

A NON-LINEAR MODEL FOR THE EXPERIMENTAL
MEASUREMENT OF UTILITY

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by

Patrick Suppes and Karol Walsh

1. The Model

The experiment described in this paper was constructed to measure the utility, to an individual, of small amounts of money**, and to predict on the basis of the measured utilities, the individual's future choices with respect to similar amounts of money. The experimental design is based upon previous work done by Davidson, Suppes and Siegel [1], but no familiarity with [1] is presupposed.

The model described here is designed to eliminate a specific criticism raised against the linear-programming model, discussed in Chapter III of [1]. The criticism, which will be discussed in detail after the basic structure of the model has been outlined, is that the linear-programming model leads to a violation of the sure-thing principle, that is, it permits the choice of an action which, whatever the state of nature, is no better than, and for at least one state of nature is not as good as, a second action which is available. Put another way, the purpose of the present paper is to present a non-linear model for measuring utility which has the following desirable property: it is compatible with imperfect discrimination but at the same time incompatible with any violation of the sure-thing principle.

* Herman Rubin contributed substantially to the formulation of the model basic to this paper. The experiment reported here was supported under Contract NR 171-034, Group Psychology Branch, Office of Naval Research.

** The model also applies when outcomes are not monetary.

The structure of the model may be most easily presented in terms of the experimental situation described in more detail in the next section. Seven values (small amounts of money) $a, b, c, d, e, f,$ and g are chosen, in ascending order, and offers are given the subject in the form of a game matrix:

A	B
x	z
y	w

where x, y, z and w range over the selected values with the restriction that $x > z > w > y$. From seven original values, 35 offers of the above form can be generated. (The general combinatorial formula is $\binom{n}{4}$ where n is the number of original values chosen). The subject is instructed to choose a column, A or B, which is called an option, and then the row value is decided by the throw of a die. It is assumed that the subject will choose that option of the two whose expected value is larger. Thus from a subject's choice, say A, an inequality of the form

$$(1) \quad 1/2 u(x) + 1/2 u(y) \geq 1/2 u(z) + 1/2 u(w)$$

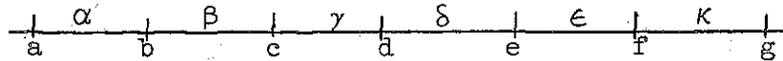
results, where u is the subject's utility function. Inequality (1) says, simply, that the expected value of option A is greater than or equal to the expected value of option B. 35 such inequalities are obtained, one for each choice of an offer. The $1/2$'s cancel out, and for further work the inequalities are converted into equivalent interval inequalities of

the form

$$(2) \quad u(x) - u(z) \geq u(w) - u(y).$$

The quantity $u(x) - u(z)$ represents a "utility interval", the difference to the subject between the utility of the offer x and the next closest value in the inequality, z .

From the seven strictly ordered values a, b, c, d, e, f and g there result six atomic intervals, $\alpha, \beta, \gamma, \delta, \epsilon$ and κ , as described in the figure below:



Thus the inequality

$$(3) \quad u(c) + u(d) \geq u(a) + u(e)$$

may be expressed as

$$(4) \quad u(c) - u(a) \geq u(e) - u(d)$$

and finally as

$$(5) \quad \alpha + \beta \geq \gamma.$$

In general the 35 inequalities resulting from the subject's responses do not have a solution. To obtain a solution for those cases for which there exists no direct one, the larger side of each inequality is multiplied

by the minimum positive quantity η , $\eta \geq 1$, such that a solution of the inequalities exists. Thus all inequalities like (5) are written as

