T.R. 234

Reprinted from J. A. Swets and L. L. Elliott (Eds.), Psychology and the Handicapped Child. Washington, D. C.: U. S. Government Printing Office, 1974.

4. COGNITION: A SURVEY

by Patrick Suppes
Institute for Mathematical Studies in the Social Sciences
Stanford University

The study of cognition in psychology is more intense now than at any previous time, and with the recognition that complex internal processing is involved in most learning and perception, a continual widening of the definition of cognitive psychology has occurred. While it is not to the point to attempt a formal definition of cognition here, a number of examples are given, and from these examples it will be clear how broad the current conception is. Currently not only are all the major academic skills, ranging from reading to mathematics and science, included under cognition, but also much that is classically considered as part of perception. In fact it has become increasingly difficult to draw any sharp line between cognition and perception.

From a theoretical standpoint there are many different approaches to cognition, but it is fair to say that none of them currently dominates the scene. As in the case of an exact definition of cognition, it is also not possible to give an exact definition or to delineate sharply the key theoretical concepts in the various approaches to cognitive theory. Without too much injustice, however, we can group the current theories into four classes, and the four main sections of this chapter are organized to represent each of the four main theoretical approaches.

In brief terms the four approaches are: behavioral, developmental, information processing, or linguistic in orientation. The behavioral approach to cognition is typically represented by stimulus-response theorists like Estes, the developmental approach by Piaget, and the information-processing approach by Newell and Simon, as well as current work in artificial intelligence. The linguistic approach has been most stimulated by Chomsky, but the large literature on semantics derives not from the linguistic tradition of Chomsky and his colleagues, but rather from that of logicians and philosophers. Some attention will be given to both of these linguistic approaches.

Without attempting anything like an adequate or complete survey, I have also tried to indicate for these approaches some of the relevant studies directly concerned with the cognitive capacities of handicapped children.

As I turn to these four theoretical approaches to cognition, it is important to emphasize that each is incomplete and unsatisfactory in any one of several ways. There are some reasons for thinking that we are perhaps on the verge of a real synthesis of theoretical ideas that have been emerging in psychology from a number of different viewpoints, but it is premature to indicate the lines of this synthesis. It is clear, however, that what once appeared as sharp conceptual differences between behavioral approaches on the one hand, and information-processing approaches on the other, has with time increasingly become less clear and less distinct. More is said about such a synthesis in the final section.

Behavioral Approach

The behavioral approach to cognition in the form of concept formation may be illustrated by the application of the simple all-or-none conditioning model. Bower (1961) and Estes (1961) showed that a simple conditioning model could give an excellent account of paired-associate learning. In paired-associate experiments, the learner is shown, for example, a nonsense syllable and is asked to learn to associate with it the response of pressing a left or right key. Given a list of, say, 20 nonsense syllables, half of them randomly assigned to the left key and half of them to the right key, the scientific problem is to give an exact account of the course of learning. The naive idea most of us have is that on each trial, with exposure to the stimulus and an indication of what is the correct response, learning will gradually occur. One traditional way of expressing this was that the connection or response strength would gradually build up from trial to trial.

The experiments reported by Bower and Estes showed that in simple paired-associate learning the situation is somewhat different. The evidence is fairly clear that in the kind of paired-associate experiment just described the learner does not improve incrementally, but rather learns the association between a stimulus and response on an all-or-none basis. There is no improvement in the probability of his making a correct response until he fully learns the association. The theory of such experiments can be stated rather explicitly within a classical stimulus-response framework. The only important concepts are those of conditioning a response to a stimulus and sampling the stimuli on a given trial, together with the reinforcement that serves as a correction procedure when incorrect responses are made or that informs the learner that a correct response has been made.

One might accept such an exceedingly simple theory for paired-associate learning but doubt its ability to account for concept learning in children. Suppes and Ginsberg (1962, 1963) showed that this same all-or-none model gives a good approximation to concept learning where now the stimulus is replaced by the concept to be learned. In order to distinguish sharply such concept experiments from paired-associate experiments, Suppes and Ginsberg define a pure concept experiment as one in which the stimulus display changes on each trial, so that there is no opportunity to account for the learning data by a simple stimulus-association model.

