

TWO FORMAL MODELS FOR MORAL PRINCIPLES

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TWO FORMAL MODELS FOR MORAL PRINCIPLES

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Patrick Suppes

1. Introduction. The present paper offers an analysis of moral principles from a viewpoint foreign to the thinking of most philosophers primarily concerned with ethics. Certainly the attempt to force the discussion within a given formal model runs counter to the dominant Oxford trend in moral philosophy. My own contention is that ethics should not primarily be the logical study of the language of morals. The results would have been paltry indeed, if seventeenth century physicists had regarded mechanics as being mainly the logical study of the language of artisans and engineers. The relative barrenness of scholastic physics may be attributed to the doctrine that the study of the language of Aristotle and his commentators was the main business of natural philosophy.

To hold that the fundamental problems of ethics are conceptual and substantive, rather than linguistic, is not to hold that academic philosophers should become moral casuists resolving all issues of right and wrong for their fellow man. Economists concerned with welfare economics do not consider it their proper task to say exactly how the welfare state should be organized. What they do is to investigate the precise formulation and consequences of various alternative assumptions. Yet the assumptions analyzed are not derived from consideration of the subtle uses of words like 'happiness', 'freedom' or 'democracy' in ordinary language. In fact, many of the terms of welfare economics have only been introduced to name new concepts invented in the course of analyzing the manifold problems of efficient production and equitable distribution. I believe moral philosophers can operate in the same

way. The mistaken notion is widely held that the plain man is really clear about practical moral issues and that philosophers need only tidy up certain wayward corners of the subject.*/ Personally I find difficult the problem of devising any general ethical rules of behavior for simple two-person games; the ethical complexities of progressive taxation, tariff barriers, or treatment of sexual psychopaths are beyond any exact conceptual analysis. That decisions are and must be made about these issues no more proves that their moral aspects are completely understood than does the fact that the Romans built bridges prove that they had any quantitative grasp of the branch of mechanics known as the theory of statics.

Finally, it is pertinent to remark that the first model used in this paper is at the basis of much recent foundational work in the most sophisticated normative science, namely, mathematical statistics (see Blackwell and Girshick [2] and Savage [11]). The considerations in the last two sections are within the more general framework of the theory of games as developed by von Neumann and others. My particular concern is the embedding in this framework of a theory of two-person justice.

2. Individual Decision Model. The structure of the first model to be considered is simple. We shall call an ordered triple $\mathcal{J} = \langle S, C, D \rangle$ an individual decision situation when S and C are sets and D is a set of functions mapping S into C . The intended interpretation is:

*/ Kant's views are typical: "... in matters which concern all men without distinction, nature cannot be accused of any partial distribution of her gifts; and that with regard to essential interests of human nature, the highest philosophy can achieve no more than that guidance which nature has vouchsafed even to the meanest understanding." [5], p. 666.

S = set of states of nature

C = set of consequences

D = set of decisions or actions.

Since the terms 'states of nature', 'consequences', 'decisions' and 'actions' are used here in a somewhat special manner, an example may help to make clearer their intended meaning.

Example 1. Suppose I come home and find a bottle of ink spilt on the rug. I know immediately that it could have been spilt either by my four year old daughter or by my cat. These two possibilities correspond to the two states of nature. I can take one of two actions, let us say: spank the child or do not spank the child. And the possible consequences are four in number, as illustrated in the following table. The rows correspond to the two states of nature, the columns to the two actions, and the entries in the table to possible consequences.

Table 1

states of nature \ actions	a ₁ - spank the child	a ₂ - do not spank the child
s ₁ - child spilt the ink	c ₁ - ink spilt by child and child spanked	c ₂ - ink spilt by child and child not spanked
s ₂ - cat spilt the ink	c ₃ - ink spilt by cat and child spanked	c ₄ - ink spilt by cat and child not spanked

Since the term 'states of nature' is not much used in moral philosophy there should be little objection to its special use here; the term 'action' is used in a way that is consonant with at least one of its major uses in ordinary contexts. But without question my use of 'consequence' is at variance with its primary uses in the writings of moral philosophers. The consequence c_1 above, for instance, ink spilt by child and child spanked, would be regarded by many as the bare beginning of consequences. It is to avoid exactly the vagueness of the consequence flowing from c_1, c_2, c_3 or c_4 , that I have adopted the restricted use. The longer term 'immediate consequence' could be used. Yet in ordinary usage there is much to defend the use adopted here when a quarterback throws an intercepted pass in the last two minutes of play it might be appropriate to remark "The consequence of that is obvious. We lose the game." It would seem pedantic to insist on saying "The immediate consequence of that is obvious. We lose the game." And it would be a classroom gambit to object that the use of the definite article is wrong, because the action could have other important consequences for the quarterback: he quarrels with his girl that night, the coach decides not to start him in the next game.

Apart from any questions of ordinary usage there is a technical device which may be used to meet the difficulty that it is almost always impossible to characterize the full set of consequences which may flow from an action. Given an individual decision situation $\langle S, C, D \rangle$, let C' be the set of all consequences which may result from some state of nature in S and some decision in D . Then C is a partition of C' , that is C is a family of non-empty pairwise disjoint sets whose union is C' . In this analysis, each c_i in our example, is a set of consequences. It is practically

impossible to say exactly what the members of c_1 , say, are, but in rough terms they are the possible consequences, proximate and remote, which would wholly or in part result from the immediate consequence of the ink being spilt by the child and the child being spanked.

