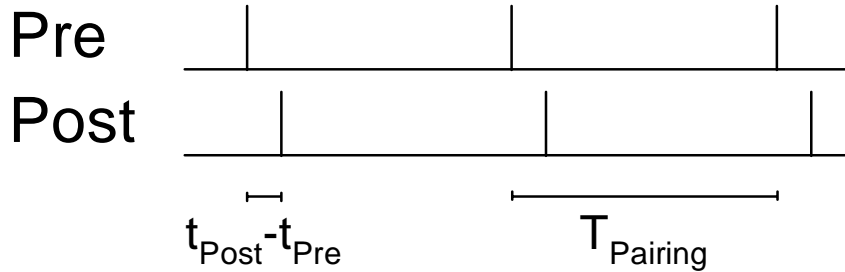


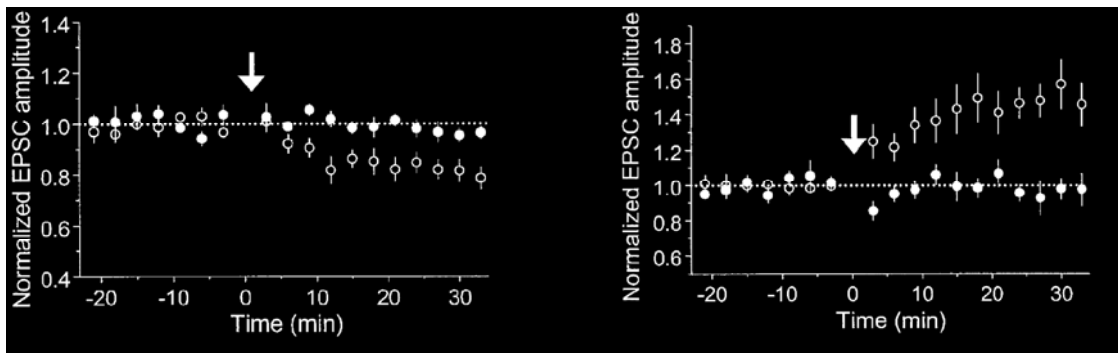
Synaptic Plasticity-II: Poisson Spike Trains



STDP is usually measured with repeated pairings with a fixed Pre to Post time difference at a fixed interstimulus period.

We used repeated pairings, neglecting rates and irregularities of spike trains.
We correlated pre- and post-synaptic activity.

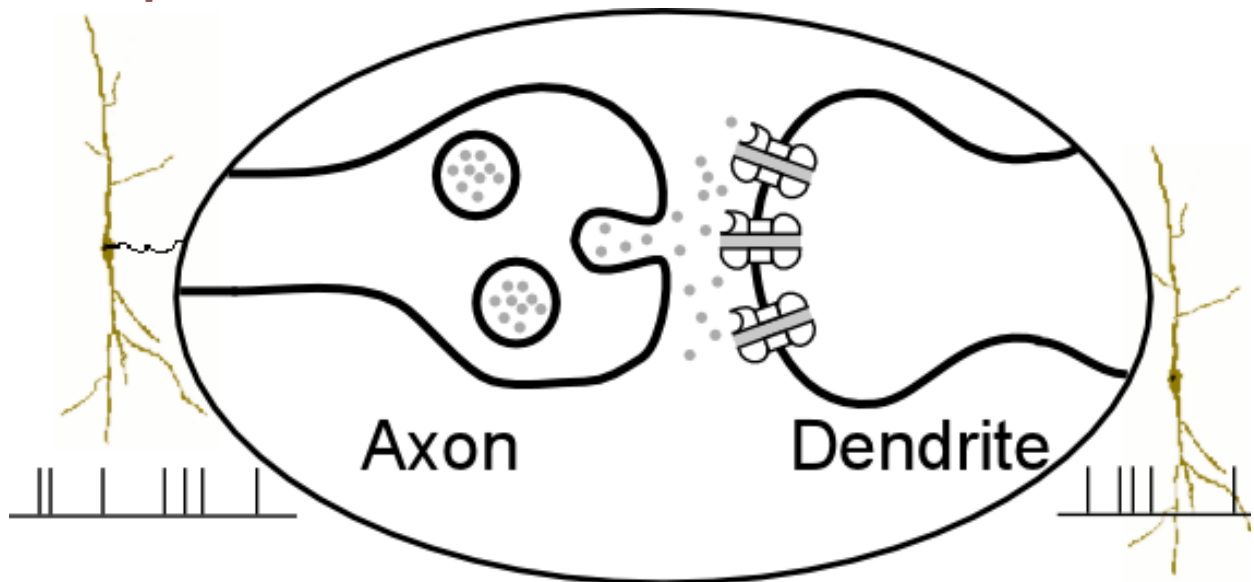
Plasticity Depends on Rates



Low rates favor depression, high rates favor potentiation.

