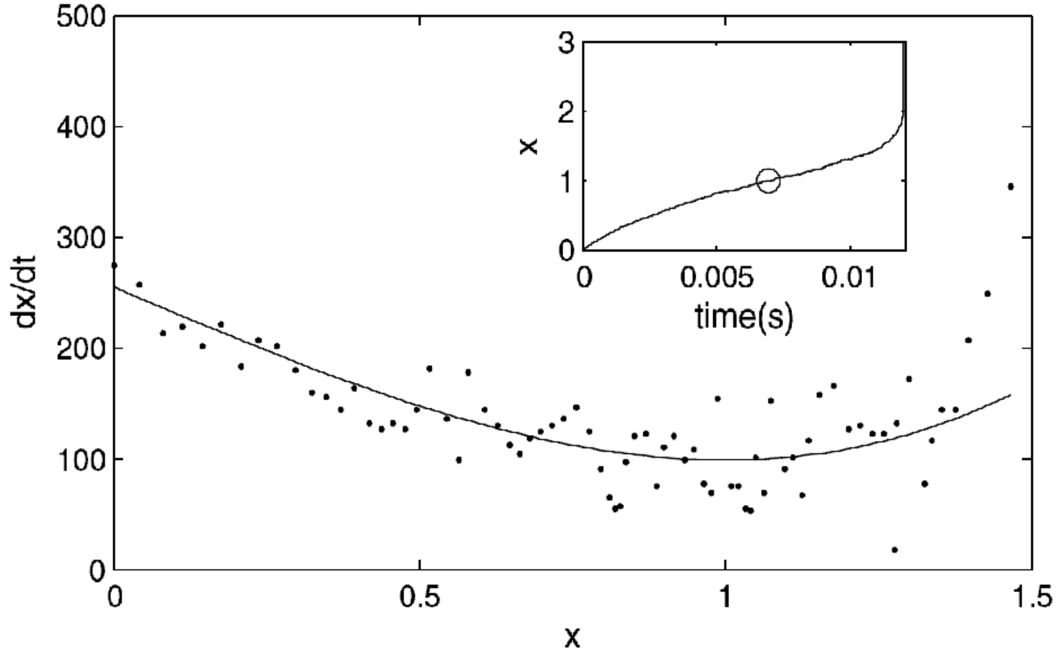
Positive-Feedback Neuron

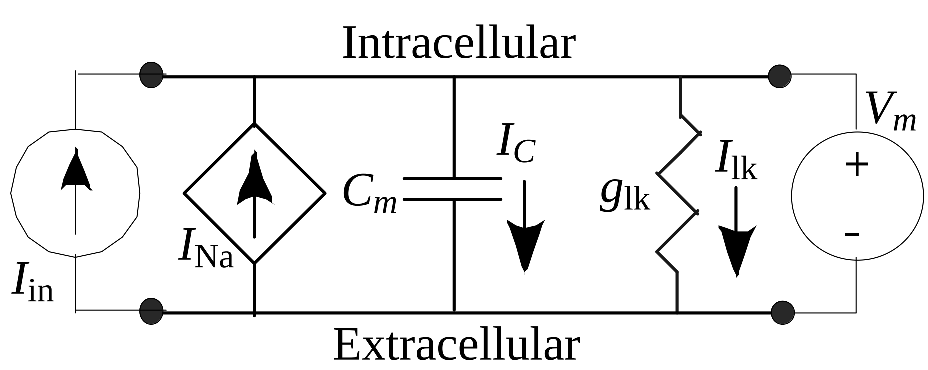


A cortical fast-spiking interneuron's phase-plot, computed from its membrane voltage trace (insert) [Izhikevich07].