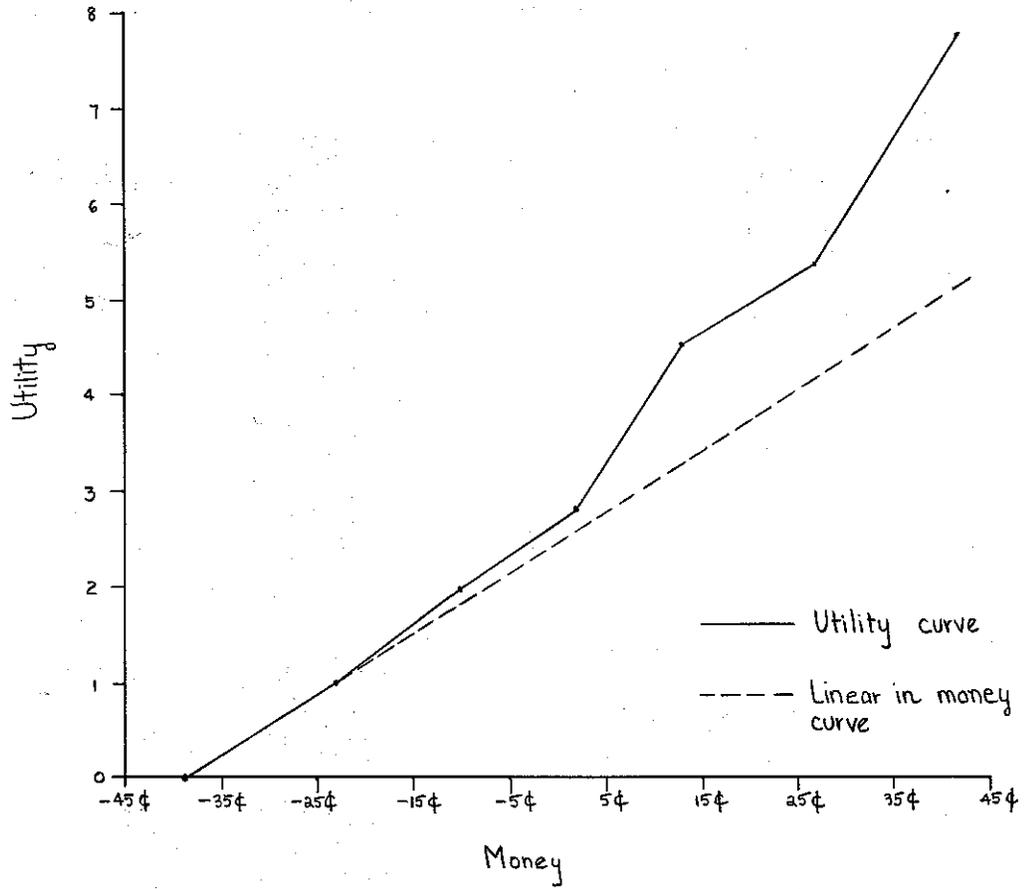
$$(6) \quad \eta\alpha + \eta\beta \geq \gamma .$$

The existence of a unique minimum η is easily proved; in the experiment discussed in this paper for each subject an η was used which was within .1 of the minimum. In finding a solution for the atomic intervals the normalizing restriction was made that at least one interval be equal to one. Unfortunately, as is usually the case with inequalities, the solution is not unique; here the set of solutions is a convex polyhedron. Due to limited computational facilities, the first solution found had to suffice, with the exception of one subject.

Using the interval values obtained, a utility curve is constructed for each subject, with monetary values plotted on the abscissa and utility values on the ordinate. Figure 1, which is the graph for Subject No. 4, is typical.* Using these graphs, predictions are made for another set of alternatives, similar to the first in form but containing many other values besides the original seven. In other words, these graphs provide a means of linear interpolation from the utilities of the seven original values to other values given in the "prediction sessions". The solutions obtained for the intervals α , β , γ , δ , ϵ and κ indicate how far apart the seven given values are in utility (with respect to an arbitrary unit) and from

* However, shapes of the graphs varied considerably from subject to subject.

FIGURE 1. UTILITY CURVE FOR SUBJECT #4



the graph obtained from this information the relative utilities of other values can easily be found.

The essential idea of the model is this: If the ratio of two intervals, say of α to β , where α is larger than β , (or of β to α if β is larger than α) is greater than η , then the option which gives rise to the first α must be chosen. If the ratio of the larger interval to the smaller is less than η , then the choice is said to be probabilistic, that is, it is predictable only in the sense that it satisfies the following two postulates:

Let $x > z > w > y$, and let the options be $A = (x,y)$ and $B = (z,w)$.