One of the basic problems of statistical decision theory is to introduce a preference ordering on the set of decisions of an individual decision situation and to consider what postulates the preference ordering of a reasonable man should satisfy. (For such an analysis see Savage [11] or Suppes [12].) The notion of reasonableness or rationality used here is an informal, intuitive one, and its application in defense of any particular postulate consists of analyzing particular examples. The problem is presumed solved if reasonable postulates can be found which are strong enough to guarantee the existence of a (subjective) probability measure on the states of nature and a utility function on the set of consequences such that one decision is to be preferred to another if and only if the expected utility of the first decision with respect to the probability measure is greater than that of the second. Once such a probability measure and utility function are constructed no further principles of action are needed. The sole maxim to be followed by the rational man is: maximize expected utility.

Historically the idea of maximizing utility is closely connected with the hedonistic ideas of Bentham, Mill, Sidgwick and their followers. However, it is an unequivocal mistake to think that the maxim: maximize expected utility, in any respect involves a commitment to hedonism. As I hope to make clear in the sequel if the utility function on consequences were guided by an ethic of duty rather than pleasure, it would still be

good advice to maximize expected utility. In this case a calculus of duty would replace a calculus of pleasure. To my mind the most important aspect of the hedonistic tradition in ethics has been the clear recognition that some principle of calculation is required for rational action in the face of other than trivial situations. The main point of this paper is to defend a thesis as to how moral principles should enter into these calculations.

Before developing these ideas further I want to say something about a major criticism that is usually made of the general maximization viewpoint adopted here. To wit, as one philosopher scornfully put it to me, who ever heard of a man making such calculations prior to making any actual decision. Naturally this philosopher had in mind the "ordinary" man in "ordinary" situations like that of buying a pint of whiskey or selecting a new tobacco. One might as well reject a whole discipline such as the physical theory of the strength of materials by remarking that no carpenter computes the load capacity of a joist before sawing and nailing it. There are situations where elaborate calculations are made in order to maximize utility; the new disciplines of management science and operations research provide numerous examples.*/ Moreover, I maintain that in many ordinary situations it is not the impossibility of detailed calculation that is relevant but rather the superfluity of it. For instance, in the simple situation schematized by Table 1, if it is definitely known that the ink

*/ In this respect it seems unfortunate that in his inaugural lecture Theory of Games as a Tool for the Moral Philosopher Professor Braithwaite picked for detailed analysis an example which would not in practice be subject to elaborate calculations. His painstaking careful presentation would apply equally well to more realistic labor-management bargaining situations.

was spilt by the child and not the cat then to take appropriate action I need only order in preference two consequences: c_1 and c_2 , according to my principles of childrearing. I need no numerical utility function. And this situation is characteristic: whenever uncertainty regarding the true state of nature is eliminated, the pertinence of a numerical utility function disappears, and the principle of maximizing expected utility assumes a very simple form: choose that action whose consequence is most preferred (for reasons of pleasure, duty, justice or what have you).

3. Definition of Moral Principles. Traditionally in ethics actions are said to be right or wrong, and consequences good or bad. If we carried over this distinction to individual decision situations then we would need moral principles governing acts and value principles arranging consequences in order of preference. But I am proposing here that the one controlling principle of action is the maxim: maximize expected utility. (Hereafter referred to as the M.E.U. maxim.) On this view it is a mistake to hold that moral principles aid us directly in distinguishing between the quality of acts. The function of moral principles is rather to aid the individual in constructing his preference relation on the set of consequences.*/ There is a simple reason why this position is not in conflict with most of the standard examples purporting to show how moral principles should regulate actions; namely, if the state of nature is known, there is an effective one-one correspondence between the set D of acts and the set C of

*/ However, second-order moral principles as ethical rules of behavior directly governing acts are introduced in the final section.

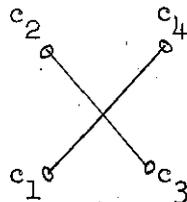
consequences, and any relation on C defines a corresponding relation on D . This point is further amplified below.

To put it baldly then, I am claiming that the proper logical status of a moral principle in an individual decision situation is as a binary relation on the set C of consequences, in fact, an asymmetric, transitive relation on C , i.e., a strict partial ordering of C .

Definition 1. Let $\mathcal{J} = \langle S, C, D \rangle$ be an individual decision situation. Then a moral principle with respect to \mathcal{J} is a strict partial ordering of C .

I have insisted that a moral principle have at least the properties of a strict partial ordering, for otherwise it would scarcely be a guide to fixing the preference relation.

Example 2. A principle of childrearing. Referring to Example 1, a tenable moral principle held by some modern parents is: never punish a child. This leads to the following strict partial ordering of C ,^{*/} which we may represent by a Hasse diagram:



It should be noted that this principle of childrearing is sufficient to

^{*/} The intuitive idea behind a Hasse diagram is simple: if point x may be reached from point y by a continually ascending, not necessarily straight line, then xMy .

determine action although the set of consequences is not completely ordered by it. For, whatever the true state of nature, the consequence of taking action a_2 is preferred to the consequence of taking action a_1 , that is, c_2 is preferred to c_1 and c_4 is preferred to c_3 .

As the following example drawn from welfare economics shows, most moral principles are not sufficient to determine action.

Example 3. Principle of unanimity. Suppose that the decision situation consists of an arbitrary set S of states of nature, and C is a set of ordered n -tuples (n -dimensional vectors) representing the distribution of some desired commodity to a group of n individuals. Administrator A , a member of the group, is to decide in a just manner which distribution vector is to be used in allotting the quantities of the commodity. The moral principle of unanimity asserts that vector $x = \langle x_1, \dots, x_n \rangle$ is to be preferred to vector $y = \langle y_1, \dots, y_n \rangle$ if for every $i=1, \dots, n$, $x_i \geq y_i$ and for some i , $x_i > y_i$. This principle, also known as the principle of efficiency or Pareto optimality, is a very weak moral principle and surely any administrator who did not satisfy it would be stoned out of office.*/ It is obvious that in general the principle of unanimity does not uniquely determine the optimal action even when there is only one state of nature.