Has an inflection in its membrane-voltage trace

Spike frequency increases sublinearly at high rates

Membrane-voltage equation

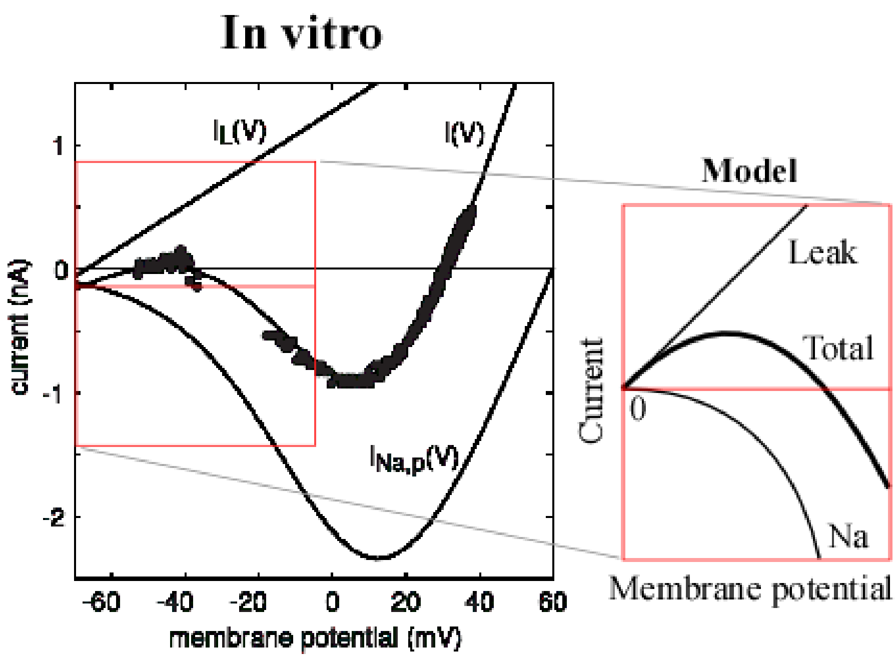


The inward currents (I_{in} plus I_{Na}) equal the outward currents (I_{lk} plus I_C).

$$C_m \frac{dV_m}{dt} + g_{lk} V_m = I_{in} + I_{Na} \quad \text{where} \quad I_{Na} = \frac{1}{3}\left(\frac{V_m}{V_{th}}\right)^2 g_{lk} V_m$$

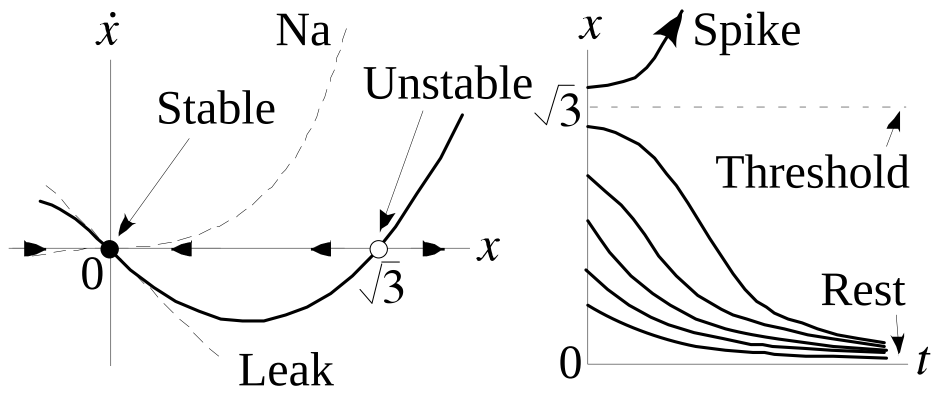
$$\Leftrightarrow \tau_m \frac{dx}{dt} + x = r + \frac{1}{3} x^3 \quad \text{where} \quad \tau_m = \frac{C_m}{g_{lk}},\ x = \frac{V_m}{V_{th}},\ r = \frac{I_{in}}{g_{lk} V_{th}}$$

Comparison



Matches neuron's behavior in -70 to -5mV range (rat cortex L5 pyramidal cell) [Izhikevich07].

Fixed points: Rest and threshold



Two equilibria (left), corresponding to rest and threshold (right); *r* is set to zero.

We solve the membrane-equation for the derivative. It is proportional to the net current (Input + Na – Leak):

$$\frac{dx}{dt} = r + \frac{1}{3} x^3 - x \text{ where } \tau_m \equiv 1 \quad (\text{time is in units of } \tau_m)$$

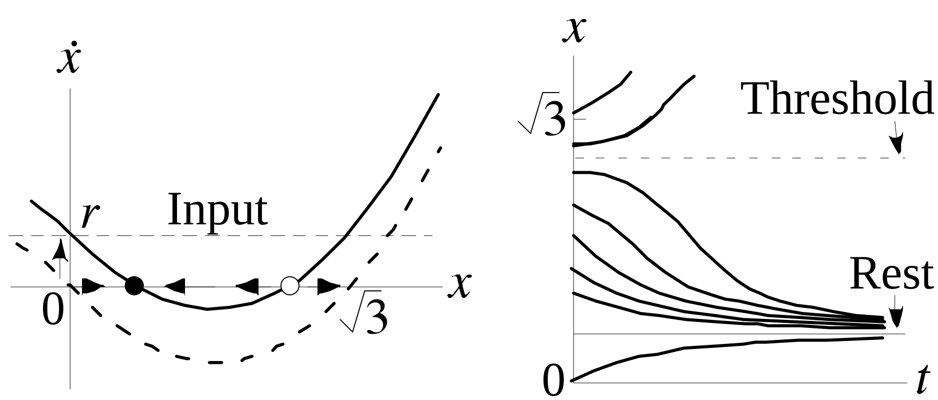
Plotting the derivative versus x for $r = 0$ reveals two fixed points:

Rest: A stable point at $x = 0$ — x moves toward 0 ($\dot{x} > 0$ when $x < 0$ and $\dot{x} < 0$ when $x > 0$).

Threshold: An unstable point at $x = \sqrt{3}$ — x moves away from $\sqrt{3}$ ($\dot{x} < 0$ when $x < \sqrt{3}$ and $\dot{x} > 0$ when $x > \sqrt{3}$).

That is, if you initialize x above $\sqrt{3}$ (i.e., peg V_m above $\sqrt{3}$ V_{th} and release it), the neuron will spike.

Adding input brings fixed points together



The equilibria move together (left); rest rises and threshold drops (right).

Increasing r shifts the phase-plot up, moving the equilibria closer. That is, the neuron rests at a higher voltage and has a lower threshold — the value above which x must be initialized to get a spike is now less than $\sqrt{3}$.

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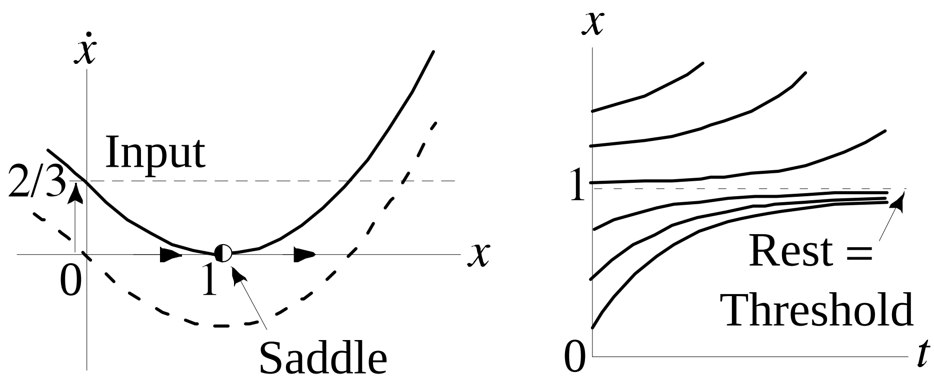
Saddle-node bifurcation



The two equilibria coalesce into a saddle point (left); x increases below it and increases above it (right).

Eventually, the fixed points meet at $x = 1$. They coalesce into a ***saddle point*** — a fixed-point that is neither stable nor unstable — x may move toward it or away from it, depending on whether x is above or below ($\dot{x} > 0$ when $x < 1$ and $\dot{x} > 0$ when $x > 1$). Thus, when x is reset to 0, it approaches 1, and sits there (similar to rest). However, with a little nudge (from noise), it takes off, producing a full-blown spike (similar to threshold). Thus, the current level at which the saddle point appears is the minimum input required for spiking.

A **bifurcation** is said to occur when the number (or nature) of fixed points changes. This particular type — where a stable and unstable point coalesce — is called a *saddle-node bifurcation*. When it occurs, the neuron goes from resting queiscently to spiking rhythmically.

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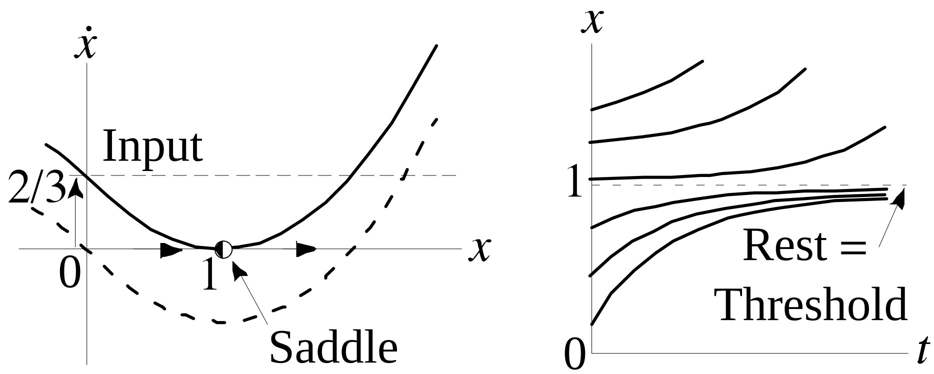
Determining the minimum input



How far up must the phase-plot move for the minimum to touch the x-axis?