Then

(i) if $u(x) - u(z) \geq u(w) - u(y)$, then $P(A) \geq P(B)$,

and

(ii) if $(u(x) - u(z)) / (u(w) - u(y)) \leq \eta$, then $P(A) < 1$,

(where $P(A)$ is the probability that option A will be chosen).

The postulates seem intuitively satisfactory. The first says, simply, that if the utility of option A is greater than or equal to the utility of option B, then the probability that option A will be chosen is greater than or equal to the probability that B will be chosen.

Postulate (ii) says that if the utility ratio of option A to option B is less than or equal to η , then the probability that option A will be chosen is less than one. It is implied that as the ratio gets larger the probability that A will be chosen approaches one.

Before discussing the rationale of making predictions a few remarks

should be made about the non-linear model just outlined. First, an important property of this model is that η is invariant under a linear transformation of the utility function. For this reason it is meaningful to compare the η values of different individuals. If, for example, $\eta_1 < \eta_2$ it makes sense to say that individual 1 is more discriminating than individual 2. In fact, not only more discriminating, but more rational; for it would not be difficult to construct a two-person game in which 1 would have a clear advantage over 2 because of his finer discriminating ability. (A similar kind of example is discussed in detail in von Neumann and Morgenstern [2], pp. 614-616.) For obvious reasons we call η a discrimination parameter. It should be obvious that discrimination is perfect if and only if $\eta = 1$.

Second, it might be remarked that a large η indicates substantial inconsistency in the individual's responses with respect to maximization of expected winnings. Since the model does not take into consideration the possible utility of gambling to the subject but is based on the simple assumption that a subject will always seek to maximize his gains, irrespective of a love of gambling, it might be conjectured that subjects with larger η 's are those with a high utility of gambling and that their choices seem inconsistent because, at times, they fail to maximize expected gains in favor of taking a lucky chance on a temptingly high offer. That is, to be very explicit, when given say, option A: (18, -10) and option B: (7, 5) a subject with a high utility of gambling might sometimes pick A, whose expected utility is 4, instead of B whose expected utility is 6, simply because he enjoys "gambling" on the outcome of the toss of the deciding die being that one with utility of 18.

Third, the non-linear model meets and eliminates the criticism raised against the linear-programming model of violating the "sure-thing" principle. An example may be used to illustrate the problem and show how the non-linear model succeeds where the linear model does not. The essential difference between the linear and non-linear models is that in order to obtain a solution for a set of inequalities the first model adds the minimum constant θ , $\theta \geq 0$, to the larger side of each inequality to make a solution possible instead of multiplying by the minimum positive constant η . Suppose, then, that the linear model is applied, and the following results are obtained (where ϕ is the subject's utility function):

$$\phi(50\text{¢}) = 49$$

$$\phi(45\text{¢}) = 46$$

$$\phi(-45\text{¢}) = -47$$

$$\phi(-50\text{¢}) = -51$$

$$\theta = 8$$

Then since

$$(7) \quad \phi(45\text{¢}) + \phi(-50\text{¢}) + 8 > \phi(50\text{¢}) + \phi(-48\text{¢})$$

the linear-programming model predicts that with probability greater than zero, the subject will choose option B when the options are:

A	B
50¢	45¢
-45¢	-50¢

where the subjective probability of each row is $1/2$. Such behavior seems extremely unlikely, since any person with sense would immediately choose more money (option A) rather than less.

But suppose the non-linear model is applied to the above example. Then if the same absurd result were to hold, it would follow that, for some η , $\eta \geq 1$

$$(8) \quad \eta(\varphi(45\text{¢}) - \varphi(50)) \geq \varphi(-45\text{¢}) - \varphi(-50\text{¢}),$$

or

$$(9) \quad \eta(46 - 49) \geq -47 - (-51)$$

or

$$(10) \quad -3\eta \geq 4,$$

which obviously cannot be true for any positive η . Thus under the non-linear model, it can be concluded that the choice of an option like B above can occur only with probability zero, which result satisfies intuitive demands.