Both LTP and LTD depend on pre- and post-synaptic rates.
When pre is high and post is low, LTD dominates.
When post is high and pre is low, LTP dominates.
When both rates are high, LTP dominates.

Poisson Spike Trains



Neuronal spike trains are noisy.

Neuronal spike trains lack regularity and are Poisson-like.



Aside: The Poisson Distribution

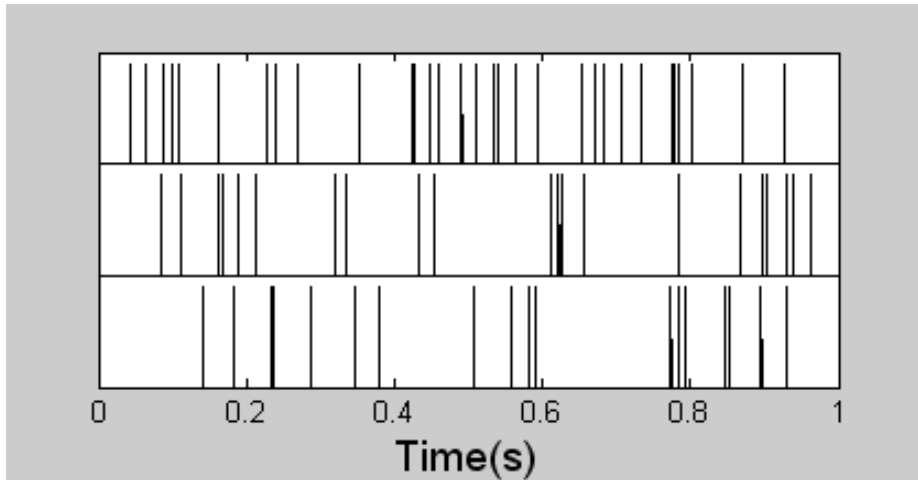


Poisson spike trains are created by dividing time into (small) bins of width Δt and generate a spike with probability $r\Delta t$, in each bin where r is the desired spike rate.

**A spike is generated in each time bin independent of other bins.
Two Poisson spike trains are independent as well; any correlation between them is a function of rate.**



Aside: The Poisson Distribution



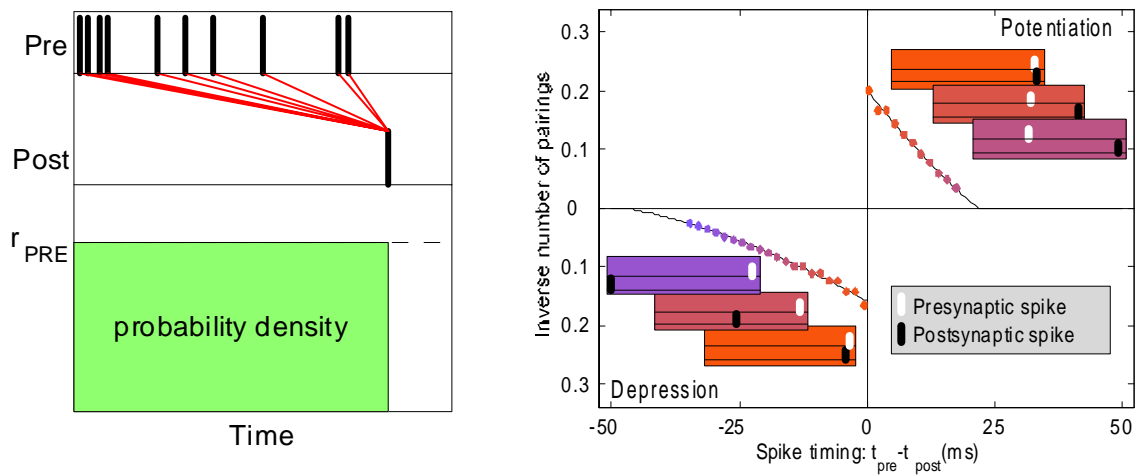
Spike trains generated by dividing time in (Δt) 0.1 ms bins and randomly placing a spike in each bin with a probability proportional to the rate, r ($r\Delta t = 25 \times 0.0001 = 0.0025$).

