VARIABILITY IN THE PROOF BEHAVIOR OF COLLEGE STUDENTS IN A CAI COURSE IN LOGIC AS A FUNCTION OF PROBLEM CHARACTERISTICS

by

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CHAPTER ONE

For centuries, teachers have been teaching and students have been doing whatever it is that students do. It is only in this century, however, that any systematic and sustained attempt has been made to study the nature and the results of the interaction between teacher and student, and, during this period, progress has been painfully slow.

The educational psychologist is faced with serious difficulties in doing research on human learning and performance. If research is to be done in a school, the cooperation of administrators and teachers must be obtained, and experiments must be tailored to fit the organizational structure of the school. Even then, it is very difficult to obtain detailed information on student performance over a long period of instruction. It may be possible to obtain an adequate description of social processes from a discreet distance, but it seems almost impossible to obtain detailed profiles of individual student responses in this way. In order to obtain the data necessary to investigate cognitive performance, it is necessary to record student behavior in great detail.

Since it is impractical to maintain teams of research workers in a classroom without completely disrupting the process to be observed, the systematic investigation of problem solving behavior has been restricted to the laboratory. Laboratory research on these issues has been hampered by the difficulty in obtaining adequate samples of subjects willing to work on problem-solving tasks over a long period of time.

The advent of computer-assisted instruction makes it possible to circumvent some of these difficulties. When a student does problems at a computer terminal, it is possible to record a complete profile of his typed responses (as well as the time to each response). Since the collection of these responses is automated, and therefore invisible to the student, it is possible to record problem solving behavior over a long period of time without disrupting the process being observed. In a semester of work in mathematics done at a computer terminal, it is relatively easy to obtain complete profiles of individual student solutions to hundreds of problems.

This implies a further advantage of using CAI for research on problem solving. In a laboratory experiment or in classroom observation, the subjects (or students) are aware that their efforts are being recorded. It has been shown that, under such conditions, subjects tend to modify their behavior to fit the expectations of the experimenter (Neisser,1967). To the extent that data collection is truly invisible, this more subtle source of possible bias in the data is also eliminated.

The use of a CAI curriculum as a context for research on cognitive processes still presents serious difficulties however. In order to exploit its full potential, we must develop techniques for analyzing and interpreting the data collected. The principal purpose of this research was to develop such techniques for examining the details of student proof behavior.

The traditional tools used to analyze the results of educational and psychological experiments are, of course, available and have been used. Regression analysis, for example, has been used extensively in investigating the effects of curriculum structure on student performance. The analysis of variance has been used to compare CAI to more traditional types of instruction, and to examine the effect produced by varying certain conditions within CAI.

It is clear that the use of such techniques can make a valuable contribution to our understanding of student behavior, but all of these studies deal with global measures of performance. They tell us how well students perform under various conditions; they do not tell us how students perform - what they actually do. If the solution to a problem requires a sequence of steps rather than a single response, then this distinction is of great importance. The total time taken to solve a problem or the number of errors may be adequate measures of a student's overall performance, but they tell us nothing about how individual students solve problems. An analysis that makes use only of summary measures of performance ignores the structure of student solutions, and, so, does not exploit the full potential of CAI as a setting for educational research.

In this study a particular type of problem solving behavior is investigated. In the following sections, some techniques for analyzing the details of student proof behavior in a complex CAI setting are developed and then used to evaluate a specific aspect of the Stanford Logic-Instructional System (LIS).

LIS is designed to allow students considerable latitude in the construction of proofs, and students work at their own pace and develop their own strategies for finding proofs. By measuring the actual variation in a sample of proofs collected under ordinary operating conditions, it is possible to characterize the effectiveness of the curriculum in encouraging diversity in the students' approaches to proof construction. This research was motivated by a desire to estimate how much variation (in the types of proofs generated) actually occurs when students work through the current LIS curriculum.

The data collection facilities for LIS store a complete record of each student's typed responses, and it is possible to examine the exact sequence of steps for every proof. It is possible, therefore, to determine the number of classes of equivalent proofs in a sample of student proofs, but first it is necessary to specify a set of criteria that separates proofs into classes, and so defines what is meant by the statement that two proofs are equivalent.

The objective of the initial phase of this study is to formulate such criteria. Five distinct procedures are developed each of which classifies any sample of proofs into a set of mutually exclusive and exhaustive subsets, thus defining a partition on the sample. The procedures are essentially definitions of what it means to say that two proofs are equivalent or not equivalent. These partitions are then shown to be nested in the sense that if two proofs are equivalent under the i-th partition, they are also equivalent under the (i+1)-th partition. A detailed development of these procedures is presented in Chapter III.

The second purpose of this study was to determine the amount of variation that actually occurs in the structure of the proofs produced by a sample of college students for the problems in the LIS curriculum. The proofs constructed by 23 Stanford University students for 125 separate derivation problems in the LIS curriculum are used for this purpose. In order to determine how this variation is distributed through the curriculum, each problem is analyzed separately.

For all of the problems included in this study and each set of criteria, the student proofs are assigned to equivalence classes. The numbers of classes for the five partitions for a problem are taken as separate measures of the variability of the student proofs for that problem.

The results indicate that there is relatively little variability for the earliest problems and considerable variability for the later problems. The increase in variability through the curriculum is not smooth. There is a gradual increase from the first problem considered to the 50-th problem (approximately), but even the last of these early problems shows relatively little variation among the proofs generated. There is then an abrupt increase in variability and subsequently a continued gradual increase. The rule, Replace Equals(RE), is introduced in the curriculum just before the abrupt increase in variability; this initial indication of the importance of RE is confirmed by the subsequent regression analysis.

Regression analysis is used to pinpoint variables defining structural properties of the problems which predict variability among the student proofs. The results indicate that relatively simple measures of structural complexity (for example, the number of steps in the standard proof for a problem) are good predictors of the amount of superficial variation in the sample of proofs, such as differences in the order of the steps, but relatively poor predictors for the more substantial variations such as differences in the rules used to construct the proof. As the importance of these measures of structural complexity systematically decreases from the first to the fifth partition, the importance of the number of theorems (and axioms), as predictors of variability, increases. This analysis is described in Chapter IV, and the results of the analyses are presented in Chapters V and VI.

The use of a nested sequence of measures, rather than a single measure, makes the detection of this trend possible. The results indicate that the regression equation which best predicts variability is quite sensitive to changes in the measure of variability. If a single measure of variability (partition) were used, there would have been no indication of the sensitivity of the results to the definition of equivalence, and it is likely that erroneous conclusions would be drawn. For example, if only the first partition had been used, it would seem that theorems are relatively poor predictors of variability; in fact, the other four partitions indicate that theorems are very important predictors of variability.

In general, the most significant kinds of variability (for example, differences in the rules used to construct a proof) depend on the number and type of rules that are available when the proof is done; Replace Equals and the theorems are especially important. Where variability in student proof behavior is desired, the more powerful rules should be introduced as soon as possible.

In a third part of this study, an attempt was made to identify patterns of proof behavior that characterized groups of students over the sample of problems. This attempt took advantage of the fact that metric functions for the set of students can be easily defined in terms of the classification procedures.

The search for patterns in student proof behavior was exploratory in nature. If definable patterns had been detected, their properties would have been investigated, and further research in this direction would have been suggested. In fact, no indication of the existence of definable patterns was detected.

The failure of this part of the study to yield the desired results was not surprising. The problems in the logic curriculum are quite heterogeneous, and differences in proofs from problem to problem are much more pronounced than the differences between students for a given problem. Since these efforts failed to reveal any sustantial results, and the questions raised here are peripheral to the main purpose of the study, this part of the study is not discussed in the main body of the text. The methods developed for this part of the study, however, make possible a more systematic analysis of problem solving behavior and should be useful in future studies dealing with problem solving behavior, so a description of the analysis is included as Appendix A.

Overall, this study indicates that the use of formally defined partitions over sets of complex behaviors (in this case, proofs) can provide an intuitively satisfying and fruitful technique for examining the details of complex behavior.

CHAPTER TWO

I have included this brief description of the operation of the Logic Instructional System (LIS) for those with no previous experience of it; some discussion of the curriculum is also included. The description is far from complete, but I hope that it is sufficiently detailed to enable the reader to follow the development in subsequent sections. Further discussion of the material included in this chapter can be found in James Moloney's dissertation (Moloney, 1972) and in several papers by Patrick Suppes (Suppes, 1965,1970,1971). A new instructional system for elementary logic, which has many features in common with the system discussed here, is described in detail in a recent paper by Adele Goldberg (Goldberg 1971).

The first part of the curriculum is designed to give a thorough introduction to sentential logic. Once the student has acquired an understanding of sentential logic, he uses this knowledge in his study of elementary algebra. In sentential logic, the approach used is a natural deduction treatment in which the students are taught rules of inference, such as modus ponens, and proof procedures (conditional proof and indirect proof). Some examples of the rules of inference are:

```
(A) Affirm the antecedent - AA
        From  (1) Q -> R
         and  (2) Q
       infer  (3) R

(B) Form a conjunction - FC
        From  (1) Q
         and  (2) R
       infer  (3) Q and R
```

Using the rules of inference, the student is asked to construct a mathematically valid proof of some specified sentence (formula) from a given set of premises. The proof consists of a sequence of steps, each of which utilizes one of the rules of inference. The computer does not interfere with the course of the student's attempt to find a proof as long as his steps are valid applications of the rules of inference; the computer does act as a proof-checker to determine if each new step is valid, and types an error message whenever a rule is used incorrectly. This gives the student the freedom to construct his own proof, subject to the constraint that each step be a correct application of some rule.

In the second part of the curriculum, the student is first taught certain rules about the identity relation (e.g. adding a term to both sides of the equation). Then he is given a set of axioms for an additive group (i.e. commutativity, associativity, and the properties of zero and negative numbers). From these axioms and the set of rules, he constructs proofs for a number of theorems about addition. In his proofs, he can use any theorems that he has already proved as well as the axioms and rules that he has learned.

The remainder of this paper deals exclusively with derivation problems, and I shall restrict the following discussion of LIS to its derivation mode, ignoring its other modes.

Each derivation problem consists of a formula to be derived and a sequence of k (with k possibly equal to 0) formulas called premises. The k premises are numbered sequentially from 1 to k. The student is required to find a sequence of valid steps that lead to the formula to be derived; when this formula is generated, LIS types CORRECT and continues with the curriculum.

Essentially what a student does at each step of a proof is to give a formal justification of the step that he wants to take. These justifications are coded as short mnemonics. Most codes require auxilliary information or parameters; the student types these as prefixes or postfixes to the code name. The prefixes are line numbers and specify the lines already in the proof that are to be operated on in order to generate the new line. For example, the left conjunct rule, LC, requires a single line reference, the line number of a conjunction already in the derivation.

Postfix numbers can be either occurrence numbers or literal numbers. For example, an occurrence number is required by the commute disjunction rule to specify which disjunction of a complex formula is to be commuted. A literal number is required by the number definition rule to specify the number for which a definition is to be generated.

Let us consider a very simple example - problem 406.6:

```
Derive:  Q
  P      (1) R
  P      (2) R -> Q
```

Q is the sentence to be derived, and lines (1) and (2) are premises. The number, 1, is the line number of the sentence, R, and the number, 2, is the line number of the sentence, R->Q.

The student generates new lines by making use of the rules available to him. If the student now types, 2.1AA, LIS generates a new sentence labeled, (3). The proof then looks like this:

```
Derive:  Q
  P      (1) R
  P      (2) R -> Q
  2.1AA  (3) Q

CORRECT
```

AA is a mnemonic for affirm the antecedent (modus ponens). The format for the use of this rule is n.mAA where n is the line number of a conditional, and m is the line number of the antecedent of the conditional in line (n). In this case, line (2) is a conditional and line (1) is the antecedent of that conditional. LIS, therefore, accepts this instruction and generates, Q, as line (3); 2.1AA is a valid step of the proof. Since, Q, is the sentence to be derived, the computer types CORRECT and the proof is complete.

If, instead of 2.1AA, the student types 1.2AA, then LIS would not accept the instruction, and no new line would be generated. An error message is typed by the computer (in this case, LINE 1 IS NOT A CONDITIONAL), and LIS then waits for the student's next instruction. Each instruction is checked to insure that every sentence generated is justified by the correct use of a rule of inference, axiom, or theorem.

The proof for this example requires only a single step, but LIS would accept any other valid step as well. If the student chooses to use the double negation rule on line 1, for example, then line 3 is generated as:

```
(3) NOT(NOT R)
```

Since this is not the formula to be derived, LIS would wait for another instruction.

As indicated above, lines are numbered consecutively as they are generated, and, with one exception, each valid instruction generates a new line. The instruction DLL, delete last line, does not generate a new line. Instead, it erases all internal references to the last line generated; for LIS, that line no longer exists (of course, the deleted line is not erased from the student's paper copy of the derivation). The next line generated will have the number of the last line deleted. A sequence of DLL's may be used to delete a sequence of lines starting from the most recently generated line and working backwards through the derivation. The student, however, cannot delete premises and he cannot delete any line in his derivation without previously deleting all subsequent lines.

In our example, the student may decide that he does not need line (3), and type a DLL as his second instruction. If he then types 2.1AA, his record of the derivation would appear as:

```
Derive:   Q
  P       (1) R
  P       (2) R -> Q
  1DN     (3) NOT(NOT R)
  DLL
  2.1AA   (3) Q

CORRECT
```

If he had typed a second DLL instead of the AA instruction, he would be told that line (2) is a premise, and cannot be deleted. He could, however, have typed 2.1AA directly after 1DN, and the derivation would then appear as:

```
Derive:   Q
  P       (1) R
  P       (2) R -> Q
  1DN     (3) NOT(NOT R)
  2.1AA   (4) Q

CORRECT
```

Line (3) in this derivation does not bring the student any closer to a solution, but it is a valid instruction and is accepted by LIS. The four lines listed do constitute an acceptable proof, but line (3) is not really used; a precise definition of 'unused line' will be given in the next chapter.

In this example, the use of DLL is a matter of convenience, but there are two situations where it may be necessary to eliminate some lines from a partially completed solution. LIS will not generate more than 31 lines for any problem. None of the problems in the curriculum require more than 31 lines, but a student can easily generate 31 lines without completing a derivation by producing one or more false starts. When this happens, it is necessary to delete some unused lines before continuing with the derivation.

The other situation that requires the deletion of lines from a partial solution involves the working premise rule, WP. Working premises must be used in conjunction with either the conditional proof rule, CP, or the indirect proof rule, IP. A brief description of these rules will be given before continuing with the discussion.

WP allows the student to introduce any formula or sentence as a working premise. He may then instruct LIS to generate new lines from this working premise until he has generated the consequent of the conditional that he wishes to prove; CP is then used to generate the conditional sentence. Alternately, the student may derive a contradiction by using a working premise, and then use IP to generate the denial of the working premise.

The use of WP begins a subsidiary derivation that must be completed before the solution is completed. The line generated by WP and all subsequent lines up to, but not including, the next line generated by a CP or an IP, are indented on the student's paper copy of the derivation to indicate that they are part of the subsidiary proof. Generating the formula to be derived in a problem within a subsidiary derivation does not constitute a proof for the problem; a different problem, with an additional premise, has been solved. While the student has a working premise that has not been referenced by a CP or an IP step, he is still in a subsidiary proof and cannot complete the proof.

The student may find that he has introduced a working premise that he does not wish to use. Any working premise which is not used (with either CP or IP) must be deleted before the proof is completed.

If many lines must be deleted for either of these reasons, it may be more convenient to end the session and then begin a new session. The same problem is presented again, and the student can then restart it.

The final point to be discussed here is the use of substitution instances for axioms and theorems. A student uses an axiom or a theorem by typing its code and then hitting the enter key. LIS then types a statement of the axiom or theorem and a list of variables in the theorem that require substitution ,and asks that a specific term be substituted for each of these variables.

To use the additive inverse axiom, the student types AI. LIS types the statement of the axiom, A+(-A)=0, on the same line, and requests the single substitution required for AI by typing A: on the following line. The student can then reply with any term. For example, if the student wishes to generate for line (n), 6+(-6)=0, he must type the number, 6, after the the computer types an A: .

```
AI  A+(-A)=0
A:  6     (n)  6+(-6)=0
```

Axioms are introduced in the same way that the other rules are introduced. Theorems are presented as derivation problems, and become available for use after they have been proved.

I shall conclude this discussion with an example of a proof for a derivation problem from the algebra part of the curriculum. A brief explanation of each step is given after the solution. Further examples are presented in Appendix C.

406.24:

```
DERIVE:   A+A=3+3 -> A+A=6
P         (1)  A=3 -> 6=A+A
P         (2)  3+3=A+A -> 3=A
WP        (3)        A=3+3
DLL
WP        (3)        A+A=3+3
3CE1      (4)        3+3=A+A
2.4AA     (5)        3=A
5CE1      (6)        A=3
1.6AA     (7)        6=A+A
3.7CP     (8)  A+A=3+3 -> 6=A+A
8CE2      (9)  A+A=3+3 -> A+A=6
```

CORRECT

Lines (1) and (2) are premises and are typed by SLAP as part of the problem. The student's first step is a working premise (first line (3)). This is a valid step and is accepted by SLAP, but it is not the working premise that the student wants. Therefore he deletes it in his next step and generates a new working premise (second line (3)). CE is then used to commute the expressions in line (3) (prefix number is 3) around the first equal sign (postfix number is 1) to generate line (4). Line (5) is generated by applying AA to lines (2) and (4). Lines (6) and (7) are generated by using CE and AA respectively. Next, conditional proof is used to generate the conditional formula in line (8). This step has two line references. The first line referred to is a working premise as it must be, and the second line referred to is the line that is to be the consequent of the conditional formula. Since line (8) terminates the subsidiary derivation begun in line (3), the indenting that began in line (3) terminates at line (8). Line (9) is generated by another application of the CE rule. Since line (9) is the formula to be derived and since the student is no longer in a subsidiary proof, the proof is accepted and LIS types CORRECT.

CHAPTER THREE

In this chapter, the classification procedures which are the basis for this study are described. In section 3.1, an informal introductory description of the criteria is presented. In section 3.2, the procedure is developed formally, and in the last section 3.3, an example is described in detail.

Given any two proofs for a derivation problem, we want to be able to decide that the proofs are equivalent (given some set of criteria) or that they are not equivalent; in order to do this, we must define a partition on the set of proofs.

It would have been possible to have trained human judges make the decisions, but I decided not to use this technique for two reasons. First, it is an onerous task to examine carefully 25 or 30 separate proofs each consisting of 20 or 30 steps. It is difficult to remain consistent for a single problem, and it is much more difficult to maintain consistency from problem to problem. Second, if this procedure were used, it would be impossible to specify precisely the criteria employed.

With these difficulties in mind, I have decided to specify in advance a precise set of criteria for classifying proofs. This eliminates the problem of maintaining consistency throughout the classification and permits an unambiguous statement of the criteria used in obtaining my results.

Five distinct sets of criteria for classifying proofs are defined in section 3.2; each of these sets of criteria is shown to define a partition (and thus an equivalence relation) on any set of proofs. It is also demonstrated that the sequence of partitions is nested in the sense that, if two proofs are equivalent under the i-th partition, they are also equivalent under the (i+1)-th partition. In the following paragraphs, these results are presented informally.

3.1 - INTRODUCTION TO THE CLASSIFICATION CRITERIA

The equivalence relations are defined in terms of specific one-to-one mappings (correspondences) of components of one proof onto components of another. If a mapping of the specified form exists between two proofs, they are equivalent, otherwise they are not. The proof elements that are mapped and the nature of the mappings vary from one equivalence relation to another, but in each case, the equivalence of two proofs depends on a mapping (correspondence) between component parts of the proofs.

I will begin the discussion with the fifth partition, where the criteria for equivalence are least stringent, and work backwards to the first partition, where the criteria are most stringent. The nesting of the partitions is a consequence of the fact that restrictions are added at each level, from the fifth partition to the first. The classification procedure is illustrated in section 3.3, where the resulting partitions for each of the five sets of criteria are presented for a small sample of proofs.

For the fifth equivalence relation, the set of elements of each proof is the set of all rules that appear at least once in the used steps of the proof. The mapping for this partition requires that the elements mapped onto each other be the same rule; two proofs are equivalent if they use exactly the same rules.

The fourth partition also requires that equivalent proofs have the same set of used rules, but imposes the additional requirement that the rules occur the same number of times in both proofs. Therefore, proofs that are equivalent under the fourth partition will also be equivalent under the fifth partition; the partitions are nested.

The elements mapped under the remaining partitions are the steps of the proofs. Under the third partition, equivalent proofs must contain the same number of steps, and the steps mapped onto each other must use the same rule. The additional requirements added at the third partition are more complicated than those for any of the other partitions. The description included here is very brief and incomplete in some details. One of the requirements of the third partition is that corresponding steps have identical arguments (arguments specify how the rule is to be applied - see Chapter II and section 3 of this chapter). The third partition also places requirements on the structure of the proof, on the relationship between the steps in the proof. The principal requirement, added at this level, is that the steps referred to by corresponding steps must correspond. If D and D′ are equivalent proofs under the third partition, d(i) in D corresponds to d′(i′) in D′, d(i) refers to d(j), and d′(i′) refers to d′(j′), then d(j) corresponds to

$d'(j')$. This condition implies a partial restriction on the order of the steps since valid steps always come after the steps that they refer to; no additional restrictions are placed on the order of steps at this level. In a sense, made explicit in section 3.2, proofs must have the same structure.

The second partition imposes all of the requirements of the third and also requires that the ordinal position of corresponding steps in equivalent proofs be the same.

The first partition is defined by the identity relation. The nesting of the partitions results from the fact that, for $i = 1,....4$, the mapping for the i-th partition imposes all of the conditions of the (i+1)-th partition along with additional conditions.

3.2 DEFINITION OF THE CLASSIFICATION PROCEDURE

The development that follows will take as primitives, the smallest units of student behavior evaluated by the Logic Instructional System (LIS); these units will be called instructions. A student constructs solutions to the derivation problems on the LIS by typing a sequence of instructions. A valid solution to a derivation problem will be called a derivation or 'proof'; a formal definition of a proof will be presented below.

An instruction is a string of characters (modified ASCII including blank spaces) followed by a carriage return or an enter character. The carriage return or enter character signals LIS that the instruction is complete.

After an instruction has been typed by the student, the system responds in one of three ways; instructions may be classified into three mutually exclusive and exhaustive categories on the basis of this response. If the response is an error message, the instruction will be called an E-instruction. If the response is a request for further information, the instruction will be called an I-instruction (intermediate instruction). If LIS responds by typing a new formula, then the instruction is called an L-instruction. If the student types 'DLL' followed by a carriage return or enter character, then the system gives no overt response, but deletes all internal references to the last formula generated. This special type of instruction is also classified as an L-instruction.

Def 1: A sequence of instructions is an L-step if and only if the last instruction in the sequence is a

L-instruction and all previous instructions in the sequence are I-instructions.

<u>Def</u> <u>1a:</u> A sequence of instructions is an E-step if and only if the last instruction in the sequence is an E-instruction and all previous instructions in the sequence are I-instructions.

<u>Def</u> <u>2:</u> The formula typed by the system after the last instruction in an L-step is said to be generated by the L-step.

<u>Def</u> <u>3:</u> In a sequence of steps, all steps between any WP step and the first IP or CP step following the WP step are called conditional steps.

<u>Def</u> <u>4:</u> The subsequence of L-steps in a sequence of steps is a proof (or derivation) of the line, L, if and only if the last L-step in the subsequence generates L and is not a conditional step.

As defined here a student's proof for a problem consists only of L-steps, and the subsequent analysis treats only these L-steps; E-steps are excluded from the definition of proof, and student errors will not be included in the following analysis. At this point in the discussion, the distinction between L-steps and E-steps will be dropped, and the term 'step' will be used to designate L-steps.

The sequence of steps that defines a proof generates a sequence of formulas with the formula to be derived as the last formula in the sequence. LIS associates with each of these formulas an integer that identifies it for subsequent reference. These integers are called labels. A proof, then, consists of a sequence of labeled steps (L-steps) in which the last step in the sequence of steps generates the formula to be proved.

For the purposes of the following discussion, it will be useful to decompose any step into three functional components. A step is then viewed as an ordered triple consisting of:

(1) a sequence, possibly null, of numerals (called references) that are the labels of some previous steps in the proof

(2) a string of letters designating one of a finite set of rules of derivation

(3) an argument list, possibly null, which provides additional information on how the rule of the step is to be applied

Further discussion of labels, rules, and argument lists can be found in Chapter II.

<u>Def 5:</u> A step d(i) is said to refer to a step d(j) if the label d(j) is equal to a reference of d(i) .

<u>Def 6:</u> There exists a chain of reference from d(i) to d(j) iff there exists a sequence of steps d′(1)...d′(k), such that:
(1) d(i) = d′(1) and d(n) = d′(k)
(2) for all i=1,...k-1 , d′(i+1) refers to d′(i)

<u>Def 7:</u> A step,d, in a derivation,D, is said to be used if d is the last step in D, or if there exists a chain of reference from d to last step.

<u>Th 2:</u> If d is a used step in D, and d refers to d′, then d′ is a used step in D.
Pf: Let d″ be the last step in D. Since d is used in D, there exists a chain of reference d...d″. But d refers to d′; so d′,d,...,d″ is also a chain. Since there exists a chain from d′ to d″, d′is a used step in D.

Let S designate a finite set of proofs for some derivation problem. S = {D,D′,D″...}.

<u>Def 8:</u> <1>D′ iff D and D′ are derivations in S, and D is identical to to D′.

<u>Th 3:</u> <1> is an equivalence relation on S.
Pf: The identity relation is an equivalence relation.

Definitions 9 and 10 are complicated by the unique properties of Indirect Proof (IP). IP is the only rule in the set of available rules that requires more than two references. For steps with rules that require two references, the interpretation of the step depends on the order of the references. The valid use of AA, for example, requires that that the first reference be the label of an implication and that the second formula referred to be the antecedent of this conditional. For IP, the first reference must be the label of a working premise, but the only requirement on the second and third references is that they be the labels of two formulas, one of which is the negation of the other. A change in the order of these two references has no effect on the validity of the step and no effect on the formula generated by the step.

The second and third sets of equivalence criteria (Def 9 and Def 10) place restrictions on the order of the references in each step, and it is desirable that the second and third references in IP steps be exceptions to these restrictions. In order to do this, a separate restriction on the order of the references is specified for IP.

<u>Def 9:</u> $D\langle 2\rangle D'$ iff D and D′ are derivations in S, and there exists a mapping of the used steps of D onto the used steps of D′, with the following properties: let $d(m)$ in D map into $d'(m')$ in D′.

(1) if $d(m) \to d'(m')$ and $d(m)$ is the n-th step in the subsequence of used steps of D, then $d'(m')$ is the n-th step in the subsequence of used steps of D′.

(2) $d(m)$ and $d'(m')$ have the same rule, and the same argument list.

(3) if $d(m)$ uses a rule that requires either one or two references and $d(m)$ refers to $d(i)$, $d(j)$, ($d(i)$ if $d(m)$ has only one reference), then $d'(m')$ refers to $d'(i')$, $d'(j')$ and $d(i) \to d'(i')$, $d(j) \to d'(j')$.

(4) if $d(m)$ has rule IP, then $d(m)$ refers to $d(i)$, $d(j)$, $d(k)$ and $d'(m')$ refers to $d'(i')$, $d'(j')$, $d'(k')$. $d(i) \to d'(i')$, and either $d(j) \to d'(j')$, $d(k) \to d'(k')$ or $d(j) \to d'(k')$, $d(k) \to d'(j')$.

<u>Th 4:</u> $\langle 2\rangle$ is an equivalence relation.

Pf: The proof consists of showing that the three properties that define an equivalence relation hold for $\langle 2\rangle$; in this proof, numerals used as subscripts designate the first, second, or third step referred to by some step.

(A) Symmetry $D\langle 2\rangle D$

Define a mapping of D onto D such that $d(m) \to d(m)$. It is clearly true that properties (1) and (2) of $\langle 2\rangle$ hold. If d refers to some sequence of steps $d(i)$, $i = 1,2,3$, then $d(i) \to d(i)$; so properties (3) and (4) also hold.

(B) Reflexivity If $D\langle 2\rangle D'$ then $D'\langle 2\rangle D$.

Assume that $D\langle 2\rangle D'$. Then there exists a mapping, d(m)->d'(m'), with properties (1) to (4). For D'<2>D, define the inverse mapping d'(m')->d(m).

Since properties (1) and (2) hold for the mapping of D onto D', they hold for the mapping of D' onto D.

Let d' be any step in D', and let d be the step in D that maps into d' under D->D'; under D'->D , d'->d. If d in D refers to d(i), i = 1,2, and d->d' in D', then d' refers to d'(i), i = 1,2 where d(i)->d'(i), i = 1,2 (by property (3) of D->D'). under the inverse mapping d'->d, d' refers to d'(i'), i' = 1,2, and d'(i)->d(i), i = 1,2 .Therefore property (3) holds for the inverse mapping.

If d has rule, IP, then d and d' have three line references. d refers to d(i), i = 1,2,3 and d' refers to d'(i), i = 1,2,3. Under D->D', either d(i)->d'(i), i = 1,2,3, or d(1)->d'(1), d(2)->d'(3) and d(3)->d'(2). If the former condition holds, then d'(i)->d(i), i = 1,2,3 under D'->D. If the latter condition occurs, then d'(1)->d(1), d'(3)->d(2), and d'(2)->d(3). In either case, property (4) holds for D'->D.

(C) Transitivity If D<2>D' and D'<2>D" then D<2>D".

Assume that D<2>D' and D'<2>D". Then there exist mappings D->D' and D'->D", with properties (1) to (4). Let d be any step in D; d->d' and d'->d". Define a new mapping from the used steps of D onto the used steps of D" such that d->d" for all d in D.

By property (1) of D->D' and D'->D", if d is the n-th step in D then d" is the n-th step in D". Property (1) holds for the new mapping.

Since d has the same rule and sequence of arguments as d' and d' has the same rule and sequence of argument as d", d and d" have the same rule and the same sequence of arguments. Property (2) holds for the new mapping.

Let d refer to d(i), i = 1,2 , d' refer to d'(i), i = 1,2, and d" refer to d"(i), i = 1,2. Under the new mapping d->d", and d(i)->d"(i), i = 1,2.

If d has rule, IP, then d,d',and d" all have three line references. Under D->D', d(1)->d'(1), and under D'->D", d(1)->d"(1); under D->D", d(1)->d"(1). For the second and third references there are four possible cases, since there are two cases for D->D' and two cases for D'->D". Assume that d(2)->d'(3), d(3)->d'(2), and d'(2)->d"(2), d'(3)->d"(3). Then d(2)->d"(3) and d(3)->d"(2), and property (4) holds in this case. In a similar fashion, it can be shown that property (4) holds in the other three cases as well.

<u>Def</u> <u>10:</u> D<3>D′ iff D and D′ are derivations in S, and there exists a mapping of the used steps of D onto the used steps of D′, with the following properties: d(m)->d′(m′)

(1) d(m) and d′(m′) have the same rule and the same argument list.

(2) if d(m) uses a rule that requires either one or two references and d(m) refers to d(i), d(j) (d(i) if d has only one reference), then d′(m′) refers to d(i′), d(j′) and d(i)->d′(i′), d(j)->d′(j′).

(3) if d has rule, IP, then d refers to d(i),d(j),d(k) and d′ refers to d′(i′), d′(j′), d′(k′). d(i)->d′(i′), and either d(j)->d′(j′), d(k)->d′(k′) or d(j)->d′(k′), d(k)->d′(j′).

<u>Th</u> <u>5:</u> <3>is an equivalence relation on S.

Pf: The proof of theorem 5 follows the same pattern as the proof of theorem 4.

Derivations on the logic program consist of a sequence of steps and each step applies one of a finite set of rules. Let R(i) be the i-th rule in the set of rules; i = 1,...,M. The order of the rules is not important.

<u>Def</u> <u>11:</u> R(i) is said to occur in D, if some used step in D applies R(i).

Since rules may occur more than once in a derivation, we will designate the number of occurrences of R(i) in D by N(i). It should be emphasized that the definition of occurrence for a rule is restricted to used steps. The sequence of numbers, N(i), is a frequency distribution over the set of rules.

<u>Def</u> <u>12:</u> D<4>D′ iff D and D′ are derivations in S, and for every rule, the frequency of occurrence in D is the same as the frequency of occurrence in D′; for i = 1,...,M, N(i) = N′(i).

<u>Th</u> <u>6:</u> <4> is an equivalence relation on S.

Pf:

(A) Identity D<4>D D has the same frequency distribution for rules as itself.

(B) Reflexivity If D<4>D′ then D′<4>D. If D<4>D′, then N(i) = N′(i), i = 1,...,M. But then D′<4>D.

(C) Transitivity If D<4>D′ and D′<4>D″, then D<4>D″. Assume that D<4>D′ and D′<4>D″. Then N(i) = N′(i), and N′(i) = N″(i), i = 1,..,M. Therefore

$N(i) = N''(i)$, $i = 1,..,M$ and $D\langle 4\rangle D''$.

Def 13: $D\langle 5\rangle D'$ iff D and D' are derivations in S, and a le rule occurs in the used steps of D iff the same r occurs in the used steps of D'; for $i = 1,...,M$ $N(i) = 0$ iff $N'(i) = 0$.

Th 7: $\langle 5\rangle$ is an equivalence relation on S.
Pf: Here the frequency distribution over the set of rules is reduced to a set of 0-1 variables, $O(i)$. Let $O(i) = 1$ if $N(i)$ is not equal to 0 and let $O(i) = 0$ if $N(i)$ is equal to 0. This theorem is then a special case of the previous theorem.

Th 8: Let D and D' be solutions to some derivation problem. For $i = 1,...,4$, if $D\langle i\rangle D'$ then $D\langle i+1\rangle D'$.
Pf:
(1) If $D\langle 1\rangle D'$, then D is identical to D'; $D=D'$. Since $\langle 2\rangle$ is an equivalence relation, $D\langle 2\rangle D$ or $D\langle 2\rangle D'$.
(2) If $D\langle 2\rangle D'$, then a mapping $D\text{->}D'$ exists with properties (2) to (4) of definition 9. The weaker mapping of definition 10 is defined by these three properties. So $D\langle 3\rangle D'$.
(3) If $D\langle 3\rangle D'$, then there exists a mapping, $D\text{->}D'$, with property (1) of definition 11. For every occurrence of $R(i)$ in D, here is a corresponding occurrence of R in D'. So $N(i) = N'(i)$ for $i = 1,..,M$, and $D\langle 4\rangle D'$.
(4) If $D\langle 4\rangle D'$, then $N(i) = N'(i)$ for $i = 1,..,M$. So $O(i) = O'(i)$ for $i = 1,..M$ and $D\langle 5\rangle D'$.

3.3 EXAMPLE OF CLASSIFICATION OF PROOFS

Eight proofs for problem 414035 are included in this section to illustrate how the nested classification procedures work. Although these proofs were selected from the data used in this study, they are not meant to be representative of the general data base or even the data for this problem. The proofs in this subsample were selected so that the number of classes would decrease by one or two from each partition to the next. Problem 414035 was selected for two reasons. First, the proofs generated for it show enough differences at each level of equivalence to illustrate the procedure. Second, the proofs are short enough to permit a relatively clear presentation of the differences without the distraction of too much detail. The statement of problem 414035 is:

414035:
'AI' STANDS FOR THE ADDITIVE INVERSE AXIOM.
DERIVE: 3+(A+(-A))=3

The eight proofs are found in Table 1, and are labeled from "A" to "H" for ease of reference. Under the first partition, each proof in the subsample defines a separate equivalence class; the eight proofs were chosen so that this would be the case.

None of the proofs in Table 1 are identical, but they all have certain things in common. Each uses the two axioms, AI (additive inverse axiom) and Z (zero axiom), and some subset of the following rules:

AE - add equal terms to both sides of an equation
CA - commute addition
CE - commute around an equal sign
LT - logical truth
RE - replace equals

The similarity in the proofs is not surprising since they are all proofs for the same formula.

The equivalence classes under the second partition are also defined by paradigm proofs for each class; the paradigm proofs for the second partition are listed in Table 2.

Proofs C and F are now equivalent. In proof F (see Table 1 for the original form of proof F), the first step is not referred to by any susequent step; the first step is an unused step and is eliminated from the proof before the comparisons for the second classification are done. The line reference numbers in proof F are also changed to reflect the elimination of the first step. When this is done proofs C and F are identical.

The paradigm proof for proof H is also changed. The DLL step and the CE step, that was deleted by the DLL, are removed. In this case, no changes in line reference numbers are required. In proof B, an unused LT step is removed and subsequent line reference numbers are changed.

The paradigm proofs for the third partition are listed in Table 3. Under this partition, proofs A and H are equivalent, and proofs C, D and F are equivalent.

If we examine proofs C and D in Table 2, we see that each has four steps. The first two steps in D are identical to the first two steps in C but the order is reversed; this change in order has no effect on the form of the lines generated. The rules for the last two steps are the same in the two proofs, but the line reference numbers are different. This difference is due to the fact that the order of the first two steps is different in the two proofs. The third step (CE1) in each proof refers to the previous AI step. The fourth step in each proof refers to the Z step and the CE step in that order. The structure of the two proofs is the same, and the apparent differences all result from the arbitrary reversal of the first two lines.

The equivalence classes for the fourth partition are defined by frequency distributions over the available rules (see Table 4). For convenience, the distributions in Table 4 and Table 5 are taken over the limited set of rules that actually appear in the subsample.

In going from the third partition to the fourth partition, three classes are combined into one class (A,H,B,E). The proofs in this class have the same number of steps and the same frequency distribution over the rules. The differences that exist between these proofs (see Table 3) are in the order in which the operations are performed, and the lines in the proof that the operations are performed on.

In order to clarify the distinction between the third and fourth partitions, I will compare two proofs (A,H and B) that are equivalent under the fourth partition but not under the third. The first three steps in the two proofs are the same. The rules for the remaining steps are also the same, but they are used somewhat differently in the two proofs.

In both cases, the objective of the third and fourth steps is to replace the term, "0+3", in line 3 by the term, "3". If some line in a proof is of the form A=B, where A and B are terms, the replace equals rule, RE, allows the substitution of B for any occurance of A in the proof. Using line 2 and RE, any occurance of, "3+0" can be replaced by "3". The term that appears in line 3 is, "0+3", and RE cannot be used with line 2 to replace this term by, "3".

The A,H proof resolves this difficulty by usin the CA1 step (commute addition around the first plus sign in the equation) on line 2 to form line 4, "0+3=3". In step 5, RE is used in conjunction with line 4 to replace, "0+3", in line 3 by, "3".

The B proof uses CA3 (commute addition around the third plus sign) on line 3, changing, "0+3", to, "3+0". RE is then used with line 2 to generate line 5.

The sixth step is the same for both proofs. This is a relatively minor variation but it does indicate a difference of approach in producing proofs.

The only change in going from the fourth partition to the fifth partition is the inclusion of proof G in the A,H,B,E class. Proof G has a useless transformation in the third step that is corrected in the fifth step. Thus there are two unnecessary uses of the CA rule in proof G.