More troublesome, at least from a psychological standpoint, is the decision situation in which two moral principles are in conflict. This state of affairs is reflected formally in our model by the fact that the

*/ The proof is immediate that the principle of unanimity yields a strict partial ordering of C .

union of two strict partial orderings is not always a strict partial ordering.

Definition 2. Let $\mathcal{J} = \langle S, C, D \rangle$ be an individual situation, and let M_1 and M_2 be moral principles with respect to \mathcal{J} . Then M_1 and M_2 are compatible if, and only if, $M_1 \cup M_2$ is a moral principle with respect to \mathcal{J} .*/

Two simple conditions with reasonable interpretations which will insure compatibility of moral principles are the following.

Theorem 1. If (i) M_1 is a subrelation of M_2 or M_2 is a subrelation of M_1 , or (ii) if the fields of M_1 and M_2 are mutually exclusive, then M_1 and M_2 are compatible.

The proof of this theorem is trivial. Some examples illustrating it may be drawn from welfare economics, where S and C are defined as in Example 3.

Example 4. Let

M_I = principle of unanimity,

M_{II} = principle of gross aggregation

M_{III} = principle of social weights $a = \langle a_1, \dots, a_n \rangle$,

where

$$x M_{II} y \text{ if, and only if, } \sum_{i=1}^n x_i > \sum_{i=1}^n y_i$$

and

*/ The symbol \cup denotes the union of two sets. A binary relation is a set of ordered couples, whence we may speak of the union of two relations.

$$xM_{III}y \text{ if, and only if, } \sum_{i=1}^n a_i x_i > \sum_{i=1}^n a_i y_i .$$

Principle M_I has already been discussed; Principle M_{II} says that one distribution x is preferred to another y if x results in a greater total quantity of the commodity for the social group; Principle M_{III} corresponds to the assignment of weights to each individual by Administrator A; presumably A would use some further principle of need or merit to aid in determining the weights.*/ As application of Theorem 1, we have that M_I and M_{II} are compatible, since M_I is a subrelation of M_2 , that is, if $xM_I y$ then $xM_2 y$. To see this, we observe that if $xM_I y$ then

$$(1) \quad \begin{aligned} x_i &\geq y_i && \text{for all } i \\ x_i &> y_i && \text{for some } i , \end{aligned}$$

whence

$$\sum x_i > \sum y_i .$$

Moreover, if each individual is given a strictly positive weight, that is, $a_i > 0$ for all i , then M_I is a subrelation of M_{III} , and hence compatible with it. The reasoning is obvious. From (1) and the hypothesis that $a_i > 0$ we have:

*/ In the literature of socialist economics, Administrator A is often the Central Planning Board, but a bureaucratic assignment of weights is not essential to the economic theory of the welfare state (cf. Lange [7]).

$$a_i x_i \geq a_i y_i \quad \text{for all } i$$

$$a_i x_i > a_i y_i \quad \text{for some } i,$$

whence by addition of inequalities

$$\sum a_i x_i > \sum a_i y_i .$$

On the other hand, when C has any abundance of different distribution vectors, M_{II} and M_{III} are incompatible. For instance, let $n=3$ and

$$x = \langle 1, 2, 4 \rangle$$

$$y = \langle 4, 1, 1 \rangle$$

$$a = \langle 2, 1/2, 1/4 \rangle .$$

Then

$$xM_{II}y$$

since

$$\sum x_i = 7 \quad \text{and} \quad \sum y_i = 6 ,$$

but

$$yM_{III}x$$

since

$$\sum a_i x_i = 4 \quad \text{and} \quad \sum a_i y_i = 8\frac{3}{4} .$$

Examples which satisfy (ii) of Theorem 1 are easy to construct but will not be considered here. The intuitive idea of (ii) is the truism that moral principles concerned with entirely different spheres of activity are

compatible.

The use of the word 'activity' in the last sentence underlines the difficulty of not speaking of moral principles as referring to acts or decisions rather than consequences. Before turning to social decision situations in the next section, something more needs to be said about the status here advocated for moral principles. One natural tendency is to formulate moral principles in the imperative mood so as to command the execution of certain acts. But Hare ([4], Part III) has cogently argued it is more appropriate to use the indicative mood and the auxiliary verb 'ought' to obtain the proper sort of universal formulation. The one further emendation required here is to add the infinitive 'to prefer' after 'ought'. Thus, we go from the imperative:

Honor thy father

to:

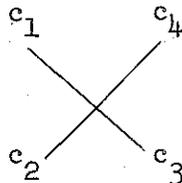
Everyone ought to honor his father,

and on to:

(1) Everyone ought to prefer to honor his father.

To recapitulate, my arguments for (1) are twofold. Positively, I hold that the maxim: maximize expected utility is the one controlling principle of action. Unfortunately, a full-scale defense of this maxim would be distracting here, and detailed discussions already exist in the literature (see particularly Arrow [1] and Savage [11]). The core of any such defense is to produce reasonable axioms of rational behavior satisfaction of which guarantees satisfaction of the maxim. Negatively, I maintain that ordinary

usage addresses moral principles directly to acts because the problem of acting without knowing the true state of nature is ignored. This point is important enough to be amplified by referring again to Example 1. Consider the moral imperative 'Punish the guilty and defend the innocent'. Suppose this is the only moral imperative guiding my choice of action a_1 or a_2 in Example 1. It seems patently obvious to me that without knowing the true state of nature I can make no direct application of the imperative to choose between a_1 and a_2 . If s_1 is the true state of nature I should choose a_1 , but if s_2 is the true state, I should choose a_2 . To be sure, I could first sum up the factual evidence for s_1 and s_2 , decide which is more likely, assume the more likely state is nearly certain to be the true state, and then take the appropriate action. But this is surely a crude way to proceed and is wholly inadequate in more complicated situations; for instance, suppose there were three states of nature to each of which I assigned a subjective probability of $1/3$. On the other hand, the imperative may be applied directly to constrain my preference relation on the set $\{c_1, c_2, c_3, c_4\}$ of consequences. The Hasse diagram of the resulting strict partial ordering is obviously:



which may be compared with the diagram for Example 2. When applied directly to consequences, application of the imperative need not be confounded with the difficult and distinct problem of weighing factual evidence regarding

the true state of nature.