The reason that the Θ solution was applicable to the linear-programming experiment of [1] and not to the present one is to be found in the fact that in the former experiment the items offered as outcomes were phonograph records, not amounts of money, and although some might feel the necessity for it, there is really no objective principle, similar to the one that a rational person must prefer a sure gain of more money to less money, which says that an option consisting of music by Bach and Corelli should be preferred to one consisting of works by Bartok and Stravinsky. However, it might reasonably

be argued against the applicability of the θ -model, even to choices among phonograph records, that if a subject strictly orders six records in preference a, b, c, d, e and f (f the highest) and if options (e, f) and (c, d) are offered him, then he should be judged just as irrational in choosing (c, d) as in choosing the corresponding $(45¢, -50¢)$ as opposed to $(50¢, -45¢)$.

Following the model, three kinds of predictions are made: (1) using η , (2) without η and (3) stochastic, or probabilistic, predictions. For illustration, again let x, y, z and w be amounts of money such that $x > z > w > y$ and let option A be (x, y) and option B be (z, w) . Predictions of the first kind are performed as follows: The ratio $(u(x) - u(z))/(u(w) - u(y))$ is found, call it α/β . Then the three mutually exclusive and exhaustive possibilities are:

- (1) if $\alpha/\beta > \eta$, option A is predicted,
- (2) if $\beta/\alpha > \eta$, option B is predicted,
- (3) if $\alpha/\beta \leq \eta$, and $\beta/\alpha \leq \eta$, then the prediction is probabilistic.

Predictions of the second kind, again using the ratio α/β , fall into the following three categories, also mutually exclusive and exhaustive:

- (1) if $\alpha/\beta > 1$, option A is predicted,
- (2) if $\beta/\alpha > 1$, option B is predicted,
- (3) if $\alpha/\beta = 1$, then the choice is indeterminate.

Predictions of the third kind are made for those cases in which the ratios

α/β and β/α are less than or equal to η . Each η is partitioned into three intervals*:

$$[1, 1 + 1/3(\eta - 1))$$

$$[1 + 1/3(\eta - 1), 1 + 2/3(\eta - 1))$$

$$[1 + 2/3(\eta - 1), \eta]$$

and the relative frequency of correct predictions to the total number of predictions in each interval is computed. (The choice predicted in each case is the option having the greater utility.) It is expected, as a result of the probabilistic postulates, that each successive interval will contain a larger number of correct predictions than the preceding one. In other words, as the ratio of α to β becomes larger (within η) it is assumed that the option with the greater utility will have a greater probability of being chosen.

2. ** Experimental Procedure

The subjects were eight randomly chosen sailors from Moffett Field Air Base. At the beginning of the first session the subject was told that he was being asked to participate in a "gambling" experiment whose purpose was to give some insight into how people make choices involving various offers of money. The subject was given \$2.00 credit to assure him that he would not lose any money in the experiment. He was told that he would be

* The choice of three intervals rather than some other number was determined by the number of observations available.

** For a detailed description of a similar experimental design see Chapter II of [1].

allowed to keep whatever amount of money he won during the six sessions of the experiment and that in previous runs the average profit to a subject for two hours of play (six sessions)* was about \$2.50.

The subject was assured that the experiment was strictly descriptive, i.e., that its purpose was to indicate how people do act under certain conditions, not to measure, according to some norm, the goodness or badness of the particular subject's actions. It was remarked, also, that the results of the experiment would remain confidential, and each of the subjects was asked not to discuss the experiment with any other person who might also be participating in it, since each person was to be run separately, and independent results were desired.

After these general remarks had been made, the subject was given a trial set of alternatives similar to the following example:

	A	B
ZOJ	6¢	-5¢
ZEJ	-4¢	6¢

He was told to pick column A or column B. Then a die with nonsense syllables ZOJ and ZEJ each on three sides was tossed from a shaker box by the subject to determine the actual outcome. This particular die was

* Actually the results of only four sessions, the first two and the last two, were used to test the above-described model; results of the other two sessions were for a utility of gambling model.