TABLE 3.1 - FIRST PARTITION

.	.	.AI.,A	(1)	A+(-A)=0	
.	.	.Z.,3	(2)	3+0=3	PROOF A
1.	.	.AE.,3	(3)	(A+(-A))+3=0+3	
2.	.	.CA1.*	(4)	0+3=3	
3.	4.	.RE1.*	(5)	(A+(-A))+3=3	
5.	.	.CA2.*	(6)	3+(A+(-A))=3	

.	.	.LT.,3	(1)	3=3	B
.	.	.AI.,A	(2)	A+(-A)=0	
.	.	.Z.,3	(3)	3+0=3	
2.	.	.AE.,3	(4)	(A+(-A))+3=0+3	
4.	.	.CA3.*	(5)	(A+(-A))+3=3+0	
5.	3.	.RE1.*	(6)	(A+(-A))+3=3	
6.	.	.CA2.*	(7)	3+(A+(-A))=3	

.	.	.Z.,3	(1)	3+0=3	C
.	.	.AI.,A	(2)	A+(-A)=0	
2.	.	.CE1.*	(3)	0=A+(-A)	
1.	3.	.RE1.*	(4)	3+(A+(-A))=3	

.	.	.AI.,A	(1)	A+(-A)=0	D
.	.	.Z.,3	(2)	3+0=3	
1.	.	.CE1.*	(3)	0=A+(-A)	
2.	3.	.RE1.*	(4)	3+(A+(-A))=3	

.	.	.AI.,A	(1)	A+(-A)=0	E
1.	.	.AE.,3	(2)	(A+(-A))+3=0+3	
2.	.	.CA2.*	(3)	3+(A+(-A))=0+3	
.	.	.Z.,3	(4)	3+0=3	
4.	.	.CA1.*	(5)	0+3=3	
3.	5.	.RE1.*	(6)	3+(A+(-A))=3	

.	.	.LT.,3	(1)	3=3	F
.	.	.Z.,3	(2)	3+0=3	
.	.	.AI.,A	(3)	A+(-A)=0	
3.	.	.CE1.*	(4)	0=A+(-A)	
2.	4.	.RE1.*	(5)	3+(A+(-A))=3	

TABLE 3.1 CONTINUED

.	.	.AI.,A	(1) A+(-A)=0	G
1.	.	.AE.,3	(2) (A+(-A))+3=0+3	
2.	.	.CA1.*	(3) ((-A)+A)+3=0+3	
3.	.	.CA3.*	(4) ((-A)+A)+3=3+0	
4.	.	.CA1.*	(5) (A+(-A))+3=3+0	
5.	.	.CA2.*	(6) 3+(A+(-A))=3+0	
.	.	.Z.,3	(7) 3+0=3	
6.	7.	.RE1.*	(8) 3+(A+(-A))=3	

.	.	.AI.,A	(1) A+(-A)=0	H
1.	.	.AE.,3	(2) (A+(-A))+3=0+3	
.	.	.Z.,3	(3) 3+0=3	
3.	.	.CE1.*	(4) 3=3+0	
.	.	.DLL.*		
3.	.	.CA1.*	(4) 0+3=3	
2.	4.	.RE1.*	(5) (A+(-A))+3=3	
5.	.	.CA2.*	(6) 3+(A+(-A))=3	

TABLE 3.2 - SECOND PARTITION

.	.	.AI.,A	(1)	A+(-A)=0	A
.	.	.Z.,3	(2)	3+0=3	
1.	.	.AE.,3	(3)	(A+(-A))+3=0+3	
2.	.	.CA1.*	(4)	0+3=3	
3.	4.	.RE1.*	(5)	(A+(-A))+3=3	
5.	.	.CA2.*	(6)	3+(A+(-A))=3	
.	.	.AI.,A	(1)	A+(-A)=0	B
.	.	.Z.,3	(2)	3+0=3	
1.	.	.AE.,3	(3)	(A+(-A))+3=0+3	
3.	.	.CA3.*	(4)	(A+(-A))+3=3+0	
4.	2.	.RE1.*	(5)	(A+(-A))+3=3	
5.	.	.CA2.*	(6)	3+(A+(-A))=3	
.	.	.Z.,3	(1)	3+0=3	C,F
.	.	.AI.,A	(2)	A+(-A)=0	
2.	.	.CE1.*	(3)	0=A+(-A)	
1.	3.	.RE1.*	(4)	3+(A+(-A))=3	
.	.	.AI.,A	(1)	A+(-A)=0	D
.	.	.Z.,3	(2)	3+0=3	
1.	.	.CE1.*	(3)	0=A+(-A)	
2.	3.	.RE1.*	(4)	3+(A+(-A))=3	
.	.	.AI.,A	(1)	A+(-A)=0	E
1.	.	.AE.,3	(2)	(A+(-A))+3=0+3	
2.	.	.CA2.*	(3)	3+(A+(-A))=0+3	
.	.	.Z.,3	(4)	3+0=3	
4.	.	.CA1.*	(5)	0+3=3	
3.	5.	.RE1.*	(6)	3+(A+(-A))=3	
.	.	.AI.,A	(1)	A+(-A)=0	G
1.	.	.AE.,3	(2)	(A+(-A))+3=0+3	
2.	.	.CA1.*	(3)	((-A)+A)+3=0+3	
3.	.	.CA3.*	(4)	((-A)+A)+3=3+0	
4.	.	.CA1.*	(5)	(A+(-A))+3=3+0	
5.	.	.CA2.*	(6)	3+(A+(-A))=3+0	
.	.	.Z.,3	(7)	3+0=3	
6.	7.	.RE1.*	(8)	3+(A+(-A))=3	
.	.	.AI.,A	(1)	A+(-A)=0	H
1.	.	.AE.,3	(2)	(A+(-A))+3=0+3	
.	.	.Z.,3	(3)	3+0=3	
3.	.	.CA1.*	(4)	0+3=3	
2.	4.	.RE1.*	(5)	(A+(-A))+3=3	
5.	.	.CA2.*	(6)	3+(A+(-A))=3	

TABLE 3.3 - THIRD PARTITION

.	.	.AI.,A	(1)	A+(-A)=0	A,H
.	.	.Z.,3	(2)	3+0=3	
1.	.	.AE.,3	(3)	(A+(-A))+3=0+3	
2.	.	.CA1.*	(4)	0+3=3	
3.	4.	.RE1.*	(5)	(A+(-A))+3=3	
5.	.	.CA2.*	(6)	3+(A+(-A))=3	
.	.	.AI.,A	(1)	A+(-A)=0	B
.	.	.Z.,3	(2)	3+0=3	
1.	.	.AE.,3	(3)	(A+(-A))+3=0+3	
3.	.	.CA3.*	(4)	(A+(-A))+3=3+0	
4.	2.	.RE1.*	(5)	(A+(-A))+3=3	
5.	.	.CA2.*	(6)	3+(A+(-A))=3	
.	.	.Z.,3	(1)	3+0=3	C,F,D
.	.	.AI.,A	(2)	A+(-A)=0	
2.	.	.CE1.*	(3)	0=A+(-A)	
1.	3.	.RE1.*	(4)	3+(A+(-A))=3	
.	.	.AI.,A	(1)	A+(-A)=0	E
1.	.	.AE.,3	(2)	(A+(-A))+3=0+3	
2.	.	.CA2.*	(3)	3+(A+(-A))=0+3	
.	.	.Z.,3	(4)	3+0=3	
4.	.	.CA1.*	(5)	0+3=3	
3.	5.	.RE1.*	(6)	3+(A+(-A))=3	
.	.	.AI.,A	(1)	A+(-A)=0	G
1.	.	.AE.,3	(2)	(A+(-A))+3=0+3	
2.	.	.CA1.*	(3)	((-A)+A)+3=0+3	
3.	.	.CA3.*	(4)	((-A)+A)+3=3+0	
4.	.	.CA1.*	(5)	(A+(-A))+3=3+0	
5.	.	.CA2.*	(6)	3+(A+(-A))=3+0	
.	.	.Z.,3	(7)	3+0=3	
6.	7.	.RE1.*	(8)	3+(A+(-A))=3	

TABLE 3.4 - THE FOURTH PARTITION

AE	AI	CA	CE	RE	Z	
1	1	2		1	1	A,H,B,E
	1		1	1	1	C,F,D
1	1	4		1	1	G

TABLE 3.5 - THE FIFTH PARTITION

AE	AI	CA	CE	RE	Z	
X	X	X		X	X	A,H,B,E,G
	X		X	X	X	C,F,D

CHAPTER FOUR

Since one objective of the logic program is to develop flexibility in the student's approach to the construction of proofs, it is desirable to avoid the inclusion of derivation problems which encourage stereotyped proof behavior. For the future development of this curriculum (and similar curricula), it would be useful to know how the attributes of derivation problems affect the degree of diversity found in proof behavior. The analysis described below is designed to identify those characteristics of derivation problems which best predict the amount of variation found in a sample of proofs for the problems. For each problem in the curriculum and for each set of criteria, student proofs were classified into equivalence classes. The number of different classes occurring for a particular problem was used as a measure of variability of student proofs for the problem.

After the sample of proofs had been partitioned, the relationship between the number of classes per problem and the structural attributes of the problems was investigated using multiple linear regression.

Since linear regression is a commonly used technique, the details of this method will not be included here. A discussion of the way in which regression analysis was used in this study and of the assumptions involved in using regression is found in section 4.5. The model assumed in all of the analyses is linear:

$$Y_j = a_1 * X_{1,j} + \ldots + a_n * X_{n,j} + e_j$$

where Y(j) is the value of the dependent variable for the j-th problem, X(i,j) is the value of the i-th independent variable for the j-th partition, the a(i) are constants, and e(j) is the error term for the j-th problem.

For each of the five measures of variation, a separate regression analysis was run, with the number of classes per problem as the dependent variable. The independent variables used are similar to those used in a previous study of the Stanford Logic- algebra curriculum (Moloney, 1971), and these variables are discussed in section 4.4.

4.2 - THE SAMPLE OF DATA

During the summer quarter of 1970, the Logic-Instructional system was used as an integral part of the introductory logic course (Philosophy 157) at Stanford University. The students were proctored during their sessions at the computer terminals by the philosophy graduate students who gave the lectures in the course. The course consisted of two hours of traditional classroom instruction each week in addition to the time spent working at the computer terminals.

The LIS curriculum emphasizes the construction of formal proofs, and it is the behavior of students in constructing such proofs that is examined in this dissertation. Four of the 27 students who enrolled in this course failed to complete some parts of the curriculum included in this study, and these students have been dropped from the analysis.

The fact that approximately fifteen percent of the original sample were dropped because they failed to complete a substantial part of the curriculum raises the possibility that the results of this study are biased by the selection of the more successful students. If we assume that there is no interaction effect (student-problem), the elimination of the data for these students would tend to affect the results for all problems in the same way, but would not bias a comparison between problems. Moreover, the inclusion of proofs by students who did only part of the curriculum would introduce bias into a comparison between problems, because the results for some problems would be affectd by these students while others would not. So, it is necessary to drop these students and accept the possibility of bias arising from selection.

A similar problem of non-random selection arises when the full set of 27 students is considered, since these students selected themselves for this study by deciding to enroll fo Philosophy 157 in the Summer of 1970. The extent to which the findings of this study can be generalized to other curricula and other student populations will depend on the extent to which the tasks and the population in this study are representative of the target tasks and population.

Even for the 23 students who completed the part of the curriculum included in this study, some data were lost because of machine failure; this problem will be discussed in the next section. A relatively complete set of data is available from these students for the problems in lessons 405 to 415, and it is the 127 proof problems in these lessons that are considered in the analysis.

4.3 DEPENDENT VARIABLES (MEASURES OF VARIABILITY)

In the analysis reported in Chapters V and VI, stepwise regression was used to relate five measures of variation in the sample of proofs to 17 variables that characterize the nature of the problem. A separate regression analysis is presented for each of the five measures of variation. In this section, the dependent variables (measures of variation) are discussed, and in the next section the independent variables are discussed.

For each of the problems under consideration there are approximately 23 proofs in the sample, and the same 23 students are used for all problems. The five sets of equivalence criteria defined in Chapter III generate a nested sequence of five partitions on the sample of proofs for each problem. The first dependent variable, C1, is defined to be the number of classes under the first partition. The variables, C2 to C5, are defined to be the number of classes under the second to the fifth partitions respectively. The full set of proofs for any problem generates a single value for each of the five dependent variables, the number of classes of proofs for the five partitions.

Even for those students who completed the lessons of the curriculum included in this study, there was some loss of data due to machine failure, and the data lost in this way cannot be recovered. Since the machine failures that cause this type of data loss are independent of the students' behavior, the loss is assumed to be random.

If no data had been lost, the sample of proofs for the 23 students and 127 problems in this study would consist of 2,921 proofs. Out of this number, 51 proofs were lost because of machine failure. Although the percentage of missing proofs is quite small (1.7 percent), this loss of data could be a serious problem.

The definitions of the dependent variables make it difficult to deal with the poblem of missing data. Failure to include the proofs of one or more students cannot increase the number of classes found, and can decrease this number. Missing data, therefore, introduces a bias toward lower values for all five dependent variables on the problems with an incomplete sample of proofs. It should be emphasized that this bias results from the nature of the dependent variables, and exists even though the loss of data is random.

There are 89 problems with no missing proofs, 28 problems with one proof missing, eight problems with two missing proofs, one problem with three missing proofs, and one problem with four missing proofs. The two problems with more than two missing proofs were not used in the analysis that follows, and the results for the other 125 problems were modified to correct for the missing proofs.

In order to correct for the missing data, some assumptions must be made about the functional relationship between the number of classes in the sample of proofs and the total number of proofs in the sample. Using the relationship assumed, the number of classes in a sample of 21 or 22 proofs can then be extrapolated to a hypothetical sample of 23 proofs.

The nature of the dependent measures being used in this research implies that they are monotonically nondecreasing functions of the number of problems in the sample because the inclusion of another proof in the set being partitioned cannot decrease the number of subsets defined by the partition but can increase this number by one. Therefore, the desired functional relationship must have a positive slope.

As the number of proofs that have been partitioned increases, the probability that an additional proof would specify a new class (not fall into a class already specified) decreases. So, an accepable candidate for the functional relationship between the number of classes and the number of proofs should have a negative second derivative.

Examination of the set of student proofs for a representative sample of problems indicated that the relationship between the number of classes in a random subset of proofs and the number of proofs in the subset is approximated by the following formula:

$$CL = A*(SL)^{B} \qquad (1)$$

Where SL is the number of problems in the subset, CL is the number of classes, and A and B are constants that depend on the problem. For A positive and B between zero and one, this function meets both of the criteria specified above. Since a sample of one proof will always have one class, A is equal to 1, and (1) formula reduces to:

$$CL = (SL)^{B} \qquad (2)$$

The value of B for any problem can be estimated from the number of classes in the available set of proofs for the problem. Taking the logarithm of both sides of (2) gives:

$$\ln(CL) = B * \ln(SL) \qquad (3)$$

and B is then given by:

$$B = \frac{\ln(CL)}{\ln(SL)} \qquad (4)$$

Since the actual values of CL and SL are available for each problem, an estimate of B for each problem can be obtained using (4). The predicted value for a full set of 23 proofs can then be calculated from formula (2).

Using this technique, tables of the predicted values of CL for the possible range of the observed values of CL have been computed, and are included in Tables 1 and 2. Since the observed values of the dependent variables (number of classes of proofs) are integers, the corrected values for these variables are rounded to integers. The final correction criteria are listed in Table 3.

As a partial check on the impact of this correction procedure on the final results, the analyses to be discussed in Chapter V were also run without the eight problems that are missing two proofs. There were no major changes in the results when this was done. The corrected values for the dependent variables are used in all of the analyses reported in this paper

4.4 - INDEPENDENT VARIABLES

The set of independent variables used in this study is very similar to the set of variables used by James Moloney in a previous study of the same curriculum (Moloney, 1971). A list of the variables used in the present study is included as Table 4.4.

The first five variables listed in Table 4.4 quantify various types of structural complexity that can appear in the problem statements. Since these variables do not play a very prominent role in the analysis that follows and since the definitions for these variables are clear, they will not be discussed further here. In the remainder of this work, these variables will be called the 'problem-structure variables'.

S13(AV RE), S17(R INF), S18(AV TH), S19(AV AX), S20(TOT R), S21 (PSLI) AND S22(POSIT) are all defined in terms of the problem's position in the curriculum. S13(AV RE) is a 0-1 variable and indicates whether the problem appears before (S13=0) or after (S13=1), the introduction of Replace Equals. S17, S18, and S19 are counts of the numbers of rules of inference, theorems, and axioms that are available when the problem is reached in the curriculum. S20(TOT R) measures the total number of rules available when the problem is encountered, and is equal to the sum of S17, S18, and S19. S22(POSIT) is defined as the ordinal position of the particular problem in the sequence of problems considered in this study; this variable is included to check for any general order effect in the curriculum. These variables are referred to as the 'rule-position variables'.

The last group of variables to be considered are those that Moloney calls the 'standard proof variables'; the variables in this group are S11(RE), S12(CP), S14(AXIOM), S15(THERM), and S16(STEPS). The values of the standard proof variables for a problem are defined in terms of a 'standard' proof for the problem. The standard proofs used in this study are those constructed by Moloney; the same set of problems were done independently by the present author and no changes were found to be necessary. The criteria used in constructing these proofs are given by Moloney:

> Several criteria were used by the author in generating the standard proofs. First, the author worked through the entire set of problems included in this study two times. The proofs generated the second time through are used as standard. An attempt was made to construct proofs with a minimal number of lines. Also, within the constraint of producing a minimal proof, an attempt was made to use rules and theorems most recently introduced, wherever possible. It is the judgement of the author that the great majority of the proofs produced are minimal in the sense of containing the least possible number of lines.

Since it is the standard proof variables that dominate the discussion in Chapters V and VI, some further discussion of these variables is appropriate.

S16(STEPS) is just the number of steps in the standard proof, and functions as a simple measure of the length of the problem. The types of steps that appear in the standard proof has no effect on this variable.

S11(RE) is the number of occurrences of the rule, Replace Equals, in the standard proof. Replace Equals is an important rule in the algebra part of the curriculum because it permits the student to replace any expression(A) in a formula by an expression(B) that has been shown to be equal to expression(A). This allows the student to develop parts of an equation independently and then to combine these partial results into a single formula, thus it provides a mechanism for the use of subsidiary derivations. The problems included in this study are all drawn from the part of the curriculum dealing with algebra.

S14(AXIOM) and S15(THERM) count the number of occurrences in the standard proof of axioms and theorems respectively. The use of any of the five axioms or six theorems is counted as an occurrence; the axioms (or theorems) have equal weight and no distinction is made between them. If an axiom (or theorem) is used more than once, each application is counted as an occurrence. If the standard proof for a problem uses a particular axiom as the rule in two separate steps, another axiom in a third step, and none of the remaining steps use axioms, then the value of S14 for the problem would be three.

4.5 REGRESSION ANALYSIS

Since regression analysis is a standard technique in educational research, the statistical theory will not be developed here; the way in which regression is to be used in this study and the assumptions made in interpreting F-ratios in regression analysis will be discussed.

The research reported here is exploratory. Its primary aim is to determine those quantifiable characteristics of proof problems in algebra that account for the amount of variation found in a sample of proofs for these problems. No attempt is made to test a preconceived hypothesis, and little attention is given to the coefficients of the linear equations that result from the regression analyses.

The analyses reported in Chapters V and VI examine in great detail the relationships found in the data. The emphasis is on determining how the variation in the sample of proofs is related to the features of the proof problems defined by the independent variables. The use of five different measures of variability makes it possible to examine how the relationship between variability and problem type changes as a function of the kind of variability measured.

If the F-ratios that appear in the results of the regression analyses are to be considered, the validity of the assumptions that underlie the usual interpretation of these F-ratios should be examined. The model being used here is a simple linear model and the assumptions are that the errors are independently and normally distributed with zero mean and constant variance. For the analyses discussed in Chapter V (using the full set of problems), there is clear indication that the assumption of homogeneity of variance is violated. The variance seems to be an increasing function of the predicted value of C1. Attempts to eliminate this nonhomogeneity by transforming the observed values of C1 failed.

Among the plots of residuals against the independent variables, the strongest indication of this lack of homogeneity of variance is found for S22(POSIT); there is an abrupt increase in variance just after the introduction of the rule, Replace Equals. This discontinuity seems to be a property of the curriculum and not a function of the scale chosen for the dependent variable. It is unlikely that any continuous transformation (change of scale) for the dependent variable will eliminate the nonhomogeneity of variance.

However, there is no serious violation of homogeneity of variance if only the problems that appear after the introduction of RE are considered. In Chapter VI, the analysis described in this chapter will be repeated, using only the problems that appear after the introduction of RE and that do not have any axioms.

TABLE 4.1

PROJECTED NUMBER OF CLASSES FOR 23 STUDENTS
WHEN 21 SOLUTIONS ARE ACTUALLY CLASSIFIED

THE MODEL USED IS GIVEN BY:

$$CL = A^*(SL)^B$$

WHERE CL IS THE NUMBER OF CLASSES
SL IS THE NUMBER OF SOLUTIONS

OBSERVED	CORRECTED	EST-B*
1.00000	1.00000	.00000
2.00000	2.04186	.22767
3.00000	3.10012	.36085
4.00000	4.16917	.45534
5.00000	5.24633	.52863
6.00000	6.32999	.58852
7.00000	7.41908	.63915
8.00000	8.51285	.68301
9.00000	9.61072	.72170
10.00000	10.71225	.75630
11.00000	11.81708	.78761
12.00000	12.92492	.81619
13.00000	14.03552	.84248
14.00000	15.14868	.86682
15.00000	16.26423	.88948
16.00000	17.38200	.91068
17.00000	18.50186	.93059
18.00000	19.62369	.94937
19.00000	20.74739	.96713
20.00000	21.87285	.98397
21.00000	23.00000	1.00000

* EST-B IS THE ESTIMATED VALUE OF B

TABLE 4.2

PROJECTED NUMBER OF CLASSES FOR 23 STUDENTS
WHEN 22 SOLUTIONS ARE ACTUALLY CLASSIFIED

THE MODEL USED IS GIVEN BY:

$$CL = A^{*}(SL)^{B}$$

WHERE CL IS THE NUMBER OF CLASSES
SL IS THE NUMBER OF SOLUTIONS

OBSERVED	CORRECTED	EST B*
1.00000	1.00000	.00000
2.00000	2.02004	.22424
3.00000	3.04777	.35542
4.00000	4.08054	.44849
5.00000	5.11707	.52068
6.00000	6.15661	.57966
7.00000	7.19865	.62953
8.00000	8.24285	.67273
9.00000	9.28892	.71084
10.00000	10.33667	.74492
11.00000	11.38594	.77576
12.00000	12.43657	.80391
13.00000	13.48847	.82980
14.00000	14.54154	.85378
15.00000	15.59568	.87610
16.00000	16.65084	.89698
17.00000	17.70695	.91659
18.00000	18.76395	.93508
19.00000	19.82180	.95257
20.00000	20.88045	.96917
21.00000	21.93986	.98495
22.00000	23.00000	1.00000

* EST-B IS THE ESTIMATED VALUE OF B

TABLE 4.3

FINAL CORRECTION CRITERIA FOR THE DEPENDENT VARIABLE

ONE MISSING PROOF

NUMBER OF CLASSES FOUND	CHANGE
0 – 13	0
14 – 22	+1

TWO MISSING PROOFS

NUMBER OF CLASSES FOUND	CHANGE
0 – 7	0
8 – 16	+1
17 – 21	+2

TABLE 4.4

LIST OF INDEPENDENT VARIABLES

S6	(WORDS)	NUMBER OF WORDS PER PROBLEM
S7	(SYMBL)	NUMBER OF SYMBOLS IN THE FORMULA TO BE DERIVED
S8	(LOGCN)	NUMBER OF LOGICAL CONNECTIVES IN THE FORMULA TO BE DERIVED
S9	(PAREN)	DEPTH OF NESTING OF THE MOST DEEPLY NESTED NESTED EXPRESSION IN THE FORMULA TO BE PROVED
S10	(PREMS)	NUMBER OF PREMISES
S11	(RE)	THE NUMBER OF OCCURRENCES OF REPLACE EQUALS
S12	(CP)	THE NUMBER OF OCCURRENCES OF CONDITIONAL PROOF (CP)
S13	(AV RE)	A 0-1 VARIABLE INDICATING THE AVAILABILITY OF REPLACE EQUALS
S14	(AXIOM)	THE NUMBER OF OCCURRENCE OF ANY AXIOM
S15	(THERM)	THE NUMBER OF OCCURRENCES OF ANY THEOREM
S16	(STEPS)	THE NUMBER OF STEPS IN THE STANDARD PROOF
S17	(R INF)	THE NUMBER OF RULES OF INFERENCE AVAILABLE
S18	(AV TH)	THE NUMBER OF THEOREMS AVAILABLE
S19	(AV AX)	THE NUMBER OF AXIOMS AVAILABLE
S20	(TOT R)	THE TOTAL NUMBER OF RULES AVAILABLE WHEN THE PROBLEM IS DONE
S21	(PSLI)	THE NUMBER OF PROBLEMS SINCE THE LAST INTRODUCTION OF A RULE
S22	(POSIT)	THE ORDINAL POSITION OF THE PROBLEM IN THE PORTION OF THE CURRICULUM BEING STUDIED

CHAPTER FIVE

In this chapter, the results of the regression analyses for the full set of problems will be examined. A separate regression analysis was run for each of the five partitions discussed in chapter 3. For the first analysis, the number of classes in the first partition of the proofs for a problem is taken as the value of the dependent variable, C1, for that problem. Separate dependent variables (C2 - C5) are defined analogously for each of the other four partitions, and the regression analyses using these dependent variables are discussed in order. In each case, the set of 17 independent variables described in chapter 4 is used.

Since the nonhomogeneity of variance discussed in chapter IV occurs for all five of the regression analyses discussed in this chapter, the F-ratios computed in these analyses will not be interpreted. The discussion here emphasizes a detailed examination of the results and ignores hypothesis testing considerations.

The means and standard deviations for the full set of 22 variables (5 dependent and 17 independent) are listed in Table 5.1, and the correlation matrix is found in Table 5.2. Variables numbered from 1 to 5 are the dependent variables, and variables numbered from 6 to 22 are the independent variables.

Examination of Table 5.2 indicates a number of interesting trends. The first five columns contain the correlations of the five dependent variables with each other. All of these correlations are high (greater than (0.69), and the partitions closest in the sequence from one to five have the highest correlations.

The remaining entries in the first five rows are the correlations between the five dependent variables and the 17 independent variables. Many of the correlations are quite high; the largest is 0.85 between C2 and S11(RE). Variable, S11, also has large correlations with the other four dependent variables, and the magnitudes of these correlations decrease monotonically as we go from C2 to C5.

Another independent variable, S16(STEPS), also has high correlations with the dependent variables, and these correlations also decrease monotonically from C2 to C5. S11 and S16 are both relatively simple measures of the structural complexity of the standard proof for a problem. S16 is the number of steps in the standard proof, while S11 is the number of occurrences of the rule, RE, in the standard proof. The correlation between these two variables is 0.79.

S15(THERM) which is also a standard proof variable, displays the opposite pattern; its correlation with the first dependent variable is relatively small (0.33) but increases rapidly from C2 to C5. The correlation of S15 with C5 is 0.68, and is larger than that for any of the other independent variables.

In Figure 5.1, the correlations of S11(RE), S16(STEPS), and S15(THERM) with the five dependent variables are plotted against the ordinal number of the dependent variable (or equivalently, against the ordinal number of the partitions that define the dependent variables). The correlations of S11 and S16 decrease most rapidly as the definition of the dependent variable changes from the third to the fifth partition, while the correlation of S15 with the dependent variable increases most rapidly from the third to the fifth partition. It should be noted that the fourth and fifth partitions are the only partitions that do not depend, at all, on the order of the steps in a proof; they depend only on the rules that are used in the proof.

Variables S13(AV RE), S17(R INF), S18(AV TH), S19(AV AX), S20(TOT R), and S22(POSIT) also have substantial correlations with the dependent variables. The pairwise correlations between these variables are generally high, and all are highly correlated (> 0.85) with S22. In the discussion that follows, these variables will be referred to as the 'rule-position' variables.

The value of the position variable, S22, for a given problem, is the ordinal position of the problem within the total set being examined. All of the rule-position variables are confounded with the position variable, hence any contribution that they make to the variance accounted for by the regression equation may be due to the ordering of the problems within the curriculum.

The correlations for the remaining variables, called 'problem-structure variables', are relatively small, and they will not be discussed in any detail. These variables show the same trend as S11 and S16, but the correlations are much smaller and the pattern is less regular.

5.2 - ANALYSIS BASED ON THE FIRST PARTITION

The first dependent variable to be considered is C1, the number of classes found in the sample when the first partition is used to define the dependent variable. The output from the steOwise regression program(BMDO2R) is presented for the first four steps of the analysis in Tables 5.3A,B,C,D.

In Table 5.3A, we see that S11(RE) is the first variable to enter the equation. S11 accounts for 55 percent of the variance in C1. The table of partial correlations that results after S11 has been partialed out is worth examining carefully.

With S11(RE) partialled out, the correlation of S16(STEPS) with C1 is only 0.31, having dropped from 0.72; S11 accounts for most of the variance that could otherwise be accounted for by S16. The correlation of S15(THERM) with the dependent variable increases from 0.33 to 0.40. This increase is partially explained by the low correlation of S15 with S11 (0.06); S11 accounts for very little of the variance that S15 is capable of predicting, while eliminating much of the variance not accounted for by S15. S18(AV TH) also shows a slight increase, but the correlations of the other rule-position variables with C1 all decrease; S11 has a correlation of 0.42 with S22(POSIT), and is taking out some of the 'rule-position' variance. The correlations of the problem-structure variables with the dependent variable increase slightly but remain relatively small.

The second variable to enter the equation is S22(POSIT), and the output for this step is found in Table 5.3B. The coefficient for S22 is positive, and the coefficient for S11(RE) decreases slightly when S22 enters the equation. The small magnitude of the coefficient for S22 is due to the fact that the position variable has a very wide range compared to the dependent variable.

S22(POSIT) is strongly correlated with the measures of the complexity of the set of rules available for any of the proof problems. It is not clear how much of the importance of this variable is due to the availability of rules and how much is due to the fact that curriculum writers tend to introduce problems of increasing complexity as the curriculum progresses(the position effect).

After the variance accounted for by S22(POSIT) has been partialed out, the correlations between C1 and all of the rule-position variables drop sharply. The correlations of S15(THERM) and S14(AXIOM) with the dependent variable decrease slightly and the correlation of S16(STEPS) with C1 increases.

S16(STEPS) is the third variable to enter the equation (Table 5.3C). The addition of S16 to the regression equation causes the coefficient of S11(RE) to drop to about one-third of its value at the previous step. S16 is now accounting for a substantial part of the variance that had previously been accounted for by S11.

At this point, the variance accounted for by S11(RE), S16(STEPS), and most of the variance accounted for by the rule-position variables has been partialed out. The largest partial correlations are now found for the variables, S6 to S10, which measure the complexity of the problem statement. The next independent variable to enter the equation is S7(SYMBL).

Rather than continue this step-by-step examination of the results of regression analysis, the nature of the relationship between the dependent variable and the first three independent variables to enter the equation will be examined more closely. The summary table for the analysis is found in Figure 5.4.

A scatterplot of C1 against S11(RE) is presented in Figure 5.2A. The relationship seems linear, but the variance of C1 for any value of S11 is large, and there is some indication that the variance is not independent of S11. The plot of residuals (calculated after all the variables have entered the equation) against S11 (Figure 5.2B) confirms these observations.

Examination of the plot of C1 against S22(POSIT) in Figure 5.3A indicates a very different situation. For values of S22 less than 50, both the mean and variance of the distribution of C1 given S22 have relatively low values and seem to be independent of S22. For values of S22 above 55, the mean and variance of the conditional distribution of C1, given S22, again appear to be independent of S22, but both have much higher values than they did for the problems with S22 less than 50. The plot of the residuals(computed after all of the variables have entered the equation) against S22 in Figure 5.3B does not contain any evidence for a departure from linearity but does show clearly the abrupt change in variance.

A possible explanation of this phenomenon becomes apparent when the curriculum is examined. Between the problem with S22(POSIT) equal to 53 and the problem with S22 equal to 54, Replace Equals (RE) is introduced. RE permits the student to substitute for any expression(A), in a formula, any other expression(B) that has been shown to be equal to the expression(A). After the formula, A=B, has been proved, A can be relaced by B in any formula within the student's partial proof. This rule greatly increases the student's flexibility in the in the order in which he uses the available rules to construct a proof; the partition that defines C1 is sensitive to these differences in order (see Chapter II for a more detailed discussion of RE).

Figures 5.4A,B contain the corresponding plots for S16(STEPS). Again there is evidence for a basically linear relationship and nonhomogeneity of variance. The indication in Figure 5.4A of a possible departure from linearity is not confirmed by Figure 5.4B. This impression of non-linearity is due to the six points in the upper right corner of Figure 5.4A. All six of these problems are long but straightforward; they do not use any of the more difficult rules, and they do not involve the recognition of any complicated sequence of the simpler rules; in spite of their length, these problems are unusually simple.

Figure 5.5 is a frequency histogram for the residuals. There is no evidence in this figure of any serious departure from normality. Figure 5.6 is a scatterplot of the residuals (after all of the independent variables have entered the equation) against the predicted value of C1; in this figure, there is clear indication that the assumption of homogeneity of variance has been violated. The variance seems to be an increasing function of the predicted value of C1. Attempts to eliminate this nonhomogeneity transforming the observed values of C1 failed; a logarithmic transformation and a square-root transformation were both used without success.

Among the plots of residuals against the independent variables, the strongest indication of this nonhomogeneity of variance is found for S22 (see Figure 5.3B). The variance in the residuals is not a smoothly varying function as Figure 5.6 indicates, instead there is an abrupt increase in variance just after the introduction of the rule, Replace Equals. This discontinuity seems to be a property of the curriculum and not a function of the scale chosen for the dependent variable. It is unlikely that any continuous transformation(change of scale) for the dependent variable would eliminate the nonhomogeneity of variance.

If the analysis is restricted to the problems that occur after the introduction of RE, the nonhomogeneity of variance is eliminated. Analyses using this restricted set of problems are reported in Chapter VI.

A full interpretation of these results must await the discussion of the analyses for the other four partitions, but some preliminary observations are appropriate here.

The first six variables to enter the regression equation account for 80 percent of the total variance in the dependent variable, and the first three variables account for over 74 percent of the variance. The simple linear model that has been assumed fits the data very well.

S11(RE) is the first variable to enter the equation, and accounts for 55 percent of the total variance in the dependent variable. The initial correlation (0.72) of S16(STEPS) with the dependent variable is almost as high as that (0.75) for S11(RE), and the correlation between these two variables is 0.79. It seems that these two variables are measuring similar properties of the problems. Both can be interpreted as relatively simple measures of the complexity of the standard proof for a problem.

Together, S11(RE) and S16(STEPS) account for almost 73 percent of the variance in the dependent variable. Since the first partition is sensitive to minor variations in the proofs, including changes in the order of the steps, it in not surprising that simple measures of comlexity account for most of the variance in C1(number of classes under the first partition). When the dependent variable is defined in terms of the fourth and fifth partitions, which are not senstive to minor variations in the proofs, the predictive power of S11(RE) and S16(STEPS) is greatly diminished.

5.3 - ANALYSIS BASED ON THE SECOND PARTITION

The pattern of results for the second partition (with C2, the number of classes of proofs under the second partition, as the dependent variable) parallels the first. The initial correlations are roughly the same. The first variable to enter is S11(RE). The pattern of partial correlations that appears after S11 has been included in the equation (see Table 5.5A) is very similar to that for the first partition (see Table 5.3A). There is one notable exception to this generalization. The correlation of S22(POSIT) with the dependent variable drops more sharply when S11 is partialled out than it did for the first partition. As a result, S15(THERM) has the highest partial correlation in Table 5.5A. The apparent importance of S15 is especially notable, because the first theorem is introduced only after eighty percent of the problems included in this study have been completed, and S15 has a very small range with only three possible values, 0, 1, and 2. The inclusion of S15 at the second step is due in part to the fact that its correlation with S11 is only 0.08.

The partial correlations after the introduction of S15 are shown in Table 5.5B. The pattern that appears is very similar to the pattern found after the introduction of S22 in the previous analysis. Since the correlation between S15(THERM) and S22(POSIT) is 0.54, it is not surprising that they have a similar effect on the partial correlations in the two analyses. The third variable to enter (Table 5.5C) is S16(STEPS) and the fourth is S12(CP). The summary table for this analysis is found in Table 5.6.

Figure 5.7 is a plot of the residuals against the the predicted value of C2, and Figure 5.8 is a plot of the observed values of C2 against S22(POSIT). The evidence for nonhomogeneity of variance is even more pronounced than it was in the previous analysis; the explanation is the same as it was there.

The only difference between the first partition and the second partition is that the unused steps in proofs are not relevant under the second partition. Since the correlation between C1(first partition) and C2(second partition) is 0.94 it is not surprising that the results for this analysis are very similar to the results for the first partition.

The substitution of S15(THERM) for S22(POSIT) is worth noting. The importance of S15 as a predictor of the dependent variable increases consistently as the dependent variable changes from the second to the fifth partition.

5.4 - ANALYSIS BASED ON THE THIRD PARTITION

Since the analysis for the third partition is very similar to the two analyses already examined, the results are only sketched. The correlation of S15(THERM) with the dependent variable increases from 0.30 to 0.36 in changing from the second to the third dependent variable. The initial correlations for the rule-position variables are larger than they were for the second partition. The initial correlations of S11(RE) and S16(STEPS) with C3 are smaller than they were for the second partition, but they are still quite large.

S11 enters the equation first and has roughly the same effect on the partial correlations as it did for the second partition. S15(THERM) and S16(STEPS) are the second and third variables to enter the equation. The fourth variable included is S14(AXIOM). The first problem structure variable is not introduced until step five, and contributes only two percent to the total variance accounted for by the regression equation. For reference, the results of this analysis are included in Tables 5 7A,B,C and 5.8.

The problem of the nonhomogeneity of variance is still present and will not be discussed here. The interpretation of this analysis will be postponed until the end of this chapter where the overall pattern of the results will be discussed.

5.5 - ANALYSIS BASED ON THE FOURTH PARTITION

For two proofs to be equivalent under the fourth partition, they must have identical frequency distributions over the set of available rules. In the regression analysis with C4 as the dependent variable the general pattern of the results changes. S20 (TOT R) is the first independent variable to enter the equation. The value of S20 for a problem is just the total number of rules, including axioms and theorems, that are available when the poblem appears in the curriculum. After the second partition, the size of the initial correlation of the dependent variable with S11(RE) and S16(STEPS) declines, while the correlations of the dependent variable with S15(THERM), S14(AXIOM), and the rule-position variables increases. The rate of increase is highest for S15 (THERM) but several of the rule-position variables had much larger correlations with C1 and C2 than S15 , and some of these still have larger correlations with the dependent variable, C4.

After S20(TOT R) has entered the regression equation, the partial correlations for all of the other rule-position variables drop sharply (see Table 5.9A). The partial correlation for S15 also decreases, but the decrease for this variable is smaller than that for the other rule-position variables.

The second variable included in the equation is S11(RE). With the introduction of S11, the partial correlation of S16(STEPS) drops dramatically, and the partial correlation of S15 increases by over 50 percent (see Table 5.9B).

The third variable to enter is S15 (Table 5.9C), and S14(AXIOM) is the fourth. A summary table for this analysis is found in Table 5.10. The homogeneity of variance assumption is again violated.

The fourth partition is the first of the five partitions for which the order of the steps in a proof is irrelevant, and it is the first partition for which S11(RE) is not the first variable to enter the regression equation. In changing from the third partition to the fourth, the correlation of S11 withe the dependent variable drops from 0.78 to 0.59. Although S11 still has a prominent position in the analysis, it does not dominate the results as it did in the previous analyses.

5.6 ANALYSIS BASED ON THE FIFTH PARTITION

In the analysis for the fifth partition, S15(THERM) enters first. Its initial correlation with the dependent variable is not much larger than the correlation for some of the rule-position variables (S20, S22). The correlations of S11 and S16 with the dependent variable are almost as large as that for S15.

After S15 has been taken into account, the partial correlations of S11 and S16 increase. The partial correlations of the other rule- position variables decrease but remain relatively large.

The second and third variables added are S11 and S14(AXIOM). The results for this analysis are found in Tables 5.11A,B,C and Table 5.12.

Figure 5.9 contains the plot of C5 against S22(POSIT). The variance in C5 for the first 50 problems is practically zero; there are only four problems in this group with more than one class in the sample of student proofs. After the point in the curriculum where RE is introduced, there is evidence for a systematic dependence of variance on problem position Figure 5.10 contains a plot of residuals against the predicted value of C5; there is again a strong indication of nonhomogeneity of variance. It would seem here that the nonhomogeneity has two components: the complete lack of variance for the problems with values of S22(POSIT) less than 50, and a gradual increase in variance with increasing values of S22 for the remaining problems.

The initial correlation matrix (Table 5.2) and the analyses display a clear pattern. As we proceed from C1 to C5, the importance of S11(RE) and S16(STEPS) diminishes and the importance of S15(THERM), S14(AXIOM), and the rule-position variables increase. The remainder of the discussion in this chapter will investigate these trends.

Figures 5.11A,B,C,D,E contain respectively the plots of the dependent variables, C1 to C5, against S15(THERM). In Figure 5.11A, there is relatively little indication of any functional relationship between C1 and S15. The impression that there is a relationship between the two variables grows from one partition to the next.