The particular homily about honoring fathers illustrates another point: it and all principles of a similar form lead to a simple and crude partial ordering of consequences, namely, consequences are divided into two classes and all members of one are preferred to all members of the other. As Examples 3 and 4 emphasize such simple-minded moral principles are not of much help in making a rational decision in a complicated situation like that generated by a labor-management dispute or the problem of pricing policy in a controlled or semi-controlled economy.

4. Social Decision Model. Against the analysis of previous sections may be brought the charge that the individual decision model unduly and unrealistically isolates the moral behavior of one man from another. In the remainder of this paper social situations shall be considered. For reasons of technical simplicity the discussion shall be restricted to two persons, although most of the concepts introduced readily generalize to n persons.

The structure of the basic model is still relatively simple. We shall call an ordered sextuple $\mathcal{J} = \langle S, C_1, C_2, D_1, D_2, f \rangle$ a two-person decision situation when S, C_1, C_2, D_1, D_2 are sets and f is a function mapping the Cartesian product $S \times D_1 \times D_2$ into $C_1 \times C_2$. The intended interpretation is:

S = set of states of nature

C_1 = set of consequences for person I

C_2 = set of consequences for person II

D_1 = set of decisions or acts available to I

D_2 = set of decisions or acts available to II

f = social decision function.

Some examples will be given in the next section in connection with the theory of two-person justice.

The definition of moral principles is an obvious generalization of the one already given for the individual case. In addition, the definition is framed so as to be independent of the first or second person.

Definition 3. Let $\mathcal{J} = \langle S, C_1, C_2, D_1, D_2, f \rangle$ be a two-person decision situation. Then a moral principle with respect to \mathcal{J} is a strict partial ordering of the Cartesian product $C_1 \times C_2$.

This definition does not require that in applying a moral principle I need consider consequences to II, but does make possible such a consideration. We could in fact use Definition 3 as a basis for defining a wholly egocentric person, namely a person, say I, whose moral principles and preference relations in all two-person situations are orderings uniquely determined by elements of C_1 (the character of C_2 being never considered).

The same arguments given previously apply to the requirement that ordinary moral principles in two-person situations be partial orderings on consequences and not on acts. On the other hand, the arguments for a rigid adherence to the M.E.U. maxim are not so persuasive, since other rules of behavior like minimax or minimax regret can be strongly defended for two-person situations. But these matters will not be gone into here; for our purposes adoption of any of these alternative rules only requires admission that a utility or value function on $C_1 \times C_2$ is needed for both

persons I and II. We want to investigate how a formal principle of justice may be introduced which will put non-trivial constraints on the utility function. Moreover, it will be of interest to investigate the adequacy of a justice maxim compared to the M.E.U. or minimax kind of maxim, as an over-all rule of behavior.

In concentrating attention on justice no claim is intended that it is the most significant moral principle for social situations, nor even that the definitions given here provide more than the merest beginning of a formal theory of justice.

To begin with we need the notion of a preference relation on $C_1 \cup C_2$, that is, on the set of consequences to both I and II. The idea is that one consequence in $C_1 \times C_2$ will be deemed more just than another relative to a preference ranking of all consequences together. How in fact would a person make such a ranking? Presumably by treating himself and the other person on an "equal" basis. A suggestion as to how this idea of equality or symmetry may be formalized will be given following the definition.

Definition 4. A system $\mathcal{J} = \langle S, C_1, C_2, D_1, D_2, f, R_1, R_2 \rangle$ is a two-person decision situation with preference rankings if, and only if, $\langle S, C_1, C_2, D_1, D_2, f \rangle$ is a two-person decision situation, and R_1 and R_2 are weak orderings of $C_1 \cup C_2$.

(A weak ordering is a relation which is transitive and strongly connected.) The intended interpretation is that R_1 is the preference ranking of person I and R_2 that of II. Formally we might say that a person's preference ranking R of $C_1 \cup C_2$ is equitable or symmetric if it remains unchanged when the two persons change positions in the decision situation.

Difficulties of making this suggestion precise will not be pursued here. It is intended that in constructing R_1 , say, I will say to himself, it is better that II have x in C_2 than that I have y in C_1 , whence yR_1x and not xR_1y , etc. For example, a man should judge it better that his neighbor of equal economic status receive a thousand dollars than that he himself should receive fifty dollars. Unfortunately, I see no way of characterizing in an adequate formal manner the intuitive notion of better than used in this example. But it would be a mistake to consider this situation peculiar to moral philosophy. The notion of preference or better than had a status in formal moral philosophy very similar to that of the notion of force in mechanics. It is not a problem of mechanics proper to classify forces according to their physical origin.*

We now define the notion of more just than (abbreviated by J) relative to each person's preference ranking.

Definition 5. If $x_1, y_1 \in C_1$ and $x_2, y_2 \in C_2$ and $x = \langle x_1, x_2 \rangle$ and $y = \langle y_1, y_2 \rangle$ then for $i=1,2$, xJ_iy if, and only if, either (i) $x_1R_1y_1$ and $x_2R_2y_2$ and not ($y_1R_1x_1$ and $y_2R_2x_2$), or (ii) $x_1R_1y_2$ and $x_2R_2y_1$ and not ($y_2R_2x_1$ and $y_1R_1x_2$).