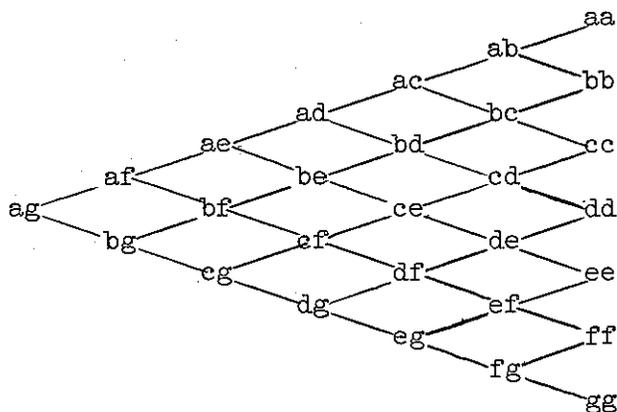
systematically alternated with two others having different nonsense syllables on their faces.

Several trial alternatives similar to the above example were given to make certain that the subject understood the game and that he had a subjective probability of $1/2$ for the chance events ZOJ and ZEJ (for the latter hypothesis choice of option A was required in the above example). Then the subject was asked if, in order to save time, he would not like to run through the rest of the alternatives (these were the real experimental ones) choosing all his options, A or B, first and tossing for his wins later. He was told that he would not be able to toss for all the options presented--this would take too long--but that he would be allowed to pick ten numbers at random from an envelope containing as many numbers as there were offers in the session, and that he could roll the die for these ten options. Thus the six sessions were run, the subject choosing his options first and tossing for them later. All eight subjects returned for their six sessions.

So much for the actual mechanics of the experiment. A word now about the values offered and how these were determined. The amounts $-39¢$, $-23¢$, $-10¢$, $2¢$, $13¢$, $27¢$ and $42¢$ were the values chosen to be paired in various combinations of offers for the first two sessions. The first two sessions formed the part of the experiment from which the utility curves and discrimination parameters were obtained. The fifth and sixth sessions formed the part whose choices of options were predicted on the basis of information obtained from the first part.

The options of the first two sessions were chosen in the following manner: The letters a, b, c, d, e, f and g were selected to represent

the values $-39¢$, $-23¢$, $-10¢$, $2¢$, $13¢$, $27¢$ and $42¢$ respectively. Then a lattice was constructed as follows on the basis of transitivity and the sure-thing principle:



and each pair of values on the lattice was combined with all other pairs of values which could not be reached by an ascending or descending line from the pair in question. (For example, ad was taken with bc and be to form the alternatives $(-39¢, 2¢)$ and $(-23¢, -10¢)$, and $(-39¢, 2¢)$ and $(-23¢, 13¢)$.) All "non-gambles", that is, offers of the form: ad with bb or ad with cc were eliminated. Seven randomly chosen offers were repeated, and all offers, totalling 42, were presented in random order to the subject. The 70 pairs of options in the "prediction" part of the experiment were composed in a manner similar to the first--no option dominated, or was greater than or equal in both values to both values of its companion--but many more values were used.

3. Results

In Table I(a) are recorded the actual number of utility predictions

which were correct, within η , and wrong for each subject. There are only 53 wrong, out of 560, as opposed to an expected 280 if predictions had been made by chance, and there are 128 correct. As might be anticipated, subjects with smaller η 's (Ss 1,2 and 4) had the greater number of right predictions. This outcome agrees with the interpretation of η as a measure of discrimination. In Table I(b) are summarized the results of predictions made without η but using the utilities read from the subjects' graphs (type 2 predictions, described earlier).

TABLE I. PREDICTIONS MADE USING SUBJECT'S UTILITY FUNCTION

Subject	η	(a): with η			(b): without η		
		Correct Predictions	wrong within η	Wrong Predictions	Correct Predictions	Equal Intervals	Wrong Predictions
1	1.4	38	30	2	61	1	8
2	1.0	35	14	21	35	14	21
3	1.9	8	57	5	31	3	36
4	1.2	42	9	19	45	0	25
5	1.9	4	65	1	45	1	24
6	1.5	0	67	3	29	4	37
7	1.62	0	70	0	28	8	34
8	1.9	1	67	2	32	1	37
TOTALS		128	379	53	304	32	224