For the system $\dot{x} = f(x, r)$, the input r_{th} at which the bifurcation occurs and the membrane voltage x_{th} at which the saddle-point appears must satisfy:

$$f[x_{th}, r_{th}] == 0 \text{ and } f'[x_{th}, r_{th}] == 0$$

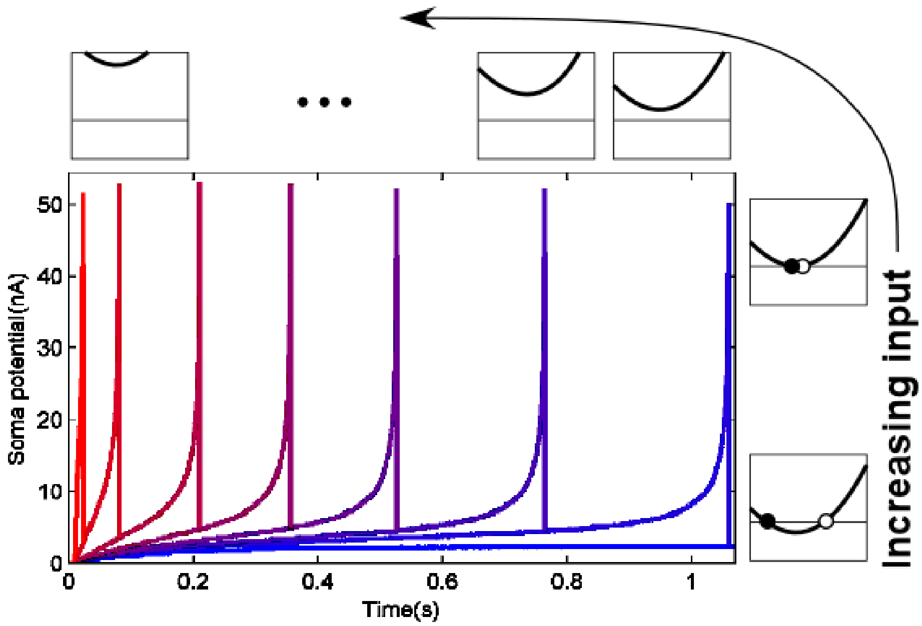
First find where $f'(x, r)$, f's derivative with respect to x, is 0:

$$\frac{d}{dx}\left(r + \frac{1}{3}x^3 - x\right) = 0 \implies x^2 - 1 = 0 \implies x_{th} = \pm 1$$

Then find the value of r that makes $f(x_{th}, r)$ equal to 0:

$$r + \frac{1}{3} - 1 = 0 \implies r_{th} = \frac{2}{3}$$

Model membrane-voltage traces



Spikes sooner wIth increasing input current—once minimum is exceeded.

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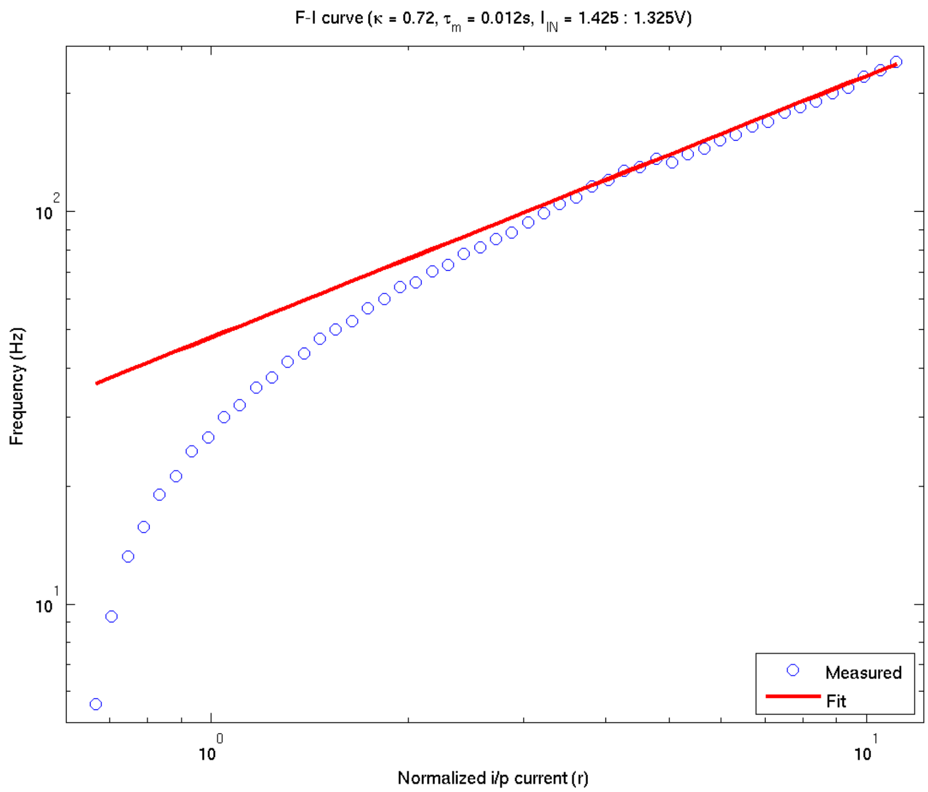
Spike frequency