This definition is simpler than it may appear at first glance. It is framed so as to make J_i (for $i=1,2$) a strict partial ordering of the Cartesian

*/ These remarks are admittedly Kantian in flavor. Cf., "And just as nothing follows from the primary formal principles of our judgments of truth except when primary material grounds are given, so also no particular definite obligation follows from these...rules except when indemonstrable material principles of practical knowledge are connected with them." ([6], pp. 283-284.)

product $C_1 \times C_2$, and yet permit the comparison of elements of C_1 with C_2 . The two "not" clauses in the definiens guarantee that J_i is asymmetric.

Examples of J_i are at the beginning of the next section. We conclude this section with the theorem:

Theorem 2. For $i=1,2$, J_i is a moral principle with respect to \mathcal{S} .

Proof: To prove that J_i is asymmetric, suppose by way of contradiction that for some $x = \langle x_1, x_2 \rangle$ and $y = \langle y_1, y_2 \rangle$ in $C_1 \times C_2$ that

$$xJ_i y \text{ and } yJ_i x .$$

From $xJ_i y$ it follows from the definition that (dropping subscript i on R for brevity) $x_1 R y_1$ or $x_1 R y_2$, and similarly from $yJ_i x$ it follows that $y_1 R x_1$ or $y_1 R x_2$. We thus have four cases to consider:

Case 1. $x_1 R y_1$ and $y_1 R x_1$.

Case 2. $x_1 R y_1$ and $y_1 R x_2$.

Case 3. $x_1 R y_2$ and $y_1 R x_1$.

Case 4. $x_1 R y_2$ and $y_1 R x_2$.

Since the proofs for all cases are similar, we shall look only at Case 2 in detail. From the hypothesis of this case, we have from (i) of the definition:

(1) $x_1 R y_1$,

(2) $x_2 R y_2$,

(3) not $y_1 R x_1$ or not $y_2 R x_2$,

and from (ii):

$$(4) \quad y_1 R x_2$$

$$(5) \quad y_2 R x_1$$

$$(6) \quad \text{not } x_2 R y_1 \quad \text{or not } x_1 R y_2.$$

From (1), (4) and the transitivity of R we infer:

$$(7) \quad x_1 R x_2$$

and from (7) and (2):

$$(8) \quad x_1 R y_2.$$

From (2) and (5) (and transitivity of R):

$$(9) \quad x_2 R x_1,$$

and from (9) and (1):

$$(10) \quad x_2 R y_1,$$

but (8) and (10) contradict (6).

To prove now that J_i is transitive, we assume

$$x J_i y \quad \text{and} \quad y J_i z,$$

which leads to four cases also. Again we shall consider only one typical case:

$$(11) \quad x_1 R y_2 \quad \text{and} \quad y_1 R z_2.$$

From (11) and (ii) of the definition, we have:

$$(12) \quad x_2 R y_1$$

$$(13) \quad y_2 R z_1$$

$$(14) \quad \text{not } y_2 R x_1 \quad \text{or not } y_1 R x_2$$

$$(15) \quad \text{not } z_2 R y_1 \quad \text{or not } z_1 R y_2.$$

From (11), (13) and transitivity of R , we get:

$$(16) \quad x_1 R z_1,$$

and similarly from (11) and (12):

$$(17) \quad x_2 R z_2.$$

It remains to show that not $z_1 R x_1$ or not $z_2 R x_2$. Suppose by way of contradiction that

$$(18) \quad z_1 R x_1 \quad \text{and} \quad z_2 R x_2.$$

From (18) and (11), we have:

$$(19) \quad z_1 R y_2,$$

and from (18) and (12)

$$(20) \quad z_2 R y_1,$$

but (19) and (20) contradict (15), which completes our proof, since for J_1 to be a moral principle with respect to \mathcal{J} , it is required by definition that J_1 be asymmetric and transitive in $C_1 \times C_2$.

5. Points of Justice and the Prisoners' Dilemma. It will be instructive to apply the ideas introduced in the last section to a simple but conceptually

troublesome example of a two-person, non-zero-sum, non-cooperative game known as the prisoners' dilemma.*/ We quote the description from Chapter 5 of Luce and Raiffa [8]:

Two suspects are taken into custody and separated. The district attorney is certain that they are guilty of a specific crime but he does not have adequate evidence to convict them at a trial. He points out to each prisoner that each has two alternatives: to confess to the crime the police are sure they have done, or not to confess. If they both do not confess, then the district attorney states he will book them on some very minor trumped up charge such as vagrancy and they will both receive minor punishment; if they both confess they will be prosecuted, but he will recommend less than the most severe sentence; but if one confesses and the other does not, then the confessor will go free while the latter will get "the book" slapped at him.

Let

n = no conviction on any charge

v = vagrancy conviction

r = reduced conviction (less than maximum)

m = maximum conviction.

Then the game may be represented by:

		II	
		confess	not confess
I	confess	$\langle r, r \rangle$	$\langle n, m \rangle$
	not confess	$\langle m, n \rangle$	$\langle v, v \rangle$

*/ A game is non-cooperative when no precommunication or bargaining between the players is permitted. The prisoners' dilemma is attributed to A. W. Tucker.

where a pair like $\langle n, m \rangle$ is interpreted so that the first member n is the outcome to I and the second member m the outcome to II. We have not distinguished n_I and n_{II} , m_I and m_{II} , etc. These consequences are treated the same for each player. Keeping this in mind, the complete two-person decision situation with preference rankings may be identified, provided we introduce the one obvious preference ranking on the set of consequences:

$S =$ one element set (trivial here)

$C_1 = C_2 = \{m, n, r, v\}$ */

$D_1 = D_2 = \{\text{confess, not confess}\}$

$f =$ function defined by above game matrix

$R_1 = R_2 =$ weak ordering arising from linear ordering
 n, v, r, m , with n most preferred.

