$, the pattern overlap.

When multiple patterns are learned, a neuron can become active erroneously if the pattern being recalled overlaps with the one the neuron belongs to.

Such errors occur when the overlap—the number of neurons that belong to both patterns—exceeds k_{th}.

Pattern overlap imposes a lower bound on k_{th}, which translates into an upperbound on r.

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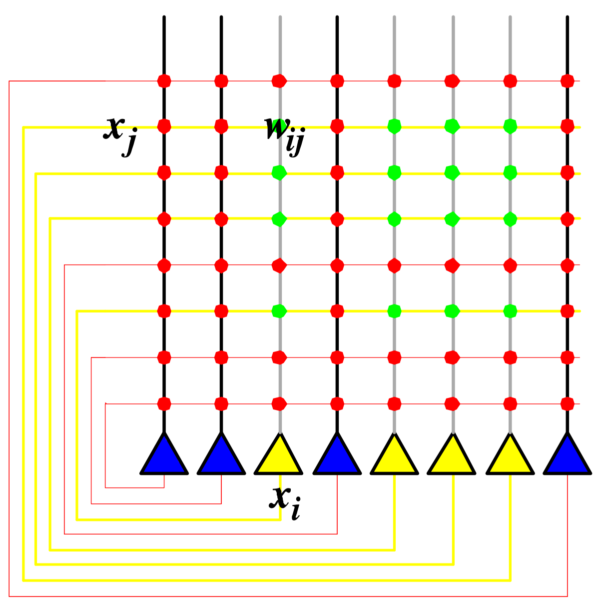
Superposition of stored patterns



Neuron j drives neuron i through a synapse with strength w_{ij}.

In the Hopfield network, neuron j's input is given by

$$y_j = \sum_{i=1}^{N} w_{ij}\, x_i \quad \text{where} \quad w_{ij} = \sum_{m=1}^{M} x_i^m\, x_j^m$$

Substituting and reversing the order of summation yields:

$$y_j = \sum_{i=1}^{N} \left(\sum_{m=1}^{M} x_i^m\, x_j^m \right) x_i = \sum_{m=1}^{M} \left(\sum_{i=1}^{N} x_i^m\, x_i \right) x_j^m$$

The inner sum is large if the neurons' activity (x_i) is similar to that invoked by pattern m (x_i^m). Thus, a pattern's similarity to the input, measure by the inner product, determines its contribution to the output. Therefore, the network recalls a superpostion of the stored patterns, each weighted by its similarity to the initial activity.

To avoid superposition, pattern overlap should be minimized, which requires that each pattern activates only a small fraction of the population (sparse coding). Then, the network will tend to recall the closest-matching pattern perfectly.

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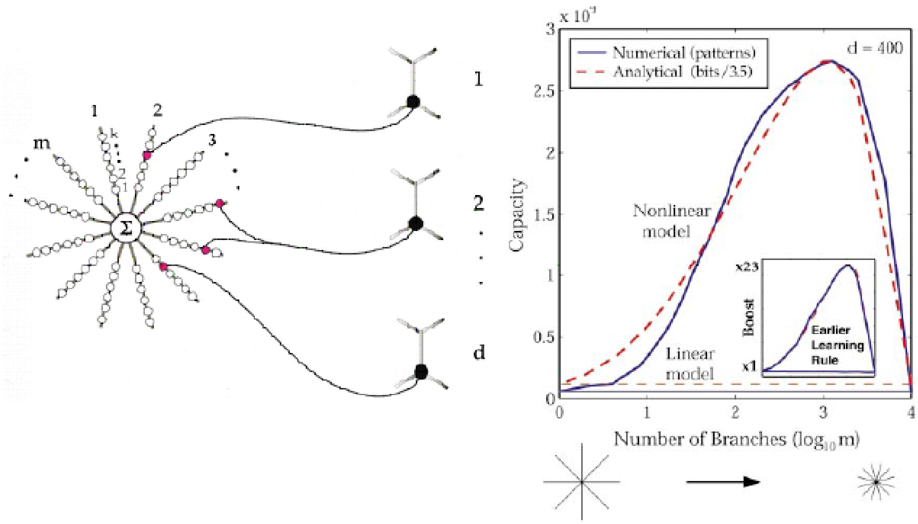
Avoiding superposition: Active dendrites



Memory capacity increases when dendritic branches act as nonlinear subunits [Mel'01].

Another way to avoid superposition is to cluster inputs onto dendritic branches and impose a threshold that must be exceeded for a branch to become active.

The capacity of a network of neurons with such active dendrites grows logarithmically with the number of distinct ways in which the inputs can be clustered (analytical curve in figure).

For neurons with 10,000 synapses, made by $d = 400$ distinct afferent axons, capacity was maximum when synapses were distributed across 1,250 branches with 8 synapses each.

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Next week: TBD