TABLE II. PREDICTIONS MADE USING ACTUARIAL VALUES

Subject	(a): with η			(b): without η		
	Correct Predictions	within η	Wrong Predictions	Correct Predictions	Equal Intervals	Wrong Predictions
1	1	69	0	31	14	25
2	35	14	21	35	14	21
3	0	70	0	32	14	24
4	3	61	6	30	14	26
5	0	70	0	33	14	23
6	0	70	0	35	14	21
7	0	70	0	32	14	24
8	0	70	0	29	14	27
TOTALS	39	494	27	257	112	191

The results of Table I are compared to what are called "actuarial" predictions, recorded in Table II. These predictions are computed in the same way as the utility predictions made with and without η (Table II(a) and Table II(b)), using as "utility values" the offered amounts of money themselves instead of the corresponding utility values interpolated from the graphs. For example, for the following offer A: (2¢, -10¢) and B: (17¢, -23¢), the differences

$$-10 - (-23) = 13 \quad \text{and} \quad 17 - 2 = 15$$

instead of

$$\mu(-10) - \mu(-23) \quad \text{and} \quad \mu(17) - \mu(2)$$

would be computed and the ratio $15/13$ compared to η , if the prediction were to be made using η , and compared to 1 if the prediction were to be made without η .

It is immediately noticeable that the use of actual monetary values as utilities fails to provide a sensitive enough scale for effective predictive results. The data of Table II(a) show that almost all predictions fall within η , (all but 10 in fact), if subject #2 is excluded. Further, a comparison of Table I(b) to Table II(b) shows that the utility model, without using η , yields 304 correct predictions to the actuarial model's 257, and 224 wrong predictions to the actuarial model's 191. This is a gain of 47 in correct predictions as opposed to a loss of 33 in wrong predictions.

In spite of the sizable discrimination parameters of some of the Ss ($\eta = 1.9$ for Ss #3, 5, 8), only one of the eight (#2) had a utility function which was linear in money. It is slightly puzzling that #2 gave perfect actuarial responses in the first two sessions but not in the final two predictive sessions, as may be seen from Table II.

For a more careful comparison of the utility and actuarial models it is natural to consider the prediction data of Tables I(b) and II(b), since the actuarial model made essentially no clear predictions when the discrimination parameter was considered. We want to test the null hypothesis

that the predictive worth of the two models is the same when η 's are ignored. The basic idea of the test we use is to tabulate the predictive disagreements of the models for each subject, then to find for each subject the significance level for rejecting the null hypothesis, and finally to combine these significance levels and apply a χ^2 test. Appropriate references to the literature for this kind of analysis are Mosteller and Bush [3] and Moses [4]. Since such tests are not completely standard, we give a certain amount of detail.

For each subject the 70 predictions for the two models were compared. Offers for which the models made the same prediction were deleted. Any remaining offer for which the utility model predicted the correct choice and the actuarial model predicted the incorrect choice or a tie was assigned a plus (+). An offer for which the utility model predicted a tie and the actuarial model a loss was also assigned a plus. Offers for which the roles of the actuarial and utility models were reversed in the above description were assigned a minus (-). The data are tabulated in Table III.

TABLE III. PREDICTIVE DISAGREEMENT OF
UTILITY AND ACTUARIAL MODELS

Subject	Plus (utility)	Minus (Actuarial)	Number of runs
1	35	8	15
2	-	-	-
3	10	16	12
4	27	18	21
5	20	14	14
6	14	26	19
7	11	21	15
8	15	19	17

Let p be the probability of a plus. For each subject an exact significance level was found for the null hypothesis that $p = .5$ against the alternative that $p > .5$. A one-sided test was used to provide a clear basis for combining significance levels. (A two-sided test does not properly take account of varying directionality of results.) A similar one-sided test was run against the hypothesis that $p < .5$.

The seven significance levels for each one-sided test were now combined. Since the distributions which gave rise to the individual significance levels $p(X_i)$ are not continuous, rather than use Fisher's statistic

$$(1) \quad \sum_{i=1}^7 -2 \log p(X_i),$$

following the suggestion of Moses [4], Lancaster's statistic

$$(2) \quad \sum_{i=1}^7 -2 \log \left(\frac{p(X_i) + p(X_i + 1)}{2} \right)$$

was used, where X_i is the observed number of +'s, and $X_i + 1$ is one more than the observed number of +'s. Statistic (2) is approximately distributed as χ^2 with 14 degrees of freedom.