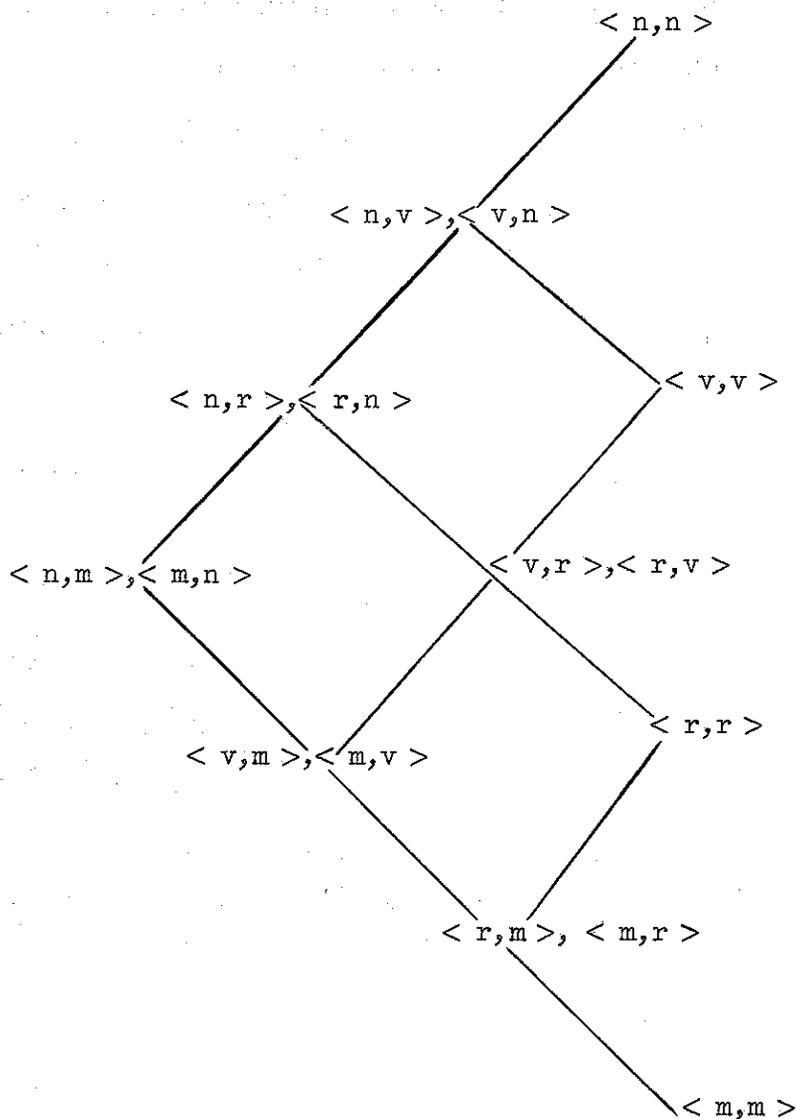
Clearly here

$$J_1 = J_2 ,$$

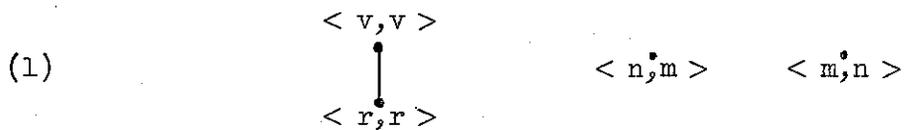
and the ordering more just than of $C_1 \times C_2$ may be represented by the following Hasse diagram, where two elements of $C_1 \times C_2$ standing at the same point in the diagram are not comparable under J_i :**/

*/ The identification of C_1 and C_2 merely simplifies the presentation and is not essential.

**/ Under the equivalence relation which "identifies" elements like $\langle r, m \rangle$ and $\langle m, r \rangle$, $C_1 \times C_2$ is a lattice with respect to J_i , but this fact is of no significance here.



Of course, only the four elements in the game matrix are of direct concern in discussing the prisoners' dilemma. The ordering induced by J_i on them may be represented by:



The important thing is that $\langle n, m \rangle$ is not related by J_1 to any of the other three elements, nor is $\langle m, n \rangle$.

The weak relation expressed by (1) would not seem to be of much help in guiding the choice of an action or strategy by either prisoner. As a direct constraint on the utility function of either it scarcely imposes any structure. Before attempting to show that considerably more can be obtained from (1) by introducing the concept of a point of justice, it will be useful briefly to review the game-theoretic solution of the prisoners' dilemma.

Two concepts of optimality for two-person non-zero-sum non-cooperative games yield the conclusion that both prisoners should choose the strategy of confessing, which leads to the outcome or consequence $\langle r, r \rangle$. One concept arises from the highly appealing sure-thing principle. A strategy or decision satisfies the sure-thing principle if no matter what your opponent does you are at least as well off, and possibly better off, with this strategy in comparison to any other available to you. Thus if I adhere to the sure-thing principle he should confess, for if II confesses I gets r rather than m , and if II does not confess I gets n rather than v ; whence for every choice of II, I is better off confessing. A similar situation obtains for II.

In many games no strategy satisfies the sure-thing principle. But every finite game of the class being discussed does have at least one equilibrium point, the second concept of optimality (introduced by John Nash [9], [10]). Roughly speaking, an equilibrium point is a set of strategies, one for each player, with the property that these strategies

provide a way of playing the game such that if all the players but one follow their given strategies, the remaining player cannot do better by following any strategy other than the one belonging to the equilibrium point. As is easily verified, the unique equilibrium point for the prisoners' dilemma is the pair of confession strategies, the same result obtained by application of the sure-thing principle.