In the Bower and Estes model, the two essential assumptions are these. First, until the single stimulus element is conditioned there is a constant guessing probability, p, that the learner responds correctly. Second, on each trial there is a constant probability, c, that the single stimulus element will be conditioned to the correct response. The only change in this model in order to apply it to concept learning is that the concept rather than the single stimulus element is now that to which the correct response is conditioned.

Without entering into statistical details, perhaps the best and most intuitive way to test this all-or-none model is to look at the probability of a correct response prior to the last error.

An experiment on learning the identity of sets in which the subjects were 48 children of

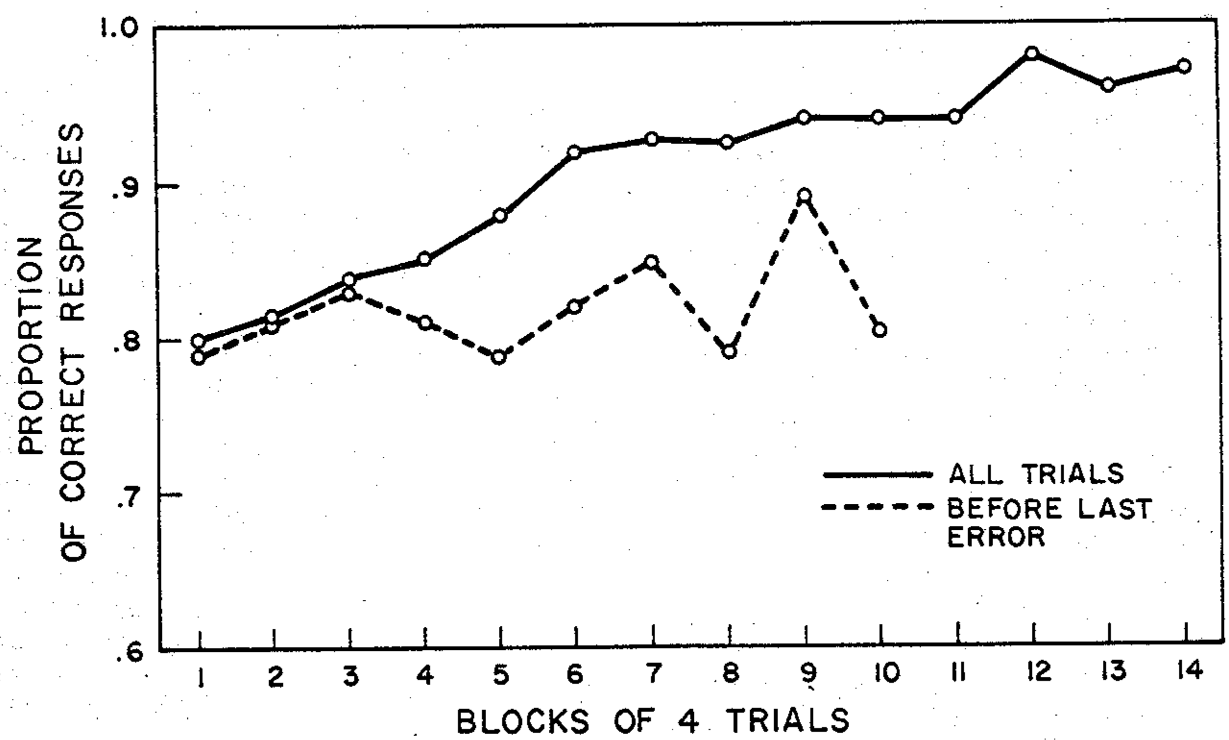

Figure 15.—Proportion of correct responses prior to last error and mean learning curve. (Identity of sets experiment.)

first-grade age is reported in Suppes and Ginsberg (1963). On each trial, the child's task was to indicate whether two sets displayed, each consisting of one, two, or three elements, were identical or not. The child was instructed to press one of two buttons when the stimulus pairs presented were "the same" and the alternative button when they were "not the same." Notice, of course, that "the same" does not mean the same from a perceptual standpoint, for permutations in the order in which the members of sets are shown do not affect the identity of the sets.