The results are as follows. Against the hypothesis that $p > .5$, $\chi^2 = 32.48$, which for a two-sided test is significant at the .01 level. Against the hypothesis that $p < .5$, $\chi^2 = 20.98$ which (for a two-sided test) is significant at the .2 level. Two-sided tests were used on the combined results because there is no real a priori basis for fixing on a directionality. The combined significance levels obtained from the two

tests indicate clear predictive superiority for the utility model. The unusually good predictions for subject #1 are the main factor in establishing this superiority. It is worth remarking that if both tests had resulted in a significance level $\leq .05$ the proper interpretation would seem to be that subjects should be divided into two subsets according to which model they most nearly satisfy, and no assessment of the over-all relative predictive worth of the two models should be attempted.*

The most obvious objection to the statistical analysis just made is that the 70 prediction observations on each S are surely not independent. On the other hand, since the 70 offers were all different, strong negative or positive recency effects should not be observed. Speculations aside, one partial test of independence is the number of runs of +'s and -'s. If Ss are using some computational or strategic device over several trials (i.e., offers), one would expect the number of runs to be less than would be anticipated on a random basis. For each S we tested the null hypothesis that the +'s and -'s are randomly distributed against the hypothesis that there are too few runs. The observed number of runs for each S is given in Table III. The significance levels obtained were combined, using statistic (2) described above. The obtained result was $\chi^2_{14} = 10.68$, which for a one-sided test is significant at the .75 level. This result supports the hypothesis of independence.

In Table IV are found the results of checking the two probabilistic

* If an external criterion can be found to provide a foundation for dividing Ss into two appropriate subsets, the comparison is more interesting. An example of this will be given in a subsequent paper reporting on a utility of gambling model.

postulates on the discrimination parameter. Unfortunately, the data do not support these postulates. The predictions in Table IV were made by dividing the interval ratios which fell within η for each subject into three sets in the manner previously described. The larger utility interval (the

TABLE IV. PROBABILISTIC PREDICTIONS

Subject	$[1, 1+1/3(\eta-1))$	$[1+1/3(\eta-1), 1+2/3(\eta-1))$	$[1+2/3(\eta-1), \eta]$
1	.70	.66	.91
2	-	-	-
3	.63	.88	.50
4	.50	-	.33
5	.39	.50	.50
6	.47	.35	.50
7	.48	.44	.50
8	.44	.59	.50

numerator, always, in these ratios) was predicted as the subject's choice. In each cell of Table IV is recorded the ratio of correct predictions to the total number falling within the indicated interval. The result expected and desired was a rise from .5 (which would be anticipated for chance) in the lowest interval $[1, 1+1/3(\eta-1))$ to some frequency close to 1 in the third interval, since at this point the numerator would be almost large

enough to make the utility ratio greater than η and hence lead to sure prediction. But only subject #1 comes close to giving an indication of the result expected, with values of: .7, .66, .91. 13 out of the 21 cell ratios were found to be greater than .5, which figure can be accounted for on the basis of chance alone.

It is worth noting that a pair of similar probabilistic postulates for the linear programming model described in Chapter III of [1] were more satisfactorily verified.

4. Summary

This study reports an experiment which tested a non-linear programming model for the measurement of utility. The model is designed to be compatible with imperfect discrimination, but incompatible with any violation of the sure-thing principle.

The Ss were eight sailors from Moffett Field Air Base. Each S participated in six sessions, which primarily consisted of choosing between pairs of options having small gains or losses of money as outcomes, depending on the throw of a die. Choices made in the first two sessions generated 42 inequalities from which a utility function was constructed. This function was used to predict choices for 70 offers of pairs of options in the last two sessions.

The predictive worth of the utility model was compared to the actuarial model (maximization of expected money value) and found to be significantly better on the basis of a χ^2 test which combined individual significance levels for each S.

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