Only three values (0,1, and 2) for S15 appear in the data. There are 107 problems with S15 equal to zero, 11 problems with S15 equal to one, and 7 problems with S15 equal to two. In Figure 5.25A, the range of the conditional distribution of C1 given that S15 is equal to zero is 22 (from 1 to 23), covering the entire possible range for the dependent variables. The ranges for the conditional distributions of C1 given that S15 equals one or two are also large, but not as large as the range for the problems that do not use theorems.

One implication of the nested character of the partitions is that the number of classes for any problem is a non-increasing function of the ordinal number of the partition. The value of the dependent variable cannot increase from any partition to the next, and can decrease (unless it is already equal to one). This property of the sets of classification criteria is reflected in the data; the means of all three of the conditional distributions of the dependent variable decrease as the dependent variable changes from one partition to the next (Note that in Figures 5.11A,B,C,D,E, the scale of the dependent variable changes).

The relationship between the means of the three conditional distributions does not change much from one partition to the next. In all five plots, the mean of the conditional distribution of the dependent variable, given S15(THERM), increases as S15 increases. The relationship seems to be nonlinear with a positive, increasing slope, but the small number of problems with two theorems in their standard proof makes this hypothesis quite unreliable. A single additional problem with S15 equal to two, and with low values for the dependent variables, would eliminate this impression of nonlinearity.

The most significant change that occurs from one partition to the next is the decrease in the variance of the dependent variable when S15 equals zero. By the fifth partition (C5), the ranges of the three conditional distributions are almost equal. For problems using theorems, the number of classes is less sensitive to the strictness of the definition of equivalence than for problems not using theorems. The large amount of variation that appears for some of the problems that do not use theorems in their standard proofs rapidly disappears for the progressively less strict sets of equivalence criteria.

Figures 5.12A,B,C,D,E, containing the plots of the five dependent variables against S11(RE), indicate the nature of these problems. A strong linear relationship between C1 and S11 in evident in Figure 5.26A; in general, problems with high values for C1 also have high values for S11. In the progression to the least strict set of equivalence criteria (the fifth partition - C5), the value of the dependent variable decreases for all of the problems, but the decrease is greater for the problems with high values for S11. In Figure 5.12B, with C2 as the dependent variable, a strong linear relationship is still apparent, but in Figure 5.12C this relationship has become obscure. By the fifth partition, Figure 5.12E, the existence of any linear relationship is not obvious.

An examination of the sample of proofs constructed for the problems in the curriculum tends to confirm the conclusions implicit in these results (specific examples will be discussed in Chapter VII). Problems that require a large number of steps (high values for S16) and involve extensive use of RE (high values for S11) tend to have a substantial number of superficial differences in the proofs generated. Variation in the order in which the rules are used is very common for these problems. The first two partitions (C1 and C2) and to a lesser extent the third partition, are sensitive to this type of variation, while the fourth and fifth partitions are not.

Problems that require the use of theorems (S15) tend to produce more basic variations in the proofs generated. The theorems chosen and the rules used in conjunction with the theorems differ from one student to another. All five sets of classification criteria are sensitive to this type of variation.

TABLE 5.1

MEANS AND STANDARD DEVIATIONS FOR FULL SET

VARIABLE		MEAN	STANDARD DEVIATION
CLAS1	1	8.80800	6.76053
CLAS2	2	5.84800	5.99201
CLAS3	3	5.00000	5.06092
CLAS4	4	3.36000	3.39449
CLAS5	5	2.89600	2.86757
WORDS	6	14.43200	7.30260
SYMBL	7	12.20000	6.72501
LOGCN	8	0.26400	0.46043
PAREN	9	0.84000	0.82696
PREMS	10	0.52000	0.84815
RE	11	0.74400	1.09915
CP	12	0.23200	0.42381
AV RE	13	0.58400	0.49488
AXIOM	14	0.27200	0.55903
THERM	15	0.20000	0.52363
STEPS	16	3.76800	2.56256
R INF	17	16.93600	2.15056
AV TH	18	0.68000	1.50054
AV AX	19	1.80000	2.18130
TOT R	20	19.41600	5.11352
PSLI	21	6.74400	5.20995
POSIT	22	64.24000	37.09847

TABLE 5.2

CORRELATION MATRIX FOR FULL SET

VARIABLE NUMBER	1	2	3	4	5
1	1.000	0.940	0.919	0.806	0.709
2		1.000	0.973	0.820	0.694
3			1.000	0.882	0.774
4				1.000	0.947
5					1.000

MATRIX CONTINUED

VARIABLE NUMBER	6	7	8	9	10
1	0.256	0.189	-0.066	0.266	-0.171
2	0.157	0.097	-0.058	0.190	-0.091
3	0.189	0.131	-0.093	0.214	-0.128
4	0.122	0.030	-0.159	0.118	-0.180
5	0.090	-0.004	-0.211	0.105	-0.230
6	1.000	0.800	0.115	0.591	-0.344
7		1.000	0.215	0.776	-0.325
8			1.000	-0.164	0.059
9				1.000	-0.398
10					1.000

MATRIX CONTINUED

VARIABLE NUMBER	11	12	13	14	15
1	0.745	-0.060	0.648	0.385	0.325
2	0.825	-0.069	0.577	0.299	0.305
3	0.776	-0.094	0.560	0.305	0.362
4	0.593	-0.137	0.546	0.335	0.558
5	0.447	-0.186	0.515	0.325	0.680
6	0.055	0.134	-0.026	0.030	-0.164
7	-0.034	0.230	-0.118	-0.032	-0.147
8	0.007	0.923	-0.116	-0.187	-0.221
9	0.043	-0.123	0.013	0.060	0.056
10	0.118	0.021	-0.115	-0.233	-0.200
11	1.000	0.025	0.574	0.219	0.076
12		1.000	-0.074	-0.166	-0.211
13			1.000	0.412	0.324
14				1.000	-0.022
15					1.000

TABLE 5.2 CONTINUED

MATRIX CONTINUED

VARIABLE NUMBER	16	17	18	19	20
1	0.717	0.609	0.315	0.511	0.567
2	0.791	0.525	0.254	0.428	0.478
3	0.731	0.537	0.295	0.468	0.512
4	0.495	0.560	0.487	0.579	0.625
5	0.333	0.547	0.603	0.630	0.676
6	0.199	0.093	−0.080	0.017	0.023
7	0.151	−0.003	−0.245	−0.170	−0.145
8	0.285	−0.178	−0.227	−0.308	−0.273
9	0.146	0.126	−0.120	0.045	0.037
10	0.004	−0.357	−0.223	−0.366	−0.372
11	0.789	0.447	0.131	0.264	0.339
12	0.258	−0.187	−0.212	−0.290	−0.264
13	0.343	0.861	0.384	0.699	0.773
14	0.168	0.444	0.153	0.534	0.459
15	−0.013	0.370	0.760	0.565	0.619
16	1.000	0.252	0.016	0.136	0.169
17		1.000	0.438	0.778	0.881
18			1.000	0.670	0.764
19				1.000	0.950
20					1.000

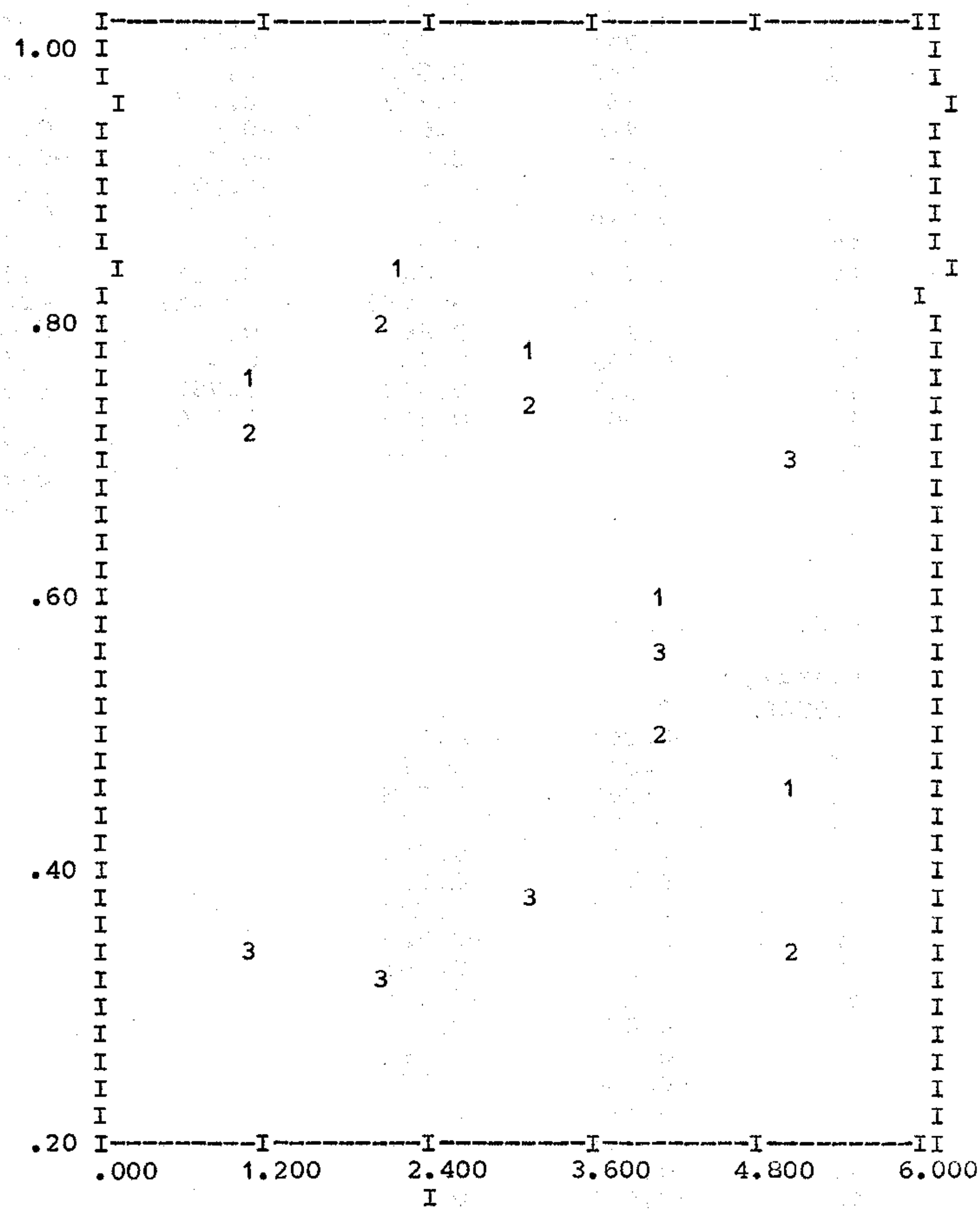
MATRIX CONTINUED

VARIABLE NUMBER	21	22
1	−0.045	0.616
2	−0.041	0.525
3	−0.060	0.548
4	−0.152	0.621
5	−0.199	0.648
6	0.196	0.051
7	0.263	−0.094
8	0.361	−0.203
9	0.041	0.055
10	0.107	−0.338
11	0.035	0.417
12	0.312	−0.197
13	−0.170	0.852
14	−0.197	0.451
15	−0.291	0.542
16	0.116	0.232
17	−0.096	0.942
18	−0.300	0.666
19	−0.385	0.896
20	−0.293	0.974
21	1.000	−0.126
22		1.000

FIGURE 5.1

CORRELATIONS BETWEEN DEPENDENT AND INDEPENDENT VARIABLES

AGAINST THE ORDINAL NUMBER OF THE DEPENDENT VARIABLE



(1) S11(RE)
(2) S16(STEPS)
(3) S15(THERM)

TABLE 5.3A

STEP NUMBER 1 FOR C1

VARIABLE ENTERED 11
MULTIPLE R 0.7454
STD. ERROR OF EST. 4.5247

ANALYSIS OF VARIANCE:

	DF	SUM OF SQUARES	MEAN SQUARE	F-RATIO
REGRESSION	1	3149.172	3149.172	153.818
RESIDUAL	123	2518.220	20.473	

VARIABLES IN EQUATION: (CONSTANT= 5.39683)

VARIABLE		COEFFICIENT	STD. ERROR	F TO REMOVE
RE	11	4.58491	0.36968	153.8182 (2)

VARIABLES NOT IN EQUATION:

VARIABLE		PARTIAL CORR.	TOLERANCE	F TO ENTER
CLAS2	2	0.86164	0.3186	351.6484 (1)
CLAS3	3	0.81121	0.3984	234.7856 (1)
CLAS4	4	0.67721	0.6479	103.3474 (1)
CLAS5	5	0.63109	0.8003	80.7507 (1)
WORDS	6	0.32348	0.9970	14.2575 (2)
SYMBL	7	0.32289	0.9988	14.1999 (2)
LOGCN	8	-0.10773	0.9999	1.4326 (2)
PAREN	9	0.35043	0.9981	17.0792 (2)
PREMS	10	-0.39108	0.9861	22.0288 (2)
CP	12	-0.11810	0.9994	1.7257 (2)
AV RE	13	0.40453	0.6710	23.8705 (2)
AXIOM	14	0.34103	0.9519	16.0561 (2)
THERM	15	0.40457	0.9943	23.8768 (2)
STEPS	16	0.31366	0.3774	13.3124 (2)
R INF	17	0.46326	0.8004	33.3360 (2)
AV TH	18	0.32916	0.9829	14.8243 (2)
AV AX	19	0.48899	0.9301	38.3384 (2)
TOT R	20	0.50090	0.8850	40.8629 (2)
PSLI	21	-0.10685	0.9988	1.4090 (2)
POSIT	22	0.50386	0.8261	41.5112 (2)

TABLE 5.3B

STEP NUMBER 2 FOR C1

VARIABLE ENTERED 22
MULTIPLE R 0.8176
STD. ERROR OF EST. 3.9244

ANALYSIS OF VARIANCE:

	DF	SUM OF SQUARES	MEAN SQUARE	F-RATIO
REGRESSION	2	3788.482	1894.241	122.995
RESIDUAL	122	1878.910	15.401	

VARIABLES IN EQUATION: (CONSTANT= 1.77608)

VARIABLE		COEFFICIENT	STD. ERROR	F TO REMOVE
RE	11	3.63703	0.35277	106.2927 (2)
POSIT	22	0.06734	0.01045	41.5112 (2)

VARIABLES NOT IN EQUATION:

VARIABLE		PARTIAL CORR.	TOLERANCE	F TO ENTER
CLAS2	2	0.84628	0.2791	305.3467 (1)
CLAS3	3	0.77260	0.3376	179.1791 (1)
CLAS4	4	0.56538	0.4789	56.8495 (1)
CLAS5	5	0.48513	0.5420	37.2435 (1)
WORDS	6	0.35670	0.9960	17.6399 (2)
SYMBL	7	0.42634	0.9912	26.8789 (2)
LOGCN	8	0.00735	0.9488	0.0065 (2)
PAREN	9	0.38235	0.9965	20.7176 (2)
PREMS	10	-0.22391	0.8041	6.3863 (2)
CP	12	-0.00345	0.9472	0.0014 (2)
AV RE	13	-0.02069	0.2165	0.0518 (2)
AXIOM	14	0.17291	0.7951	3.7292 (2)
THERM	15	0.16912	0.6787	3.5626 (2)
STEPS	16	0.47127	0.3661	34.5456 (2)
R INF	17	-0.01520	0.1095	0.0279 (2)
AV TH	18	-0.01999	0.5305	0.0484 (2)
AV AX	19	0.09758	0.1829	1.1632 (2)
TOT R	20	0.05289	0.0466	0.3394 (2)
PSLI	21	-0.03409	0.9750	0.1408 (2)

TABLE 5.3C

STEP NUMBER 3 FOR C1

VARIABLE ENTERED	16
MULTIPLE R	0.8615
STD. ERROR OF EST.	3.4756

ANALYSIS OF VARIANCE:

	DF	SUM OF SQUARES	MEAN SQUARE	F-RATIO
REGRESSION	3	4205.775	1401.925	116.058
RESIDUAL	121	1461.617	12.079	

VARIABLES IN EQUATION: (CONSTANT= -1.57704)

VARIABLE		COEFFICIENT	STD. ERROR	F TO REMOVE
RE	11	1.32601	0.50221	6.9716 (2)
STEPS	16	1.18310	0.20129	34.5456 (2)
POSIT	22	0.07691	0.00940	66.9611 (2)

VARIABLES NOT IN EQUATION:

VARIABLE		PARTIAL CORR.	TOLERANCE	F TO ENTER
CLAS2	2	0.79913	0.2082	212.0490 (1)
CLAS3	3	0.71970	0.2802	128.9437 (1)
CLAS4	4	0.55845	0.4651	54.3861 (1)
CLAS5	5	0.51081	0.5389	42.3653 (1)
WORDS	6	0.27328	0.9269	9.6852 (2)
SYMBL	7	0.34745	0.9132	16.4753 (2)
LOGCN	8	-0.24721	0.7712	7.8108 (2)
PAREN	9	0.33706	0.9596	15.3808 (2)
PREMS	10	-0.12535	0.7547	1.9155 (2)
CP	12	-0.21275	0.8219	5.6890 (2)
AV RE	13	0.04882	0.2126	0.2867 (2)
AXIOM	14	0.15938	0.7913	3.1275 (2)
THERM	15	0.20591	0.6782	5.3130 (2)
R INF	17	0.01525	0.1091	0.0279 (2)
AV TH	18	-0.00390	0.5298	0.0018 (2)
AV AX	19	0.07102	0.1819	0.6083 (2)
TOT R	20	0.06583	0.0466	0.5223 (2)
PSLI	21	-0.10357	0.9609	1.3011 (2)

TABLE 5.3D

STEP NUMBER 4 FOR C1

VARIABLE ENTERED	7
MULTIPLE R	0.8793
STD. ERROR OF EST.	3.2726

ANALYSIS OF VARIANCE:

	DF	SUM OF SQUARES	MEAN SQUARE	F-RATIO
REGRESSION	4	4382.222	1095.556	102.295
RESIDUAL	120	1285.170	10.710	

VARIABLES IN EQUATION: (CONSTANT= -3.40709)

VARIABLE		COEFFICIENT	STD. ERROR	F TO REMOVE
SYMBL	7	0.18562	0.04573	16.4753 (2)
RE	11	1.75889	0.48475	13.1654 (2)
STEPS	16	0.95828	0.19746	23.5509 (2)
POSIT	22	0.07832	0.00886	78.2004 (2)

VARIABLES NOT IN EQUATION:

VARIABLE		PARTIAL CORR.	TOLERANCE	F TO ENTER
CLAS2	2	0.80379	0.2030	217.2378 (1)
CLAS3	3	0.70265	0.2663	116.0447 (1)
CLAS4	4	0.56703	0.4620	56.3933 (1)
CLAS5	5	0.52462	0.5372	45.1883 (1)
WORDS	6	-0.00504	0.3414	0.0030 (2)
LOGCN	8	-0.29910	0.7646	11.6915 (2)
PAREN	9	0.11373	0.3805	1.5593 (2)
PREMS	10	-0.00155	0.6586	0.0003 (2)
CP	12	-0.27704	0.8082	9.8928 (2)
AV RE	13	0.07042	0.2121	0.5930 (2)
AXIOM	14	0.17299	0.7912	3.6708 (2)
THERM	15	0.26313	0.6695	8.8519 (2)
R INF	17	-0.09389	0.1002	1.0583 (2)
AV TH	18	0.09157	0.4966	1.0064 (2)
AV AX	19	0.16595	0.1721	3.3700 (2)
TOT R	20	0.17178	0.0435	3.6184 (2)
PSLI	21	-0.20193	0.9092	5.0587 (2)

TABLE 5.4

SUMMARY TABLE FOR C1 ON THE FULL SET OF PROBLEMS

STEP NUM		VARIABLE ENT	REM	MULTIPLE R	RSQ	INCREASE IN RSQ	F VALUE FOR DEL	LAST REG COEFFICNTS
1	RE	11		0.74540	0.55562	0.55562	153.8182	1.18711
2	POSIT	22		0.81760	0.66847	0.11285	41.5112	-0.08912
3	STEPS	16		0.86150	0.74218	0.07371	34.5456	1.22277
4	SYMBL	7		0.87930	0.77317	0.03099	16.4753	0.24390
5	LOGCN	8		0.89080	0.79352	0.02036	11.6915	-1.30055
6	THERM	15		0.89760	0.80569	0.01216	7.4174	3.95143
7	AXIOM	14		0.90450	0.81812	0.01243	7.9252	1.95110
8	AV RE	13		0.90890	0.82610	0.00798	5.3051	4.61910
9	PAREN	9		0.91100	0.82992	0.00382	2.6769	-0.99653
10	WORDS	6		0.91180	0.83138	0.00146	0.9327	0.08359
11	CP	12		0.91260	0.83284	0.00146	0.9891	-1.56661
12	PSLI	21		0.91300	0.83357	0.00073	0.5098	0.11952
13	R INF	17		0.91330	0.83412	0.00055	0.4201	0.33589
14	TOT R	20		0.91350	0.83448	0.00037	0.1295	0.40774
15	PREMS	10		0.91350	0.83448	0.00000	0.1035	-0.14266
16	AV TH	18		0.91360	0.83466	0.00018	0.0532	-0.11983

FIGURE 5.2A C1 VS S11(RE)

```
      I---------I---------I---------I---------I---------II
23.00 I            1                        1           2I
      I                                                  I
      I            1                                    2I
       I                                                  I
      I            1            1           1           1I
      I                                                  I
      I                         3                        I
      I                                                  I
      I                                                  I
      I            3            2                        I
18.60 I                                                  I
      I                                     1            I
      I                                                  I
      I            2            1                        I
      I                                                  I
      I1           3                                     I
      I                                                  I
      I                                                  I
      I                         1           1            I
      I                                                  I
14.20 I2                                                1I
      I                                                  I
      I            3            1                        I
      I                                                  I
      I1           1                        1            I
      I                                                  I
      I                                                  I
      I1           4            1                        I
      I                                                  I
       I2           1                                     I
 9.80 I                                                  I
      I1           1                                     I
       I                                                  I
      I                                                  I
      I2           1                                     I
      I                                                  I
       I5           1            2                        I
      I                                                  I
      I8                                                 I
      I                                                  I
 5.40 I5           3                                     I
      I                                                  I
      I                                                  I
      I*           1                                     I
      I                                                  I
      I*           2                                     I
      I                                                  I
      I6           1                                     I
      I                                                  I
      I*                                                 I
 1.00 I---------I---------I---------I---------I---------II
      .000      .800     1.600     2.400     3.200     4.000
```

FIGURE 5.2B C1 RESIDUALS(Y-AXIS) VS S11 (X-AXIS)

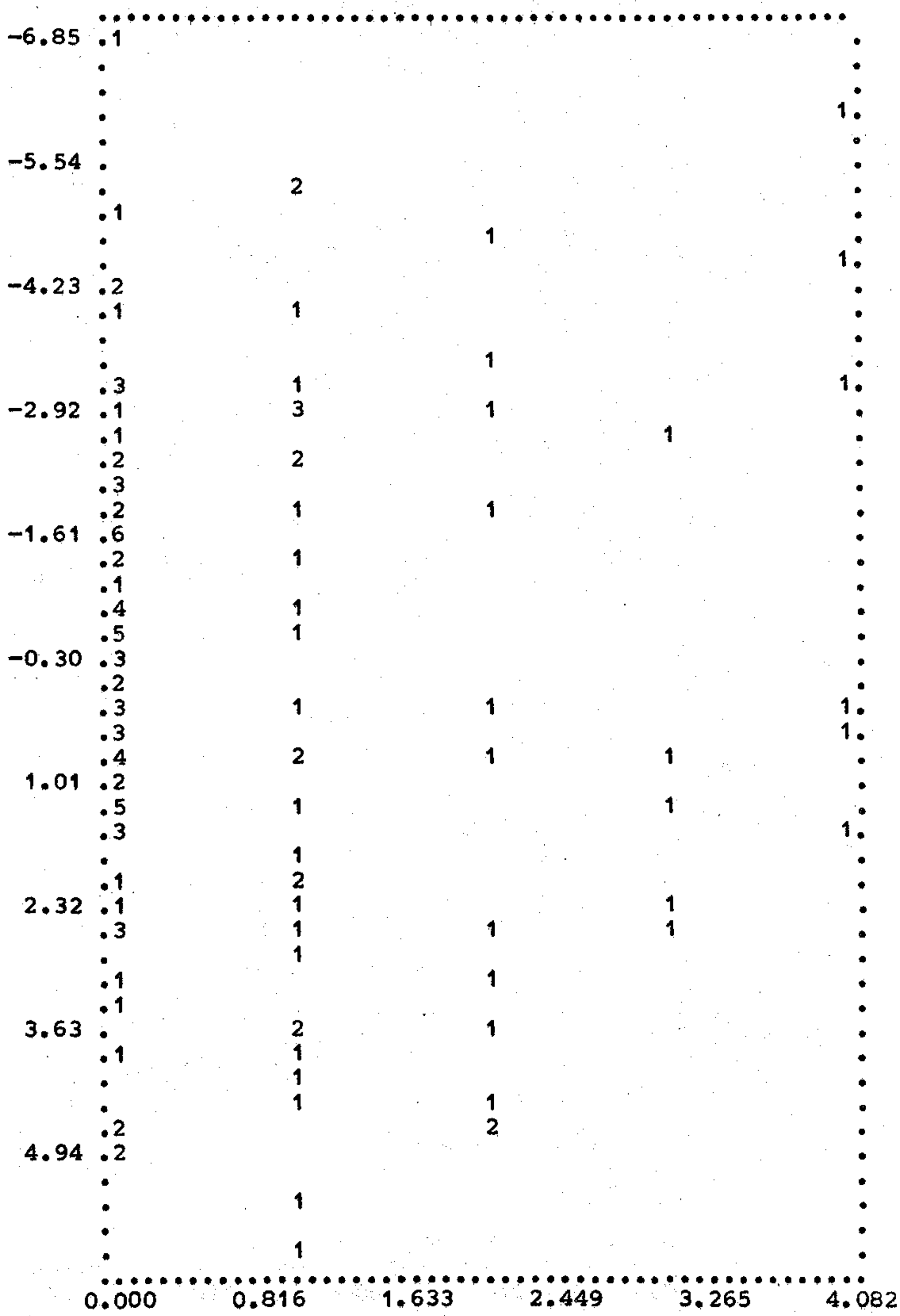


FIGURE 5.3A C1 VS S22(POSIT)

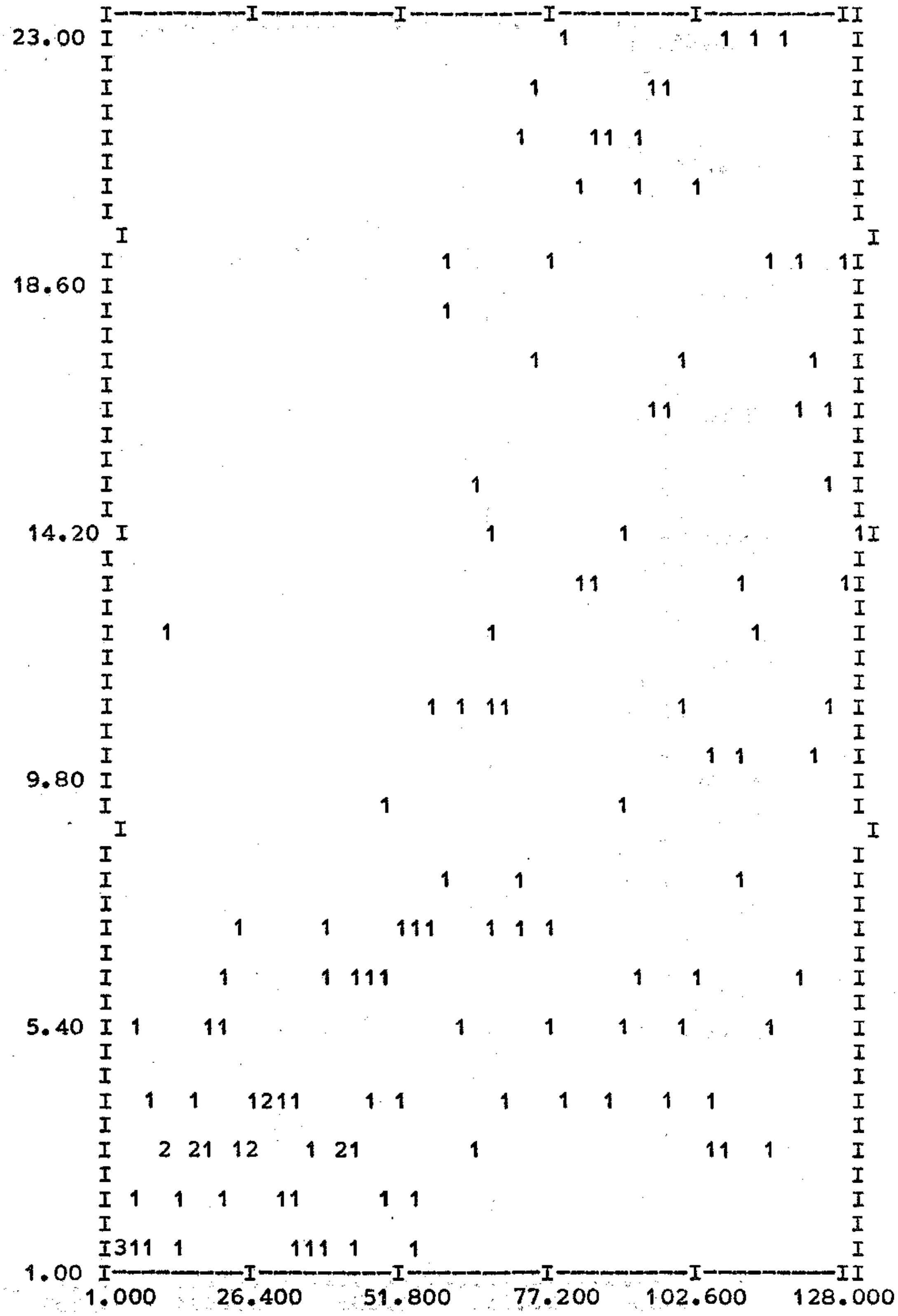
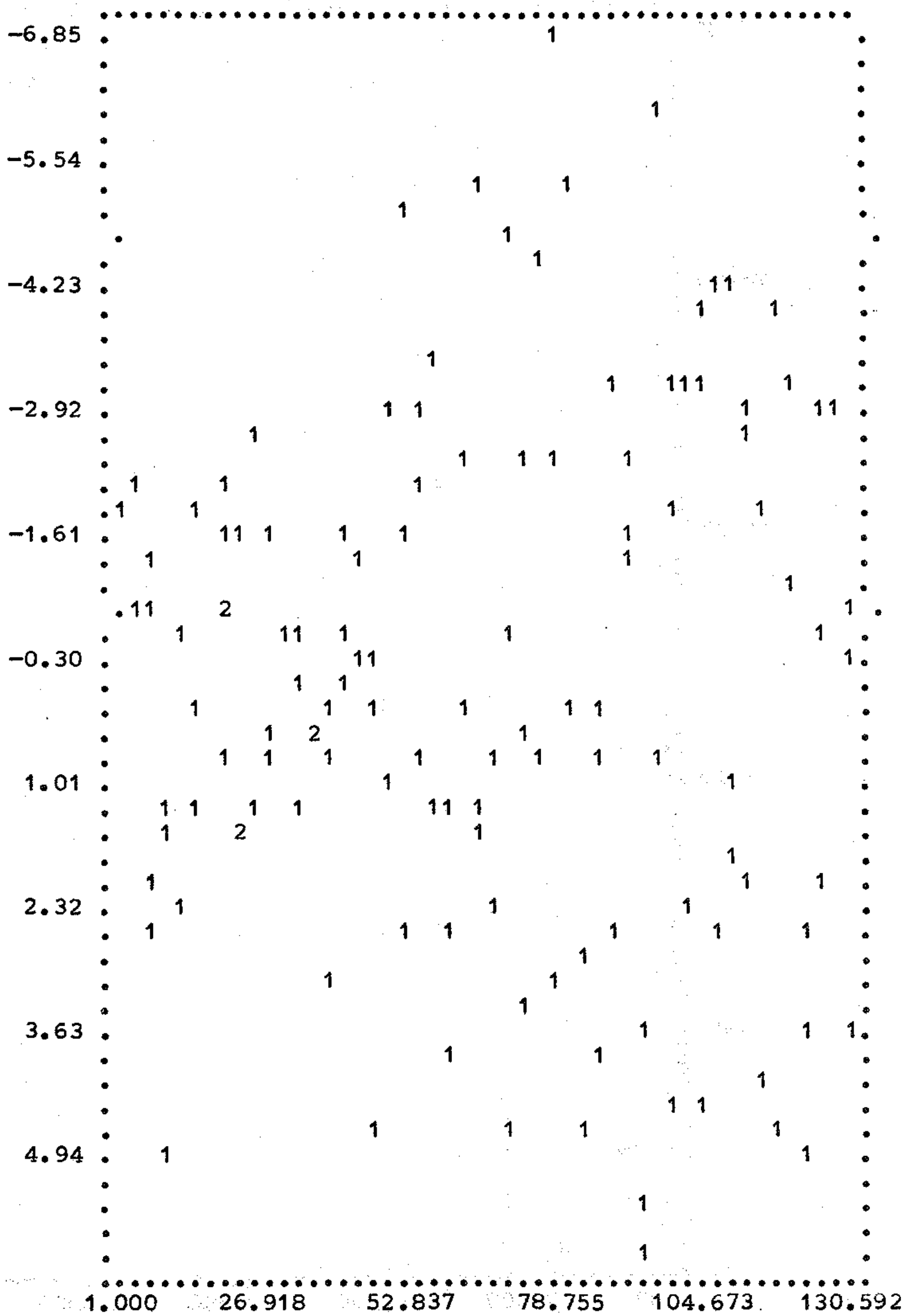


FIGURE 5.3B C1 RESIDUALS(Y-AXIS) VS S22 (X-AXIS)



<u>FIGURE 5.4A C1 VS S16(STEPS)</u>

```
      I---------I---------I---------I---------I---------II
23.00 I               1              1           1      1I
      I                                                  I
      I                   1                  1   1       I
      I                                                  I
      I               1                  2   1           I
      I                                                  I
      I               1   1   1                          I
      I                                                  I
      I                                                  I
      I           3   1       1                          I
18.60 I                                                  I
      I                          1                       I
      I                                                  I
      I       1       2                                  I
      I                                                  I
      I       2   1       1                              I
      I                                                  I
      I                                                  I
      I               2                                  I
      I                                                  I
14.20 I1          1   1                                  I
      I                                                  I
      I       1   2           1                          I
      I                                                  I
      I   1   1               1                          I
      I                                                  I
      I                                                  I
      I       2   1   1   2                              I
      I                                                  I
      I1  1   1                                          I
 9.80 I                                                  I
      I               2                                  I
      I                                                  I
      I                                                  I
      I1  1   1                                          I
      I                                                  I
      I   1   2       4   1                              I
      I                                                  I
      I3  4               1                              I
      I                                                  I
 5.40 I   3   1   4                                      I
      I                                                  I
      I                                                  I
      I3  4   4   3                                      I
      I                                                  I
      I3  5   6   2                                      I
      I                                                  I
      I4  1       2                                      I
      I                                                  I
      I5  1   3   2                                      I
 1.00 I---------I---------I---------I---------I---------II
     1.000     3.600     6.200     8.800    11.400    14.000
```

FIGURE 5.4B C1 RESIDUALS(Y-AXIS) VS S16 (X-AXIS)

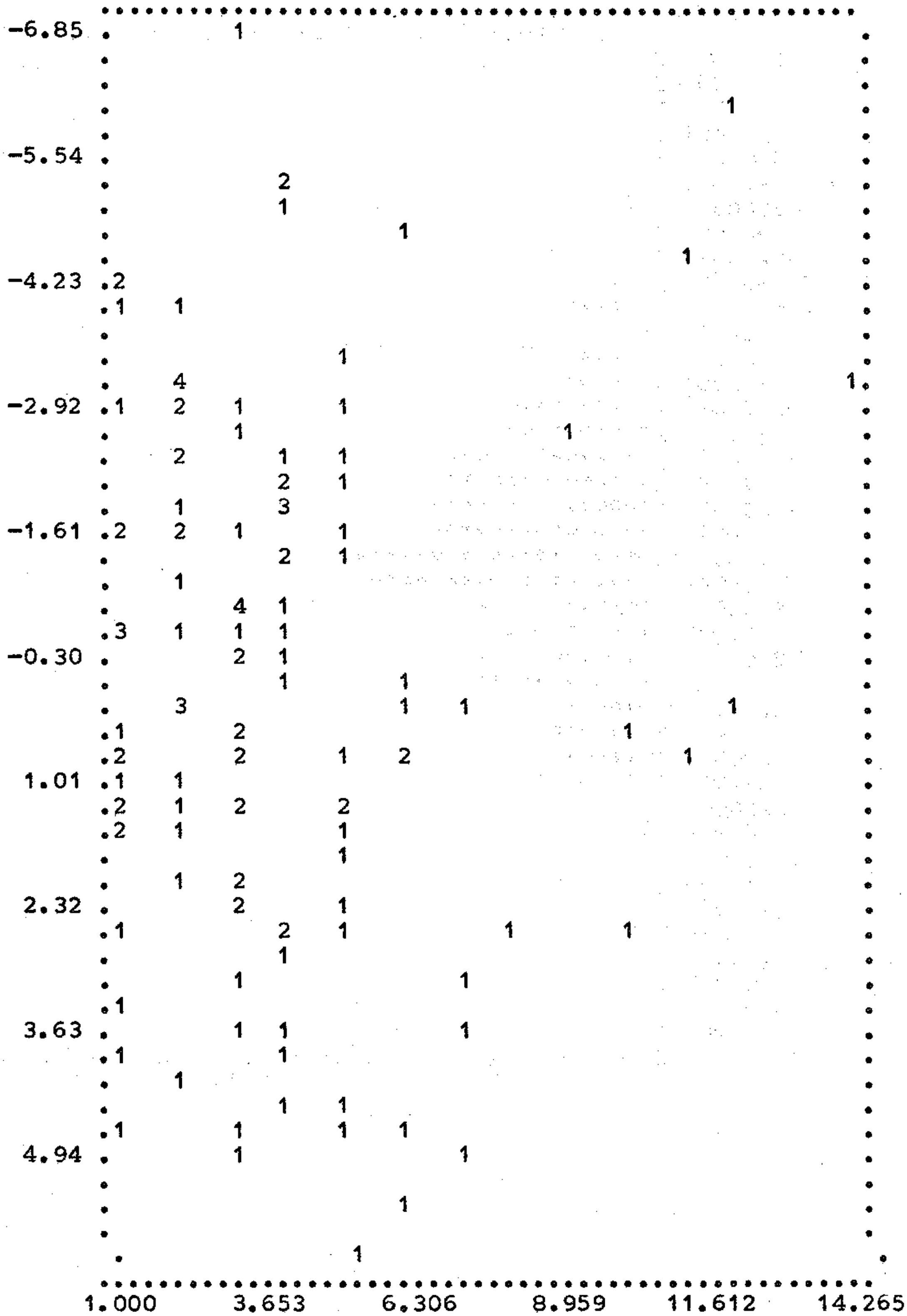


FIGURE 5.5 RESIDUALS FOR C1 ON FULL SET

```
RANGE      0         10        20        30        40        50
           I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I
-10.000-   I
   -9.001  I
 -9.000-   I
   -8.001  I
 -8.000-   I
   -7.001  I
 -7.000-   I**
   -6.001  I**
 -6.000-   I***
   -5.001  I***
 -5.000-   I******
   -4.001  I******
 -4.000-   I********
   -3.001  I********
 -3.000-   I************
   -2.001  I************
 -2.000-   I**************
   -1.001  I**************
 -1.000-   I****************
    -.001  I****************
   .000-   I********************
     .999  I********************
  1.000-   I************
    1.999  I************
  2.000-   I*************
    2.999  I*************
  3.000-   I********
    3.999  I********
  4.000-   I********
    4.999  I********
  5.000-   I***
    5.999  I***
  6.000-   I
    6.999  I
  7.000-   I
    7.999  I
  8.000-   I
    8.999  I
  9.000-   I
   10.000  I
           I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I
           0         10        20        30        40        50
```

FIGURE 5.6 RESIDUALS(Y-AXIS) VS COMPUTED C1 (X-AXIS)