A total of 48 subjects were run through individual sessions of 56 trials on which 28 of the stimuli displayed showed identical sets and the remaining 28 showed nonidentical sets. No stimulus displayed on any trial was repeated for an individual subject. The learning data, taken from Suppes and Ginsberg (1963), are shown in figure 15. The solid curve shows the mean learning curve and the dotted curve the curve for learning prior to last error. As the all-or-none model would predict, the curve for learning prior to last error is nearly horizontal and provides approximate confirmation of the all-or-none model.

A second experiment on geometric forms was reported in detail in Stoll (1962) and was also discussed in Suppes and Ginsberg (1963). The subjects were 32 kindergarten children divided into two groups. For one group the problem was to discriminate among triangles, quadrilaterals, and pentagons, and for the other to discriminate among acute, right, and obtuse angles. From a discrimination learning standpoint, the experiment was a successive discrimination, three-response situation. For all subjects a typical case of each form was shown just above the appropriate response key and, as in the previous experiment, no single stimulus display was repeated for any

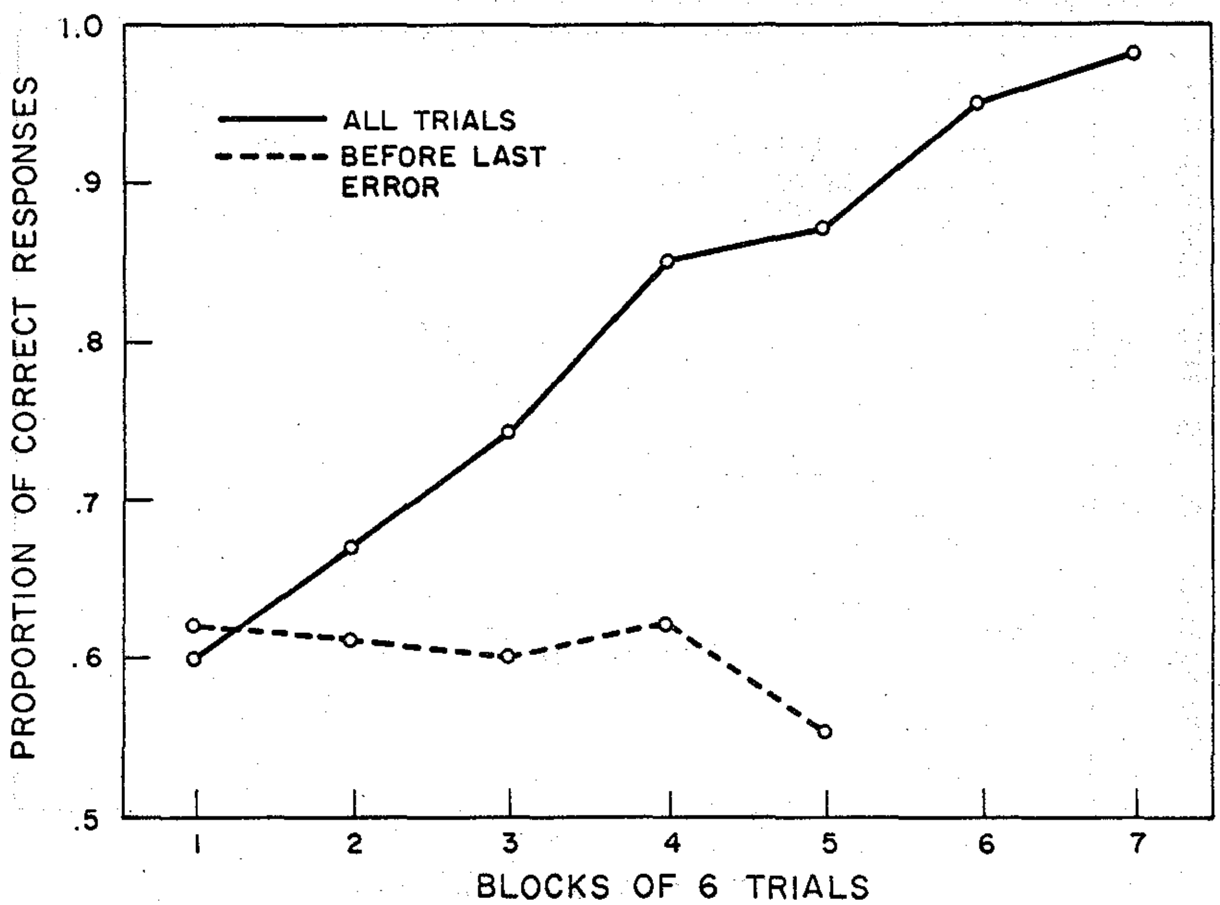

Figure 16.—Proportion of correct responses prior to last error and mean learning curve. (Quadrilateral and pentagon concepts, Stoll experiment.)

one subject. The subjects were run to a criterion of nine correct responses in any one session.