2/3-power law (red) holds above five times the minimum input ($r = 2/3$).

The period T is obtained as:

$$\int_0^T dt = \int_0^\infty \frac{dx}{\dot{x}} = \tau_m \int_0^\infty \frac{1}{r - x + x^3/3}\, dx$$

$$\approx \tau_m \int_0^\infty \frac{1}{r + x^3/3}\, dx \quad \text{when} \quad r \gg 1$$

$$\approx \frac{2\pi}{3^{7/6}} \frac{\tau_m}{r^{2/3}}$$

This result produced the red fit.

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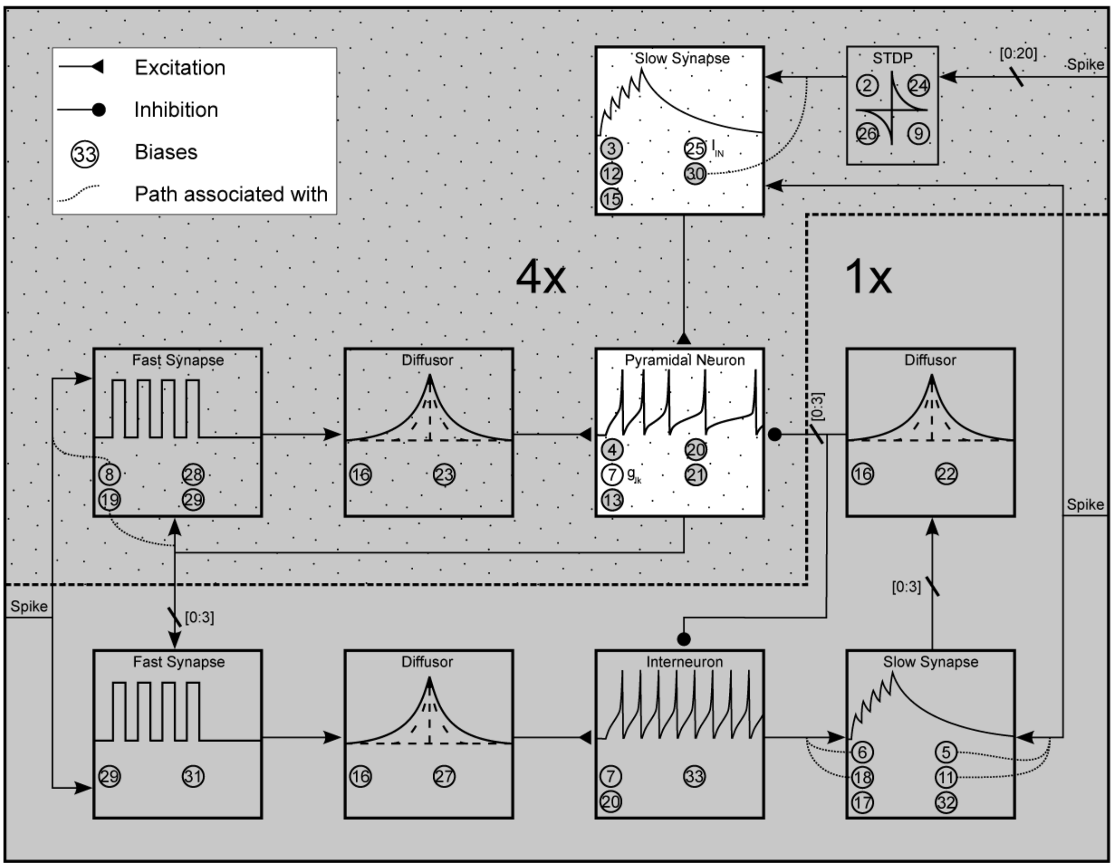
Lab Set-up