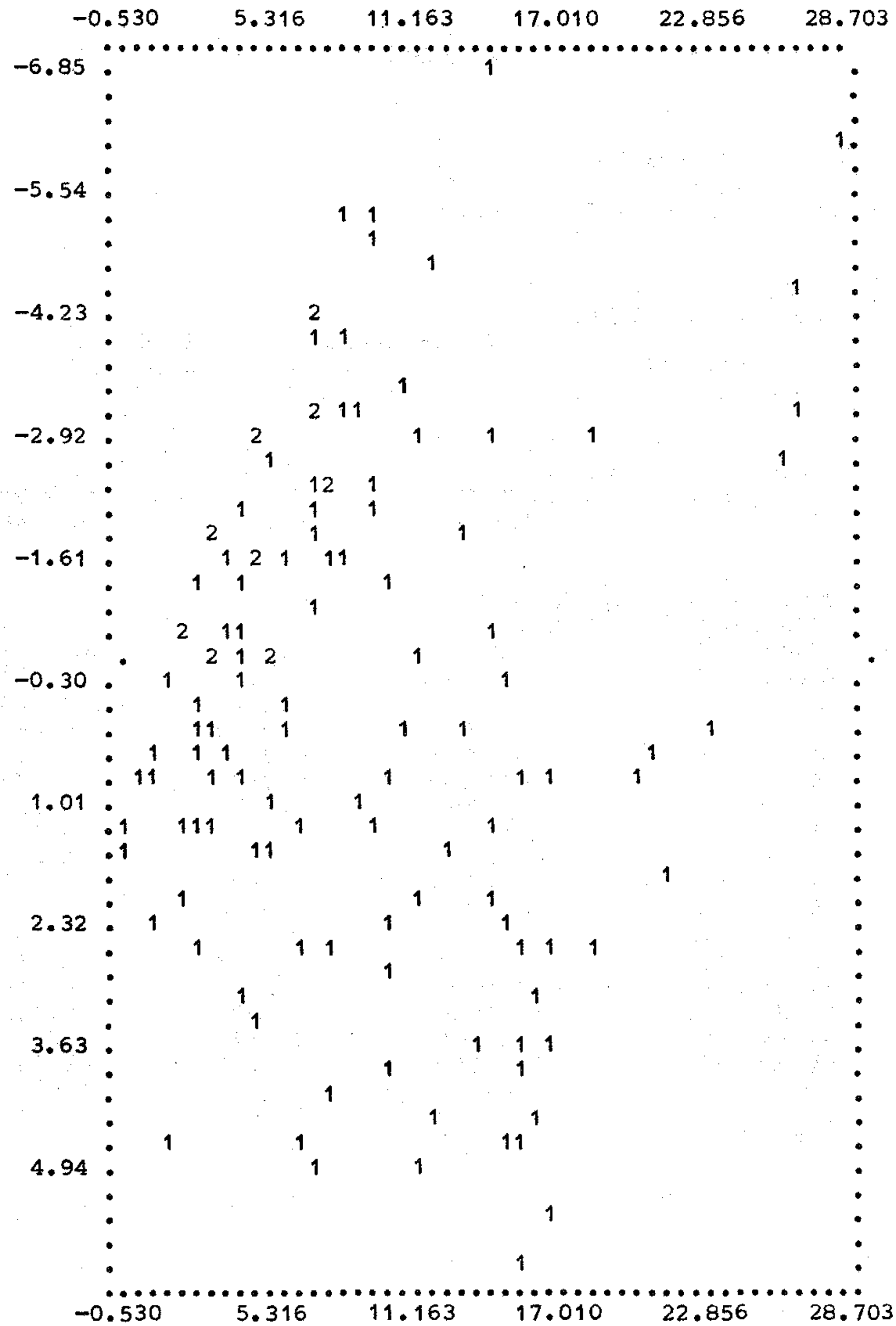


TABLE 5.5A

STEP NUMBER 1 FOR C2

VARIABLE ENTERED	11
MULTIPLE R	0.8255
STD. ERROR OF EST.	3.3960

ANALYSIS OF VARIANCE:

	DF	SUM OF SQUARES	MEAN SQUARE	F-RATIO
REGRESSION	1	3033.612	3033.612	263.048
RESIDUAL	123	1418.500	11.533	

VARIABLES IN EQUATION: (CONSTANT= 2.50000)

VARIABLE		COEFFICIENT	STD. ERROR	F TO REMOVE
RE	11	4.50000	0.27746	263.0485 (2)

VARIABLES NOT IN EQUATION:

VARIABLE		PARTIAL CORR.	TOLERANCE	F TO ENTER
CLAS1	1	0.86164	0.4443	351.6484 (1)
CLAS3	3	0.93263	0.3984	815.0219 (1)
CLAS4	4	0.72735	0.6479	137.0459 (1)
CLAS5	5	0.64321	0.8003	86.0897 (1)
WORDS	6	0.19866	0.9970	5.0125 (2)
SYMBL	7	0.22315	0.9988	6.3933 (2)
LOGCN	8	-0.11393	0.9999	1.6044 (2)
PAREN	9	0.27417	0.9981	9.9159 (2)
PREMS	10	-0.33548	0.9861	15.4720 (2)
CP	12	-0.15758	0.9994	3.1065 (2)
AV RE	13	0.22351	0.6710	6.4153 (2)
AXIOM	14	0.21421	0.9519	5.8670 (2)
THERM	15	0.43154	0.9943	27.9194 (2)
STEPS	16	0.40362	0.3774	23.7427 (2)
R INF	17	0.30920	0.8004	12.8968 (2)
AV TH	18	0.26045	0.9829	8.8783 (2)
AV AX	19	0.38593	0.9301	21.3512 (2)
TOT R	20	0.37298	0.8850	19.7140 (2)
PSLI	21	-0.12433	0.9988	1.9155 (2)
POSIT	22	0.35226	0.8261	17.2837 (2)

TABLE 5.5B

STEP NUMBER 2 FOR C2

VARIABLE ENTERED	15
MULTIPLE R	0.8607
STD. ERROR OF EST.	3.0760

ANALYSIS OF VARIANCE:

	DF	SUM OF SQUARES	MEAN SQUARE	F-RATIO
REGRESSION	2	3297.779	1648.889	174.269
RESIDUAL	122	1154.333	9.462	

VARIABLES IN EQUATION: (CONSTANT= 2.01589)

VARIABLE		COEFFICIENT	STD. ERROR	F TO REMOVE
RE	11	4.39924	0.25204	304.6665 (2)
THERM	15	2.79542	0.52905	27.9194 (2)

VARIABLES NOT IN EQUATION:

VARIABLE		PARTIAL CORR.	TOLERANCE	F TO ENTER
CLAS1	1	0.83282	0.3716	273.8946 (1)
CLAS3	3	0.91680	0.3058	637.7576 (1)
CLAS4	4	0.65084	0.3831	88.9216 (1)
CLAS5	5	0.53160	0.3804	47.6637 (1)
WORDS	6	0.30545	0.9685	12.4510 (2)
SYMBL	7	0.31983	0.9780	13.7872 (2)
LOGCN	8	-0.02064	0.9507	0.0516 (2)
PAREN	9	0.27906	0.9953	10.2185 (2)
PREMS	10	-0.27730	0.9423	10.0796 (2)
CP	12	-0.07435	0.9539	0.6726 (2)
AV RE	13	0.08903	0.5920	0.9668 (2)
AXIOM	14	0.25665	0.9504	8.5322 (2)
STEPS	16	0.50798	0.3721	42.0828 (2)
R INF	17	0.17564	0.6870	3.8516 (2)
AV TH	18	-0.11355	0.4177	1.5806 (2)
AV AX	19	0.19027	0.6315	4.5451 (2)
TOT R	20	0.14303	0.5307	2.5270 (2)
PSLI	21	0.00352	0.9118	0.0015 (2)
POSIT	22	0.14642	0.5638	2.6509 (2)

TABLE 5.5C

STEP NUMBER 3 FOR C2

VARIABLE ENTERED 16
MULTIPLE R 0.8987
STD. ERROR OF EST. 2.6605

ANALYSIS OF VARIANCE:

	DF	SUM OF SQUARES	MEAN SQUARE	F-RATIO
REGRESSION	3	3595.649	1198.550	169.330
RESIDUAL	121	856.463	7.078	

VARIABLES IN EQUATION: (CONSTANT= -0.42479)

VARIABLE		COEFFICIENT	STD. ERROR	F TO REMOVE
RE	11	2.56248	0.35733	51.4245 (2)
THERM	15	3.15129	0.46086	46.7565 (2)
STEPS	16	0.99152	0.15284	42.0828 (2)

VARIABLES NOT IN EQUATION:

VARIABLE		PARTIAL CORR.	TOLERANCE	F TO ENTER
CLAS1	1	0.79795	0.3126	210.3244 (1)
CLAS3	3	0.90031	0.2524	513.4358 (1)
CLAS4	4	0.66405	0.3717	94.6522 (1)
CLAS5	5	0.57592	0.3784	59.5561 (1)
WORDS	6	0.21999	0.9131	6.1030 (2)
SYMBL	7	0.21586	0.9024	5.8648 (2)
LOGCN	8	-0.31725	0.7650	13.4296 (2)
PAREN	9	0.21581	0.9594	5.8619 (2)
PREMS	10	-0.22143	0.9130	6.1869 (2)
CP	12	-0.33090	0.8205	14.7554 (2)
AV RE	13	0.21921	0.5707	6.0573 (2)
AXIOM	14	0.30548	0.9503	12.3505 (2)
R INF	17	0.29589	0.6715	11.5139 (2)
AV TH	18	-0.08413	0.4150	0.8555 (2)
AV AX	19	0.26132	0.6287	8.7949 (2)
TOT R	20	0.24136	0.5224	7.4228 (2)
PSLI	21	-0.06375	0.8999	0.4896 (2)
POSIT	22	0.24837	0.5544	7.8890 (2)

TABLE 5.6

SUMMARY TABLE FOR C2 ON THE FULL SET OF PROBLEMS

STEP NUM	VARIABLE ENT	VARIABLE REM	MULTIPLE R	MULTIPLE RSQ	INCREASE IN RSQ	F VALUE FOR DEL	LAST REG COEFFICNTS
1	RE 11		0.82550	0.68145	0.68145	263.0485	1.96598
2	THERM 15		0.86070	0.74080	0.05935	27.9194	4.09484
3	STEPS 16		0.89870	0.80766	0.06686	42.0828	1.18386
4	CP 12		0.91030	0.82865	0.02098	14.7554	-2.31862
5	R INF 17		0.91750	0.84181	0.01316	9.8915	0.13185
6	SYMBL 7		0.92270	0.85138	0.00957	7.5359	0.18167
7	AXIOM 14		0.92540	0.85637	0.00499	4.0936	0.91066
8	AV TH 18		0.92760	0.86044	0.00408	3.3821	-0.68172
9	PAREN 9		0.93020	0.86527	0.00483	4.1235	-1.20807
10	AV RE 13		0.93080	0.86639	0.00112	0.9606	2.02866
11	PREMS 10		0.93160	0.86788	0.00149	1.3115	-0.32617
12	R INF	17	0.93160	0.86788	0.00000	0.0000	
13	WORDS 6		0.93180	0.86825	0.00037	0.3128	0.02218
14	LOGCN 8		0.93200	0.86862	0.00037	0.2403	-0.67460
15	TOT R 20		0.93200	0.86862	0.00000	0.0443	0.43478
16	PSLI 21		0.93200	0.86862	0.00000	0.0161	0.07729
17	POSIT 22		0.93210	0.86881	0.00019	0.2098	-0.07002
18	R INF 17		0.93220	0.86900	0.00019	0.0541	0.13185

FIGURE 5.7 RESIDUALS(Y-AXIS) VS COMPUTED C2 (X-AXIS)

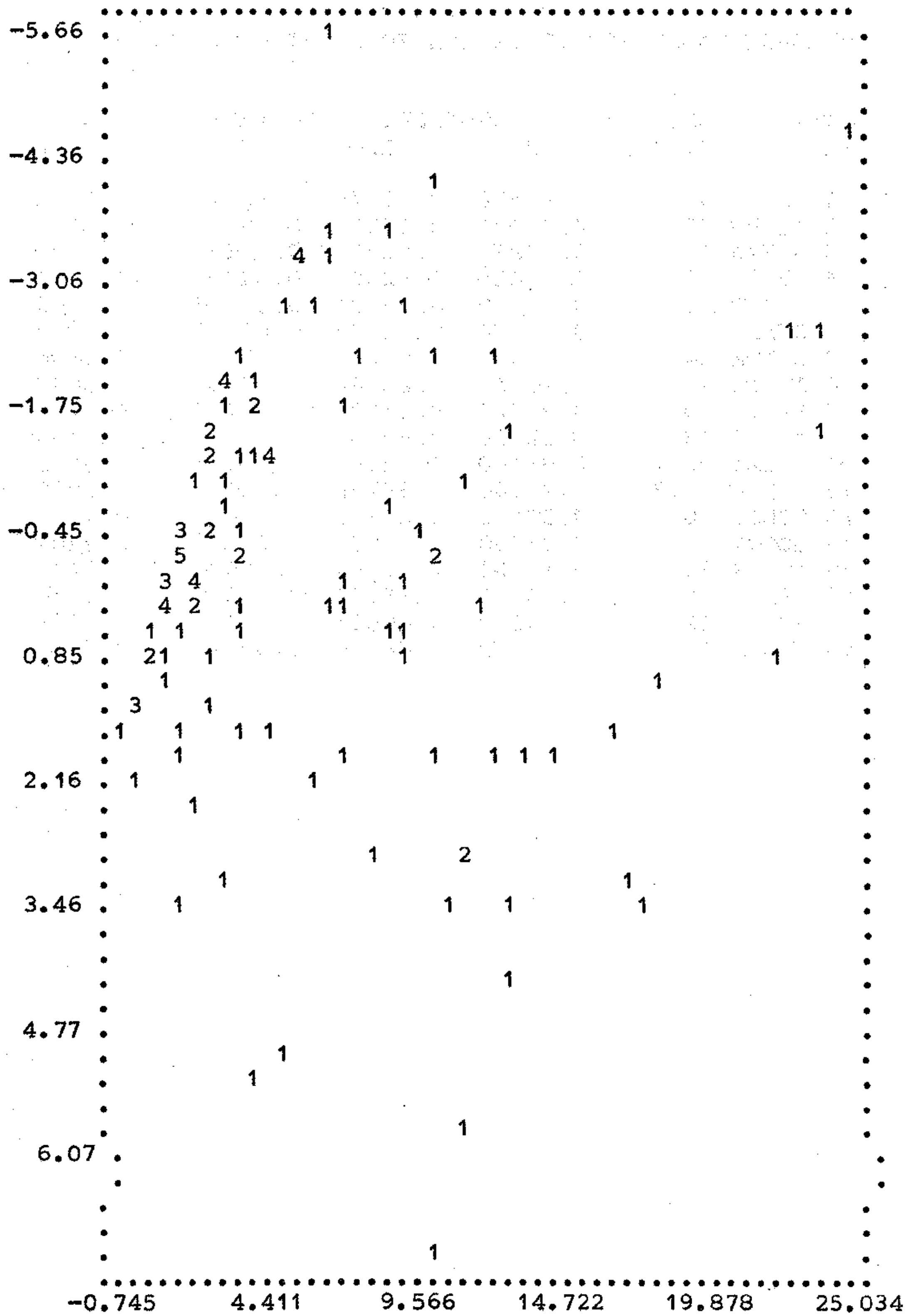


FIGURE 5.8 C2 VS S22(POSIT)

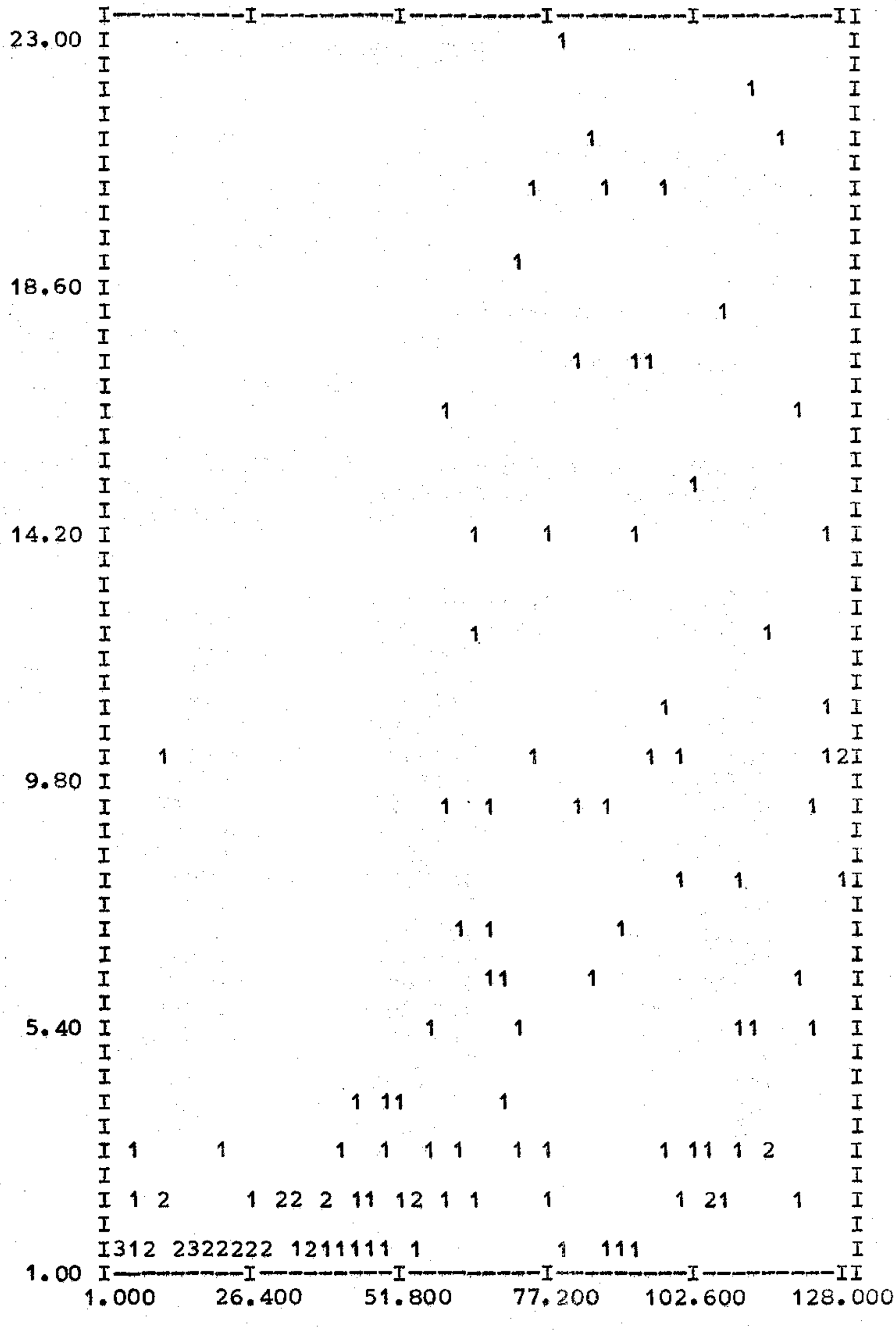


TABLE 5.7A

STEP NUMBER 1 FOR C3

VARIABLE ENTERED	11
MULTIPLE R	0.7756
STD. ERROR OF EST.	3.2074

ANALYSIS OF VARIANCE:

	DF	SUM OF SQUARES	MEAN SQUARE	F-RATIO
REGRESSION	1	1910.612	1910.612	185.718
RESIDUAL	123	1265.388	10.288	

VARIABLES IN EQUATION: (CONSTANT= 2.34300)

VARIABLE		COEFFICIENT	STD. ERROR	F TO REMOVE
RE	11	3.57124	0.26205	185.7180 (2)

VARIABLES NOT IN EQUATION:

VARIABLE		PARTIAL CORR.	TOLERANCE	F TO ENTER
CLAS1	1	0.81121	0.4443	234.7856 (1)
CLAS2	2	0.93263	0.3186	815.0219 (1)
CLAS4	4	0.83027	0.6479	270.7244 (1)
CLAS5	5	0.75699	0.8003	163.7349 (1)
WORDS	6	0.23239	0.9970	6.9648 (2)
SYMBL	7	0.25048	0.9988	8.1666 (2)
LOGCN	8	-0.15681	0.9999	3.0757 (2)
PAREN	9	0.28592	0.9981	10.8615 (2)
PREMS	10	-0.34984	0.9861	17.0132 (2)
CP	12	-0.17927	0.9994	4.0508 (2)
AV RE	13	0.22317	0.6710	6.3948 (2)
AXIOM	14	0.21914	0.9519	6.1542 (2)
THERM	15	0.48212	0.9943	36.9456 (2)
STEPS	16	0.30604	0.3774	12.6072 (2)
R INF	17	0.33766	0.8004	15.6997 (2)
AV TH	18	0.30959	0.9829	12.9327 (2)
AV AX	19	0.43118	0.9301	27.8611 (2)
TOT R	20	0.41934	0.8850	26.0303 (2)
PSLI	21	-0.13847	0.9988	2.3850 (2)
POSIT	22	0.39085	0.8261	21.9981 (2)

TABLE 5.7B

STEP NUMBER 2 FOR C3

VARIABLE ENTERED	15
MULTIPLE R	0.8332
STD. ERROR OF EST.	2.8215

ANALYSIS OF VARIANCE:

	DF	SUM OF SQUARES	MEAN SQUARE	F-RATIO
REGRESSION	2	2204.741	1102.371	138.469
RESIDUAL	122	971.259	7.961	

VARIABLES IN EQUATION: (CONSTANT= 1.83217)

VARIABLE		COEFFICIENT	STD. ERROR	F TO REMOVE
RE	11	3.46491	0.23119	224.6214 (2)
THERM	15	2.94969	0.48528	36.9456 (2)

VARIABLES NOT IN EQUATION:

VARIABLE		PARTIAL CORR.	TOLERANCE	F TO ENTER
CLAS1	1	0.76904	0.3716	175.1466 (1)
CLAS2	2	0.91680	0.2593	637.7576 (1)
CLAS4	4	0.77492	0.3831	181.8814 (1)
CLAS5	5	0.67509	0.3804	101.3209 (1)
WORDS	6	0.36348	0.9685	18.4193 (2)
SYMBL	7	0.36927	0.9780	19.1047 (2)
LOGCN	8	-0.05831	0.9507	0.4128 (2)
PAREN	9	0.29772	0.9953	11.7679 (2)
PREMS	10	-0.28985	0.9423	11.0976 (2)
CP	12	-0.08929	0.9539	0.9724 (2)
AV RE	13	0.07017	0.5920	0.5988 (2)
AXIOM	14	0.27219	0.9504	9.6822 (2)
STEPS	16	0.41779	0.3721	25.5870 (2)
R INF	17	0.19246	0.6870	4.6544 (2)
AV TH	18	-0.09804	0.4177	1.1744 (2)
AV AX	19	0.21889	0.6315	6.0894 (2)
TOT R	20	0.16844	0.5307	3.5331 (2)
PSLI	21	0.00457	0.9118	0.0025 (2)
POSIT	22	0.16471	0.5638	3.3741 (2)

TABLE 5.7C

STEP NUMBER 3 FOR C3

VARIABLE ENTERED	16
MULTIPLE R	0.8646
STD. ERROR OF EST.	2.5741

ANALYSIS OF VARIANCE:

	DF	SUM OF SQUARES	MEAN SQUARE	F-RATIO
REGRESSION	3	2374.276	791.425	119.446
RESIDUAL	121	801.724	6.626	

VARIABLES IN EQUATION: (CONSTANT= -0.00914)

VARIABLE		COEFFICIENT	STD. ERROR	F TO REMOVE
RE	11	2.07922	0.34573	36.1688 (2)
THERM	15	3.21817	0.44589	52.0915 (2)
STEPS	16	0.74803	0.14788	25.5870 (2)

VARIABLES NOT IN EQUATION:

VARIABLE		PARTIAL CORR.	TOLERANCE	F TO ENTER
CLAS1	1	0.72310	0.3126	131.5064 (1)
CLAS2	2	0.90031	0.1924	513.4358 (1)
CLAS4	4	0.78554	0.3717	193.3721 (1)
CLAS5	5	0.71158	0.3784	123.0865 (1)
WORDS	6	0.29879	0.9131	11.7629 (2)
SYMBL	7	0.29004	0.9024	11.0216 (2)
LOGCN	8	-0.29810	0.7650	11.7034 (2)
PAREN	9	0.24474	0.9594	7.6457 (2)
PREMS	10	-0.24173	0.9130	7.4475 (2)
CP	12	-0.29142	0.8205	11.1367 (2)
AV RE	13	0.16750	0.5707	3.4641 (2)
AXIOM	14	0.30546	0.9503	12.3493 (2)
R INF	17	0.28416	0.6715	10.5409 (2)
AV TH	18	-0.07074	0.4150	0.6035 (2)
AV AX	19	0.27260	0.6287	9.6334 (2)
TOT R	20	0.24455	0.5224	7.6331 (2)
PSLI	21	-0.04786	0.8999	0.2755 (2)
POSIT	22	0.24282	0.5544	7.5186 (2)

TABLE 5.8

SUMMARY TABLE FOR C3 ON THE FULL SET OF PROBLEMS

STEP NUM	VARIABLE ENT REM		MULTIPLE R	RSQ	INCREASE IN RSQ	F VALUE FOR DEL	LAST REG COEFFICNTS
1	RE	11	0.77560	0.60156	0.60156	185.7180	1.72670
2	THERM	15	0.83320	0.69422	0.09267	36.9456	4.12214
3	STEPS	16	0.86460	0.74753	0.05331	25.5870	0.88129
4	AXIOM	14	0.87810	0.77106	0.02353	12.3493	0.82494
5	SYMBL	7	0.89090	0.79370	0.02264	13.0033	0.22774
6	CP	12	0.90120	0.81216	0.01846	11.6125	-1.17221
7	PAREN	9	0.90480	0.81866	0.00650	4.2352	-1.41810
8	AV TH	18	0.90720	0.82301	0.00435	2.8131	-0.24658
9	AV AX	19	0.91090	0.82974	0.00673	4.5963	0.58843
10	LOGCN	8	0.91210	0.83193	0.00219	1.4453	-1.49248
11	PREMS	10	0.91270	0.83302	0.00109	0.7322	-0.19960
12	R INF	17	0.91290	0.83339	0.00037	0.2761	0.64315
13	WORDS	6	0.91310	0.83375	0.00037	0.2529	0.02520
14	PSLI	21	0.91310	0.83375	0.00000	0.0155	0.07453
15	POSIT	22	0.91320	0.83393	0.00018	0.0595	-0.06885
16	AV RE	13	0.91340	0.83430	0.00037	0.2126	0.89661

TABLE 5.9A

STEP NUMBER 1 FOR C4

VARIABLE ENTERED	20
MULTIPLE R	0.6255
STD. ERROR OF EST.	2.6592

ANALYSIS OF VARIANCE:

	DF	SUM OF SQUARES	MEAN SQUARE	F-RATIO
REGRESSION	1	558.996	558.996	79.048
RESIDUAL	123	869.804	7.072	

VARIABLES IN EQUATION: (CONSTANT= -4.70182)

VARIABLE		COEFFICIENT	STD. ERROR	F TO REMOVE
TOT R	20	0.41522	0.04670	79.0482 (2)

VARIABLES NOT IN EQUATION:

VARIABLE		PARTIAL CORR.	TOLERANCE	F TO ENTER
CLAS1	1	0.70182	0.6787	118.4180 (1)
CLAS2	2	0.76067	0.7716	167.5221 (1)
CLAS3	3	0.83827	0.7379	288.3557 (1)
CLAS5	5	0.91114	0.5436	596.3591 (1)
WORDS	6	0.13812	0.9995	2.3725 (2)
SYMBL	7	0.15679	0.9788	3.0747 (2)
LOGCN	8	0.01529	0.9254	0.0285 (2)
PAREN	9	0.12225	0.9986	1.8509 (2)
PREMS	10	0.07217	0.8616	0.6387 (2)
RE	11	0.51945	0.8850	45.0829 (2)
CP	12	0.03775	0.9301	0.1741 (2)
AV RE	13	0.12590	0.4021	1.9650 (2)
AXIOM	14	0.06831	0.7889	0.5720 (2)
THERM	15	0.27869	0.6166	10.2733 (2)
STEPS	16	0.50709	0.9716	42.2307 (2)
R INF	17	0.02415	0.2239	0.0712 (2)
AV TH	18	0.01788	0.4167	0.0390 (2)
AV AX	19	-0.06170	0.0969	0.4663 (2)
PSLI	21	0.04150	0.9144	0.2105 (2)
POSIT	22	0.06814	0.0520	0.5691 (2)

TABLE 5.9B

STEP NUMBER 2 FOR C4

VARIABLE ENTERED	11
MULTIPLE R	0.7453
STD. ERROR OF EST.	2.2816

ANALYSIS OF VARIANCE:

	DF	SUM OF SQUARES	MEAN SQUARE	F-RATIO
REGRESSION	2	793.689	396.845	76.231
RESIDUAL	122	635.111	5.206	

VARIABLES IN EQUATION: (CONSTANT= -3.80896)

VARIABLE		COEFFICIENT	STD. ERROR	F TO REMOVE
RE	11	1.33047	0.19815	45.0829 (2)
TOT R	20	0.31825	0.04259	55.8293 (2)

VARIABLES NOT IN EQUATION:

VARIABLE		PARTIAL CORR.	TOLERANCE	F TO ENTER
CLAS1	1	0.55316	0.3328	53.3476 (1)
CLAS2	2	0.67451	0.2743	101.0027 (1)
CLAS3	3	0.79170	0.3284	203.2142 (1)
CLAS5	5	0.92200	0.4899	686.1566 (1)
WORDS	6	0.13115	0.9969	2.1175 (2)
SYMBL	7	0.17381	0.9786	3.7691 (2)
LOGCN	8	-0.04940	0.9142	0.2960 (2)
PAREN	9	0.12321	0.9976	1.8652 (2)
PREMS	10	-0.08903	0.7943	0.9667 (2)
CP	12	-0.03268	0.9153	0.1294 (2)
AV RE	13	-0.19930	0.2925	5.0047 (2)
AXIOM	14	0.03388	0.7844	0.1391 (2)
THERM	15	0.44406	0.5962	29.7202 (2)
STEPS	16	0.18510	0.3664	4.2930 (2)
R INF	17	-0.18443	0.1991	4.2609 (2)
AV TH	18	0.15258	0.3982	2.8843 (2)
AV AX	19	0.04883	0.0931	0.2892 (2)
PSLI	21	-0.04256	0.8940	0.2196 (2)
POSIT	22	-0.18213	0.0435	4.1512 (2)

TABLE 5.9C

STEP NUMBER 3 FOR C4

VARIABLE ENTERED	15
MULTIPLE R	0.8020
STD. ERROR OF EST.	2.0528

ANALYSIS OF VARIANCE:

	DF	SUM OF SQUARES	MEAN SQUARE	F-RATIO
REGRESSION	3	918.926	306.309	72.691
RESIDUAL	121	509.874	4.214	

VARIABLES IN EQUATION: (CONSTANT= -1.12523)

VARIABLE		COEFFICIENT	STD. ERROR	F TO REMOVE
RE	11	1.51015	0.18130	69.3844 (2)
THERM	15	2.48564	0.45595	29.7202 (2)
TOT R	20	0.14754	0.04949	8.8880 (2)

VARIABLES NOT IN EQUATION:

VARIABLE		PARTIAL CORR.	TOLERANCE	F TO ENTER
CLAS1	1	0.55736	0.3272	54.0763 (1)
CLAS2	2	0.64216	0.2540	84.2096 (1)
CLAS3	3	0.76820	0.2971	172.7806 (1)
CLAS5	5	0.90518	0.3443	544.2899 (1)
WORDS	6	0.26274	0.9480	8.8981 (2)
SYMBL	7	0.22974	0.9737	6.6866 (2)
LOGCN	8	-0.03065	0.9119	0.1128 (2)
PAREN	9	0.11340	0.9952	1.5633 (2)
PREMS	10	-0.14879	0.7866	2.7168 (2)
CP	12	-0.01658	0.9138	0.0330 (2)
AV RE	13	-0.09770	0.2730	1.1565 (2)
AXIOM	14	0.28085	0.6365	10.2758 (2)
STEPS	16	0.21363	0.3663	5.7383 (2)
R INF	17	0.01678	0.1596	0.0338 (2)
AV TH	18	-0.12131	0.2785	1.7924 (2)
AV AX	19	0.12385	0.0913	1.8692 (2)
PSLI	21	0.01364	0.8805	0.0223 (2)
POSIT	22	-0.05951	0.0397	0.4264 (2)

TABLE 5.10

SUMMARY TABLE FOR C4 ON THE FULL SET OF PROBLEMS

STEP NUM	VARIABLE ENT	VARIABLE REM	MULTIPLE R	MULTIPLE RSQ	INCREASE IN RSQ	F VALUE FOR DEL	LAST REG COEFFICNTS
1	TOT R 20		0.62550	0.39125	0.39125	79.0482	0.89661
2	RE 11		0.74530	0.55547	0.16422	45.0829	1.01207
3	THERM 15		0.80200	0.64320	0.08773	29.7202	4.28564
4	AXIOM 14		0.81930	0.67125	0.02805	10.2758	1.25623
5	WORDS 6		0.83730	0.70107	0.02982	11.8369	0.06957
6	STEPS 16		0.84040	0.70627	0.00520	2.0728	0.29581
7	PAREN 9		0.84320	0.71099	0.00471	1.9620	-1.19005
8	LOGCN 8		0.84710	0.71758	0.00659	2.6536	-1.63819
9	TOT R	20	0.84710	0.71758	0.00000	0.0023	
10	SYMBL 7		0.85110	0.72437	0.00679	2.8960	0.10362
11	AV TH 18		0.85320	0.72795	0.00358	1.5032	0.78169
12	R INF 17		0.85490	0.73085	0.00290	1.2176	1.80194
13	POSIT 22		0.85540	0.73171	0.00086	0.3643	-0.20106
14	AV AX 19		0.85600	0.73274	0.00103	0.4798	1.14620
15	PSLI 21		0.85790	0.73599	0.00326	1.2987	0.19715
16	AV RE 13		0.86010	0.73977	0.00378	1.6328	1.94833
17	CP 12		0.86050	0.74046	0.00069	0.2912	0.53331
18	PREMS 10		0.86060	0.74063	0.00017	0.0415	-0.05619

TABLE 5.11A

STEP NUMBER 1 FOR C5

VARIABLE ENTERED	15
MULTIPLE R	0.6799
STD. ERROR OF EST.	2.1112

ANALYSIS OF VARIANCE:

	DF	SUM OF SQUARES	MEAN SQUARE	F-RATIO
REGRESSION	1	471.399	471.399	105.759
RESIDUAL	123	548.249	4.457	

VARIABLES IN EQUATION: (CONSTANT= 2.15129)

VARIABLE		COEFFICIENT	STD. ERROR	F TO REMOVE
THERM	15	3.72353	0.36207	105.7586 (2)

VARIABLES NOT IN EQUATION:

VARIABLE		PARTIAL CORR.	TOLERANCE	F TO ENTER
CLAS1	1	0.70420	0.8942	120.0150 (1)
CLAS2	2	0.69615	0.9068	114.7198 (1)
CLAS3	3	0.77228	0.8689	180.2896 (1)
CLAS4	4	0.93227	0.6886	810.2264 (1)
WORDS	6	0.27808	0.9731	10.2250 (2)
SYMBL	7	0.13139	0.9785	2.1433 (2)
LOGCN	8	-0.08534	0.9513	0.8951 (2)
PAREN	9	0.09174	0.9969	1.0354 (2)
PREMS	10	-0.13052	0.9601	2.1142 (2)
RE	11	0.54088	0.9943	50.4507 (2)
CP	12	-0.05913	0.9556	0.4281 (2)
AV RE	13	0.42484	0.8952	26.8693 (2)
AXIOM	14	0.46331	0.9995	33.3454 (2)
STEPS	16	0.46577	0.9998	33.7991 (2)
R INF	17	0.43381	0.8634	28.2824 (2)
AV TH	18	0.18192	0.4231	4.1758 (2)
AV AX	19	0.40594	0.6810	24.0704 (2)
TOT R	20	0.44211	0.6166	29.6405 (2)
PSLI	21	-0.00168	0.9150	0.0003 (2)
POSIT	22	0.45377	0.7060	31.6352 (2)

TABLE 5.11B

STEP NUMBER 2 FOR C5

VARIABLE ENTERED 11
MULTIPLE R 0.7872
STD. ERROR OF EST. 1.7830

ANALYSIS OF VARIANCE:

	DF	SUM OF SQUARES	MEAN SQUARE	F-RATIO
REGRESSION	2	631.790	315.895	99.364
RESIDUAL	122	387.858	3.179	

VARIABLES IN EQUATION: (CONSTANT= 1.41221)

VARIABLE		COEFFICIENT	STD. ERROR	F TO REMOVE
RE	11	1.03769	0.14610	50.4507 (2)
THERM	15	3.55872	0.30666	134.6666 (2)

VARIABLES NOT IN EQUATION:

VARIABLE		PARTIAL CORR.	TOLERANCE	F TO ENTER
CLAS1	1	0.53615	0.3716	48.8150 (1)
CLAS2	2	0.53160	0.2593	47.6637 (1)
CLAS3	3	0.67509	0.3058	101.3209 (1)
CLAS4	4	0.91176	0.3831	596.3046 (1)
WORDS	6	0.28717	0.9685	10.8751 (2)
SYMBL	7	0.17151	0.9780	3.6673 (2)
LOGCN	8	-0.11727	0.9507	1.6871 (2)
PAREN	9	0.08390	0.9953	0.8577 (2)
PREMS	10	-0.24507	0.9423	7.7314 (2)
CP	12	-0.09717	0.9539	1.1535 (2)
AV RE	13	0.16089	0.5920	3.2154 (2)
AXIOM	14	0.41875	0.9504	25.7290 (2)
STEPS	16	0.07249	0.3721	0.6392 (2)
R INF	17	0.25235	0.6870	8.2296 (2)
AV TH	18	0.14447	0.4177	2.5792 (2)
AV AX	19	0.32129	0.6315	13.9278 (2)
TOT R	20	0.30788	0.5307	12.6709 (2)
PSLI	21	-0.04050	0.9118	0.1988 (2)
POSIT	22	0.28077	0.5638	10.3548 (2)

TABLE 5.11C

STEP NUMBER 3 FOR C5

VARIABLE ENTERED 14
MULTIPLE R 0.8284
STD. ERROR OF EST. 1.6258

ANALYSIS OF VARIANCE:

	DF	SUM OF SQUARES	MEAN SQUARE	F-RATIO
REGRESSION	3	699.801	233.267	88.246
RESIDUAL	121	319.847	2.643	

VARIABLES IN EQUATION: (CONSTANT= 1.14557)

VARIABLE		COEFFICIENT	STD. ERROR	F TO REMOVE
RE	11	0.88414	0.13661	41.8856 (2)
AXIOM	14	1.35888	0.26790	25.7290 (2)
THERM	15	3.61508	0.27985	166.8704 (2)

VARIABLES NOT IN EQUATION:

VARIABLE		PARTIAL CORR.	TOLERANCE	F TO ENTER
CLAS1	1	0.44565	0.3149	29.7381 (1)
CLAS2	2	0.48323	0.2422	36.5586 (1)
CLAS3	3	0.64213	0.2832	84.1981 (1)
CLAS4	4	0.89684	0.3300	493.2515 (1)
WORDS	6	0.31066	0.9684	12.8181 (2)
SYMBL	7	0.20321	0.9770	5.1688 (2)
LOGCN	8	-0.03412	0.9097	0.1398 (2)
PAREN	9	0.06757	0.9924	0.5504 (2)
PREMS	10	-0.14585	0.8675	2.6080 (2)
CP	12	-0.02018	0.9198	0.0489 (2)
AV RE	13	-0.00619	0.4989	0.0046 (2)
STEPS	16	0.08569	0.3720	0.8877 (2)
R INF	17	0.08151	0.5514	0.8027 (2)
AV TH	18	0.04849	0.3931	0.2829 (2)
AV AX	19	0.07521	0.3711	0.6827 (2)
TOT R	20	0.09045	0.3554	0.9899 (2)
PSLI	21	0.06441	0.8625	0.5000 (2)
POSIT	22	0.08181	0.4121	0.8086 (2)