The mean learning curve and the mean learning curve prior to last error are shown in figure 16 for the quadrilateral and pentagon concepts (the learning of triangles was sufficiently fast to make the data uninteresting). The combined learning data for acute, right, and obtuse angles are shown in figure 17. In both cases there is good support in first approximation for the all-or-none conditioning model, although as Suppes and Ginsberg (1963) show, in a more detailed analysis of the data, statistically significant evidence for slight deviation from the all-or-none model can be found.

The experiments just reported indicate how an exceedingly simple behavioral model can give in first approximation an excellent account of data on concept-learning experiments with children. It should be obvious, of course, that a model as simple as the all-or-none model does not begin to give a full account of the processing that takes place in the child's learning of the concepts in question. What the model does is abstract certain features of the learning and give a good account of those features.

From a behavioral standpoint an excellent review with clear theoretical orientation of learning in retarded children is found in Estes (1970). To indicate how a more complicated behavioral model can be applied to the conceptual learning of handicapped children, it may be useful to review Estes' analysis of the Zeaman and House two-stage intentional model for discriminational learning. The Zeaman and House model is applicable to concept identification or simple concept formation and is an extension of the one-element model just described. The Zeaman and House work is almost unique in being one of the few cases in which a theoretically detailed set of assumptions has been applied to problems of concept formation