1	Vrefd2a	reference voltage - leave at 2.530	18	VMAGGABA	(-) increases inhibitory synapse strength (inhibitory interneuron input)
2	VQAPRE	(-) increases LTP-side of STDP curve's height	19	VQAAMPA2	(-) increases fast excitatory synapse strength pulse-width (pyramidal neuron input)
3	VLEAKDNMDA	(+) increases slow excitatory synapse rise-time	20	VLEAKREFRACT	(+) increases absolute refractory period
4	VMAGK	(-) increases m-type potassium channel strength	21	VLEAKK	(+) increases m-type potassium decay-constant (and strength)
5	VLEAKDGABA2	(+) increases inhibitory synapse rise-time (external input)	22	VRI	(+) increases spread of inhibition
6	VLEAKDGABA	(+) increases inhibitory synapse rise-time (inhibitory interneuron input)	23	VRRC	(+) increases spread of fast excitation to pyramidal neurons
7	VLEAKSOMA	(+) increases somatic leak current	24	VLEAKPOST	(+) increases LTP-side of STDP curve's decay
8	VQAAMPA	(-) increases fast excitatory synapse strength pulse-width (external input)	25	VMAGNMDA	(-) increases slow excitatory synapse strength
9	VQAPOST	(-) increases LTD-side of STDP curve's height	26	VLEAKPRE	(+) increases LTD-side of STDP curve's decay
10	VLEAKLTP	not used	27	VRE	(+) increases spread of fast excitation to interneurons
11	VMAGGABA2	(-) increases inhibitory synapse strength (external input)	28	VMAGAMPARC	(+) increases fast excitatory synapse strength to pyramidal neurons
12	VQADNMDA2	(+) increases slow excitatory synapse rise-time	29	VLEAKAMPA	(-) increases fast excitatory synapse strength pulse-width
13	VANP	(+) increases pyramidal neuron sodium threshold	30	VQANMDA	(+) increases slow excitatory synapse rise-time
14	VLEAKLTD	not used	31	VMAGAMPAINT	(+) increases fast excitatory synapse strength to interneurons
15	VLEAKNMDA	(+) increases slow excitatory synapse decay-constant (and strength)	32	VQADGABA	(+) increases inhibitory synapse rise-time
16	VG	leave at 1.250	33	VANI	(+) increases interneuron sodium threshold
17	VLEAKGABA	(+) increases inhibitory synapse decay-constant (and strength)			



In this lab, you will use the slow synapse to drive the positive-feedback neuron.

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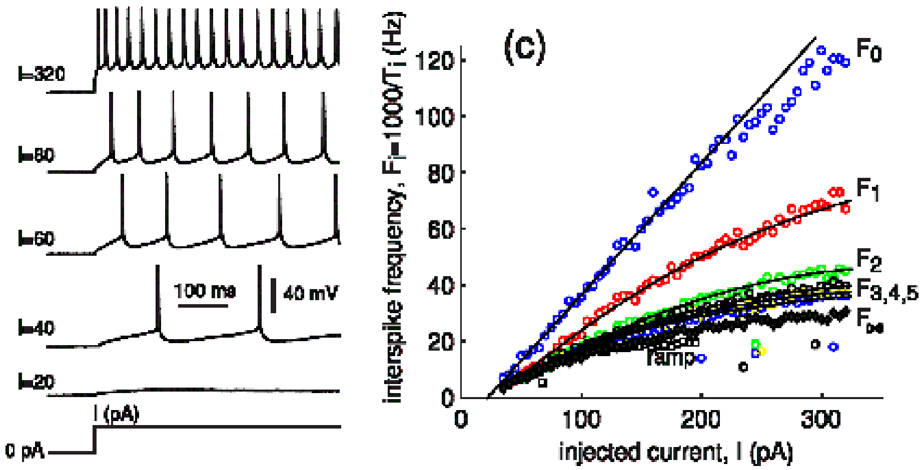
Next lecture: Adaptive neuron



Frequency adaptation (rat cortex L5 pyramidal cell) due to M-current.

Requires an outward current that is proportional to spike frequency