The interspike interval (ISI) distribution (T_{ISI}) of a Poisson spike train is exponential, i.e., the probability of no spikes arriving for a long period is very low.

$$P[T] = r e^{-rT_{ISI}}$$

Biological spike trains are variable, but usually less than a Poisson spike train (characterized by the CV of their ISIs).

STDP with Poisson Spikes



Poisson spike trains result in a uniform distribution of pre-before-post pairings

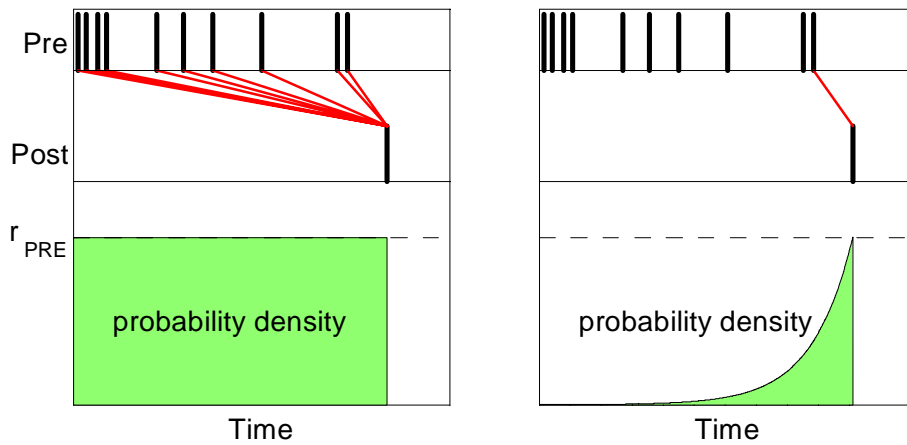
All-to-all pairings results in a uniform timing difference distribution.

To calculate potentiation's efficacy, we add up its contribution at each timing difference: Multiply the timing difference distribution by the STDP curve's LTP side, summing across all timing differences.

For LTD, the process is the same, we add up the contribution of each timing difference, and multiply this distribution by the LTD side of the STDP curve and sum across all timing differences.

In this model, STDP will always favor the side of the STDP curve with greater area under its curve, independent of the pre and post rates.

Latest Neighbor Pairs

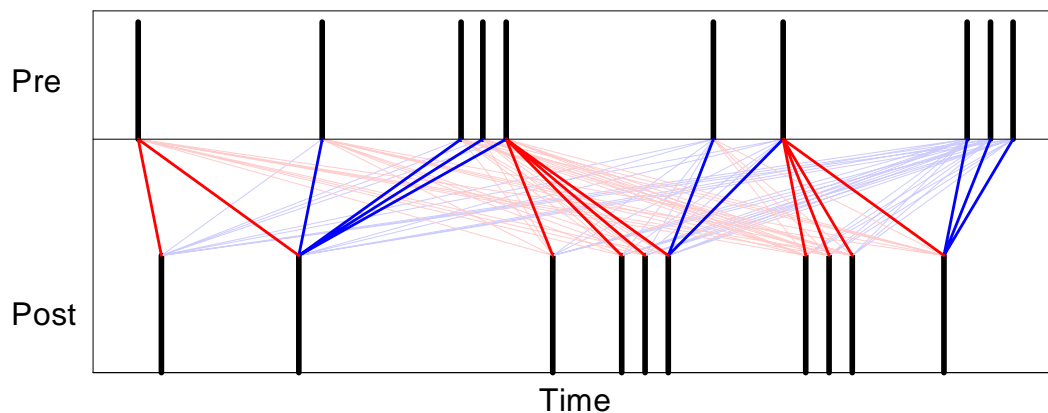


Poisson spike trains result in an exponential distribution of pre-before-post pairings if only the most recent presynaptic spike is considered.

If LTP considers the most recent pre-before-post pairing, the distribution is exponential, decaying with an exponential equal to the presynaptic rate.