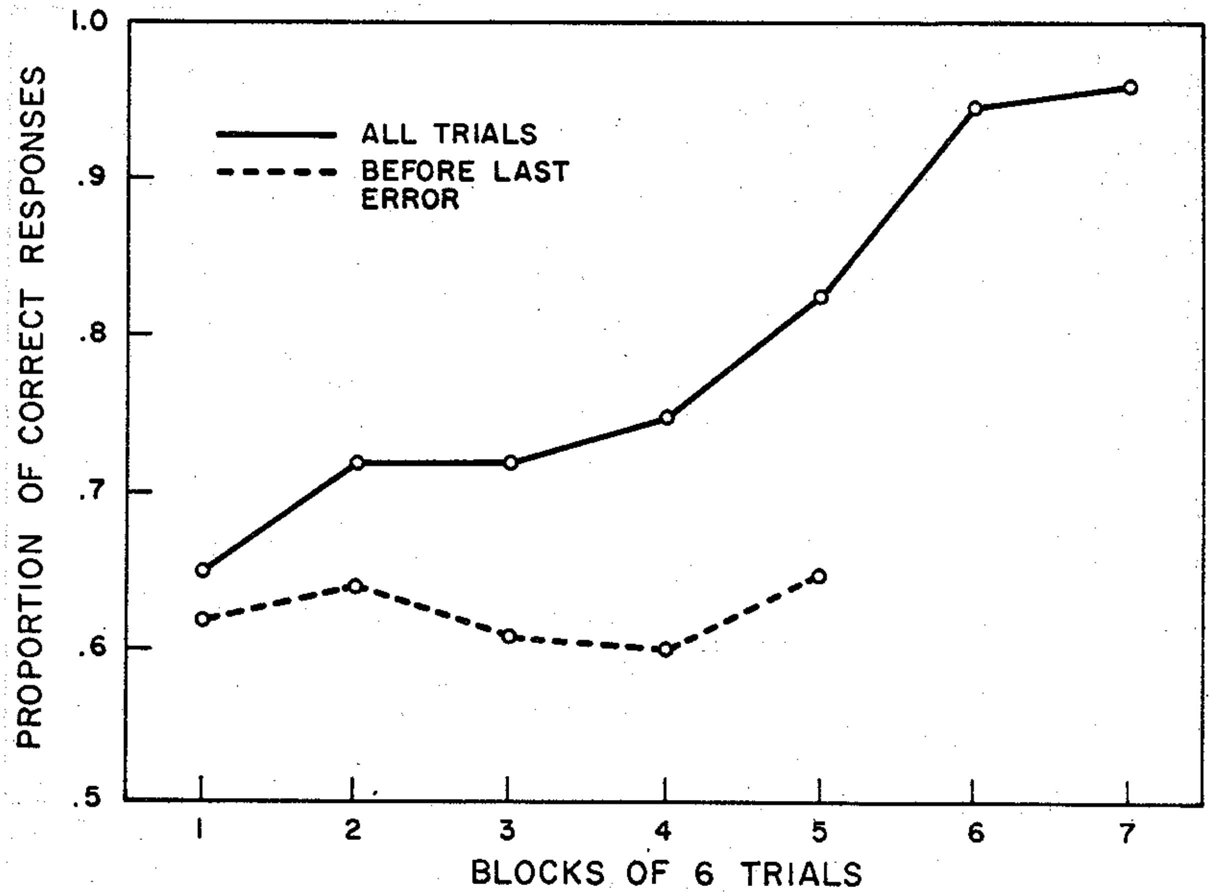

Figure 17.—Proportion of correct responses prior to last error and mean learning curve. (Acute, right, and obtuse angle concepts, Stoll experiment.)

or identification in retarded children, for example, in color-form discriminations. The two stages in their model represent an attentional process and a learning process.

What is surprising and almost paradoxical in the theory is that the main differences in learning for subjects of different mental ages are reflected in the initial attentional process, which primarily consists of learning to attend to the correct or relevant dimensions of a problem. Very small differences are reflected in the learning of the appropriate associations once the proper dimensions are attended to. In one analysis, for example, groups of children with mean mental ages of 2 years 4 months and 4 years 6 months, respectively, were compared. The curve for the higher group rose steeply from chance to nearly 100 percent correct responses over about 40 trials. The curve for the lower group differed only in that it hovered around the chance level of 50 percent correct, responding with no obvious trend for about 180 trials before beginning to rise. Then, like the curve for the higher group, the trend rose steeply to virtually 100 percent correct responses over about 40 trials.

Backward or Vincent learning curves for the data prior to last error were used in this study to detect learning trends. As Estes points out, it is hard to accept that the only differences in learning of retarded children can be identified simply as the probability of attending to the correct dimension. Since the attentional function is a probabilistic function and sums to one, this means that if the theory were pushed relentlessly, on some dimensions the performance of retarded children should be better than that of normal children, because they must have a higher probability of attending to these dimensions.

In principle individual parameters can be estimated in the model, but in practice this has not been done. In fact, I have been unable to identify any studies of concept formation or identification in retarded students, or even for groups of subjects stratified according to mental age, that actually work out models in sufficient detail to estimate in standard statistical fashion learning parameters for individual subjects. In view of the extensive work that has been devoted in mathematical psychology to the development of such models over the past two decades, it would seem especially desirable to push the detailed analysis of data by the application of such models and the identification of various phases of learning at a more abstract level in terms of the estimation of parameters. It would also be interesting to then regress the estimated parameters for individual subjects or stratified groups of subjects on variables of mental age, chronological age and other features of overall performance.

Moreover, in those experiments for which the all-or-none model fits fairly well, by assuming a beta distribution for individual differences in the conditioning parameter c, more exact and quantitative comparisons between normal and retarded children could be made by estimating such beta distributions for the two populations. It would be anticipated that in many studies the mean for the beta distribution of the retarded children would be significantly lower than that for the normal children, but the overlap in the two distributions, as well as in the scatter plots of the individual estimated parameters, would provide information to deepen our summary view on the differences and similarities of the two populations with respect to different conceptual tasks.