TABLE 5.12

SUMMARY TABLE FOR C5 ON THE FULL SET OF PROBLEMS

STEP NUM	VARIABLE ENT	VARIABLE REM	MULTIPLE R	MULTIPLE RSQ	INCREASE IN RSQ	F VALUE FOR DEL	LAST REG COEFFICNTS
1 THERM	15		0.67990	0.46226	0.46226	105.7586	4.05968
2 RE	11		0.78720	0.61968	0.15742	50.4507	0.67997
3 AXIOM	14		0.82840	0.68625	0.06656	25.7290	1.25372
4 WORDS	6		0.84650	0.71656	0.03032	12.8181	0.07214
5 PAREN	9		0.85090	0.72403	0.00747	3.1787	−0.87724
6 LOGCN	8		0.85280	0.72727	0.00324	1.4294	−1.10059
7 SYMBL	7		0.85450	0.73017	0.00290	1.2940	0.06478
8 STEPS	16		0.85610	0.73291	0.00274	1.1375	0.11336
9 PREMS	10		0.85680	0.73411	0.00120	0.5597	−0.11963
10 AV TH	18		0.85720	0.73479	0.00069	0.2695	−0.10586
11 R INF	17		0.85740	0.73513	0.00034	0.1253	0.32683
12 PSLI	21		0.85750	0.73531	0.00017	0.0910	0.11460
13 POSIT	22		0.85770	0.73565	0.00034	0.1139	−0.10662
14 TOT R	20		0.85810	0.73634	0.00069	0.3230	0.58693
15 AV RE	13		0.85930	0.73840	0.00206	0.8699	1.19229
16 CP	12		0.85940	0.73857	0.00017	0.0841	0.27553

FIGURE 5.9 C5 VS S22(POSIT)

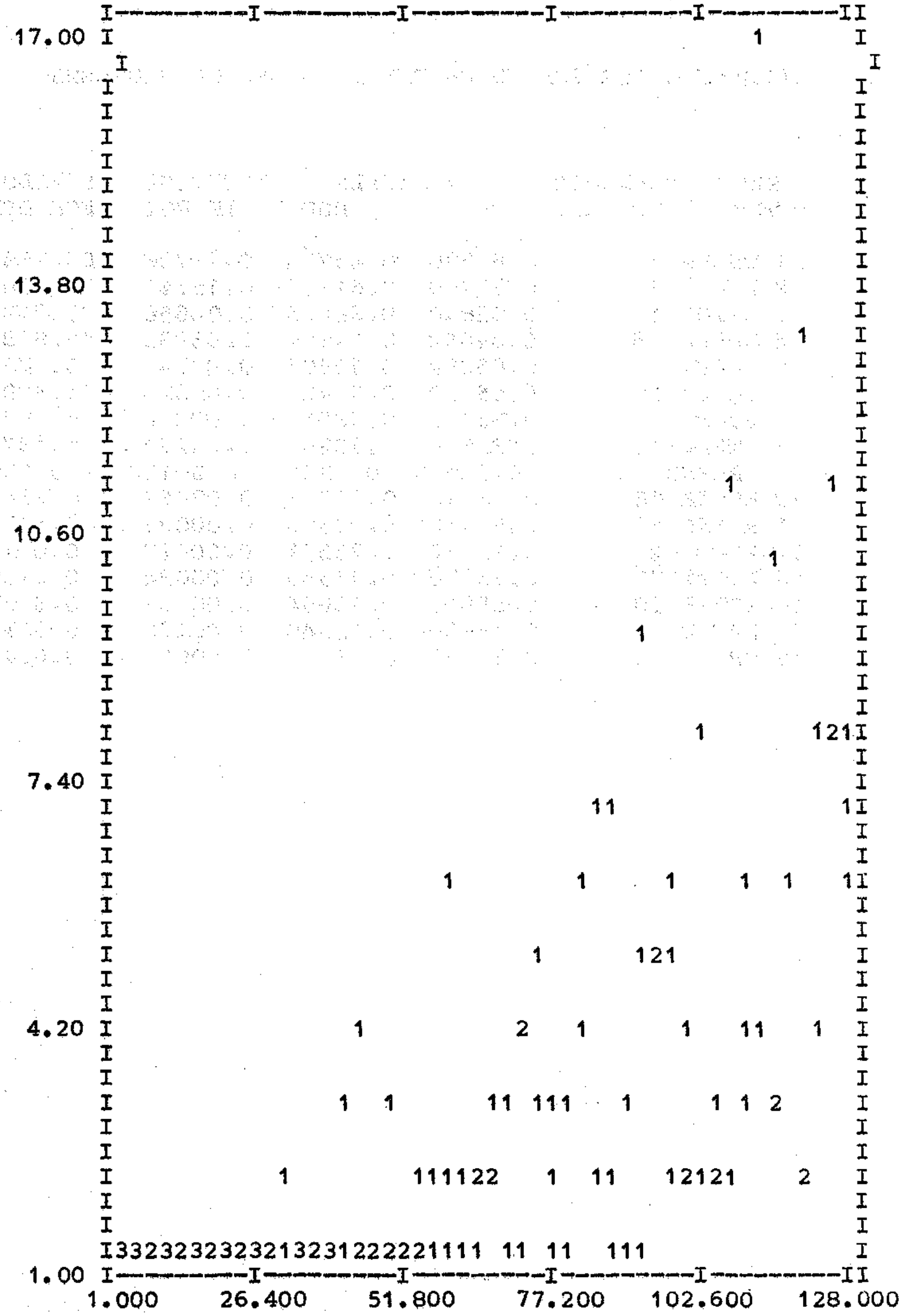


FIGURE 5.10 RESIDUALS(Y-AXIS) VS COMPUTED C5(X-AXIS)

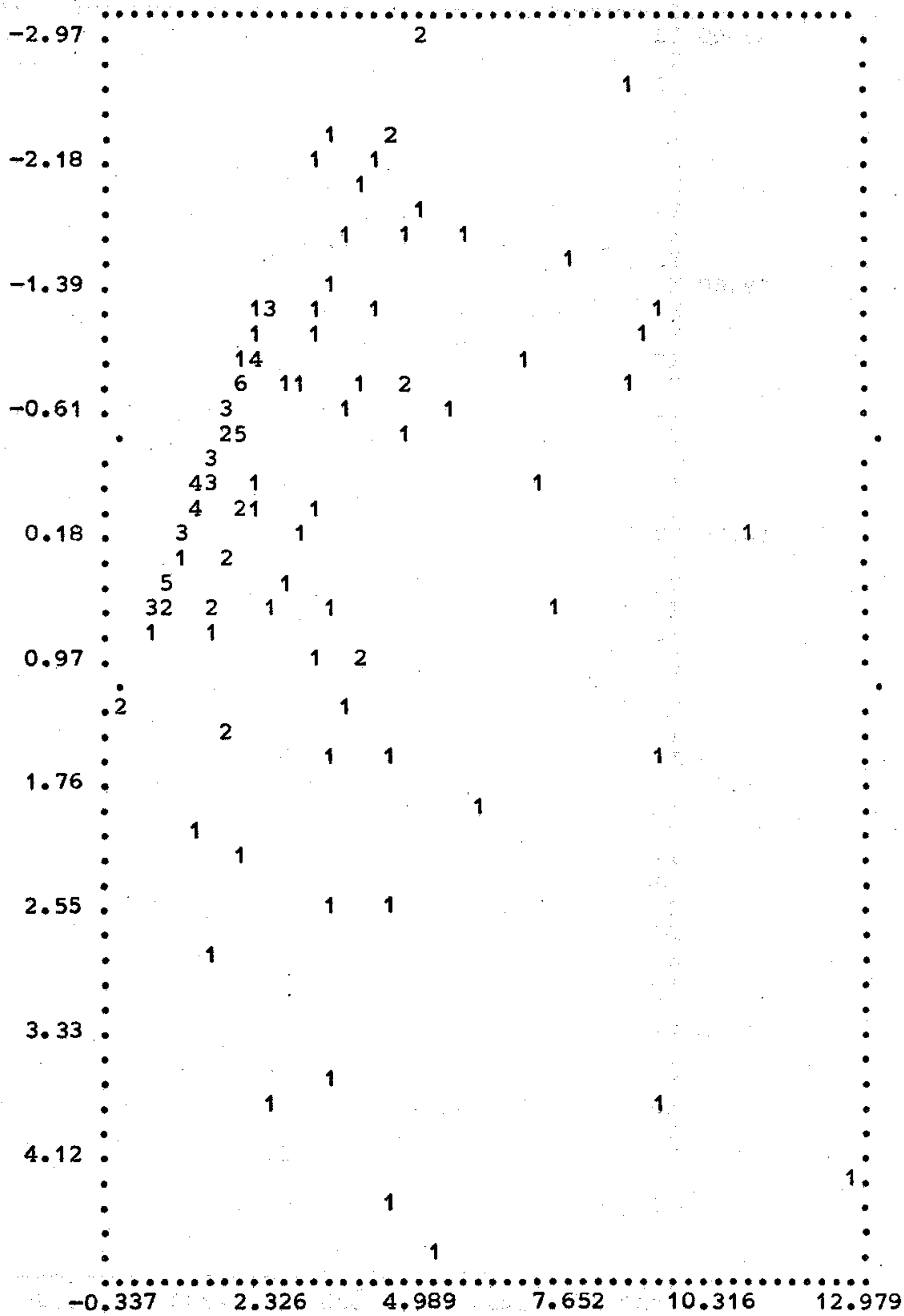


FIGURE 5.11A C1 VS S15(THERM)

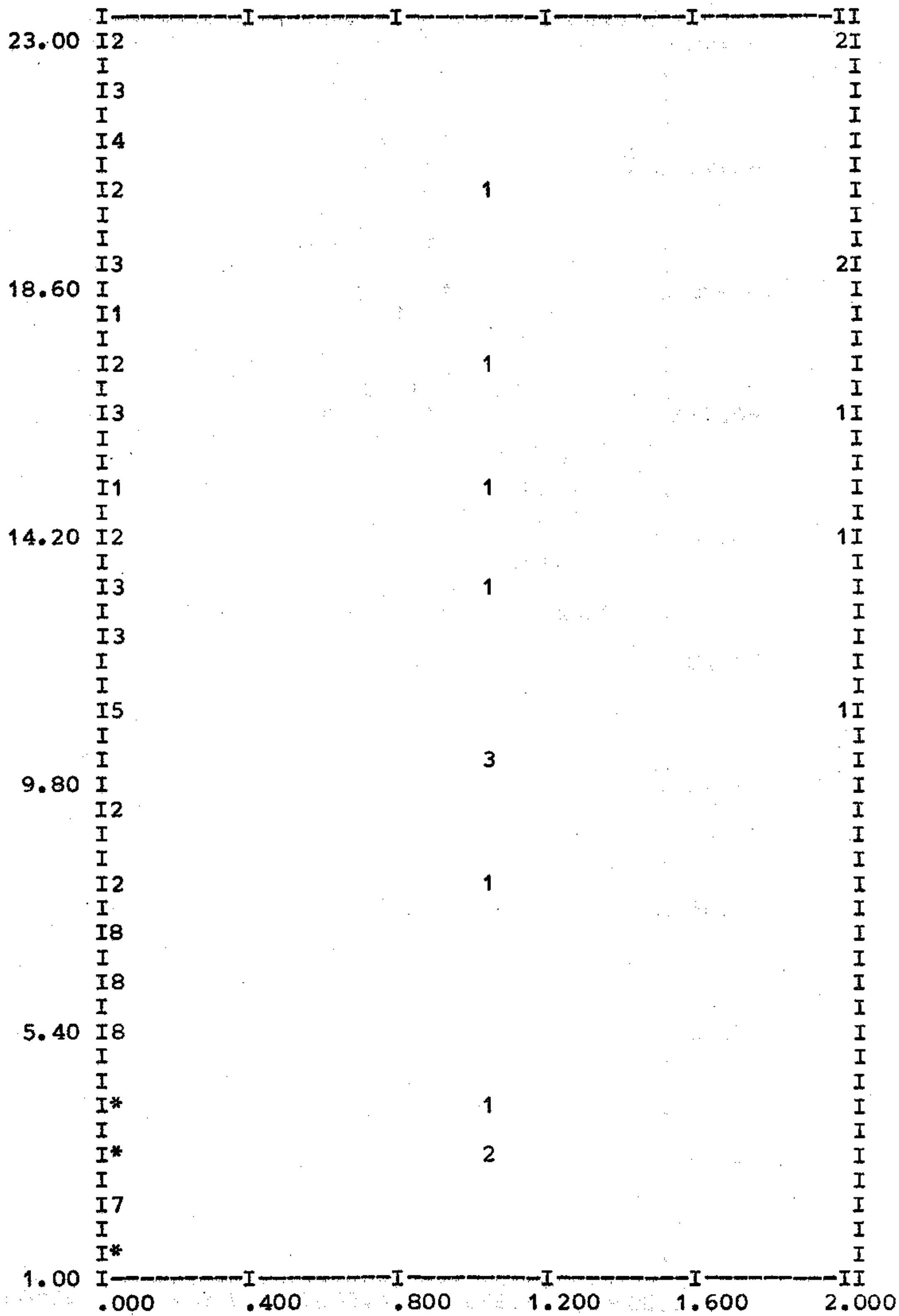


FIGURE 5.11B   C2 VS S15(THERM)

```
      I---------I---------I---------I---------I---------II
23.00 I1                                                 I
      I                                                  I
      I                                                 1I
      I                                                  I
      I2                                                 I
      I                                                  I
      I3                                                 I
      I                                                  I
      I                                                  I
      I1                                                 I
18.60 I                                                  I
      I                                                 1I
      I                                                  I
      I3                                                 I
      I                                                  I
      I1                                                1I
      I                                                  I
      I                                                  I
      I                         1                        I
      I                                                  I
14.20 I3                                                1I
      I                                                  I
      I                                                  I
      I                                                  I
      I2                                                 I
      I                                                  I
      I                                                  I
      I1                        1                        I
      I                                                  I
      I4                                                3I
 9.80 I                                                  I
      I4                        1                        I
      I                                                  I
      I                                                  I
      I2                        1                        I
      I                                                  I
      I3                                                 I
      I                                                  I
      I4                                                 I
      I                                                  I
 5.40 I3                        2                        I
      I                                                  I
      I                                                  I
      I4                                                 I
      I                                                  I
      I*                        2                        I
      I                                                  I
      I*                        3                        I
      I                                                  I
      I*                                                 I
 1.00 I---------I---------I---------I---------I---------II
      .000      .400      .800     1.200     1.600     2.000
```

FIGURE 5.11C C3 VS S15(THERM)

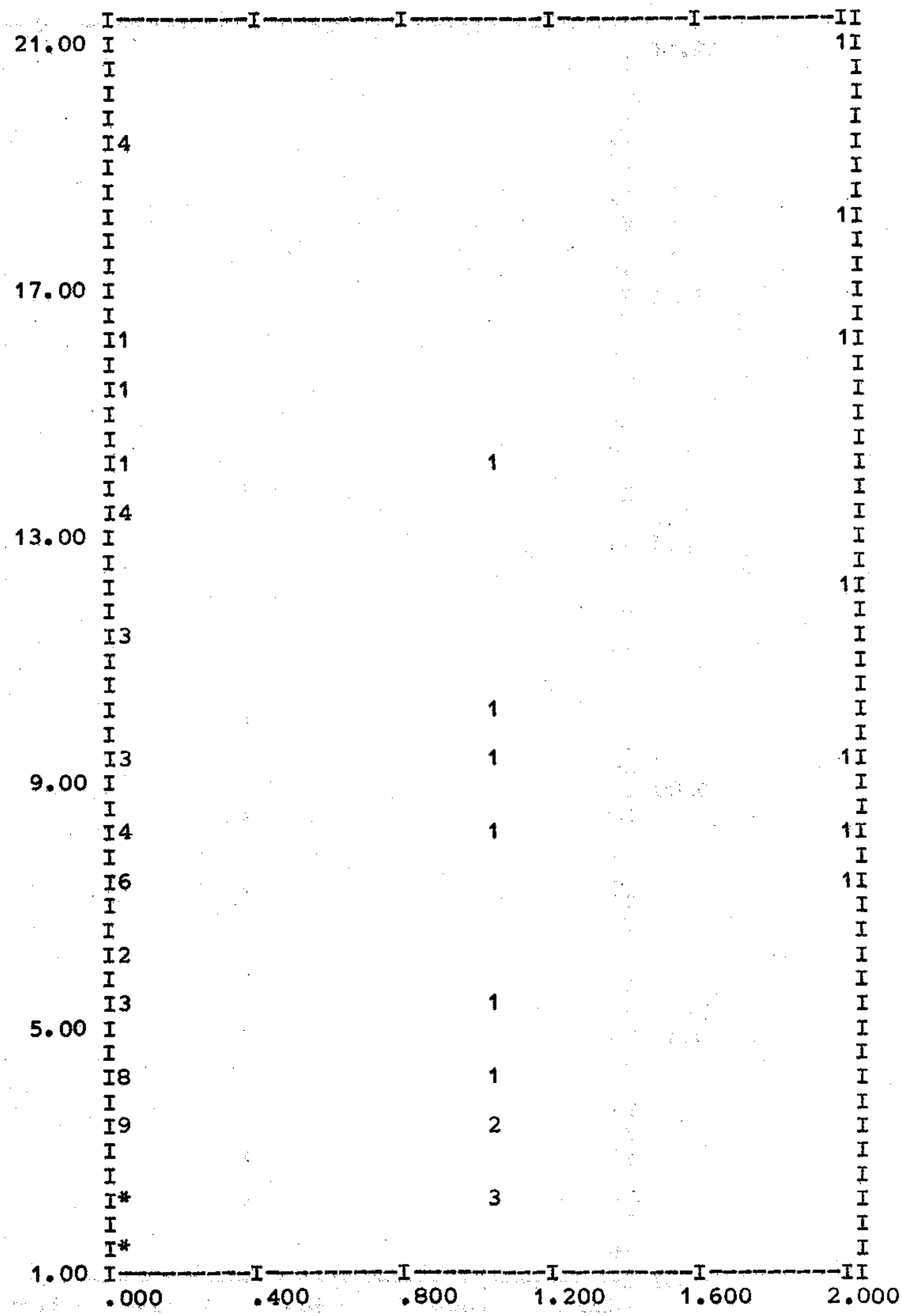


FIGURE 5.11D C4 VS S15(THERM)

```
      I---------I---------I---------I---------I---------II
17.00 I                                                 1I
      I                                                  I
      I                                                  I
      I                                                  I
      I                                                  I
      I                                                  I
      I                                                  I
      I                                                  I
      I                                                  I
      I1                                                1I
13.80 I                                                  I
      I                                                  I
      I                                                 1I
      I                                                  I
      I                                                  I
      I1                                                 I
      I                                                  I
      I                                                  I
      I1                                                1I
      I                                                  I
10.60 I                                                  I
      I1                                                 I
      I                                                  I
      I                                                  I
      I3                        2                        I
      I                                                  I
      I                                                  I
      I                                                  I
      I1                        1                       2I
      I                                                  I
 7.40 I                                                  I
      I2                        1                        I
      I                                                  I
      I                                                  I
      I3                                                1I
      I                                                  I
      I                                                  I
      I8                                                 I
      I                                                  I
      I                                                  I
 4.20 I6                        2                        I
      I                                                  I
      I                                                  I
      I9                        2                        I
      I                                                  I
      I                                                  I
      I*                        3                        I
      I                                                  I
      I                                                  I
      I*                                                 I
 1.00 I---------I---------I---------I---------I---------II
      .000      .400      .800     1.200     1.600     2.000
```

FIGURE 5.11E C5 VS S15(THERM)

```
        I---------I---------I---------I---------I---------II
  17.00 I                                                 1I
        I                                                  I
        I                                                  I
        I                                                  I
        I                                                  I
        I                                                  I
        I                                                  I
        I                                                  I
        I                                                  I
        I                                                  I
  13.80 I                                                  I
        I                                                  I
        I                                                 1I
        I                                                  I
        I                                                  I
        I                                                  I
        I                                                  I
        I                                                  I
        I                                                 2I
        I                                                  I
  10.60 I                                                  I
        I1                                                 I
        I                                                  I
        I                                                  I
        I1                                                 I
        I                                                  I
        I                                                  I
        I                                                  I
        I                         3                       2I
        I                                                  I
   7.40 I                                                  I
        I2                        1                        I
        I                                                  I
        I                                                  I
        I5                                                1I
        I                                                  I
        I                                                  I
        I5                                                 I
        I                                                  I
        I                                                  I
   4.20 I6                        2                        I
        I                                                  I
        I                                                  I
        I*                        2                        I
        I                                                  I
        I                                                  I
        I*                        3                        I
        I                                                  I
        I                                                  I
        I*                                                 I
   1.00 I---------I---------I---------I---------I---------II
        .000      .400      .800     1.200     1.600     2.000
```

FIGURE 5.12A   C1 VS S11(RE)

```
      I---------I---------I---------I---------I---------II
23.00 I            1                        1           2I
      I                                                  I
      I            1                                    2I
      I                                                  I
      I            1            1           1           1I
      I                                                  I
      I                         3                        I
      I                                                  I
      I                                                  I
      I            3            2                        I
18.60 I                                                  I
      I                                     1            I
      I                                                  I
      I            2            1                        I
      I                                                  I
      I1           3                                     I
      I                                                  I
      I                                                  I
      I                         1           1            I
      I                                                  I
14.20 I2                                                1I
      I                                                  I
      I            3            1                        I
      I                                                  I
      I1           1                        1            I
      I                                                  I
      I                                                  I
      I1           4            1                        I
      I                                                  I
      I2           1                                     I
 9.80 I                                                  I
       I1           1                                     I
      I                                                  I
      I                                                  I
      I2           1                                     I
      I                                                  I
      I5           1            2                        I
      I                                                  I
      I8                                                 I
      I                                                  I
 5.40 I5           3                                     I
      I                                                  I
      I                                                  I
      I*           1                                     I
      I                                                  I
      I*           2                                     I
      I                                                  I
      I6           1                                     I
      I                                                  I
      I*                                                 I
 1.00 I---------I---------I---------I---------I---------II
      .000      .800     1.600     2.400     3.200     4.000
```

FIGURE 5.12B C2 VS S11(RE)

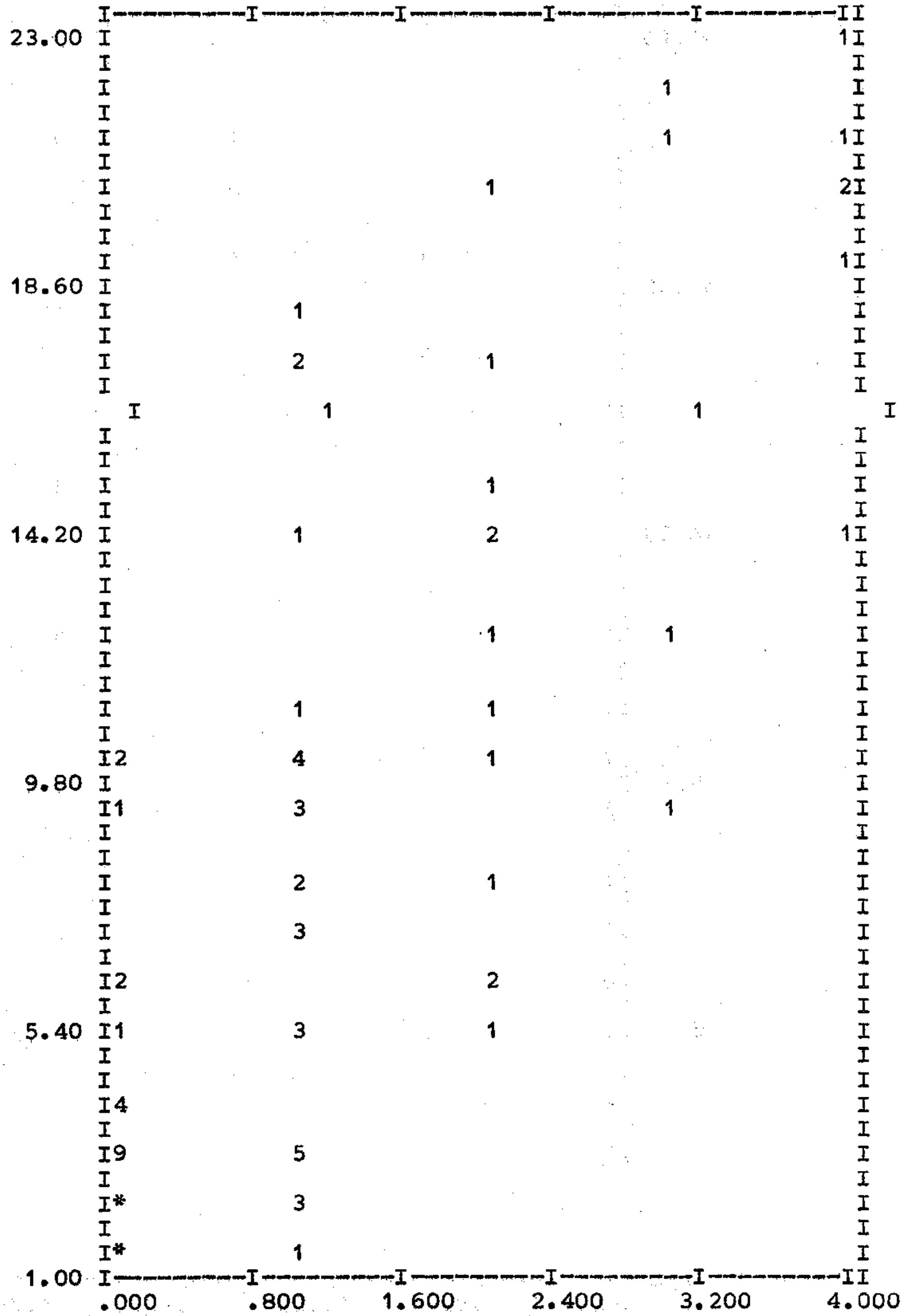


FIGURE 5.12C   C3 VS S11(RE)

```
        I---------I---------I---------I---------I---------II
  21.00 I                                     1            I
        I                                                  I
        I                                                  I
        I                                                  I
        I                                     1           3I
        I                                                  I
        I                                                  I
        I           1                                      I
        I                                                  I
        I                                                  I
  17.00 I                                                  I
        I                                                  I
        I           1                                     1I
        I                                                  I
        I           1                                      I
        I                                                  I
        I                                                  I
        I           1            1                         I
        I                                                  I
        I                        3                        1I
  13.00 I                                                  I
        I                                                  I
        I           1                                      I
        I                                                  I
        I                        2                        1I
        I                                                  I
        I                                                  I
        I                        1                         I
        I                                                  I
        I1          3                         1            I
   9.00 I                                                  I
        I                                                  I
        I           5            1                         I
        I                                                  I
        I2          3                         2            I
        I                                                  I
        I                                                  I
        I2                                                 I
        I                                                  I
        I1          2            1                         I
   5.00 I                                                  I
        I                                                  I
        I4          2            3                         I
        I                                                  I
        I7          4                                      I
        I                                                  I
        I                                                  I
        I*          4                                      I
        I                                                  I
        I*          2                                      I
   1.00 I---------I---------I---------I---------I---------II
        .000      .800     1.600     2.400     3.200     4.000
```

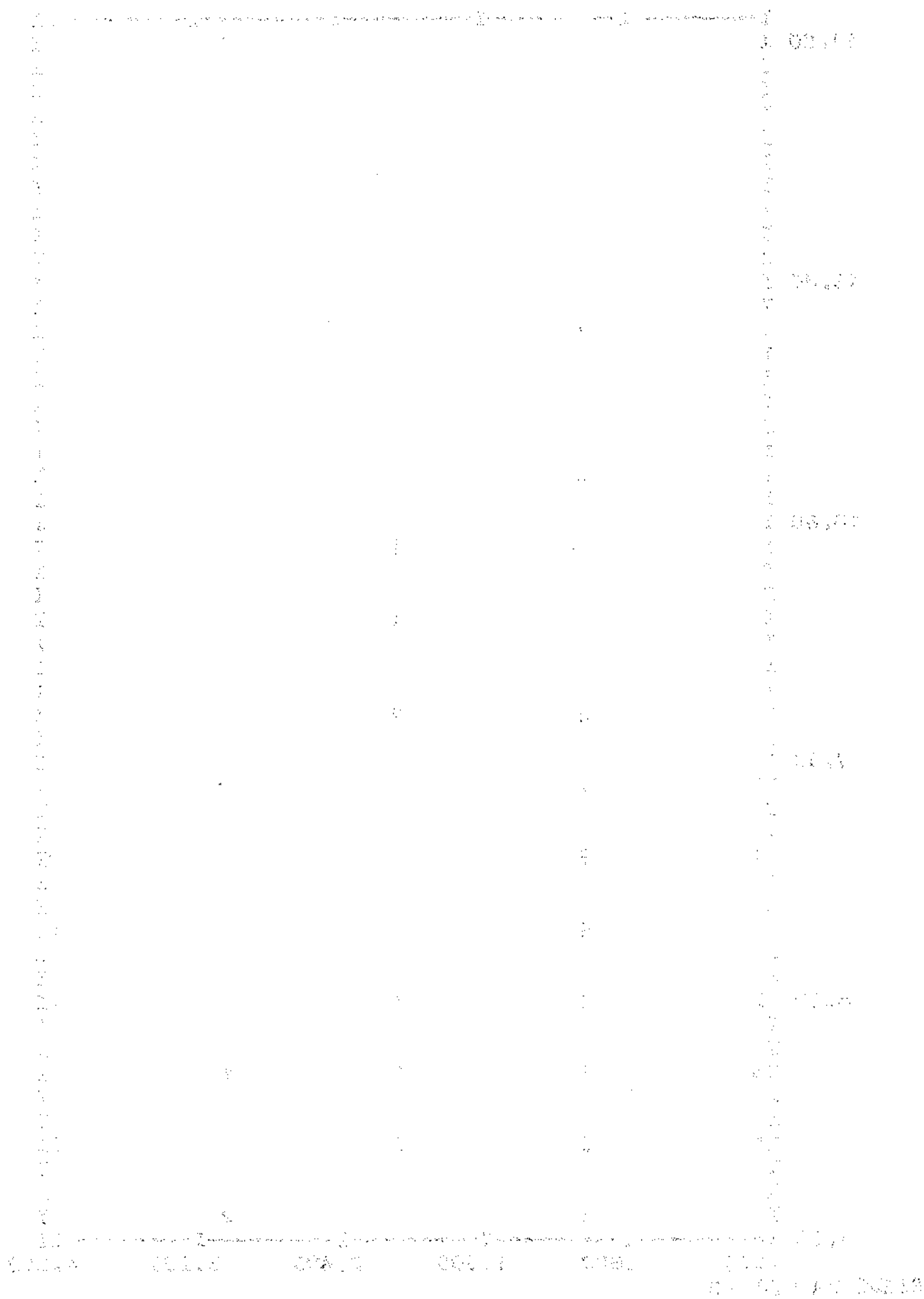
FIGURE 5.12D   C4 VS S11(RE)

```
         I---------I---------I---------I---------I---------II
   17.00 I                                     1            I
         I                                                  I
         I                                                  I
         I                                                  I
         I                                                  I
         I                                                  I
         I                                                  I
         I                                                  I
         I                                                  I
         I            1                        1            I
   13.80 I                                                  I
         I                                                  I
         I            1                                     I
         I                                                  I
         I                                                  I
         I                         1                        I
         I                                                  I
         I                                                  I
         I            1            1                        I
         I                                                  I
   10.60 I                                                  I
         I                                                 1I
         I                                                  I
         I                                                  I
         I1           1            2                       1I
         I                                                  I
         I                                                  I
         I                                                  I
         I            3                                    1I
         I                                                  I
    7.40 I                                                  I
         I            2            1                        I
         I                                                  I
         I                                                  I
         I1           3                                     I
         I                                                  I
         I                                                  I
         I            3            1           1           3I
         I                                                  I
         I                                                  I
    4.20 I2           3            2           1            I
         I                                                  I
         I                                                  I
         I5           4            2                        I
         I                                                  I
         I                                                  I
         I*           4            2                        I
         I                                                  I
         I                                                  I
         I*           4                        1            I
    1.00 I---------I---------I---------I---------I---------II
         .000      .800     1.600     2.400     3.200     4.000
```

FIGURE 5.12E C5 VS S11(RE)

```
      I---------I---------I---------I---------I---------II
17.00 I                                     1            I
      I                                                  I
      I                                                  I
      I                                                  I
      I                                                  I
      I                                                  I
      I                                                  I
      I                                                  I
      I                                                  I
      I                                                  I
13.80 I                                                  I
      I                                                  I
      I            1                                     I
      I                                                  I
      I                                                  I
      I                                                  I
      I                                                  I
      I                                                  I
      I            2                                     I
      I                                                  I
10.60 I                                                  I
      I                         1                        I
      I                                                  I
      I                                                  I
      I                         1                        I
      I                                                  I
      I                                                  I
      I                                                  I
      I            3            2                        I
      I                                                  I
 7.40 I                                                  I
      I1           1                        1            I
      I                                                  I
      I                                                  I
      I1           3                                    2I
      I                                                  I
      I                                                  I
      I            4                                    1I
      I                                                  I
      I                                                  I
 4.20 I2           4            1                       1I
      I                                                  I
      I                                                  I
      I5           3            2           1           1I
      I                                                  I
      I                                                  I
      I*           5            5                       1I
      I                                                  I
      I                                                  I
      I*           4                        2            I
 1.00 I---------I---------I---------I---------I---------II
      .000      .800     1.600     2.400     3.200     4.000
```

RIGHT MARGIN 60



CHAPTER SIX

6.1 - INTRODUCTION

In this chapter, the results of regression analyses over the restricted set of problems, which was discussed in Chapter IV, is be examined. This restricted set consists of the 54 problems that appear after the introduction of Replace Equals and do not have any premises. As in the previous chapter, a separate analysis was run for each of the five partitions. The results of these analyses do not differ sharply from those discussed in chapter five, and the discussion here will be brief. For the sake of completeness, however, a full set of tables of results is included.

Table 6.1 lists the means and standard deviations for all 22 variables, using the restricted set of problems, and Table 6.2 is the correlation matrix for these variables. A plot of the correlations of S11(RE), S16(STEPS), and S15(THERM) with the dependent variable against the ordinal number of the dependent variable for the restricted set of problems is found in Figure 6.1. The pattern in Figure 6.1 is very similar to that in Figure 5.1.

6.2 - REGRESSION WITH C1 AS THE DEPENDENT VARIABLE

For the first regression analysis, C1 is again the dependent variable. The results for the first three variables to enter the regression equation and a summary for the complete analysis are found in Tables 6.3A,B,C,D.

Together, the first four variables to enter the equation account for over 70 percent of the total variance in the dependent variable. This is somewhat less than the 77 percent that was accounted for by the first four variables when the full set of problems was used, but the fit is still quite good. Since the predictive power of several of the independent variables (S11(RE) and S22(POSIT) for example) was enhanced by the inclusion of the first fifty problems, the slight decrease in the variance accounted for by the regression equation is not surprising.

The first variable to enter the equation is S16(STEPS). Using the full set of problems and C1 as the dependent variable, S11(RE) was the first variable to enter. It has already been observed that S11 and S16 serve very similar function as measures of relatively superficial structural complexity. In the analysis of the first set of problems, the predictive power of S11 was enhanced by the fact that the first 53 problems had uniformly low values for the dependent variable and had value zero for S11. It is not surprising then that S16(STEPS) replaces S11(RE) as the first variable to enter the equation.

The second variable entering the equation, using the restricted set, is S15(THERM). Using the full set, S22(POSIT) was the second variable to enter the equation. The overall predictive power of S22 was also enhanced by the inclusion of the first 53 problems in the analysis. With this effect eliminated, S15 is prominent even for the first partition. The relative predictive power of S15 is greater for the restricted set of problems, because the percentage of problems with values of S15 greater than zero is much larger than it was for the full set.

The third and fourth variables to enter are S14(AXIOM) and S7(SYMBL). These are the same variables that entered as the third and fourth variables for the full set of problems.

At this point it is appropriate to discuss the assumptions in the model for regression, specifically normality of the distibution of errors and homogeneity of their variance.

Figure 6.2A contains a histogram for the residuals after all of the variables have entered the equation. The distribution does not indicate any serious violations of the normality asumption. A plot of the residuals against the computed value of C1 is found in Figure 6.2B. From this plot, it appears that the homogeniety-of- variance assumption is not seriously violated for the restricted set of problems. The highly significant values for the F-ratios in this analysis and in the other four analyses presented in this chapter provide reassurance that the results obtained are not due to chance.

The equation as a whole is significant at the .01 level for all fourteen steps in the stepwise regression analysis presented. The F-ratios for adding each of the first four variables in the equation are also significant at the .01 level. Lastly, in Table 6.3D, it can be seen that the F-ratios for deleting any of the first four variables are also significant.

The question of statistical significance has been discussed only very briefly here, and will not be discussed for the other analyses in this chapter. The reason for this omission has already been explained.

6.3 - REGRESSION WITH C2 AS THE DEPENDENT VARIABLE

In the regression analysis for the second dependent variable, S16(STEPS) and S15(THERM) are the first two variables to enter; again the patterns found in the tables of partial correlations (Table 6.4A,B), for this analysis, are generally similar to those for corresponding analysis of the full set of problems (Tables 5.5A,B). The differences that do appear in these tables are principally due to the diminished predictive value of S11(RE) and the rule-position variables.

The third variable to enter the equation is S12(CP), one of the problem-structure variables (Table 6.4C). A summary of the results for all variables in this analysis is included as Table 6.4D. Using the full set of problems and the second partition, the regression equation accounted for 82 percent of the variance in the dependent variable; with the restricted set used here, the regression equation accounts for 79 percent of the variance.

Figure 6.3A contains a histogram of the residuals for this analysis and Figure 6.3B contains a plot of the residuals against the predicted value of C2. Neither of these figures indicates a serious violation of the assumptions.

6.4 - DISCUSSION

The results for the regression analyses using C3, C4, and C5 as dependent variables follow the pattern established in the last chapter and will not be discussed in detail here. For completeness, the results are included in Tables 6.5, 6.6 and 6.7, and Figures 6.4, 6.5, and 6.6. There are no serious violations of the assumptions in any of these analyses.

The evidence for the restricted set of problems tends to confirm the general conclusions indicated by the analysis of the full set. There are two types of variation that appear in the sample of proofs. The first type of variation consists of the relatively superficial differences that appear in the proofs. Variation in the order in which rules are used in proofs is one example of this type of difference. The definitions of equivalence for the first two partitions are very sensitive to changes in order. The definition for the third partition is sensitive to some differences in order but not to all; the last two partitions completely ignore difference in order.

Both S11(RE) and S16(STEPS) are good predictors of the dependent variables, C1 and C2, for the first two partitions; their importance systematically declines for the last three partitions C3, C4, and C5. It appears that both of these variables are good predictors of sources of variation such as changes in the order in which rules are used, but are relatively ineffective in predicting more significant sources of variation such as differences in the rules used in a proof.

The second principal source of variation found in this study is in the rules used to form the proofs. This type of variation is much more fundamental and important. It is predicted best by S15(THERM) and to a lesser degree by S14(AXIOM). All five of the partitions are sensitive to differences in the rules used in a proof. The relative importance of the set of rules used increases as we move from the first partition to the fifth, because other types of difference are successively being eliminated from consideration as we move from one partition to the next. The fifth partition is defined only by the particular rules used in the proofs. So, it is not surprising that the importance of rule-position variables increases from parition to partition. These observations will be developed in Chapters VII and VIII.