Again, we add up the contribution of each timing difference. We multiple this distribution by the LTP side of the STDP curve and sum across all timing differences.

Latest Neighbor Pairs



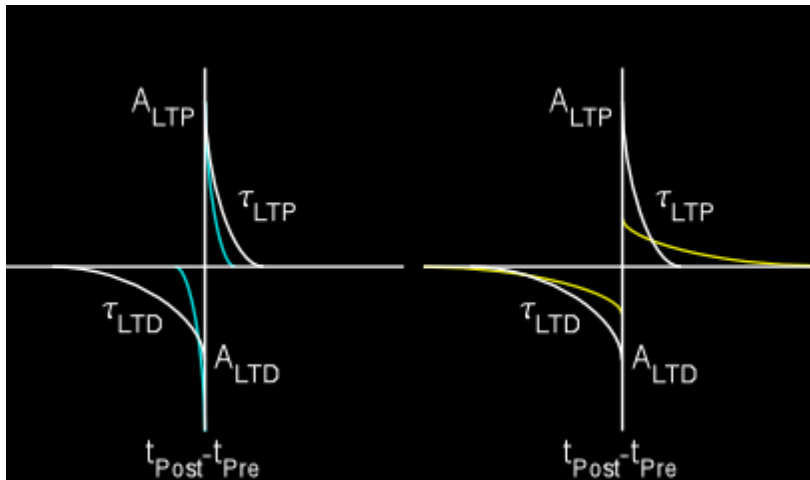
Latest neighbor (thick lines) pairs consider the most recent Pre spike for LTP (red) and the most recent Post spike for LTD (blue).

**Only the most recent Presynaptic spike influences the NMDAR [Ca] influx.
Only the most recent Postsynaptic spike influence the dendritic potential.**

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Latest Neighbor STDP



LTP and LTD for high rates (red probability density) depend on the peaks of the curve, whereas for low rates (blue probability density), they depend on the area under each curve.

**High rates (pre & post) favor LTP because its peak is greater than that of LTD.
Low rates favor LTD since the area under its curve is greater than that of LTP.**

$$r_{\text{LTP}} = r_{\text{post}} r_{\text{pre}} \frac{A_{\text{LTP}}}{\frac{1}{\tau_{\text{LTP}}} + r_{\text{pre}}}$$

$$r_{\text{LTD}} = r_{\text{post}} r_{\text{pre}} \frac{A_{\text{LTD}}}{\frac{1}{\tau_{\text{LTD}}} + r_{\text{post}}}$$

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Latest Neighbor STDP

For high frequency, the side of the STDP curve with the higher peak is favored.

$$r = r_{\text{post}} = r_{\text{pre}}, \quad r \gg \frac{1}{\tau_{\text{LTP}}}, \quad r \gg \frac{1}{\tau_{\text{LTD}}}$$

$$r_{\text{LTP}} = r A_{\text{LTP}}$$

$$r_{\text{LTD}} = r A_{\text{LTD}}$$

$$r_{\text{LTP}} - r_{\text{LTD}} = r (A_{\text{LTP}} - A_{\text{LTD}})$$

For low frequency, the side of the STDP curve with the higher peak and decay constant product is favored.

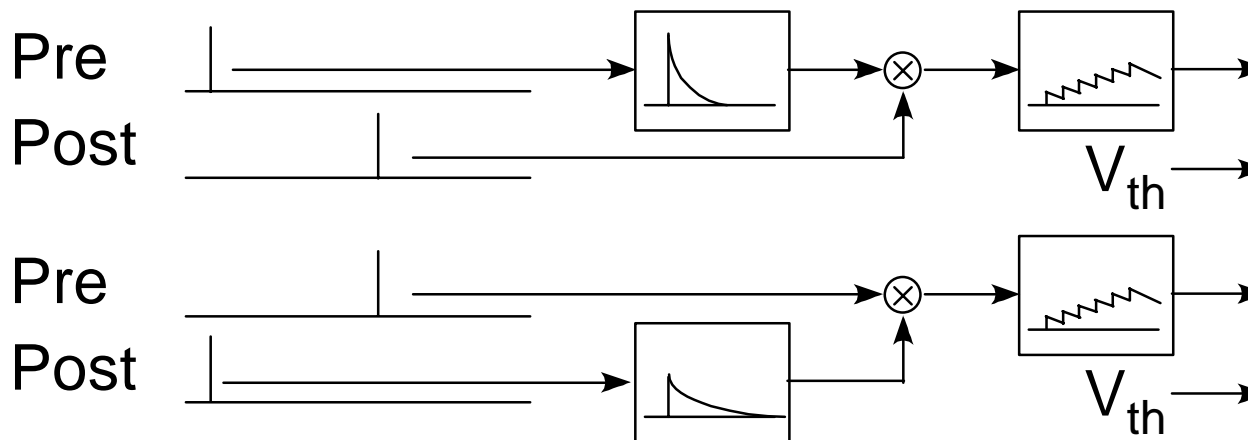
$$r = r_{\text{post}} = r_{\text{pre}}, \quad r \ll \frac{1}{\tau_{\text{LTP}}}, \quad r \ll \frac{1}{\tau_{\text{LTD}}}$$