In spite of the weight of these optimality principles there are several unsatisfactory aspects of the recommended solution. If both prisoners completely trust each other it seems more reasonable for both of them to adopt the strategy of not confessing. Moreover, the act of confessing might from a moral standpoint be distasteful. The various game-theoretical principles of behavior like the two just discussed are aimed at satisfying intuitive ideas of rational rather than moral behavior -- the notion of rationality being that of acting in one's own best interest without direct concern for others. The point of the remainder of this paper is to contrast moral and rational behavior, with special reference to the prisoners' dilemma.

In Section 3 I have argued that moral principles should be addressed to consequences rather than decisions or acts. I now want to suggest that a (first-order) moral principle concerned with consequences may lead to a second-order moral principle which is a direct guide to action. Such second-order moral principles may be termed ethical rules of behavior, in contrast to game-theoretical rational principles of behavior.*/ I shall

*/ I still insist that the familiar homilies like "Know thyself," "No man ought to commit murder," "Do the most perfect thing that can be done by you" are first-order moral principles properly applied to consequences.

use the justice relation on consequences to formulate one such ethical rule of behavior. First we define a (J_i) admissible element as an element of $C_1 \times C_2$ which is not dominated under the relation J_i by any other element. In diagram (1) elements $\langle v, v \rangle$, $\langle n, m \rangle$ and $\langle m, n \rangle$ are (J_i) admissible. In the preceding diagram only $\langle n, n \rangle$ is. Next, in analogy with the definition of an equilibrium point, let us define a (J_i) point of justice as a set of strategies, one for each player, such that adoption of these strategies leads to an admissible element as outcome.

The simplest justice oriented rule of behavior is then:

- (I) If $J_1 = J_2$ and there is a unique point of justice, the strategy belonging to this point ought to be chosen.

Unfortunately, (I) is not applicable to the prisoners' dilemma, for the requirement that there be a unique point of justice is not satisfied.*

A more complicated, but still relatively simple ethical rule of behavior may be introduced in terms of the notion of a justice saturated strategy. A strategy for player i is justice saturated (with respect to J_i) if whatever strategies are picked by the other players the resulting set of strategies is a (J_i) point of justice. The rule of behavior is then:

- (II) If for any player his set of justice saturated strategies is non-empty, he ought to choose one.

In the prisoners' dilemma each player has a unique justice saturated strategy,

*/ It is perhaps useful to mention that in general a game of the type being considered here does not have a unique equilibrium point; the prisoners' dilemma is a happy exception.

namely, the strategy of not confessing, joint use of which leads to the reasonable outcome $\langle v, v \rangle$.

To be sure, when a person's set of justice saturated strategies contains more than one element, (II) does not lead to a unique action, and some supplementary ethical rule of behavior may be needed. A similar problem arises for rational game-theoretical rules of behavior and should surprise only those who believe that satisfactory categorical rules of action are easily come by.

If neither (I) nor (II) is applicable (and simple two-person decision situations exist for which this is the case), the theory of justice outlined here is of no use in determining what action to take, except in so far as the relation J_i is a constraint on the person's utility function.*

But this last problem of applicability is one of the least difficulties that face an adequate formal theory of justice. For example, even when (II) is applicable, an "ethical" man using it may be at great competitive disadvantage against a "rational" man. In the prisoners' dilemma if prisoner I adopts his justice saturated strategy and prisoner II his equilibrium point strategy, then I will receive the maximum conviction. It is not easy, for me at least, to decide if this is an intuitive argument against the formal theory of justice set forth here, or if it is an intuitively reasonable instance of a just man getting the worst of a situation. If the latter is the case, I think it may be claimed that a

*/ In general finite games only have equilibrium points when mixed strategies (i.e., probability mixtures of pure strategies) are admitted. A discussion of Rules (I) and (II) with respect to mixed strategies would take us too far afield.

man who in all situations acts according to ethical rules of behavior may fare as well in the long run as the rational man, provided knowledge of his standards of actions are known to his fellow man. It is an interesting problem to determine how far this claim may be made to stick by detailed formal analysis.

Another difficulty with the present theory is its structural weakness. It is a priori certain that no very elaborate theory of action can be built on the simple notion of a strict partial ordering. A major step in the development of rational theories of behavior has been the quantification of value (i.e., utility) and of subjective probability (i.e., reasonable degree of belief). Plausible assumptions which will lead to quantification of the theory of justice seem hard to find.

Making the theory of justice depend on the individual preference rankings is very much in the spirit of modern welfare economics, but may seem highly unsatisfactory to many philosophers. And I think it may be rightly objected that the intuitive success of the theory depends upon these individual preference rankings themselves satisfying certain criteria of justice. To admit this objection is not to accede to a charge of circularity, for moral principles of justice, logically independent of the theory developed here, can be consistently introduced as constraints on individual preference rankings of $C_1 \cup C_2$. I simply do not have at the present any such interesting formal principles to suggest.

However, it may be appropriate to mention an alternative way of treating the theory developed here. The one detailed application has been to a non-cooperative game. In a cooperative game, for instance, an

arbitration situation, it might be reasonable for the two participants who are in conflict but who are upholders of ethical rules of behavior, to appoint an arbitrator they both trust. The arbitrator is then asked to make what he considers the fairest preference ranking of $C_1 \cup C_2$ in terms of his knowledge of the participants' needs and wants. Rules (I) and (II), if applicable, might then determine the outcome of arbitration. The immediate objection to this seems to be that if the arbitrator is going to do the ranking, why not simply let him rank the outcomes, and then agree on the one he considers fairest as the negotiated outcome. There is a simple answer to such an objection. It may be easy to rank $C_1 \cup C_2$, but very difficult to rank $C_1 \times C_2$. For example, let

$$C_1 = \{\text{trip to Hawaii, trip to N.Y.}\}$$

$$C_2 = \{\text{trip to Florida, trip to Chicago}\}$$

and the arbitrator, knowing I and II, may find it easy to rank $C_1 \cup C_2$:

trip to Hawaii
trip to Florida
trip to N.Y.
trip to Chicago ,

but he finds it very difficult to compare elements of $C_1 \times C_2$ like $\langle \text{trip to Hawaii, trip to Chicago} \rangle$ and $\langle \text{trip to N.Y., trip to Florida} \rangle$.