TABLE 6.1

MEANS AND STANDARD DEVIATIONS FOR RESTRICTED SET

VARIABLE		MEAN	STANDARD DEVIATION
CLAS1	1	13.42593	6.40048
CLAS2	2	9.31481	6.51797
CLAS3	3	7.92593	5.63966
CLAS4	4	5.37037	3.66161
CLAS5	5	4.53704	2.96974
WORDS	6	15.12963	6.55325
SYMBL	7	12.38889	6.02954
LOGCN	8	0.20370	0.40653
PAREN	9	1.03704	0.91038
PREMS	10	0.00000	0.00000
RE	11	1.16667	1.17762
CP	12	0.18519	0.39210
AV RE	13	1.00000	0.00000
AXIOM	14	0.55556	0.69137
THERM	15	0.42593	0.68960
STEPS	16	4.66667	3.15032
R INF	17	18.74074	0.55577
AV TH	18	1.40741	1.95727
AV AX	19	3.55556	1.72295
TOT R	20	23.70370	3.66848
PSLI	21	5.00000	3.49123
POSIT	22	95.92593	19.84834

TABLE 6.2

CORRELATION MATRIX FOR RESTRICTED SET

VARIABLE NUMBER	1	2	3	4	5
1	1.000	0.934	0.914	0.739	0.588
2		1.000	0.972	0.747	0.556
3			1.000	0.824	0.654
4				1.000	0.913
5					1.000

MATRIX CONTINUED

VARIABLE NUMBER	6	7	8	9	10
1	0.462	0.387	-0.019	0.324	0.000
2	0.402	0.322	-0.018	0.275	0.000
3	0.413	0.347	-0.100	0.284	0.000
4	0.239	0.117	-0.140	0.098	0.000
5	0.127	0.004	-0.249	0.055	0.000
6	1.000	0.626	0.025	0.429	0.000
7		1.000	0.190	0.747	0.000
8			1.000	-0.174	0.000
9				1.000	0.000
10					0.000

MATRIX CONTINUED

VARIABLE NUMBER	11	12	13	14	15
1	0.726	-0.077	0.000	0.116	0.035
2	0.804	-0.097	0.000	0.015	0.028
3	0.758	-0.147	0.000	0.040	0.096
4	0.467	-0.128	0.000	0.066	0.370
5	0.265	-0.217	0.000	0.036	0.559
6	0.381	0.049	0.000	0.142	-0.305
7	0.254	0.208	0.000	0.060	-0.236
8	0.125	0.943	0.000	-0.209	-0.315
9	0.223	-0.125	0.000	0.027	0.004
10	0.000	0.000	0.000	0.000	0.000
11	1.000	0.095	0.000	-0.023	-0.205
12		1.000	0.000	-0.178	-0.297
13			0.000	0.000	0.000
14				1.000	-0.347
15					1.000

TABLE 6.2 CONTINUED

MATRIX CONTINUED

VARIABLE NUMBER	16	17	18	19	20
1	0.736	0.122	0.016	0.048	0.050
2	0.808	0.148	-0.027	-0.006	0.006
3	0.727	0.210	0.023	0.084	0.084
4	0.402	0.354	0.297	0.311	0.358
5	0.151	0.349	0.491	0.427	0.515
6	0.444	-0.198	-0.285	-0.112	-0.235
7	0.311	-0.330	-0.423	-0.339	-0.435
8	0.275	-0.430	-0.296	-0.488	-0.452
9	0.221	-0.130	-0.273	-0.098	-0.211
10	0.000	0.000	0.000	0.000	0.000
11	0.926	-0.019	-0.112	-0.167	-0.141
12	0.204	-0.468	-0.272	-0.462	-0.433
13	0.000	0.000	0.000	0.000	0.000
14	-0.009	0.186	-0.087	0.243	0.096
15	-0.290	0.294	0.736	0.528	0.685
16	1.000	-0.061	-0.201	-0.229	-0.224
17		1.000	0.342	0.764	0.693
18			1.000	0.614	0.874
19				1.000	0.913
20					1.000

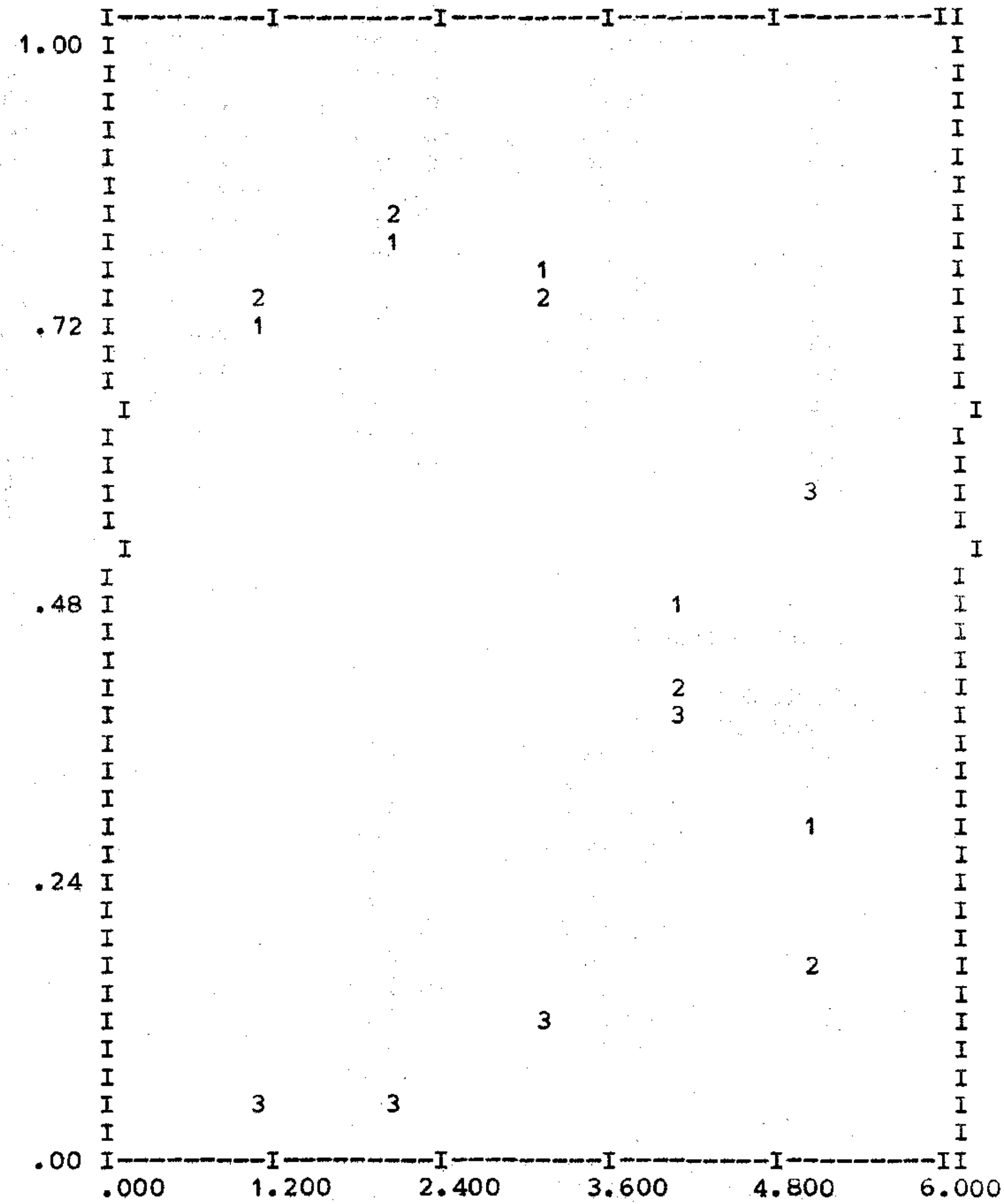
MATRIX CONTINUED

VARIABLE NUMBER	21	22
1	0.192	0.073
2	0.159	0.025
3	0.106	0.101
4	-0.028	0.365
5	-0.111	0.521
6	0.319	-0.193
7	0.187	-0.418
8	0.306	-0.447
9	0.059	-0.201
10	0.000	0.000
11	0.110	-0.139
12	0.221	-0.439
13	0.000	0.000
14	-0.102	0.084
15	-0.384	0.667
16	0.259	-0.211
17	-0.224	0.710
18	-0.353	0.853
19	-0.420	0.910
20	-0.420	0.990
21	1.000	-0.310
22		1.000

FIGURE 6.1

CORRELATIONS BETWEEN DEPENDENT AND INDEPENDENT VARIABLES

AGAINST THE ORDINAL NUMBER OF THE DEPENDENT VARIABLE



(1) S11(RE)
(2) S16(STEPS)
(3) S15(THERM)

TABLE 6.3A

STEP NUMBER 1 FOR C1

VARIABLE ENTERED	16
MULTIPLE R	0.7361
STD. ERROR OF EST.	4.3736

ANALYSIS OF VARIANCE:

	DF	SUM OF SQUARES	MEAN SQUARE	F-RATIO
REGRESSION	1	1176.510	1176.510	61.505
RESIDUAL	52	994.693	19.129	

VARIABLES IN EQUATION: (CONSTANT= 6.44663)

VARIABLE		COEFFICIENT	STD. ERROR	F TO REMOVE
STEPS	16	1.49556	0.19070	61.5049 (2)

VARIABLES NOT IN EQUATION:

VARIABLE		PARTIAL CORR.	TOLERANCE	F TO ENTER
CLAS2	2	0.85137	0.3466	134.3401 (1)
CLAS3	3	0.81412	0.4713	100.2426 (1)
CLAS4	4	0.71580	0.8385	53.5881 (1)
CLAS5	5	0.71352	0.9773	52.8930 (1)
WORDS	6	0.22308	0.8024	2.6708 (2)
SYMBL	7	0.24619	0.9033	3.2906 (2)
LOGCN	8	-0.34100	0.9244	6.7106 (2)
PAREN	9	0.24430	0.9509	3.2370 (2)
PREMS	10	0.00000	1.0000	0.0000 (2)
RE	11	0.17565	0.1432	1.6237 (2)
CP	12	-0.34264	0.9585	6.7838 (2)
AV RE	13	0.00000	1.0000	0.0000 (2)
AXIOM	14	0.18092	0.9999	1.7257 (2)
THERM	15	0.38307	0.9162	8.7709 (2)
R INF	17	0.24683	0.9963	3.3086 (2)
AV TH	18	0.24723	0.9596	3.3202 (2)
AV AX	19	0.32965	0.9474	6.2177 (2)
TOT R	20	0.32552	0.9497	6.0446 (2)
PSLI	21	0.00151	0.9329	0.0001 (2)
POSIT	22	0.34553	0.9553	6.9142 (2)

TABLE 6.3B

STEP NUMBER 2 FOR C1

VARIABLE ENTERED	15
MULTIPLE R	0.7804
STD. ERROR OF EST.	4.0794

ANALYSIS OF VARIANCE:

	DF	SUM OF SQUARES	MEAN SQUARE	F-RATIO
REGRESSION	2	1322.473	661.237	39.734
RESIDUAL	51	848.730	16.642	

VARIABLES IN EQUATION: (CONSTANT= 4.63224)

VARIABLE		COEFFICIENT	STD. ERROR	F TO REMOVE
THERM	15	2.51418	0.84894	8.7709 (2)
STEPS	16	1.65489	0.18583	79.3066 (2)

VARIABLES NOT IN EQUATION:

VARIABLE		PARTIAL CORR.	TOLERANCE	F TO ENTER
CLAS2	2	0.82327	0.2715	105.1697 (1)
CLAS3	3	0.77770	0.3691	76.5257 (1)
CLAS4	4	0.65486	0.5807	37.5408 (1)
CLAS5	5	0.65929	0.5813	38.4422 (1)
WORDS	6	0.33375	0.7686	6.2674 (2)
SYMBL	7	0.33730	0.8802	6.4188 (2)
LOGCN	8	-0.27201	0.8637	3.9952 (2)
PAREN	9	0.23464	0.9458	2.9133 (2)
PREMS	10	0.00000	1.0000	0.0000 (2)
RE	11	0.12015	0.1389	0.7324 (2)
CP	12	-0.27452	0.8966	4.0751 (2)
AV RE	13	0.00000	1.0000	0.0000 (2)
AXIOM	14	0.37329	0.8663	8.0952 (2)
R INF	17	0.15402	0.9132	1.2149 (2)
AV TH	18	-0.04632	0.4586	0.1075 (2)
AV AX	19	0.17447	0.7152	1.5697 (2)
TOT R	20	0.10280	0.5303	0.5340 (2)
PSLI	21	0.14881	0.8287	1.1323 (2)
POSIT	22	0.13856	0.5550	0.9788 (2)

TABLE 6.3C

STEP NUMBER 3 FOR C1

VARIABLE ENTERED 14
MULTIPLE R 0.8146
STD. ERROR OF EST. 3.8222

ANALYSIS OF VARIANCE:

	DF	SUM OF SQUARES	MEAN SQUARE	F-RATIO
REGRESSION	3	1440.739	480.246	32.873
RESIDUAL	50	730.465	14.609	

VARIABLES IN EQUATION: (CONSTANT= 2.68057)

VARIABLE		COEFFICIENT	STD. ERROR	F TO REMOVE
AXIOM	14	2.32139	0.81589	8.0952 (2)
THERM	15	3.40299	0.85455	15.8578 (2)
STEPS	16	1.71563	0.17542	95.6551 (2)

VARIABLES NOT IN EQUATION:

VARIABLE		PARTIAL CORR.	TOLERANCE	F TO ENTER
CLAS2	2	0.81228	0.2543	95.0327 (1)
CLAS3	3	0.75434	0.3384	64.6947 (1)
CLAS4	4	0.60134	0.5054	27.7551 (1)
CLAS5	5	0.60360	0.4990	28.0841 (1)
WORDS	6	0.32246	0.7615	5.6861 (2)
SYMBL	7	0.36018	0.8801	7.3045 (2)
LOGCN	8	-0.16531	0.7623	1.3766 (2)
PAREN	9	0.22901	0.9424	2.7120 (2)
PREMS	10	0.00000	1.0000	0.0000 (2)
RE	11	0.11934	0.1388	0.7079 (2)
CP	12	-0.18254	0.8142	1.6891 (2)
AV RE	13	0.00000	1.0000	0.0000 (2)
R INF	17	0.03668	0.8159	0.0660 (2)
AV TH	18	-0.16474	0.4251	1.3669 (2)
AV AX	19	-0.02959	0.5145	0.0429 (2)
TOT R	20	-0.09793	0.4041	0.4745 (2)
PSLI	21	0.27278	0.7741	3.9391 (2)
POSIT	22	-0.03659	0.4415	0.0657 (2)

TABLE 6.3D

SUMMARY TABLE FOR C1:

STEP NUM	VARIABLE ENT REM		MULTIPLE R	RSQ	INCREASE IN RSQ	F VALUE FOR DEL	LAST REG COEFFICNTS
1	STEPS	16	0.73610	0.54184	0.54184	61.5049	0.75218
2	THERM	15	0.78040	0.60902	0.06718	8.7709	4.44347
3	AXIOM	14	0.81460	0.66357	0.05455	8.0952	2.15535
4	SYMBL	7	0.84100	0.70728	0.04371	7.3045	0.41046
5	PSLI	21	0.85310	0.72778	0.02050	3.6330	0.56270
6	RE	11	0.86410	0.74667	0.01889	3.4873	2.20050
7	CP	12	0.87090	0.75847	0.01180	2.2580	-2.99584
8	PAREN	9	0.87700	0.76913	0.01066	2.0799	-1.37648
9	R INF	17	0.87890	0.77247	0.00334	0.6434	1.90201
10	AV TH	18	0.87960	0.77370	0.00123	0.2448	0.92563
11	AV AX	19	0.88010	0.77458	0.00088	0.1563	1.51330
12	POSIT	22	0.88140	0.77687	0.00229	0.4215	-0.22560
13	WORDS	6	0.88160	0.77722	0.00035	0.0474	0.03198
14	LOGCN	8	0.88160	0.77722	0.00000	0.0183	0.61787

FIGURE 6.2A - RESIDUALS FOR C1 ON RESTRICTED SET

```
RANGE      0        10        20        30        40        50
           I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I
-10.000-   I
   -9.001  I
 -9.000-   I
   -8.001  I
 -8.000-   I
   -7.001  I
 -7.000-   I**
   -6.001  I**
 -6.000-   I*
   -5.001  I*
 -5.000-   I**
   -4.001  I**
 -4.000-   I****
   -3.001  I****
 -3.000-   I******
   -2.001  I******
 -2.000-   I****
   -1.001  I****
 -1.000-   I*****
    -.001  I*****
    .000-  I************
     .999  I************
  1.000-   I****
    1.999  I****
  2.000-   I*****
    2.999  I*****
  3.000-   I*****
    3.999  I*****
  4.000-   I***
    4.999  I***
  5.000-   I**
    5.999  I**
  6.000-   I
    6.999  I
  7.000-   I
    7.999  I
  8.000-   I
    8.999  I
  9.000-   I
   10.000  I
           I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I
           0        10        20        30        40        50
```

FIGURE 6.2B RESIDUALS(Y-AXIS) VS COMPUTED C1 (X-AXIS)

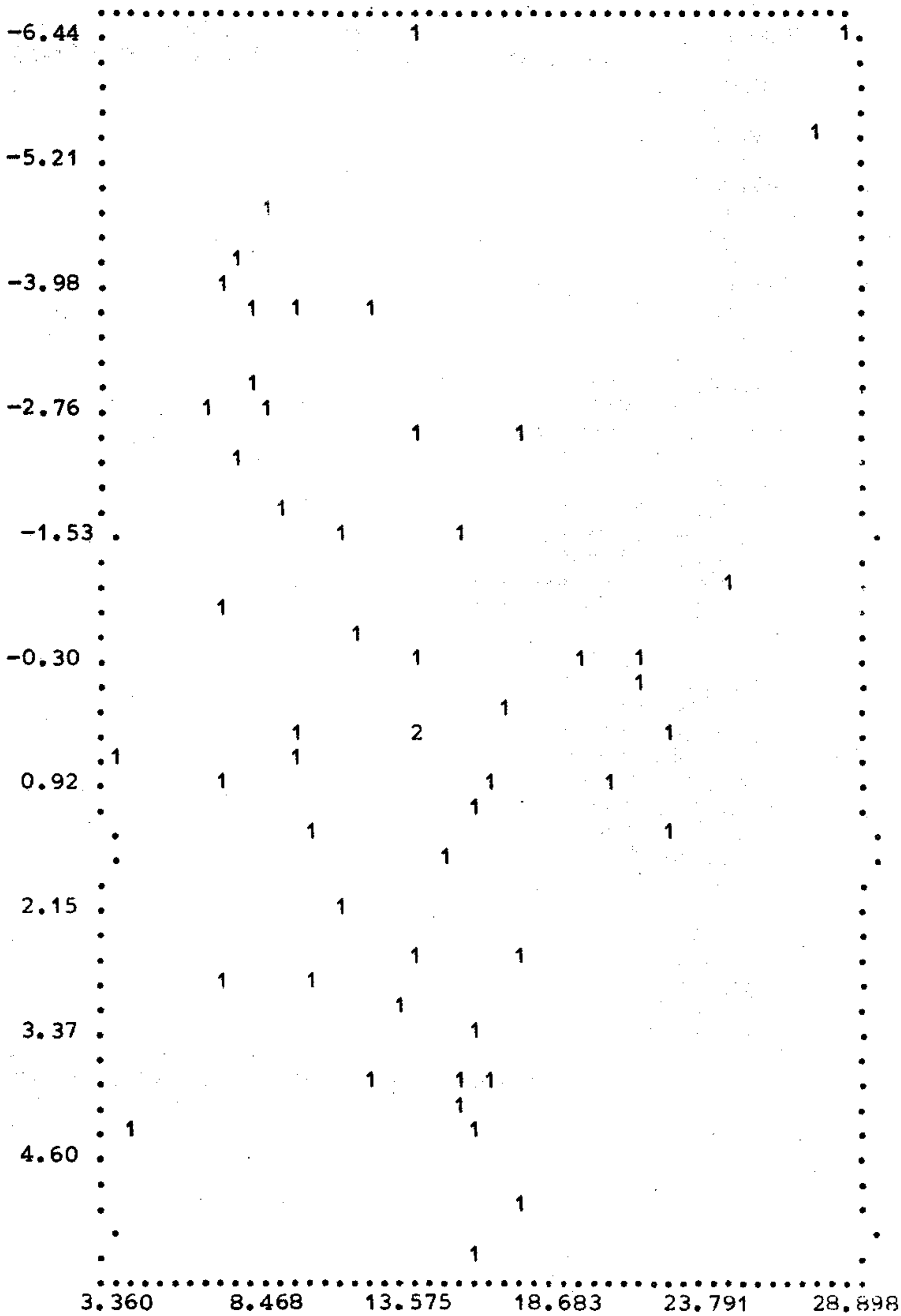


TABLE 6.4A

STEP NUMBER 1 FOR C2

VARIABLE ENTERED 16
MULTIPLE R 0.8083
STD. ERROR OF EST. 3.8743

ANALYSIS OF VARIANCE:

	DF	SUM OF SQUARES	MEAN SQUARE	F-RATIO
REGRESSION	1	1471.128	1471.128	98.010
RESIDUAL	52	780.520	15.010	

VARIABLES IN EQUATION: (CONSTANT= 1.51042)

VARIABLE		COEFFICIENT	STD. ERROR	F TO REMOVE
STEPS	16	1.67237	0.16893	98.0099 (2)

VARIABLES NOT IN EQUATION:

VARIABLE		PARTIAL CORR.	TOLERANCE	F TO ENTER
CLAS1	1	0.85137	0.4581	134.3401 (1)
CLAS3	3	0.95144	0.4713	487.1914 (1)
CLAS4	4	0.78282	0.8385	80.7189 (1)
CLAS5	5	0.74691	0.9773	64.3511 (1)
WORDS	6	0.08080	0.8024	0.3351 (2)
SYMBL	7	0.12606	0.9033	0.8236 (2)
LOGCN	8	-0.42367	0.9244	11.1571 (2)
PAREN	9	0.16652	0.9509	1.4545 (2)
PREMS	10	0.00000	1.0000	0.0000 (2)
RE	11	0.25148	0.1432	3.4432 (2)
CP	12	-0.45399	0.9585	13.2407 (2)
AV RE	13	0.00000	1.0000	0.0000 (2)
AXIOM	14	0.03718	0.9999	0.0706 (2)
THERM	15	0.46558	0.9162	14.1148 (2)
R INF	17	0.33577	0.9963	6.4806 (2)
AV TH	18	0.23564	0.9596	2.9984 (2)
AV AX	19	0.31351	0.9474	5.5590 (2)
TOT R	20	0.32554	0.9497	6.0453 (2)
PSLI	21	-0.08825	0.9329	0.4003 (2)
POSIT	22	0.33972	0.9553	6.6540 (2)

TABLE 6.4B

STEP NUMMBER 2 FOR C2

VARIABLE ENTERED	15
MULTIPLE R	0.8535
STD. ERROR OF EST.	3.4622

ANALYSIS OF VARIANCE:

	DF	SUM OF SQUARES	MEAN SQUARE	F-RATIO
REGRESSION	2	1640.320	820.160	68.422
RESIDUAL	51	611.328	11.987	

VARIABLES IN EQUATION: (CONSTANT= -0.44301)

VARIABLE		COEFFICIENT	STD. ERROR	F TO REMOVE
THERM	15	2.70686	0.72049	14.1148 (2)
STEPS	16	1.84391	0.15771	136.6924 (2)

VARIABLES NOT IN EQUATION:

VARIABLE		PARTIAL CORR.	TOLERANCE	F TO ENTER
CLAS1	1	0.82327	0.3909	105.1697 (1)
CLAS3	3	0.93799	0.3691	366.0775 (1)
CLAS4	4	0.71237	0.5807	51.5183 (1)
CLAS5	5	0.66009	0.5813	38.6080 (1)
WORDS	6	0.20364	0.7686	2.1631 (2)
SYMBL	7	0.22967	0.8802	2.7842 (2)
LOGCN	8	-0.35585	0.8637	7.2493 (2)
PAREN	9	0.14991	0.9458	1.1495 (2)
PREMS	10	0.00000	1.0000	0.0000 (2)
RE	11	0.19601	0.1389	1.9977 (2)
CP	12	-0.39212	0.8966	9.0847 (2)
AV RE	13	0.00000	1.0000	0.0000 (2)
AXIOM	14	0.25175	0.8663	3.3832 (2)
R INF	17	0.23761	0.9132	2.9919 (2)
AV TH	18	-0.16474	0.4586	1.3949 (2)
AV AX	19	0.10800	0.7152	0.5900 (2)
TOT R	20	0.02441	0.5303	0.0298 (2)
PSLI	21	0.08077	0.8287	0.3283 (2)
POSIT	22	0.05684	0.5550	0.1621 (2)

TABLE 6.4C

STEP NUMBER 3 FOR C2

VARIABLE ENTERED	12
MULTIPLE R	0.8776
STD. ERROR OF EST.	3.2166

ANALYSIS OF VARIANCE:

	DF	SUM OF SQUARES	MEAN SQUARE	F-RATIO
REGRESSION	3	1734.316	578.105	55.874
RESIDUAL	50	517.332	10.347	

VARIABLES IN EQUATION: (CONSTANT= 0.17969)

VARIABLE		COEFFICIENT	STD. ERROR	F TO REMOVE
CP	12	-3.58702	1.19009	9.0847 (2)
THERM	15	2.17648	0.69213	9.8886 (2)
STEPS	16	1.90122	0.14775	165.5704 (2)

VARIABLES NOT IN EQUATION:

VARIABLE		PARTIAL CORR.	TOLERANCE	F TO ENTER
CLAS1	1	0.80901	0.3614	92.8201 (1)
CLAS3	3	0.92749	0.3171	301.5748 (1)
CLAS4	4	0.73022	0.5730	55.9761 (1)
CLAS5	5	0.66932	0.5721	39.7655 (1)
WORDS	6	0.17763	0.7602	1.5965 (2)
SYMBL	7	0.30331	0.8674	4.9644 (2)
LOGCN	8	0.03721	0.1041	0.0679 (2)
PAREN	9	0.09363	0.9198	0.4334 (2)
PREMS	10	0.00000	1.0000	0.0000 (2)
RE	11	0.12320	0.1323	0.7552 (2)
AV RE	13	0.00000	1.0000	0.0000 (2)
AXIOM	14	0.15161	0.7867	1.1528 (2)
R INF	17	0.08547	0.7487	0.3606 (2)
AV TH	18	-0.21655	0.4552	2.4108 (2)
AV AX	19	-0.04307	0.6177	0.0910 (2)
TOT R	20	-0.12015	0.4731	0.7177 (2)
PSLI	21	0.13171	0.8201	0.8650 (2)
POSIT	22	-0.08777	0.4914	0.3804 (2)

TABLE 6.4D

SUMMARY TABLE FOR C2:

STEP NUM	VARIABLE ENT	VARIABLE REM	MULTIPLE R	MULTIPLE RSQ	INCREASE IN RSQ	F VALUE FOR DEL	LAST REG COEFFICNTS
1	STEPS 16		0.80830	0.65335	0.65335	98.0099	1.07026
2	THERM 15		0.85350	0.72846	0.07511	14.1148	4.74007
3	CP 12		0.87760	0.77018	0.04172	9.0847	-4.37355
4	SYMBL 7		0.88960	0.79139	0.02121	4.9644	0.33321
5	PAREN 9		0.89700	0.80461	0.01322	3.2381	-1.65153
6	AV TH 18		0.90060	0.81108	0.00647	1.6413	-1.65153
7	AXIOM 14		0.90500	0.81903	0.00794	2.0218	1.31265
8	RE 11		0.90930	0.82683	0.00780	1.9887	2.06043
9	PSLI 21		0.91510	0.83741	0.01058	2.8831	0.30025
10	R INF 17		0.91810	0.84291	0.00550	1.5033	2.27328
11	POSIT 22		0.91890	0.84438	0.00147	0.4185	-0.11116
12	LOGCN 8		0.91900	0.84456	0.00018	0.0423	0.94639
13	AV AX 19		0.91910	0.84474	0.00018	0.0444	0.35906
14	AV TH	18	0.91910	0.84474	0.00000	0.0001	
15	WORDS 6		0.91920	0.84493	0.00018	0.0146	0.01311

FIGURE 6.3A RESIDUALS FOR C2 ON RESTRICTED SET

```
RANGE      0        10        20        30        40        50
           I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I
-10.000-   I
   -9.001  I
  -9.000-  I
   -8.001  I
  -8.000-  I
   -7.001  I
  -7.000-  I
   -6.001  I
  -6.000-  I**
   -5.001  I**
  -5.000-  I**
   -4.001  I**
  -4.000-  I*
   -3.001  I*
  -3.000-  I*******
   -2.001  I*******
  -2.000-  I*********
   -1.001  I*********
  -1.000-  I******
    -.001  I******
    .000-  I**********
     .999  I**********
   1.000-  I*****
    1.999  I*****
   2.000-  I****
    2.999  I****
   3.000-  I****
    3.999  I****
   4.000-  I***
    4.999  I***
   5.000-  I
    5.999  I
   6.000-  I*
    6.999  I*
   7.000-  I
    7.999  I
   8.000-  I
    8.999  I
   9.000-  I
   10.000  I
           I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I
           0        10        20        30        40        50
```

FIGURE 6.3B - RESIDUALS(Y-AXIS) VS COMPUTED C2 (X-AXIS)

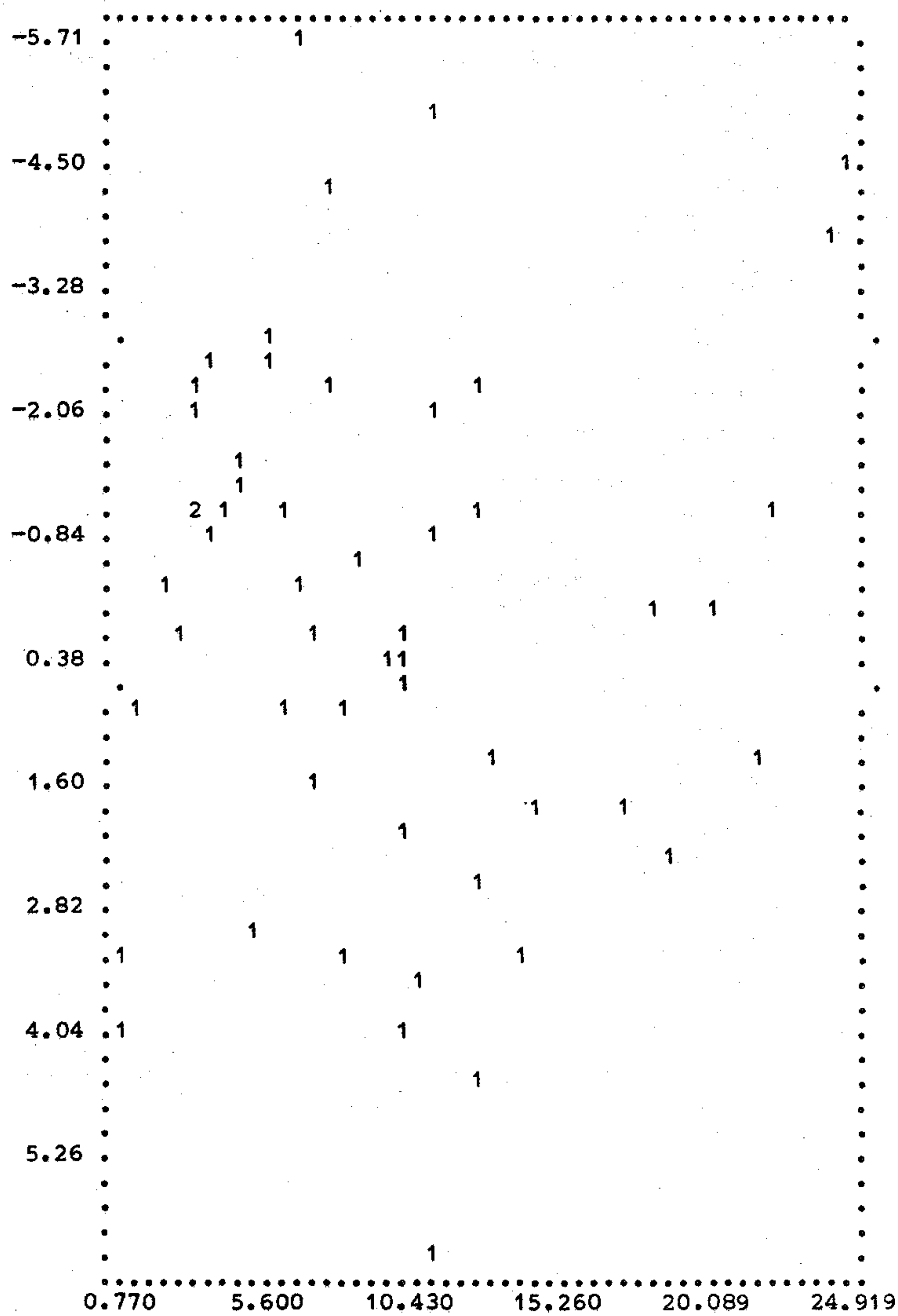


TABLE 6.5A

STEP NUMBER 1 FOR C3

VARIABLE ENTERED 11
MULTIPLE R 0.7576
STD. ERROR OF EST. 3.7164

ANALYSIS OF VARIANCE:

	DF	SUM OF SQUARES	MEAN SQUARE	F-RATIO
REGRESSION	1	967.498	967.498	70.049
RESIDUAL	52	718.206	13.812	

VARIABLES IN EQUATION: (CONSTANT= 3.69312)

VARIABLE		COEFFICIENT	STD. ERROR	F TO REMOVE
RE	11	3.62812	0.43349	70.0494 (2)

VARIABLES NOT IN EQUATION:

VARIABLE		PARTIAL CORR.	TOLERANCE	F TO ENTER
CLAS1	1	0.80970	0.4724	97.0869 (1)
CLAS2	2	0.93578	0.3532	359.2754 (1)
CLAS4	4	0.81431	0.7822	100.3802 (1)
CLAS5	5	0.71918	0.9296	54.6369 (1)
WORDS	6	0.20568	0.8548	2.2528 (2)
SYMBL	7	0.24438	0.9356	3.2392 (2)
LOGCN	8	-0.30083	0.9844	5.0748 (2)
PAREN	9	0.18015	0.9503	1.7106 (2)
PREMS	10	0.00000	1.0000	0.0000 (2)
CP	12	-0.33782	0.9909	6.5699 (2)
AV RE	13	0.00000	1.0000	0.0000 (2)
AXIOM	14	0.08788	0.9995	0.3969 (2)
THERM	15	0.39302	0.9579	9.3169 (2)
STEPS	16	0.10468	0.1432	0.5651 (2)
R INF	17	0.34481	0.9996	6.8820 (2)
AV TH	18	0.16658	0.9875	1.4557 (2)
AV AX	19	0.32748	0.9720	6.1262 (2)
TOT R	20	0.29514	0.9801	4.8664 (2)
PSLI	21	0.03534	0.9879	0.0638 (2)
POSIT	22	0.31943	0.9806	5.7951 (2)

TABLE 6.5B

STEP NUMBER 2 FOR C3

VARIABLE ENTERED 15
MULTIPLE R 0.7998
STD. ERROR OF EST. 3.4507

ANALYSIS OF VARIANCE:

	DF	SUM OF SQUARES	MEAN SQUARE	F-RATIO
REGRESSION	2	1078.436	539.218	45.285
RESIDUAL	51	607.268	11.907	

VARIABLES IN EQUATION: (CONSTANT= 2.47952)

VARIABLE		COEFFICIENT	STD. ERROR	F TO REMOVE
RE	11	3.88574	0.41125	89.2764 (2)
THERM	15	2.14364	0.70229	9.3169 (2)

VARIABLES NOT IN EQUATION:

VARIABLE		PARTIAL CORR.	TOLERANCE	F TO ENTER
CLAS1	1	0.79388	0.4370	85.2225 (1)
CLAS2	2	0.92838	0.3142	312.0335 (1)
CLAS4	4	0.77776	0.5559	76.5551 (1)
CLAS5	5	0.66357	0.5371	39.3372 (1)
WORDS	6	0.34154	0.8013	6.6028 (2)
SYMBL	7	0.35542	0.9004	7.2296 (2)
LOGCN	8	-0.20917	0.8968	2.2877 (2)
PAREN	9	0.17366	0.9477	1.5548 (2)
PREMS	10	0.00000	1.0000	0.0000 (2)
CP	12	-0.25620	0.9104	3.5124 (2)
AV RE	13	0.00000	1.0000	0.0000 (2)
AXIOM	14	0.26730	0.8700	3.8474 (2)
STEPS	16	0.23743	0.1329	2.9872 (2)
R INF	17	0.26014	0.9121	3.6293 (2)
AV TH	18	-0.19412	0.4571	1.9578 (2)
AV AX	19	0.16016	0.7180	1.3163 (2)
TOT R	20	0.04303	0.5311	0.0927 (2)
PSLI	21	0.21244	0.8515	2.3631 (2)
POSIT	22	0.08760	0.5554	0.3866 (2)

TABLE 6.5C

STEP NUMBER 3 FOR C3

VARIABLE ENTERED	7
MULTIPLE R	0.8278
STD. ERROR OF EST.	3.2575

ANALYSIS OF VARIANCE:

	DF	SUM OF SQUARES	MEAN SQUARE	F-RATIO
REGRESSION	3	1155.150	385.050	36.288
RESIDUAL	50	530.554	10.611	

VARIABLES IN EQUATION: (CONSTANT= -0.00639)

VARIABLE		COEFFICIENT	STD. ERROR	F TO REMOVE
SYMBL	7	0.21028	0.07821	7.2296 (2)
RE	11	3.65488	0.39760	84.4980 (2)
THERM	15	2.49612	0.67580	13.6423 (2)

VARIABLES NOT IN EQUATION:

VARIABLE		PARTIAL CORR.	TOLERANCE	F TO ENTER
CLAS1	1	0.76201	0.3740	67.8519 (1)
CLAS2	2	0.92249	0.2875	279.8288 (1)
CLAS4	4	0.79091	0.5473	81.8544 (1)
CLAS5	5	0.68238	0.5338	42.6985 (1)
WORDS	6	0.18041	0.5394	1.6486 (2)
LOGCN	8	-0.26918	0.8850	3.8278 (2)
PAREN	9	-0.15661	0.4039	1.2319 (2)
PREMS	10	0.00000	1.0000	0.0000 (2)
CP	12	-0.33260	0.8914	6.0946 (2)
AV RE	13	0.00000	1.0000	0.0000 (2)
AXIOM	14	0.28652	0.8700	4.3825 (2)
STEPS	16	0.19408	0.1293	1.9180 (2)
R INF	17	0.40983	0.8315	9.8914 (2)
AV TH	18	-0.05984	0.3832	0.1761 (2)
AV AX	19	0.27572	0.6727	4.0315 (2)
TOT R	20	0.21371	0.4481	2.3450 (2)
PSLI	21	0.18915	0.8425	1.8183 (2)
POSIT	22	0.25172	0.4798	3.3149 (2)

TABLE 6.5D

SUMMARY TABLE FOR C3:

STEP NUM	VARIABLE ENT REM		MULTIPLE R	RSQ	INCREASE IN RSQ	F VALUE FOR DEL	LAST REG COEFFICNTS
1	RE	11	0.75760	0.57396	0.57396	70.0494	2.63458
2	THERM	15	0.79980	0.63968	0.06572	9.3169	4.62393
3	SYMBL	7	0.82780	0.68525	0.04557	7.2296	0.47907
4	R INF	17	0.85910	0.73805	0.05280	9.8914	3.04043
5	PSLI	21	0.86900	0.75516	0.01711	3.3242	0.43077
6	AXIOM	14	0.87780	0.77053	0.01537	3.1591	1.08338
7	PAREN	9	0.88590	0.78482	0.01429	3.0652	-2.34706
8	CP	12	0.89790	0.80622	0.02141	4.9519	-0.68024
9	AV TH	18	0.90590	0.82065	0.01443	3.5415	0.58327
10	STEPS	16	0.90970	0.82755	0.00690	1.7355	0.51950
11	LOGCN	8	0.91180	0.83138	0.00383	0.9394	-2.93745
12	POSIT	22	0.91200	0.83174	0.00036	0.0761	-0.23139
13	AV AX	19	0.91360	0.83466	0.00292	0.7454	1.38585

FIGURE 6.4A - RESIDUALS FOR C3 ON RESTRICTED SET

```
RANGE      0         10        20        30        40        50
           I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I
-10.000-   I
   -9.001  I
 -9.000-   I
   -8.001  I
 -8.000-   I
   -7.001  I
 -7.000-   I
   -6.001  I
 -6.000-   I
   -5.001  I
 -5.000-   I***
   -4.001  I***
 -4.000-   I**
   -3.001  I**
 -3.000-   I***
   -2.001  I***
 -2.000-   I***********
   -1.001  I***********
 -1.000-   I**********
    -.001  I**********
    .000-  I*********
     .999  I*********
  1.000-   I******
    1.999  I******
  2.000-   I***
    2.999  I***
  3.000-   I*****
    3.999  I*****
  4.000-   I*
    4.999  I*
  5.000-   I*
    5.999  I*
  6.000-   I
    6.999  I
  7.000-   I
    7.999  I
  8.000-   I
    8.999  I
  9.000-   I
   10.000  I
           I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I
           0         10        20        30        40        50
```