$$r_{\text{LTP}} = r^2 A_{\text{LTP}} \tau_{\text{LTP}}$$

$$r_{\text{LTD}} = r^2 A_{\text{LTD}} \tau_{\text{LTD}}$$

$$r_{\text{LTP}} - r_{\text{LTD}} = r^2 (A_{\text{LTP}} \tau_{\text{LTP}} - A_{\text{LTD}} \tau_{\text{LTD}})$$

A Model of STDP



The LTP and LTD implementation rely on decay elements and integrators.

The STDP model can be tuned to only consider the most recent presynaptic spike for LTP and the most recent postsynaptic spike for LTD.



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Plasticity versus Rate

All synapses start depressed (or potentiated).

We measure the fraction of synapses that potentiate, P_{LTP} (or depress P_{LTD}) in a period, T , which we approximate as:

$$P_{LTP} \approx r_{LTP} T - r_{LTP} r_{LTD} T^2$$

$$P_{LTD} \approx r_{LTD} T - r_{LTP} r_{LTD} T^2$$

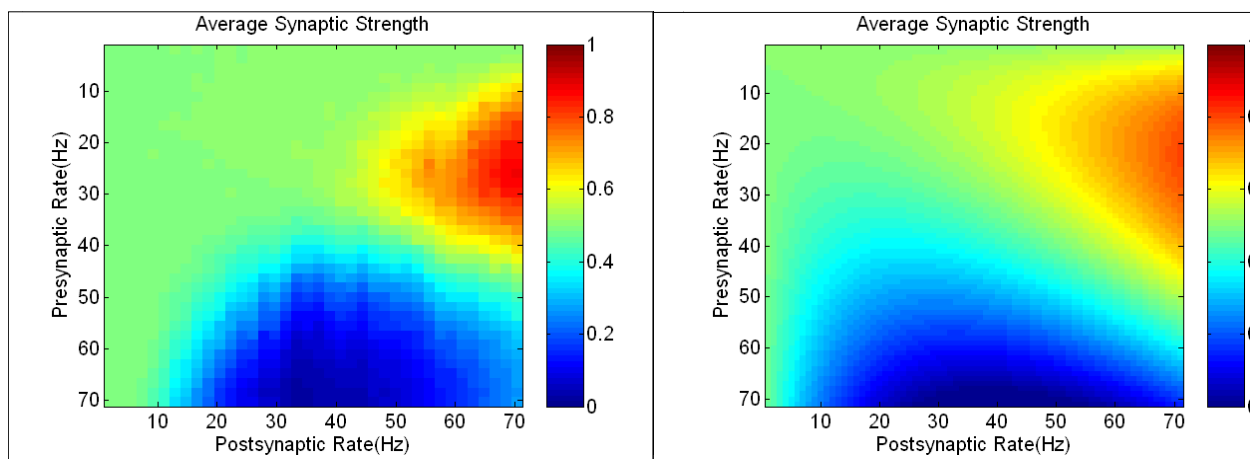
The expected synaptic weight, W , is:

$$W = \frac{1}{2} (P_{LTP} + (1 - P_{LTD})) \approx \frac{1 + (r_{LTP} - r_{LTD}) T}{2}$$



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A Model of STDP



High postsynaptic and low presynaptic rates result in potentiation (red), whereas low postsynaptic and high presynaptic rates result in depression (blue).