In conclusion an example may be constructed for which equilibrium point analysis seems to lead to a more equitable and just solution of a non-cooperative game than the theory of justice outlined here. Let

$$C_1 = \{a, b, c\}$$

$$C_2 = \{\alpha, \beta, \gamma, \delta\}$$

let $R_1 = R_2 =$ the ranking:

a

α

b

β

c

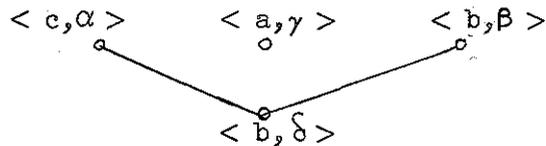
γ

δ

and let the game matrix be:

		II	
		1	2
I			
1		$\langle a, \gamma \rangle$	$\langle b, \beta \rangle$
2		$\langle c, \alpha \rangle$	$\langle b, \delta \rangle$

Then $J_1 = J_2$, and we have as Hasse diagram of the justice partial ordering of $C_1 \times C_2$:

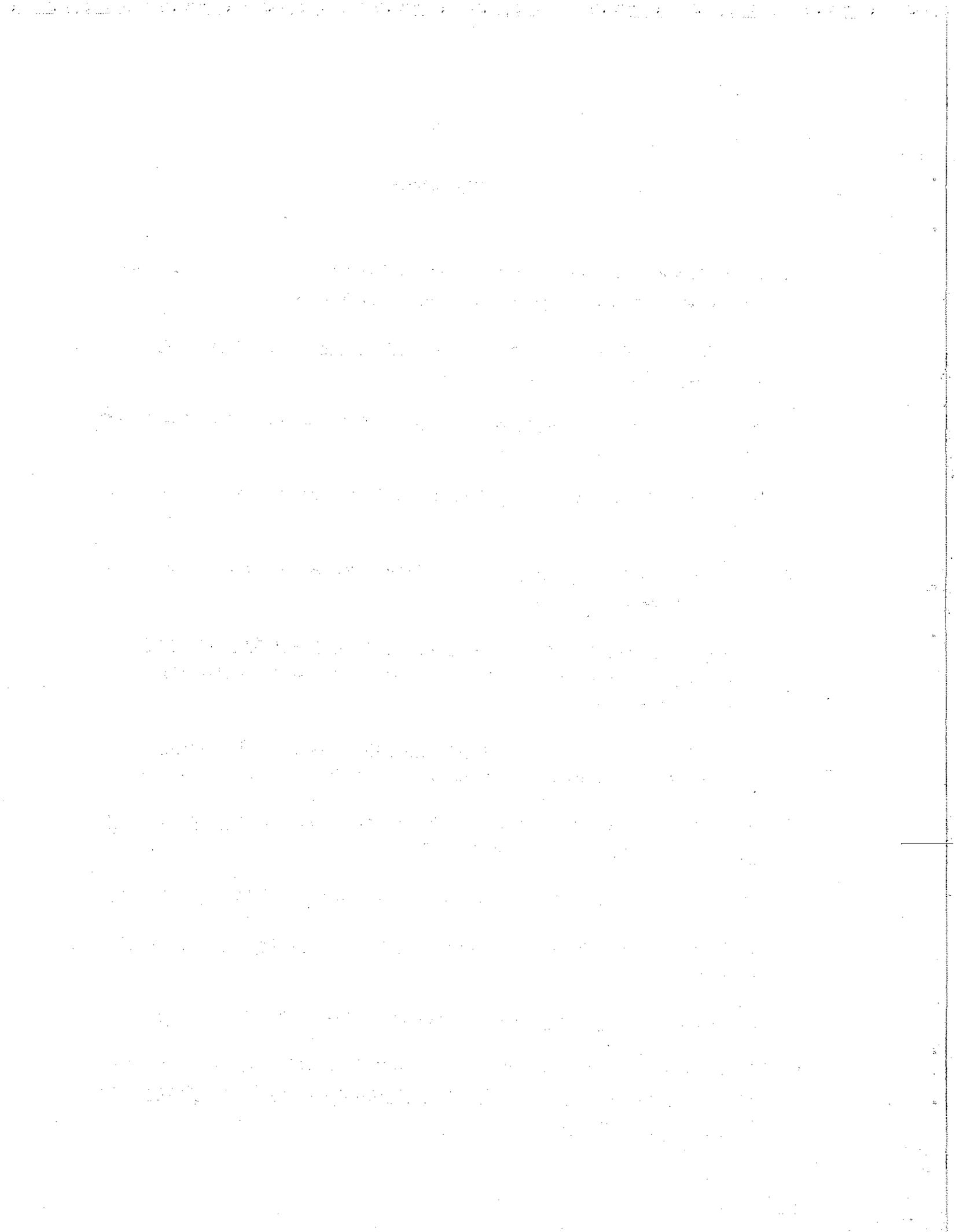


It is easily checked that decision 1 is the unique justice saturated strategy for each player, yielding the outcome $\langle a, \gamma \rangle$, whereas the unique equilibrium

point strategies yield $\langle b, \beta \rangle$ as the outcome. In terms of the ordinal properties of the consequences at least, outcome $\langle b, \beta \rangle$ seems fairer than $\langle a, \gamma \rangle$. I conclude that the theory of justice developed here satisfactorily solves only a certain perhaps small proper subset of two-person non-cooperative games.

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