FIGURE - RESIDUALS(Y-AXIS) VS COMPUTED C3 (X-AXIS)

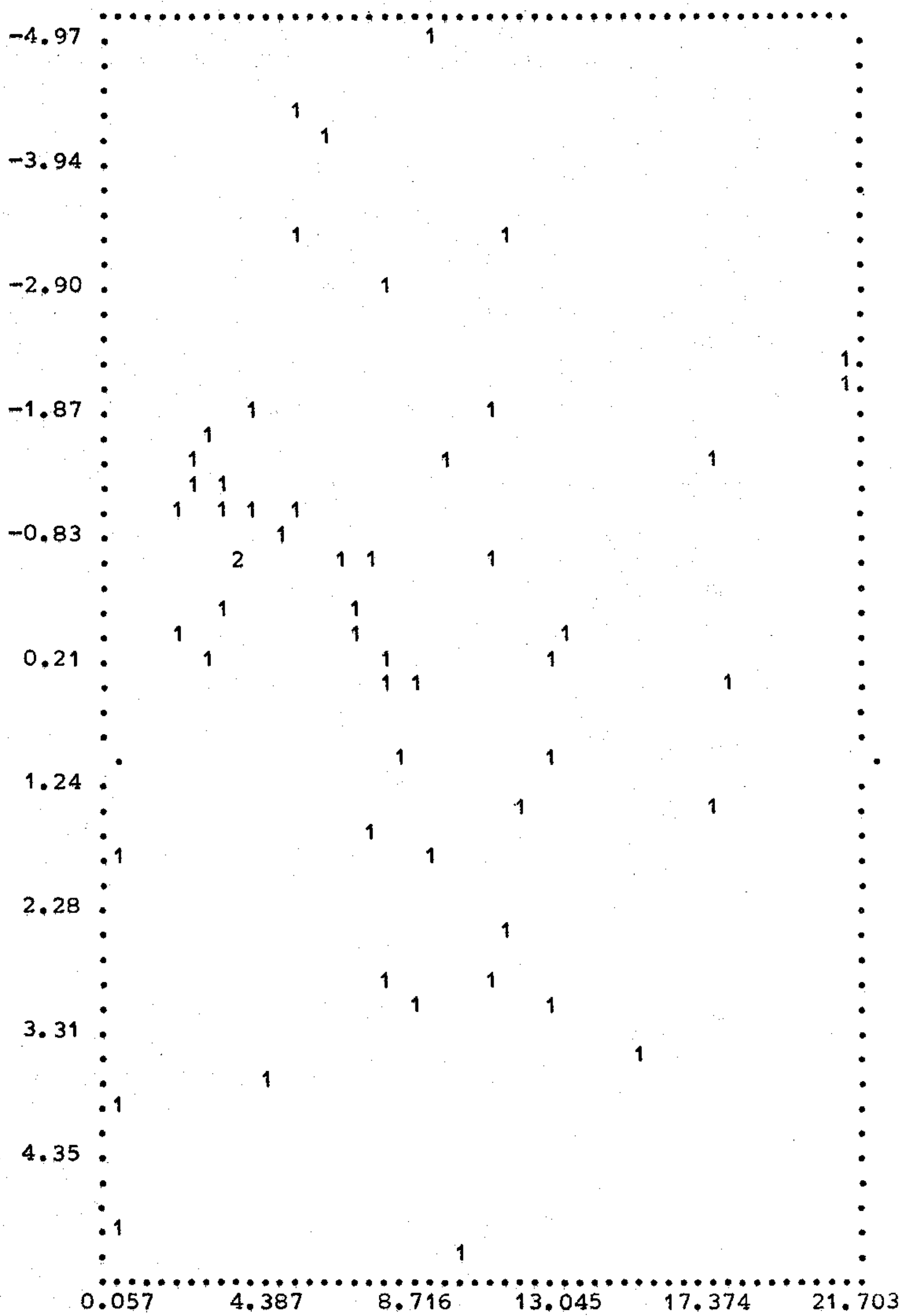


TABLE 6.6A

STEP NUMBER 1 FOR C4

VARIABLE ENTERED 11
MULTIPLE R 0.4667
STD. ERROR OF EST. 3.2693

ANALYSIS OF VARIANCE:

	DF	SUM OF SQUARES	MEAN SQUARE	F-RATIO
REGRESSION	1	154.800	154.800	14.483
RESIDUAL	52	555.793	10.688	

VARIABLES IN EQUATION: (CONSTANT= 3.67725)

VARIABLE		COEFFICIENT	STD. ERROR	F TO REMOVE
RE	11	1.45125	0.38134	14.4831 (2)

VARIABLES NOT IN EQUATION:

VARIABLE		PARTIAL CORR.	TOLERANCE	F TO ENTER
CLAS1	1	0.65876	0.4724	39.1000 (1)
CLAS2	2	0.70676	0.3532	50.9010 (1)
CLAS3	3	0.81431	0.4261	100.3802 (1)
CLAS5	5	0.92566	0.9296	305.2680 (1)
WORDS	6	0.07525	0.8548	0.2904 (2)
SYMBL	7	-0.00137	0.9356	0.0000 (2)
LOGCN	8	-0.22635	0.9844	2.7540 (2)
PAREN	9	-0.00737	0.9503	0.0028 (2)
PREMS	10	0.00000	1.0000	0.0000 (2)
CP	12	-0.19540	0.9909	2.0246 (2)
AV RE	13	0.00000	1.0000	0.0000 (2)
AXIOM	14	0.08716	0.9995	0.3904 (2)
THERM	15	0.53784	0.9579	20.7570 (2)
STEPS	16	-0.09022	0.1432	0.4185 (2)
R INF	17	0.41054	0.9996	10.3381 (2)
AV TH	18	0.39748	0.9875	9.5693 (2)
AV AX	19	0.44595	0.9720	12.6601 (2)
TOT R	20	0.48427	0.9801	15.6243 (2)
PSLI	21	-0.09039	0.9879	0.4201 (2)
POSIT	22	0.49136	0.9806	16.2321 (2)

TABLE 6.6B

STEP NUMBER 2 FOR C4

VARIABLE ENTERED	15
MULTIPLE R	0.6664
STD. ERROR OF EST.	2.7831

ANALYSIS OF VARIANCE:

	DF	SUM OF SQUARES	MEAN SQUARE	F-RATIO
REGRESSION	2	315.573	157.786	20.371
RESIDUAL	51	395.020	7.745	

VARIABLES IN EQUATION: (CONSTANT= 2.21628)

VARIABLE		COEFFICIENT	STD. ERROR	F TO REMOVE
RE	11	1.76139	0.33168	28.2006 (2)
THERM	15	2.58059	0.56642	20.7570 (2)

VARIABLES NOT IN EQUATION:

VARIABLE		PARTIAL CORR.	TOLERANCE	F TO ENTER
CLAS1	1	0.63086	0.4370	33.0537 (1)
CLAS2	2	0.66398	0.3142	39.4253 (1)
CLAS3	3	0.77776	0.3602	76.5551 (1)
CLAS5	5	0.89915	0.5371	211.0452 (1)
WORDS	6	0.25715	0.8013	3.5403 (2)
SYMBL	7	0.12449	0.9004	0.7871 (2)
LOGCN	8	-0.08188	0.8968	0.3375 (2)
PAREN	9	-0.04238	0.9477	0.0899 (2)
PREMS	10	0.00000	1.0000	0.0000 (2)
CP	12	-0.05213	0.9104	0.1363 (2)
AV RE	13	0.00000	1.0000	0.0000 (2)
AXIOM	14	0.35691	0.8700	7.2988 (2)
STEPS	16	0.06689	0.1329	0.2247 (2)
R INF	17	0.31214	0.9121	5.3973 (2)
AV TH	18	0.00579	0.4571	0.0017 (2)
AV AX	19	0.23603	0.7180	2.9499 (2)
TOT R	20	0.19374	0.5311	1.9499 (2)
PSLI	21	0.13982	0.8515	0.9970 (2)
POSIT	22	0.21623	0.5554	2.4523 (2)

TABLE 6.6C

STEP NUMBER 3 FOR C4

VARIABLE ENTERED	14
MULTIPLE R	0.7176
STD. ERROR OF EST.	2.6256

ANALYSIS OF VARIANCE:

	DF	SUM OF SQUARES	MEAN SQUARE	F-RATIO
REGRESSION	3	365.891	121.964	17.691
RESIDUAL	50	344.702	6.894	

VARIABLES IN EQUATION: (CONSTANT= 1.03759)

VARIABLE		COEFFICIENT	STD. ERROR	F TO REMOVE
RE	11	1.84887	0.31459	34.5392 (2)
AXIOM	14	1.51097	0.55928	7.2988 (2)
THERM	15	3.13748	0.57276	30.0069 (2)

VARIABLES NOT IN EQUATION:

VARIABLE		PARTIAL CORR.	TOLERANCE	F TO ENTER
CLAS1	1	0.58273	0.3907	25.1952 (1)
CLAS2	2	0.64743	0.3016	35.3613 (1)
CLAS3	3	0.75805	0.3345	66.1977 (1)
CLAS5	5	0.88348	0.4550	174.2755 (1)
WORDS	6	0.24514	0.7960	3.1329 (2)
SYMBL	7	0.13382	0.9004	0.8934 (2)
LOGCN	8	0.05078	0.7846	0.1267 (2)
PAREN	9	-0.06662	0.9448	0.2184 (2)
PREMS	10	0.00000	1.0000	0.0000 (2)
CP	12	0.06675	0.8218	0.2193 (2)
AV RE	13	0.00000	1.0000	0.0000 (2)
STEPS	16	0.09859	0.1322	0.4810 (2)
R INF	17	0.22129	0.3144	2.5230 (2)
AV TH	18	-0.10233	0.4229	0.5185 (2)
AV AX	19	0.05844	0.5150	0.1679 (2)
TOT R	20	0.02276	0.4031	0.0254 (2)
PSLI	21	0.26233	0.7897	3.6213 (2)
POSIT	22	0.06511	0.4409	0.2086 (2)

TABLE 6.6D

SUMMARY TABLE FOR C4:

STEP NUM		VARIABLE ENT	REM	MULTIPLE R	RSQ	INCREASE IN RSQ	F VALUE FOR DEL	LAST REG COEFFICNTS
1	RE	11		0.46670	0.21781	0.21781	14.4831	1.97847
2	THERM	15		0.66640	0.44409	0.22628	20.7570	4.71216
3	AXIOM	14		0.71760	0.51495	0.07086	7.2988	1.60187
4	PSLI	21		0.74050	0.54834	0.03339	3.6213	0.40789
5	R INF	17		0.75800	0.57456	0.02622	2.9664	3.13453
6	WORDS	6		0.77420	0.59939	0.02482	2.9168	0.11472
7	AV AX	19		0.78530	0.61670	0.01731	2.0714	0.11472
8	CP	12		0.79590	0.63346	0.01676	2.0570	3.62206
9	PAREN	9		0.80290	0.64465	0.01119	1.3767	-1.47970
10	SYMBL	7		0.80860	0.65383	0.00919	1.1491	0.18363
11	LOGCN	8		0.81700	0.66749	0.01366	1.7186	-3.38808
12	POSIT	22		0.82280	0.67700	0.00951	1.2225	-0.29187
13	AV AX		19	0.82280	0.67700	0.00000	0.0005	
14	TOT R	20		0.82720	0.68426	0.00726	0.9382	1.28037
15	STEPS	16		0.82800	0.68558	0.00132	0.1589	-0.14571

FIGURE 6.5A - RESIDUALS FOR C4 ON RESTRICTED SET

```
RANGE      0         10        20        30        40        50
           I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I
-10.000-   I
   -9.001  I
  -9.000-  I
   -8.001  I
  -8.000-  I
   -7.001  I
  -7.000-  I
   -6.001  I
  -6.000-  I
   -5.001  I
  -5.000-  I**
   -4.001  I**
  -4.000-  I*
   -3.001  I*
  -3.000-  I****
   -2.001  I****
  -2.000-  I**********
   -1.001  I**********
  -1.000-  I**************
    -.001  I**************
    .000-  I********
     .999  I********
   1.000-  I******
    1.999  I******
   2.000-  I***
    2.999  I***
   3.000-  I****
    3.999  I****
   4.000-  I**
    4.999  I**
   5.000-  I
    5.999  I
   6.000-  I
    6.999  I
   7.000-  I
    7.999  I
   8.000-  I
    8.999  I
   9.000-  I
   10.000  I
           I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I
           0         10        20        30        40        50
```

FIGURE 6.5B - RESIDUALS(Y-AXIS) VS COMPUTED C4 (X-AXIS)

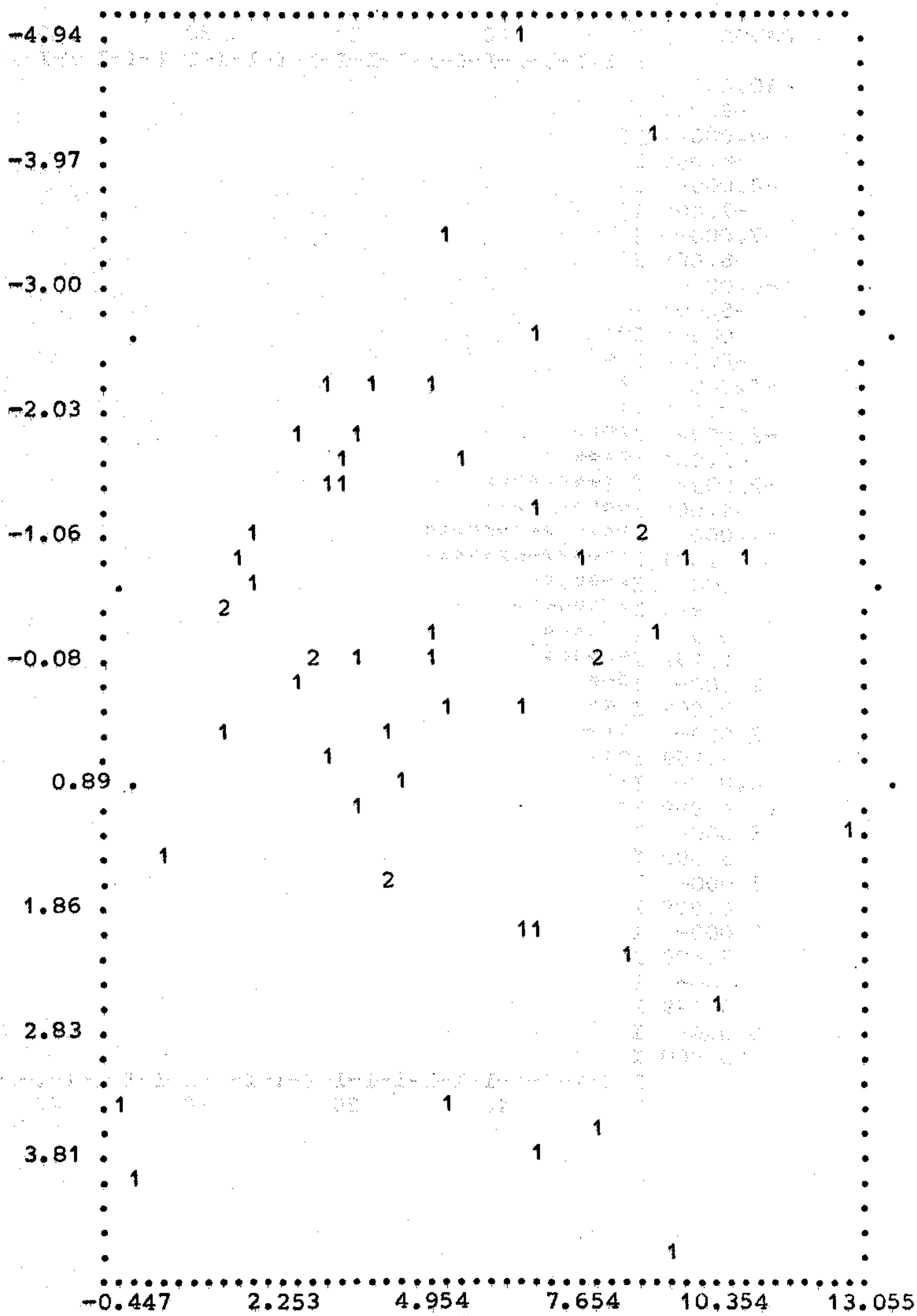


TABLE 6.7A

STEP NUMBER 1 FOR C5

VARIABLE ENTERED	15
MULTIPLE R	0.5588
STD. ERROR OF EST.	2.4865

ANALYSIS OF VARIANCE:

	DF	SUM OF SQUARES	MEAN SQUARE	F-RATIO
REGRESSION	1	145.939	145.939	23.605
RESIDUAL	52	321.487	6.182	

VARIABLES IN EQUATION: (CONSTANT= 3.51212)

VARIABLE		COEFFICIENT	STD. ERROR	F TO REMOVE
THERM	15	2.40632	0.49528	23.6054 (2)

VARIABLES NOT IN EQUATION:

VARIABLE		PARTIAL CORR.	TOLERANCE	F TO ENTER
CLAS1	1	0.68615	0.9988	45.3724 (1)
CLAS2	2	0.65212	0.9992	37.7357 (1)
CLAS3	3	0.72700	0.9909	57.1707 (1)
CLAS4	4	0.91691	0.8633	269.2067 (1)
WORDS	6	0.37663	0.9072	8.4304 (2)
SYMBL	7	0.16829	0.9444	1.4864 (2)
LOGCN	8	-0.09201	0.9006	0.4354 (2)
PAREN	9	0.06370	1.0000	0.2078 (2)
PREMS	10	0.00000	1.0000	0.0000 (2)
RE	11	0.46809	0.9579	14.3100 (2)
CP	12	-0.06387	0.9117	0.2089 (2)
AV RE	13	0.00000	1.0000	0.0000 (2)
AXIOM	14	0.29555	0.8793	4.8812 (2)
STEPS	16	0.39348	0.9162	9.3425 (2)
R INF	17	0.23316	0.9138	2.9320 (2)
AV TH	18	0.14183	0.4587	1.0469 (2)
AV AX	19	0.18814	0.7217	1.8715 (2)
TOT R	20	0.21967	0.5311	2.5858 (2)
PSLI	21	0.13525	0.8525	0.9503 (2)
POSIT	22	0.24041	0.5554	3.1286 (2)

TABLE 6.7B

STEP NUMBER 2 FOR C5

VARIABLE ENTERED 11
MULTIPLE R 0.6804
STD. ERROR OF EST. 2.2187

ANALYSIS OF VARIANCE:

	DF	SUM OF SQUARES	MEAN SQUARE	F-RATIO
REGRESSION	2	216.380	108.190	21.979
RESIDUAL	51	251.046	4.922	

VARIABLES IN EQUATION: (CONSTANT= 2.19584)

VARIABLE		COEFFICIENT	STD. ERROR	F TO REMOVE
RE	11	1.00026	0.26442	14.3100 (2)
THERM	15	2.75689	0.45155	37.2760 (2)

VARIABLES NOT IN EQUATION:

VARIABLE		PARTIAL CORR.	TOLERANCE	F TO ENTER
CLAS1	1	0.57328	0.4370	24.4764 (1)
CLAS2	2	0.53388	0.3142	19.9325 (1)
CLAS3	3	0.66357	0.3602	39.3372 (1)
CLAS4	4	0.89915	0.5559	211.0452 (1)
WORDS	6	0.26095	0.8013	3.6534 (2)
SYMBL	7	0.07789	0.9004	0.3052 (2)
LOGCN	8	-0.13868	0.8968	0.9804 (2)
PAREN	9	-0.05040	0.9477	0.1273 (2)
PREMS	10	0.00000	1.0000	0.0000 (2)
CP	12	-0.09181	0.9104	0.4250 (2)
AV RE	13	0.00000	1.0000	0.0000 (2)
AXIOM	14	0.39105	0.8700	9.0265 (2)
STEPS	16	-0.11690	0.1329	0.6927 (2)
R INF	17	0.24086	0.9121	3.0792 (2)
AV TH	18	0.12946	0.4571	0.8523 (2)
AV AX	19	0.25120	0.7180	3.3676 (2)
TOT R	20	0.24909	0.5311	3.3074 (2)
PSLI	21	0.13477	0.8515	0.9249 (2)
POSIT	22	0.27371	0.5554	4.0491 (2)

TABLE 6.7C

STEP NUMBER 3 FOR C5

VARIABLE ENTERED 14
MULTIPLE R 0.7383
STD. ERROR OF EST. 2.0623

ANALYSIS OF VARIANCE:

	DF	SUM OF SQUARES	MEAN SQUARE	F-RATIO
REGRESSION	3	254.770	84.923	19.967
RESIDUAL	50	212.656	4.253	

VARIABLES IN EQUATION: (CONSTANT= 1.16628)

VARIABLE		COEFFICIENT	STD. ERROR	F TO REMOVE
RE	11	1.07668	0.24710	18.9861 (2)
AXIOM	14	1.31979	0.43928	9.0265 (2)
THERM	15	3.24332	0.44987	51.9762 (2)

VARIABLES NOT IN EQUATION:

VARIABLE		PARTIAL CORR.	TOLERANCE	F TO ENTER
CLAS1	1	0.51253	0.3907	17.4574 (1)
CLAS2	2	0.50526	0.3016	16.7969 (1)
CLAS3	3	0.63034	0.3345	32.3053 (1)
CLAS4	4	0.88348	0.4851	174.2755 (1)
WORDS	6	0.24994	0.7960	3.2649 (2)
SYMBL	7	0.08524	0.9004	0.3586 (2)
LOGCN	8	-0.00040	0.7846	0.0000 (2)
PAREN	9	-0.07841	0.9448	0.3031 (2)
PREMS	10	0.00000	1.0000	0.0000 (2)
CP	12	0.03457	0.8218	0.0586 (2)
AV RE	13	0.00000	1.0000	0.0000 (2)
STEPS	16	-0.09751	0.1322	0.4703 (2)
R INF	17	0.12979	0.8144	0.8396 (2)
AV TH	18	0.02529	0.4229	0.0314 (2)
AV AX	19	0.05548	0.5150	0.1513 (2)
TOT R	20	0.07122	0.4031	0.2498 (2)
PSLI	21	0.27094	0.7897	3.8821 (2)
POSIT	22	0.11727	0.4409	0.6833 (2)

TABLE 6.7D

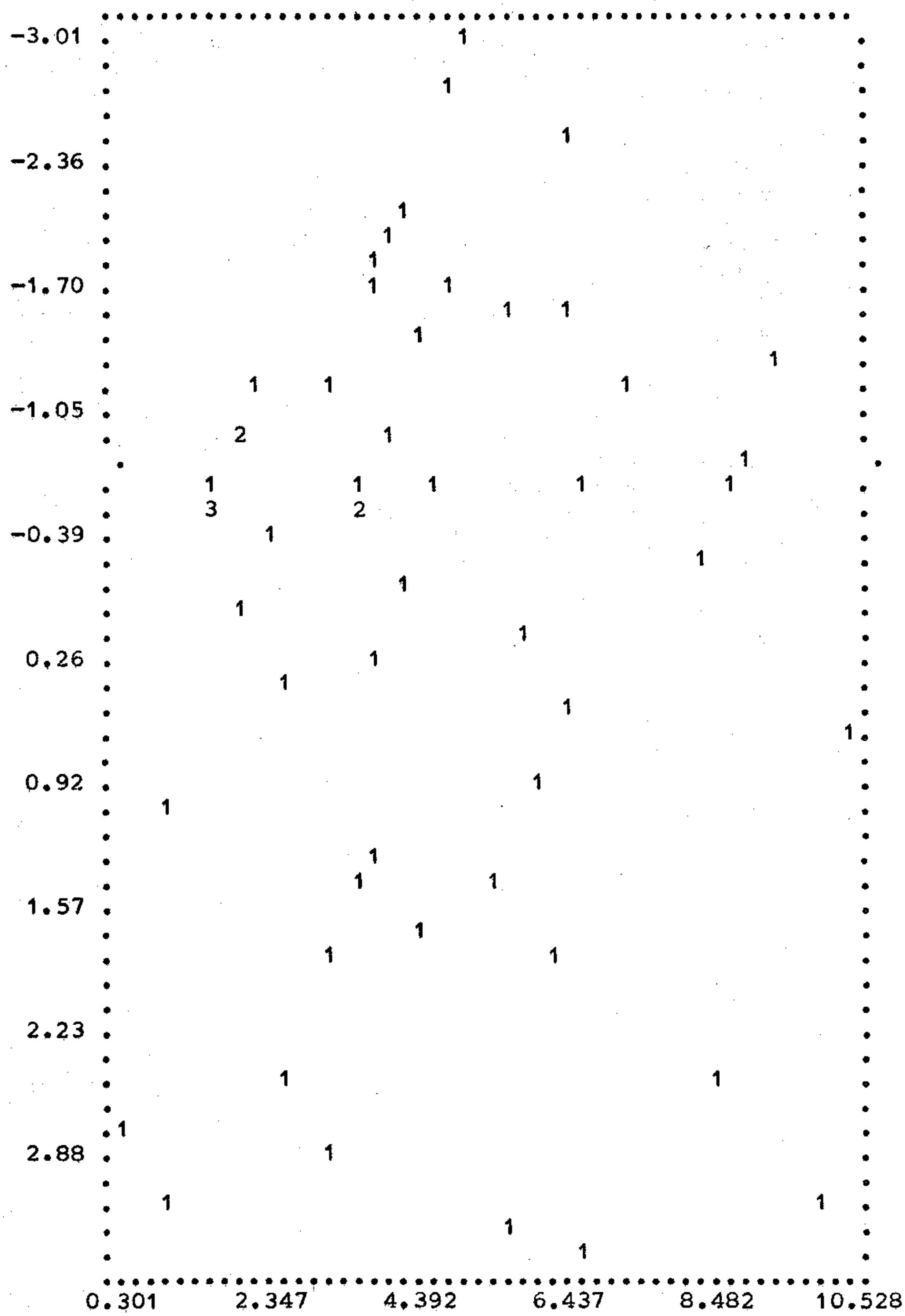
SUMMARY TABLE FOR C5:

STEP NUM	VARIABLE ENT	REM	MULTIPLE R	RSQ	INCREASE IN RSQ	F VALUE FOR DEL	LAST REG COEFFICNTS
1	THERM 15		0.55880	0.31226	0.31226	23.6054	3.93706
2	RE 11		0.68040	0.46294	0.15069	14.3100	2.14152
3	AXIOM 14		0.73830	0.54509	0.08214	9.0265	1.37732
4	PSLI 21		0.76060	0.57851	0.03343	3.8821	0.33648
5	STEPS 16		0.77310	0.59768	0.01917	2.2905	-0.52878
6	WORDS 6		0.78660	0.61874	0.02106	2.5907	0.10568
7	PAREN 9		0.79740	0.63585	0.01711	2.1760	-0.87813
8	R INF 17		0.80470	0.64754	0.01170	1.4924	1.69602
9	SYMBL 7		0.81400	0.66260	0.01505	1.9577	0.10182
10	CP 12		0.81630	0.66635	0.00375	0.4876	1.98407
11	LOGCN 8		0.81890	0.67060	0.00425	0.5385	-1.60871
12	POSIT 22		0.82090	0.67388	0.00328	0.4165	-0.18563
13	TOT R 20		0.82540	0.68129	0.00741	0.9287	0.86059
14	AV TH 18		0.82560	0.68162	0.00033	0.0415	0.11391

FIGURE 6.6A - RESIDUALS FOR C5 ON RESTRICTED SET

```
RANGE      0        10        20        30        40        50
           I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I
-10.000-   I
   -9.001  I
 -9.000-   I
   -8.001  I
 -8.000-   I
   -7.001  I
 -7.000-   I
   -6.001  I
 -6.000-   I
   -5.001  I
 -5.000-   I
   -4.001  I
 -4.000-   I*
   -3.001  I*
 -3.000-   I***
   -2.001  I***
 -2.000-   I***********
   -1.001  I***********
 -1.000-   I*****************
    -.001  I*****************
    .000-  I*******
     .999  I*******
  1.000-   I*******
    1.999  I*******
  2.000-   I****
    2.999  I****
  3.000-   I****
    3.999  I****
  4.000-   I
    4.999  I
  5.000-   I
    5.999  I
  6.000-   I
    6.999  I
  7.000-   I
    7.999  I
  8.000-   I
    8.999  I
  9.000-   I
   10.000  I
           I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I-I
            0        10        20        30        40        50
```

FIGURE 6.6B - RESIDUALS(Y-AXIS) VS COMPUTED C5 (X-AXIS)



CHAPTER SEVEN

A subset of the set of proofs for problem 414035 was presented in Chapter III, to illustrate the classification procedure. In this chapter, the full set of proofs for three problems will be presented, to provide additional insight into the nature of the differences in the sample as a whole. The first problem discussed is drawn from the early part of the curriculum, before the intoduction of RE, and exibits very little variation for all of the five partitions. The second problem comes after the intoduction of RE, but before the introduction of the first theorem. It shows considerable variation under the first two partitions but very little under the last three. The last problem occurs when four theorems are available and shows considerable variation under all five partitions.

As in Chapter III, paradigm proofs identify the different classes under each of the first three partitions. All of the proofs in a class are equivalent to the paradigm proof, up to the differences allowed under the partition being discussed. For the last two partitions, individual classes are identified by the distribution which defines the class. All of the classes under a partition are referred to by letters of the alphabet; the numbers that appear after these letters are the number of student proofs included in the class.

7.1 - PROBLEM 407010

Problem 407010 is fairly typical of those occurring before the introduction of RE. The statement of this problem is:

```
407010:
DERIVE 6+A = (5+1)+A
```

There are three classes under the first partition (Table 7.1) and only one class under each subsequent partition.

The first partition is defined by the identity relation; even under this strict definition of equivalence, there are only three classes of proofs in the sample of twenty three. For the second partition, there is just one class. All twenty three proofs are equivalent up to defferences in unused steps. There is little variation in the set of proofs for this problem, and the variations which do occur are relatively superficial.

7.2 PROBLEM 411051

The statement of this problem is:

411051:
DERIVE 10 = A -> A-7 = 10-(6+1)

Problem 411051 occurs when RE is available. The first partition for this problem contains eleven classes of proofs. The paradigm proofs for these eleven classes are listed in Table 7.2A.

All of the proofs in Table 7.2A use the same six rules: WP, SE, ND, CE, RE, and CP; these rules are used in a consistent way from proof to proof. In each case, WP is used to generate the formula, A = 10, and ND is used to generate 7 = 6+1. CE and SE are used to modify A = 10, and RE is used to combine the formula derived from A = 10 and the formula, 7 = 6+1. Finally, CP is used to generate the required conditional, 10 = A -> A-7 = 10-(6+1). The proofs differ in the order in which these rules are used and in the presence, in some proofs, of unused steps. For example, the only difference between proofs A and D is in the postion of the step employing ND. Proofs B and D are the same, except for the two unused steps, (5) and (6), in proof B.

Under the second partition, there are seven classes of proofs. The paradigm proofs are found in Table 7.2B. The criteria which define the second partition ignore unused steps, therefore no unused steps appear in the paradigm proofs for this partition. All of these proofs contain six steps, using the same six rules. The order in which these rules are used, however, changes from one proof to another.

The three paradigm proofs for the third partition are contained in Table 7.2C. Again, the only differences between these proofs are in the order of rule use. Under the second partition any difference in order results in a separate classification; the third partition is sensitive to some differences in order but not to all. In this problem, 411051, the third partition ignores the position of the step using ND, but does not ignore changes in the order in which CE, SE, and RE are introduced. These three rules are used, in some order, to successively modify 10 = A; various sequences of these three steps constitute the core of the proofs. The ND-step is introduced only to be used with RE and can occur anywhere before RE. Most of the variation observed for the first two partitions is likewise due to the differences in the position of the ND-step.

All of the proofs in the sample for this problem are in the same class under the third and fourth partitions, since they all use the same six rules and use each of them only once.

7.3 PROBLEM 415044

The final example to be discussed is problem 415044. The statement for this problem is:

415044: HERE IS THEORM 5
DERIVE: 0 = 0

Under the first partition there are sixteen classes in the sample of twenty three proofs; a list of the paradigm proofs for each of these classes is presented in Table 7.3A.

The proof labeled D, in Table 7.3A, is the standard proof for this problem; it uses two theorems, TH3 and TH4. Six students constructed the standard proof; this is the largest number of proofs in any of the sixteen classes. C is the class with the second largest number of student proofs.

The differences between C and D are worth discussing in detail. The first step in C is identical to the first step in D. The second step in proof D uses TH3 to generate the formula, 0-0 = 0. Proof C uses three steps to generate the same formula; these three steps are a special case of the proof of TH3. Both C and D then use RE to complete the proof.

The students who constructed proof C recognized that they needed the formula, 0-0 = 0, but did not realize that this formula could be generated in one step by using TH3. So they proved this instance of TH3, using the axioms AI and N and the rule, RE, which form the standard proof of TH3. A slightly different version of this proof for the necessary instance of TH3 is found in proof J, while proof E uses TH3 and includes a derivation of the needed instance of TH4. Since every theorem in the curriculum may be proved using the axioms and rules of inference, it is never necessary to use a theorem; any instance of a theorem can be proved using the axioms and rules of inference.

Proofs F, O, and P use no theorems (The single occurrence of TH4 in proof P is in an unused step.). Proof K, on the other hand, uses TH1 and TH2 with CA and RE.

In addition to this basic variation in the rules used, there are differences in the order in which the rules are used and in the presence of unused steps. Proof H, for example, is the same as proof D except for its unused second step.

The paradigm proofs for the second partition are listed in Table 7.3B. Unused lines are ignored under the second partition, so the number of classes decreases from sixteen to fourteen. The paradigm proofs for the third partition form Table 7.3C. Here some variation in the order of steps is allowed, and the number of classes is reduced to twelve.

Moving from the third partition to the fourth, two of these twelve classes are combined, leaving a total of eleven classes (Table 7.3D). None of these merge under the fifth partition, which also contains eleven classes. For this last problem, then, the decrease in the number of classes from one partition to the next is very gradual. The reason for this was indicated in the discussion of the first are combined, leaving a total of eleven classes (Table 7.3D). None of these merge under the fifth partition, which also contains eleven classes (Table 7.3E). For this last problem, then, the decrease in the number of classes from one partition to the next is very gradual. The reason for this was indicated in the discussion of the first partition. The proofs differ principally in the set of rules employed. Since all five partitions are sensitive to such differences, this component of variation does not disappear for the later partitions.

Two additional types of variation appear under the first partition, the presence of unused steps in some proofs and the variations in the order of steps. These types of variation become irrelevant for the later partitions and disappear. In the previous examples discussed here, differences in the order of steps accounted for most of the differences observed, so the number of classes decreased rapidly from the first partition to the fifth.

TABLE 7.1

FIRST PARTITION FOR PROBLEM 407010

.	.	.ND6	(1) 6 = 5+1	A (1)*
.	.	.AE[A]	(2) 6+A = (5+1)+A	
.	.	.ND6	(1) 6 = 5+1	B (20)
1.	.	.AE[A]	(2) 6+A =(5+1)+A	
.	.	.ND5	(1) 5 = 4+1	C (1)
.	.	.DLL		
.	.	.ND6	(1) 6 = 5+1	
1.	.	.AE[A]	(2) 6+A =(5+1)+A	

* The numbers in parentheses to the right of each proof are the number of proofs in the class.

TABLE 7.2A

FIRST PARTITION FOR PROBLEM 411051

```
 .  .  .WP[10=A]    (1)   10 = A                A (4)

 .  .  .ND7         (2)   7 = 6+1
1.  .  .CE1         (3)   A = 10
3.  .  .SE[7]       (4)   A-7 = 10-7
4. 2.  .RE2         (5)   A-7 = 10-(6+1)
1. 5.  .CP          (6) 10 = A -> A-7 = 10-(6+1)

 .  .  .WP[10=A]    (1)   10 = A                B (1)
1.  .  .CE1         (2)   A = 10
2.  .  .SE[7]       (3)   A-7 = 10-7
 .  .  .ND7         (4)   7 = 6+1
4.  .  .CE1         (5)   6+1 = 7
3. 4.  .RE1         (6)   A-(6+1) = 7
3. 4.  .RE2         (7)   A-7 = 10-(6+1)
1. 7.  .CP          (8) 10 = A -> A-7 =10-(6+1)

 .  .  .WP[10=A]    (1)   10 = A                C (1)
 .  .  .ND7         (2)   7 = 6+1
1.  .  .SE[7]       (3)   10-7 = A-7
3. 2.  .RE2         (4)   10-7 = A-(6+1)
 .  .  .DLL
3.  .  .CE1         (4)   A-7 = 10-7
4. 2.  .RE2         (5)   A-7 = 10-(6+1)
1. 5.  .CP          (6) 10 = A -> A-7 =10-(6+1)

 .  .  .WP[10=A]    (1)   10 = A                D (9)
1.  .  .CE1         (2)   A = 10
2.  .  .SE[7]       (3)   A-7 = 10-7
 .  .  .ND7         (4)   7 = 6+1
3. 4.  .RE2         (5)   A-7 = 10-(6+1)
1. 5.  .CP          (6) 10 = A -> A-7 =10-(6+1)

 .  .  .WP[10=A]    (1)   10 = A                E (1)
 .  .  .CE1         (2)   A = 10
 .  .  .ND7         (3)   7 = 6+1
2.  .  .SE[7]       (4)   A-7 = 10-7
4. 3.  .RE2         (5)   A-7 = 10-(6+1)
1. 5.  .CP          (6) 10 = A -> A-7 =10-(6+1)
```

```
 .   .   .WP[10=A]      (1)   10 = A                   F (1)
1.   .   .CE1           (2)   A = 10
 .   .   .ND7           (3)   7 = 6+1
3.   .   .CE1           (4)   6+1 = 7
2.   .   .SE[7]         (5)   A-7 = 10-7
5.  3.   .RE2           (6)   A-7 = 10-(6+1)
1.  6.   .CP            (7) 10 = A -> A-7 =10-(6+1)

 .   .   .WP[10=A]      (1)   10 = A                   G (2)
1.   .   .SE[7]         (2)   10-7 = A-7
 .   .   .ND7           (3)   7 = 6+1
2.  3.   .RE1           (4)   10-(6+1) =A-7
4.   .   .CE1           (5)   A-7 = 10-(6+1)
1.  5.   .CP            (6) 10 = A -> A-7 =10-(6+1)

 .   .   .WP[10=A]      (1)   10 = A                   H (1)
 .   .   .CE1           (2)   A = 10
2.   .   .SE[7]         (3)   A-7 = 10-7
 .   .   .ND7           (4)   7 = 6+1
3.  4.   .RE2           (5)   A-7 = 10-(6+1)
1.  5.   .CP            (6) 10 = A -> A-7 =10-(6+1)

 .   .   .WP[10=A]      (1)   10 = A                   I (1)
 .   .   .ND7           (2)   7 = 6+1
1.   .   .SE[7]         (3)   10-7 = A-7
3.  2.   .RE1           (4)   10-(6+1) = A-7
4.   .   .CE1           (5)   A-7 = 10-(6+1)
1.  5.   .CP            (6) 10 = A -> A-7 =10-(6+1)

 .   .   .WP[10=A]      (1)   10 = A                   J (1)
1.   .   .SE[7]         (2)   10-7 = A-7
 .   .   .ND7           (3)   7 = 6+1
2.  3.   .RE2           (4)   10-7 = A-(6+1)
1.  4.   .CP            (5) 10 = A -> A-7 = A-(6+1)
     .   .   .DLL
 .   .   .DLL
2.   .   .CE1           (4)   A-7 = 10-7
4.  3.   .RE2           (5)   A-7 = 10-(6+1)
1.  5.   .CP            (6) 10 = A -> A-7 =10-(6+1)

 .   .   .WP[10=A]      (1)   10 = A                   K (1)
1.   .   .CE1           (2)   A = 10
 .   .   .ND7           (3)   7 = 6+1
 .   .   .DLL
 .   .   .SE[7]         (3)   A-7 = 10-7
```

```
 .  .   .ND7          (4)    7 = 6+1
3. 4.   .RE2          (5)    A-7 = 10-(6+1)
1. 5.   .CP           (6) 10 = A -> A-7 =10-(6+1)
```

TABLE 7.2B

SECOND PARTITION FOR PROBLEM 411051

```
 .  .  .WP[10=A]     (1)   10 = A                     A (4)

 .  .  .ND7          (2)   7 = 6+1
1.  .  .CE1          (3)   A = 10
3.  .  .SE[7]        (4)   A-7 = 10-7
4. 2.  .RE2          (5)   A-7 = 10-(6+1)
1. 5.  .CP           (6) 10 = A -> A-7 = 10-(6+1)

 .  .  .WP[10=A]     (1)   10 = A              B,D,H,K (12)
1.  .  .CE1          (2)   A = 10
2.  .  .SE[7]        (3)   A-7 = 10-7
  .  .  .ND7           (4)   7 = 6+1
3. 4.  .RE2          (5)   A-7 = 10-(6+1)
1. 5.  .CP           (6) 10 = A -> A-7 =10-(6+1)

 .  .  .WP[10=A]     (1)   10 = A                     C (1)
 .  .  .ND7          (2)   7 = 6+1
1.  .  .SE[7]        (3)   10-7 = A-7
3.  .  .CE1          (4)   A-7 = 10-7
4. 2.  .RE2          (5)   A-7 = 10-(6+1)
1. 5.  .CP           (6) 10 = A -> A-7 =10-(6+1)

 .  .  .WP[10=A]     (1)   10 = A                   E,F (2)
 .  .  .CE1          (2)   A = 10
 .  .  .ND7          (3)   7 = 6+1
2.  .  .SE[7]        (4)   A-7 = 10-7
4. 3.  .RE2          (5)   A-7 = 10-(6+1)
1. 5.  .CP           (6) 10 = A -> A-7 =10-(6+1)

 .  .  .WP[10=A]     (1)   10 = A                     G (2)
1.  .  .SE[7]        (2)   10-7 = A-7
 .  .  .ND7          (3)   7 = 6+1
2. 3.  .RE1          (4)   10-(6+1) =A-7
4.  .  .CE1          (5)   A-7 = 10-(6+1)
1. 5.  .CP           (6) 10 = A -> A-7 =10-(6+1)

 .  .  .WP[10=A]     (1)   10 = A                     I (1)
 .  .  .ND7          (2)   7 = 6+1
```

```
1.  .  .SE[7]          (3)   10-7 = A-7
3. 2.  .RE1            (4)   10-(6+1) = A-7
4.  .  .CE1            (5)   A-7 = 10-(6+1)
1. 5.  .CP             (6) 10 = A -> A-7 =10-(6+1)
```

```
 .  .  .WP[10=A]       (1)   10 = A                    J (1)
1.  .  .SE[7]          (2)   10-7 = A-7
 .  .  .ND7            (3)   7 = 6+1
2.  .  .CE1            (4)   A-7 = 10-7
4. 3.  .RE2            (5)   A-7 = 10-(6+1)
1. 5.  .CP             (6) 10 = A -> A-7 =10-(6+1)
```

TABLE 7.2C

THIRD PARTITION FOR PROBLEM 411051

.	.	.WP[10=A]	(1)	10 = A	A,B,D,H,K,E,F (18)
.	.	.ND7	(2)	7 = 6+1	
1.	.	.CE1	(3)	A = 10	
3.	.	.SE[7]	(4)	A-7 = 10-7	
4.	2.	.RE2	(5)	A-7 = 10-(6+1)	
1.	5.	.CP	(6)	10 = A -> A-7 = 10-(6+1)	
.	.	.WP[10=A]	(1)	10 = A	C,J (2)
.	.	.ND7	(2)	7 = 6+1	
1.	.	.SE[7]	(3)	10-7 = A-7	
3.	.	.CE1	(4)	A-7 = 10-7	
4.	2.	.RE2	(5)	A-7 = 10-(6+1)	
1.	5.	.CP	(6)	10 = A -> A-7 =10-(6+1)	
.	.	.WP[10=A]	(1)	10 = A	G,I (3)
1.	.	.SE[7]	(2)	10-7 = A-7	
.	.	.ND7	(3)	7 = 6+1	
2.	3.	.RE1	(4)	10-(6+1) =A-7	
4.	.	.CE1	(5)	A-7 = 10-(6+1)	
1.	5.	.CP	(6)	10 = A -> A-7 =10-(6+1)	

TABLE 7.3A

FIRST PARTITION FOR PROBLEM 415044

.	.	.N[A,A]	(1)	A+(-A) = A-A	A (1)
.	.	.AI[A]	(2)	A+(-A) = 0	
1.	2.	.RE1	(3)	0 = A-A	
.	.	.TH1[0]	(4)	0+0 = 0	
.	.	.AI[0]	(5)	0+(-0) = 0	
5.	.	.CE1	(6)	0 = 0+(-0)	
4.	6.	.RE3	(7)	0+0 = 0+(-0)	
7.	.	.SE[0]	(8)	(0+0)-0 = (0+(-0))-0	
.	.	.TH3[0]	(9)	0-0 = 0	
8.	4.	.RE1	(10)	0-0 = (0+(-0))-0	
10.	9.	.RE1	(11)	0 = (0+(-0))-0	
11.	.	.CA1	(12)	0 = ((-0)+0)-0	
.	.	.TH4[0]	(13)	0-0 = -0	
9.	.	.CE1	(14)	0 = 0-0	
13.	9.	.RE1	(15)	0 = -0	

.	.	.TH3[0]	(1)	0-0 = 0	B (1)
.	.	.TH4[0]	(2)	0-0 = -0	
1.	2.	.RE1	(3)	-0 = 0	
3.	.	.CE1	(4)	0 = -0	

.	.	.TH4[0]	(1)	0-0 = -0	C (3)
.	.	.AI[0]	(2)	0+(-0) = 0	
.	.	.N[0,0]	(3)	0+(-0) = 0-0	
2.	3.	.RE1	(4)	0-0 = 0	
1.	4.	.RE1	(5)	0 = -0	

.	.	.TH4[0]	(1)	0-0 = -0	D (6)
.	.	.TH3[0]	(2)	0-0 = 0	
1.	2.	.RE1	(3)	0 = -0	

.	.	.Z[-0]	(1)	(-0)+0 = -0	E (1)
.	.	.CA1	(2)	0+(-0) = -0	
.	.	.N[0,0]	(3)	0+(-0) = 0-0	
2.	3.	.RE1	(4)	0-0 = -0	
.	.	.TH3[0]	(5)	0-0 = 0	
4.	5.	.RE1	(6)	0 = -0	

```
 .  .  .LT[0]      (1) 0 =0                 F (1)
 .  .  .AI[0]      (2) 0+(-0) = 0
2.  .  .CE1        (3) 0 = 0+(-0)
1. 3.  .RE2        (4) 0 = 0+(-0)
 .  .  .Z[-0]      (5) (-0)+0 = -0
5.  .  .CA1        (6) 0+(-0) = -0
4. 6.  .RE1        (7) 0 = -0

 .  .  .TH1[0]     (1) 0+0 = 0              G (1)
 .  .  .TH2[0]     (2) (-0)+0 = 0
 .  .  .TH3[0]     (3) 0-0 = 0
 .  .  .TH4[0]     (4) 0-0 = -0
3.  .  .CE1        (5) 0 = 0-0
1. 5.  .RE3        (6) 0+0 = 0-0
6. 4.  .RE1        (7) 0+0 = -0
7. 1.  .RE1        (8) 0 = -0

 .  .  .TH4[0]     (1) 0-0 = -0             H (1)
 .  .  .TH1[0]     (2) 0+0 = 0
 .  .  .TH3[0]     (3) 0-0 = 0
1. 3.  .RE1        (4) 0 = -0

 .  .  .TH3[0]     (1) 0-0 = 0              I (1)
 .  .  .DLL
 .  .  .TH4[0]     (1) 0-0 = -0
 .  .  .TH3[0]     (2) 0-0 = 0
1. 2.  .RE1        (3) 0 = -0

 .  .  .TH4[0]     (1) 0-0 = -0             J (1)
 .  .  .N[0,0]     (2) 0+(-0) = 0-0
 .  .  .AI[0]      (3) 0+(-0) = 0
3. 2.  .RE1        (4) 0-0 = 0
1. 4.  .RE1        (5) 0 = -0

 .  .  .TH1[-0]    (1) 0+(-0) = -0          K (1)
1.  .  .CA1        (2) (-0)+0 = -0
 .  .  .TH2[0]     (3) (-0)+0 = 0
    2. 3.  .RE1        (4) 0 = -0

 .  .  .N[0,0]     (1) 0+(-0) = 0-0         L (1)
 .  .  .Z[0]       (2) 0+0 = 0
 .  .  .Z[-0]      (3) (-0)+0 = 0
3.  .  .CA1        (4) 0+(-0) = 0
1. 4.  .RE1        (5) -0 = 0-0
```

```
 .   .   .TH3[0]        (6) 0-0 = 0
5.  6.   .RE1           (7) -0 = 0
7.   .   .CE1           (8) 0 = -0

 .   .   .LT[0]         (1) 0 = 0                        M (1)
 .   .   .TH4[0]        (2) 0-0 = -0
 .   .   .TH3[0]        (3) 0-0 = 0
1.   .   .SE[0]         (4) 0-0 = 0-0
4.  3.   .RE1           (5) 0 = 0-0
5.  2.   .RE1           (6) 0 = -0

 .   .   .LT[0]         (1) 0 = 0                        N (1)
 .   .   .AE[0]         (2) 0+0 = 0+0
 .   .   .DLL
 .   .   .AE[-0]        (2) 0+(-0) = 0+(-0)
     .   .   .TH4[0]        (3) 0-0 = -0
 .   .   .TH2[0]        (4) (-0)+0 = 0
4.   .   .CA1           (5) 0+(-0) = 0
2.  5.   .RE1           (6) 0 = 0+(-0)
 .   .   .Z[0]          (7) 0+0 = 0
 .   .   .DLL
 .   .   .Z[-0]         (7) (-0)+0 = -0
7.   .   .CA1           (8) 0+(-0) = -0
6.  8.   .RE1           (9) 0 = -0

 .   .   .LT[-0]        (1) -0 = -0                      O (1)
 .   .   .N[0,0]        (2) 0+(-0) = 0-0
 .   .   .Z[-0]         (3) (-0)+0 = -0
  3.   .   .CE1           (4) -0 = (-0)+0
1.  4.   .RE1           (5) (-0)+0 = -0
 .   .   .AI[0]         (6) 0+(-0) = 0
6.   .   .CA1           (7) (-0)+0 = 0
       5.  7.   .RE1           (8) 0 = -0

 .   .   .TH4[0]        (1) 0-0 = -0                     P (1)
 .   .   .Z[-0]         (2) (-0)+0 = -0
2.   .   .CA1           (3) 0+(-0) = -0
 .   .   .AI[0]         (4) 0+(-0) = 0
3.  4.   .RE1           (5) 0 = -0
```

TABLE 7.3B

SECOND PARTITION FOR PROBLEM 415044

.	.	.TH3[0]	(1)	0-0 = 0	A (1)
.	.	.TH4[0]	(2)	0-0 = -0	
2.	1.	.RE1	(3)	0 = -0	
.	.	.TH3[0]	(1)	0-0 = 0	B (1)
.	.	.TH4[0]	(2)	0-0 = -0	
1.	2.	.RE1	(3)	-0 = 0	
3.	.	.CE1	(4)	0 = -0	
.	.	.TH4[0]	(1)	0-0 = -0	C (3)
.	.	.AI[0]	(2)	0+(-0) = 0	
.	.	.N[0,0]	(3)	0+(-0) = 0-0	
2.	3.	.RE1	(4)	0-0 = 0	
1.	4.	.RE1	(5)	0 = -0	
.	.	.TH4[0]	(1)	0-0 = -0	D,H,I (8)
.	.	.TH3[0]	(2)	0-0 = 0	
1.	2.	.RE1	(3)	0 = -0	
.	.	.Z[-0]	(1)	(-0)+0 = -0	E (1)
.	.	.CA1	(2)	0+(-0) = -0	
.	.	.N[0,0]	(3)	0+(-0) = 0-0	
2.	3.	.RE1	(4)	0-0 = -0	
.	.	.TH3[0]	(5)	0-0 = 0	
4.	5.	.RE1	(6)	0 = -0	
.	.	.LT[0]	(1)	0 =0	F (1)
.	.	.AI[0]	(2)	0+(-0) = 0	
2.	.	.CE1	(3)	0 = 0+(-0)	
1.	3.	.RE2	(4)	0 = 0+(-0)	
.	.	.Z[-0]	(5)	(-0)+0 = -0	
5.	.	.CA1	(6)	0+(-0) = -0	
4.	6.	.RE1	(7)	0 = -0	
.	.	.TH1[0]	(1)	0+0 = 0	G (1)
.	.	.TH3[0]	(2)	0-0 = 0	
.	.	.TH4[0]	(3)	0-0 = -0	

```
2    .    .CE1          (4) 0 = 0-0
1.   4.   .RE3          (5) 0+0 = 0-0
5.   3.   .RE1          (6) 0+0 = -0
6.   1.   .RE1.*        (7) 0 = -0

 .   .    .TH4[0]       (1) 0-0 = -0                  J (1)
 .   .    .N[0,0]       (2) 0+(-0) = 0-0
 .   .    .AI[0]        (3) 0+(-0) = 0
3.   2.   .RE1          (4) 0-0 = 0
1.   4.   .RE1          (5) 0 = -0

 .   .    .TH1[-0]      (1) 0+(-0) = -0               K (1)
1.   .    .CA1          (2) (-0)+0 = -0
 .   .    .TH2[0]       (3) (-0)+0 = 0
2.   3.   .RE1          (4) 0 = -0

 .   .    .N[0,0]       (1) 0+(-0) = 0-0              L (1)
 .   .    .Z[-0]        (2) (-0)+0 = 0
2.   .    .CA1          (3) 0+(-0) = 0
1.   3.   .RE1          (4) -0 = 0-0
 .   .    .TH3[0]       (5) 0-0 = 0
4.   5.   .RE1          (6) -0 = 0
6.   .    .CE1          (7) 0 = -0

 .   .    .LT[0]        (1) 0 = 0                     M (1)
 .   .    .TH4[0]       (2) 0-0 = -0
 .   .    .TH3[0]       (3) 0-0 = 0
1.   .    .SE[0]        (4) 0-0 = 0-0
4.   3.   .RE1          (5) 0 = 0-0
5.   2.   .RE1          (6) 0 = -0

 .   .    .LT[0]        (1) 0 = 0                     N (1)
 .   .    .AE[-0]       (2) 0+(-0) = 0+(-0)
 .   .    .TH2[0]       (3) (-0)+0 = 0
3.   .    .CA1          (4) 0+(-0) = 0
2.   4.   .RE1          (5) 0 = 0+(-0)
 .   .    .Z[-0]        (6) (-0)+0 = -0
5.   .    .CA1          (7) 0+(-0) = -0
4.   6.   .RE1          (8) 0 = -0

 .   .    .LT[-0]       (1) -0 = -0                   O (1)
 .   .    .Z[-0]        (2) (-0)+0 = -0
2.   .    .CE1          (3) -0 = (-0)+0
1.   3.   .RE1          (4) (-0)+0 = -0
```

.	.	.AI[0]	(5) 0+(−0) = 0	
5.	.	.CA1	(6) (−0)+0 = 0	
4.	6.	.RE1	(7) 0 = −0	

.	.	.Z[−0]	(1) (−0)+0 = −0	P (1)
1.	.	.CA1	(2) 0+(−0) = −0	
.	.	.AI[0]	(3) 0+(−0) = 0	
2.	3.	.RE1	(4) 0 = −0	

TABLE 7.3C

THIRD PARTITION FOR PROBLEM 415044

.	.	.TH3[0]	(1) 0-0 = 0	A,D,H,I (9)
.	.	.TH4[0]	(2) 0-0 = -0	
2.	1.	.RE1	(3) 0 = -0	

.	.	.TH3[0]	(1) 0-0 = 0	B (1)
.	.	.TH4[0]	(2) 0-0 = -0	
1.	2.	.RE1	(3) -0 = 0	
3.	.	.CE1	(4) 0 = -0	

.	.	.TH4[0]	(1) 0-0 = -0	C,J (3)
.	.	.AI[0]	(2) 0+(-0) = 0	
.	.	.N[0,0]	(3) 0+(-0) = 0-0	
2.	3.	.RE1	(4) 0-0 = 0	
1.	4.	.RE1	(5) 0 = -0	

.	.	.Z[-0]	(1) (-0)+0 = -0	E (1)
.	.	.CA1	(2) 0+(-0) = -0	
.	.	.N[0,0]	(3) 0+(-0) = 0-0	
2.	3.	.RE1	(4) 0-0 = -0	
.	.	.TH3[0]	(5) 0-0 = 0	
4.	5.	.RE1	(6) 0 = -0	

.	.	.LT[0]	(1) 0 =0	F (1)
.	.	.AI[0]	(2) 0+(-0) = 0	
2.	.	.CE1	(3) 0 = 0+(-0)	
1.	3.	.RE2	(4) 0 = 0+(-0)	
.	.	.Z[-0]	(5) (-0)+0 = -0	
5.	.	.CA1	(6) 0+(-0) = -0	
4.	6.	.RE1	(7) 0 = -0	

.	.	.TH1[0]	(1) 0+0 = 0	G (1)
.	.	.TH3[0]	(2) 0-0 = 0	
.	.	.TH4[0]	(3) 0-0 = -0	
2	.	.CE1	(4) 0 = 0-0	
1.	4.	.RE3	(5) 0+0 = 0-0	
5.	3.	.RE1	(6) 0+0 = -0	
6.	1.	.RE1	(7) 0 = -0	

```
 .   .   .TH1[-0]     (1) 0+(-0) = -0          K (1)
1.   .   .CA1         (2) (-0)+0 = -0
 .   .   .TH2[0]      (3) (-0)+0 = 0
2.  3.   .RE1         (4) 0 = -0

 .   .   .N[0,0]      (1) 0+(-0) = 0-0         L (1)
 .   .   .Z[-0]       (2) (-0)+0 = 0
2.   .   .CA1         (3) 0+(-0) = 0
1.  3.   .RE1         (4) -0 = 0-0
 .   .   .TH3[0]      (5) 0-0 = 0
4.  5.   .RE1         (6) -0 = 0
6.   .   .CE1         (7) 0 = -0

 .   .   .LT[0]       (1) 0 = 0                M (1)
 .   .   .TH4[0]      (2) 0-0 = -0
 .   .   .TH3[0]      (3) 0-0 = 0
1.   .   .SE[0]       (4) 0-0 = 0-0
4.  3.   .RE1         (5) 0 = 0-0
5.  2.   .RE1         (6) 0 = -0

 .   .   .LT[0]       (1) 0 = 0                N (1)
 .   .   .AE[-0]      (2) 0+(-0) = 0+(-0)
 .   .   .TH2[0]      (3) (-0)+0 = 0
3.   .   .CA1         (4) 0+(-0) = 0
2.  4.   .RE1         (5) 0 = 0+(-0)
 .   .   .Z[-0]       (6) (-0)+0 = -0
5.   .   .CA1         (7) 0+(-0) = -0
    4. 6.   .RE1          (8) 0 = -0
```

TABLE 7.3D

FOURTH PARTITION FOR PROBLEM 415044

Z	N	LT	AE	SE	CE	RE	CA	AI	TH1	TH2	TH3	TH4	
0	0	0	0	0	0	1	0	0	0	0	1	1	A,D,H,I (9)
0	0	0	0	0	1	1	0	0	0	0	1	1	B (1)
0	1	0	0	0	0	2	0	1	0	0	0	1	C,J (4)
1	1	0	0	0	0	2	1	0	0	0	1	0	E (1)
1	0	1	0	0	1	2	1	1	0	0	0	0	F,O (2)
0	0	0	0	0	1	3	0	0	1	0	1	1	G (1)
0	0	0	0	0	0	1	1	0	1	1	0	0	K (1)
1	1	0	0	0	1	2	1	0	0	0	1	0	L (1)
0	0	1	0	1	0	2	0	0	0	0	1	1	M (1)
1	0	1	1	0	0	2	2	0	0	1	0	0	N (1)
1	0	0	0	0	0	1	1	1	0	0	0	0	P (1)

CHAPTER EIGHT

8.1 - INTRODUCTION

The study discussed in this dissertation was essentially exploratory. The initial purpose was to evaluate the LIS curriculum along one of its dimensions, variability in student proofs. In order to do this, a classification procedure was developed and used to measure variability in a set of student proofs.

The classification procedures described in Chapter III allow us to compare student proofs at five levels of detail. These techniques have proven adequate for this study, and should be useful in a wide range of related studies

The classification procedure was also used to investigate the relationship between the variability (number of classes of equivalent proofs) in a sample of proofs for a problem and the characteristics of the problem. The results for this part of the study provided increased understanding of both the sources of variation within the curriculum and the properties of the classification procedure

8.2 VARIABILITY OF PROOF BEHAVIOR IN THE CURRICULUM

The derivation problems in the algebra part of the Stanford Logic-Instructional System (LIS) curriculum have been used in this study. The measured variability within this set of problems is high for all five partitions, and increases from one lesson to the next.

Even for the fifth partition, which requires that two proofs use different sets of rules if they are to be put into distinct classes, there is a substantial amount of variation in the final lessons considered. Under the first partition , identity of the proofs (except for error steps) is required; using these criteria there are a large number of proof classes for almost all of the problems studied.

LIS will accept any valid proof for a problem. It checks the validity of each step rather than comparing the student's proof against a preset standard. In investigating the extent to which the curriculum makes use of the system's ability to recognize any valid proof, all variations in student proofs are relevant, including the existence of unused steps and differences in the order of steps. The first partition is sensitive to these variations and under it there were a large number of classes for most problems. The current LIS curriculum certainly encourages a large amount of variation at this level; it continues to encourage a reasonable amount of variation even as the criteria for equivalence are relaxed from the second to the fifth partitions.

8.3 REMARKS ON THE CLASSIFICATION PROCEDURE

The ambiguity in the notion of "different proofs" had to be resolved to conduct this study. The differences relevant to the evaluation of the curriculum are defined by the differences allowed by LIS, but there is no unique definition of "different" for a general investigation of variation in proof behavior.

To some extent, any instrument (the classification criteria), that is used to measure variability in proof behavior, will determine in advance the character and extent of variation found in a given set of data. A formalized classification has been employed in this study to insure consistency, but automation of the decision criteria, however, does not eliminate any bias resulting from selective sensitivity to certain differences between proofs and insensitivity to all other differences. In fact, the results of this study show that both the amount of variation found for the curriculum as a whole and the relationship between variation and problem characteristics are quite sensitive to the criteria chosen; there are marked changes in the results of the regression analyses from the first partition to the fifth.

The use of a nested sequence of partitions rather than a single partition limits the possibility that the variation observed was the result of an unpropitious choice of critera. The first partition requires that proofs be identical, except for errors. The only requirement for equivalence under the fifth partition is that proofs use the same set of rules. These five partitions use a wide range of criteria; it is very unlikely that the results are due to a peculiarity of the classification procedures.

However, it is possible that equivalence criteria defined along some other dimension would show a different pattern of results; for example, the latencies to various steps of the occurrence of certain types of errors might be used to study additional aspects of proof behavior. Since these dimensions of variability are not relevant to the present evaluation of the LIS curriculum, they are not considered here.

The criteria defined for this study depend on the form of the completed student proofs. Examination of the data indicates that the assignments of individual proofs to equivalence classes are reasonable. The pattern of change in the results from one partition to the next is clear, and it is unlikely that any small change in the definition of equivalence would significantly modify this pattern.

The definitions of equivalence developed here have turned out to be highly satisfactory for two reasons. First, examination of the partitions (over sets of student proofs) generated for a sample of problems indicates that the formal definitions of equivalence match intuitive notions of equivalence quite well. Second, the analysis that used the formal definitions confirmed the general expectations about the curriculum, but led to a much deeper and more detailed understanding of the nature of the variability found in student proofs, and,in addition several unexpected properties of the relationship between curriculum structure and variability in student proofs were discovered using these techniques. Although the equivalence criteria used in this study are defined explicitly for the Stanford Logic-Instructional System, the general technique would be applicable to most formal problem-solving tasks. The development of this new technique for analyzing student behavior is probably the most important contribution of this research.

8.4 VARIATION IN THE SAMPLE OF PROOFS

The results discussed in Chapters V and VI indicate that variability in proof behavior can be predicted quite well from the known characteristics of a derivation problem. The first four variables to enter the equations generally account for about seventy-five percent of the variance in the dependent variable. These results must be interpreted with caution, since the study described here is exploratory and non-experimental. There is no control group and neither the subjects nor the problems were selected at random from a specified population. Thus, statistical inference to a larger population is not appropriate. Strictly speaking, the results apply to the population of students included in the analysis.

However, the results may tentatively be extrapolated to other student populations and other curricula. The criteria for reasonable extrapolation should be the extent to which the tasks and the population in this study are representative of the target tasks and population. Decisions about the reasonableness of such extrapolations will depend on the characteristics of the particular target population and curriculum.

There seem to be two distinct types of variation in the sample of proofs. The first type of variation involoves differences in the order in which rules are used. The number of steps in the standard proof for a problem and the extent to which these steps are interdependent are good predictors of the extent of this kind of variation for a given problem.

The second type of variation involves the rules used to prove a formula. The magnitude of this type of variations for a paticular problem is best predicted by the number of theorems in its standard proof. The number of axioms in the standard proof and the number of rules available when the proof is reached in the curriculum are also good predictors for this second kind of variation.

The importance of both the number of theorems and axioms used in the standard proof and the number of rules available increases systematically from the first set of equivalence criteria, which is the most stringent, to the fifth set of criteria, which is least stringent; in this progression the partitions become more and more sensitive to the second type of variation, involving the rules used to prove a formula.

8.5 CONCLUDING REMARKS

The most generally useful aspect of this study is probably the development of the classification procedures. The use of a nested sequence of measures provides a much more complete description of the variability found in the data than any single measure could provide. The classification criteria described in this study are specific to LIS, but the general properties of the technique depend only on the existence of behavior (proofs in this case) that can be segmented into discrete components(steps) chosen from some finite set. Hence, similar procedures could be developed for tasks requiring such behavior.

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APPENDIX A

The material presented here describes the attempts that were made to identify patterns of proof behavior that characterize groups of students within the total sample. Two general types of analysis were used to answer two questions. First, do students exhibit definite patterns of behavior in the construction of proofs; and second, if they do, what are the defining characteristics of these patterns? The first analysis was based on the classification criteria, and required the development of a metric function over the set of students; the second analysis was based on new variables.

The identification of a clustering of students into sub-groups would be of general interest in the study of human problem solving, and would also have important practical implications. Attempts are now being made to tailor instruction on LIS to fit the needs of individual students. The task of individualizing instruction might be greatly simplified if sequences of instruction were tailored for groups of students rather than for each individual.

SECTION 1 - INTRODUCTION

In the first analysis to be discussed, a distance matrix was defined for each of the five partitions. For each partition and each pair of students (Si and Sj), the distance was defined as:

$$M(i,j) = \frac{D(i,j)}{N}$$

where D(i,j) is the number of problems for which Si and Sj constructed proofs that were not equivalent, and N is the number of problems for which both Si and Sj constructed proofs. Hierarchical clustering (HICLUS) was then used to group students on the basis of this metric, and the proofs for each student in each cluster were examined to determine the characteristics of individual proof behavior that explain the clusterings. Using these techniques, no clear indication of the existence of proof styles was detected.

For the second analysis, pattern variables (such as the frequency of theorem use) and efficiency variables (such as the number of lines per proof) were defined and computed for each student by averaging over the problems. For both sets of variables, attempts were made to cluster students on each variable and then on all of these variables together. These analyses indicated that strong individual differences do exist between students but there was no clear pattern in the differences observed.

These results were not unexpected. The problems in the logic curriculum are too heterogeneous for this type of analysis, and differences in proofs from problem to problem were much more pronounced than the differences between students for a given problem. The methods developed for this part of the study, however, make possible a more systematic analysis of problem solving behavior and should be useful in future studies dealing with problem solving behavior. The results indicate that a more homogeneous set of problems must be used if interpretable patterns of behavior are to be identified.

For the benefit of those who might wish to undertake a similar analysis, a description of the techniques that were used is included here.

SECTION 2 - METRIC ANALYSIS

A natural extension of the procedures which partition the set of proofs for derivation problems allowed a systematic examination of the data for indications that students could be characterized by the patterns of their proof behavior. The criteria(partitions) developed in Chapter III specify whether or not the proofs produced by any two students, for a particular problem, are equivalent. These techniques have been developed further in an attempt to determine whether the methods employed by any two students in constructing proofs to a sequence of problems are, in part, the same.

It was possible, of course, to examine the student proofs looking for evidence that indicates the existence of such patterns and this was, in fact, done. Unfortunately, the fact that a large number of rules were available to the students provided the opportunity for many minor variations and tended to obscure any general patterns in the proofs constructed by the students. It was hoped that an automatic procedure that focused attention on the possible existence of such patterns would facilitate the search. The procedure which was used for this purpose is described below.

Assume that we have a set of n problems, $P = \{p(1),\ldots,p(n)\}$, and a set of t students, $S = \{s(1),\ldots,s(t)\}$; for every p(i) in P and every s(j) in

S, s(j) constructs a proof for p(i). We also assume that there exists a partition on the set of t proofs for each of the n problems. The metric matrix defined below was computed separately for each of the five sets of classification criteria.

Let s(i) and s(j) be any two students in S. Let D(i,j) be the number of problems in P for which the proofs of s(i) and s(j) are not equivalent, and let M(i,j) = D(i,j)/n. It is clear that for all s(i),s(j) in S, M(i,j) is greater than or equal to 0, and M(i,i) = 0.

If the proofs for s(i) and s(j) are not equivalent in D(i,j) cases, and the proofs of s(j) and s(k) are not equivalent in D(j,k) cases, then the maximum number of problems where the proofs of s(i) and s(k) are not equivalent is D(i,j) + D(j,k):

$$D(i,k) \text{ leq } D(i,j) + D(j,k)$$

or

$$M(i,k) \text{ leq } M(i,j) + M(j,k)$$

The five matrices defined here (one for each set of classification criteria) are metrics defined on the set of students. M(i,j) is a measure of the distance between the student, s(i), and the student, s(j). It has its minimum value when s(i) and s(j) fall into the same equivalence class for all problems; then M(i,j) = 0. It has its maximum value when when s(i) and s(j) are in different classes for all n problems, and in that case M(i,j) = n/n = 1. M(i,j) is a metric on the set S.

A measure of distance which takes into account the number of different proofs for each problem is:

$$M(i,j) = \frac{w(p)*e(p,i,j)}{w(p)}$$

where e(p,i,j) is equal to 0 if Si and Sj gave the same proof for problem p , and otherwise is equal to 1, and w(p) is the number of different proofs constructed for problem p . To corrrect for missing data, w(p) is set equal to 0 if either Si's or Sj's proof for problem p is missing. This improved definition of the distance matrix was suggested by Stanley Sclove.

The HICLUS program, developed by S. C. Johnson (Johnson, 1967) was used to analyze the metric matrices for the full set of problems and for the subsample of problems that appear after the introduction of RE and that do not contain premises. The input data for this program consist of a metric matrix, $M(i,j)$. The output is a sequence of stages or levels of clustering. At the first level, each student constitutes a distinct cluster. At each subsequent stage the two clusters with the shortest distance between them are combined into a single cluster until all of the students are in a single cluster.

After each stage of clustering, it is necessary to redefine the distance matrix unambiguously, since the number of clusters decreases by one at each stage. The properties of the clustering algorithm are determined by the way in which this new matrix is formed.

For the analysis described here, Johnson's "Maximum Method" was used to form the new matrix at each stage. This method insures that the largest of the distances (defined in terms of the original metric matrix) between any two points in any cluster is a minimum. If we restrict ourselves to three dimensions and think of each cluster of points as being enclosed in a sphere with the smallest possible radius, we have n-k spheres after the k-th stage of clustering. The diameter of the largest of these spheres is less than the diameter of the largest sphere for any other set of n-k spheres that enclose all the points of the sample. A more detailed discussion of HICLUS is found in Appendix B.

This method generates n stages of clustering for any distance matrix and it was necessary to decide which, if any, of these clusterings should be the basis for subsequent analysis. There are two conflicting criteria that must be resolved in choosing the appropriate clustering. First, the intracluster distances should be small compared to the intercluster distances; the clusters are then geometrically well-defined. Second, the number of clusters should be small compared to the number of students; if the number of clusters is not much smaller than the number of points, clustering does not contribute to the analysis.

HICLUS provides information on both of these criteria at each stage of clustering; it gives us the membership of each cluster and the diameter of the largest cluster. The value, a(k), is the diameter of the largest sphere at the k-th stage of clustering, and is a monotone increasing function of k. A sharp increase in a(k) between the i-th stage and the (i+1)-th stage indicates that the i-th stage of clustering is a promising candidate for further analysis since we must accept much less compact clusters in order to decrease the number of clusters beyond the i-th stage.

The results of this analysis were not encouraging. Since HICLUS would have generated clusters even if the distance matrix had been randomly generated, the clusters that it did generate for the data in this study could not be accepted without further justification. None of the clusterings generated met the two criteria mentioned above, and none of these clusterings were readily interpretable in terms of the actual proofs in the data.

To facilitate the interpretation of the output of HICLUS, a complementary technique, multidimensional scaling, was also used. The objective of multidimensional scaling is to find a distribution of n points in k-dimensional Euclidean space that gives the best approximation to the n by n distance matrix. MDSCAL (the multidimensional scaling program) accepts as input an n by n distance matrix and a specification of the number of dimensions to be used.

Therefore MDSCAL can be used to generate a two dimensional representation (K=2) of a distribution of points that yields the best approximation to our distance matrix. This approximation, however, may be a poor one because, in general, it requires an n-1 dimensional distribution of points to reproduce exactly an n by n distance matrix. If the distance matrix can be reproduced from a distribution of 26 points on a two dimensional hyperplane, then a graphic representation of the clusters can be prepared from the results and the data can be examined visually for evidence of clustering. While determining the hyperplane that gives the best fit, MDSCAL also calculates how good the approximation is, and this measure, the stress, can be used to decide whether the two dimensional approximation is good enough to be taken seriously.

The two dimensional representation of the data obtained in this way did not indicate the existence of any clusters. If geometrically well-defined clusters had existed, then I would have attempted to determine the characteristics of individual behavior that accounted for the existence of these clusters. This would have been done by examining the proofs for the students in each group for similarities of structure. The important consideration here was not just the existence of clusters, but the interpretability of the clusters in terms of student behavior.

In this part of the analysis, an attempt was made to cluster students without using any predetermined characteristics of their proofs. Instead, the metric analysis was based on a distance matrix where the distance between any two students is defined in terms of the number of problems for which they generated equivalent proofs. It was anticipated that the interpretation of any clustering found in this way would be difficult because the clustering was not explicitly grounded in the characteristics of student proofs. In order to facilitate the identification of the defining characteristics of the clusters, a second analysis was used that clustered students in terms of explicitly defined pattern variables. The results of this analysis were to serve as a guide to the metric analysis and as a check on that analysis.

SECTION 3 - PATTERN ANALYSIS

The second analysis of the pattern of student performance concentrated on specific aspects of the proofs, defined by the pattern variables. For each of these variables averages were taken over the two sets of problems described earlier. The pattern variables are listed below:

P1 - the number of theorem steps per proof

P2 - the ratio of the number of theorem steps to the total number of steps

P3 - the number of axiom steps per proof

P4 - the ratio of the number of axiom

P5 - the number of Logical Truth steps per proof

P6 - the ratio of the number of Logical Truth steps to the total number of steps

P7 - the ratio of the latency to the first step to the average latency of all steps in the proof

The data for these variables were first examined individually for indications of clustering. Their correlation matrix was computed and frequency histograms were prepared for each. This initial examination of the data did not indicate the existence of any distinct groups of students, where the differences between the students in a group were small compared to the differences between groups.

The analysis was then extended to the multivariate case by using principle components analysis. The values, for each student, of the first two principal components were used to plot the distribution of students in two dimensions. Again, there was no indication of clustering.

The same analyses were also applied to a second set of variables called efficiency variables. These variables were also averages over problems for each student. The efficiency variables are listed below:

E1 - the number of unused lines per proof

E2 - the ratio of the number of unused lines to the total number of lines

E3 - the number of lines per proof

E4 - the total latency (time) per proof

Using the efficiency variables, there was again no reliable basis for clustering the students.

SECTION 4 - DISCUSSION

Although all of the attempts to cluster students failed, the analysis discussed here did highlight one interesting artifact in the data. There were three students who had unusually poor performances as measured by all of the efficiency variables. Examination of the proofs constructed by these students revealed a consistently poor performance starting very early in the curriculum.

These same students also tend to have extreme values for the pattern variables. On P5(LT/problem) and P6(LT/step), these three students have very high values. On P2(theorems/step), they have very low values.

The students who did most poorly in the curriculum show a marked tendency to use Logical Truth even when an appropriate theorem is available. Logical Truth is a conceptually simple rule that is introduced early in the curriculum. As the more powerful rules, especially the theorems, become available, most of the students learn to use them where they are appropriate. The three students being considered here did not make this transition.

Since their performance was poor relative to the average of the other students even before the introduction of any theorems, it cannot be concluded that the failure to incorporate theorems into their working set of rules caused the poor performance. This failure, however, did widen the gap between the poorest students and the average and superior students.

It would seem then that the pace of LIS is too fast for some of the students who are using the system. A more thorough investigation of the characteristics of these students should be conducted in order to determine the causes of their failure.

APPENDIX B

HICLUS

The HICLUS program, developed by S. C. Johnson (Johnson,1967) was used to anslyze the metric matrices for the third stage of the analysis. The input data for this program consists of a metric matrix, M(i,j). The output is a sequence of stages or levels of clustering. At the first level, each student constitutes a distinct cluster. At each subsequent stage the two clusters with the smallest distance between them are combined into a single cluster until all of the students are in a single cluster.

HICLUS begins its analysis with the weak clustering, C(0), in which each student defines a separate cluster. If a(1) is the smallest non-zero entry in the distance matrix, then the two clusters that are separated by the distance, a(1), in C(0) are combined to form a single cluster in C(1). The value of C(1) is defined to be a(1).

If the distance between any two clusters in C(1) is defined unambiguously, a new (n-1)X(n-1) distance matrix is defined for the n-1 clusters in C(1). The clustering process can then be continued by combining the closest clusters in C(1) to form C(2), with value, a(2). After n steps, all of the students have been combined into a single cluster, C(n), with value a(n).

The problem is to define the new (n-k)X(n-k) distance matrix that results after the k-th stage in clustering. If X and Y are the two clusters in C(k) that are combined into a single cluster, [X,Y], in C(k), what is the distance between [X,Y] and any other cluster, Z, in C(k)? Johnson offers two possible answers to this question.

For the, "minimum method", the distance, in C(k), from [X,Y] to Z is defined to be the minimum of the distances from [X,Y] to Z and from [X,Y] to Z in C(k-1):

$$d([X,Y],Z) = \min[d(X,Z),d(Y,Z)].$$

For the, "maximum method", the distance is defined as:

$$d([X,Y],Z) = \max[d(X,Z),d(Y,Z)].$$

Each of these definitions has a clear geometric interpretation. Johnson (op.cit. p249) outlines this interpretation in the following way:

> If we are given a clustering obtained by the Maximum Method, we may present the value of the clustering as follows: for each cluster in the clustering, compute the diameter of the cluster (the largest intra-cluster distance). For a given Maximum Method clustering, the value of the clustering is the maximum diameter of the clusters in the clustering. At any stage, the distance from the object/cluster x to the object/clustr y is exactly the diameter of the set x union y. This gives us a simple means of visualizing the clusterings-the Maximum Method attempts at each stage to minimize the diameter of the clusters.

The geometric properties of the "Minimum Method" are slightly more complicated, and are discussed in some detail by Johnson. Since I did not use this method, and since Johnson discusses it in detail, I will not describe it here.

HICLUS has two additional advantages that should be mentioned. First, the input consists of the n(n-1)/2 distances between the n objects; the algorithm does not require that the n points be represented in Euclidean space, and will accept the metric matrices defined in Chiapter iv without further processing. Second, the results are invariant under monotone transformations of the metric data.