Using the more complex Zeaman and House model or applying the concept of individual differences to the all-or-none model still leaves us a long way from a theory adequate to account fully for the processing obviously required of the learner in mastering even a simple cognitive concept like that of identity of sets. This then is the weakness of the behavioral approach. It has not been able to develop a sufficiently complex apparatus satisfying the rigorous standards it has imposed in the analysis of data and for that part of the processing it can account for. The situation is complex and needs to be stated with some care. The theoretical issues are more subtle than we can explore in detail here, but the following points are relevant. First, it is sometimes claimed by those advocating other approaches to cognition that behavioral or stimulus-response theories cannot in principle account for complex behavior, for example, the complex processing required in language learning. These negative claims about the behavioral theory are almost always incorrect, incorrect in the sense that the claims are asserted in dogmatic fashion. No proof is formally given that some well-defined version of stimulus-response theory cannot account in principle for complex learning. As shown in Suppes (1969), it is not difficult to give a stimulus-response theory of finite automata, and without too much trouble this account can be extended to more complex automata that are in principle adequate for language learning.

On the other hand, even though the formal criticisms are incorrect, it is certainly true that the behavioral approach is not at present able to provide an adequately detailed theory of something as complex as language learning.

Developmental Approach

A major approach to cognition has been to describe in explicit terms the sequence of concept development in children from birth to adolescence. Without question, the outstanding effort has been that of Piaget and his collaborators. The studies have ranged over most of the topics one would like to see included in a broad theory of cognition and have covered more conceptual ground than the behavioral approach just discussed. There are, for example, within the Piagetian developmental approach major studies on the following concepts: the child's understanding of spatial concepts, including both two- and three-dimensional concepts; the development of geometrical concepts; the development of the concept of distance conservation and the spatial coordinate system. Extensive and controversial studies on the concepts of conservation have also dominated much of the literature in recent years and range through the conservation of mass, weight and volume. Additional studies have been concerned with the development of number concepts and set concepts closely related to those of number concepts; for example, the notion of two sets being equivalent, that is, having the same cardinality. Still other studies have been devoted to the development of logical operations and the development of the concepts of causality and also of morality in children.

Those who want to get a deeper feeling for the Piagetian approach to cognition can look at either some of the many books of Piaget that have been translated into English or at some of the excellent readers composed of shorter articles that have appeared in recent years. The book edited by Sigel and Hooper (1968) provides an excellent survey and is recommended.

The enormous body of research studies generated by Piaget and his collaborators has given us an overview of the cognitive development of the child unequaled even approximately by any of the other approaches to cognition. The attempt has been to map out in broad terms the cognitive development along every major dimension of intellectual or perceptual skill. To a lesser extent than one might expect, this conceptual apparatus and approach to cognition has not been extensively applied to handicapped children. An example of work in this area is Woodward (1961), who considered one-to-one correspondence and equivalency of sets, as well as seriation and conservation of continuous quantity. She found that the performance of retarded adults whose chronological age was 19 and retarded children whose chronological age was 12.9 was at about a level similar to an average normal child of from 4 to 7 years.

Granted that the developmental approach of Piaget has given by far the most extensive analysis of the whole range of cognitive concepts, it is natural to ask why this approach has not been uniformly adopted by most investigators and conceded to be the soundest approach to cognition. There are, I think, three reasons for reservations about the Piagetian approach to cognition. These reasons can be given and seriously held to without at the same time denigrating the great value of the work that Piaget and his collaborators have done.

One objection to the developmental Piagetian approach to cognition is the lack of emphasis and attention given to language development. The linguistic approach discussed below emphasizes the overwhelming importance of language development for the cognitive development of a child and its advocates find far too little attention paid to the problems of language development in the Piagetian viewpoint.

The second objection has been a methodological one by many experimental psychologists to the quality of the experimental data reported by Piaget and his collaborators. The standard objection has been that well-designed experiments have not been used as a basis for the conclusions drawn, but rather empirical methods have been based too much on anecdotal methods, or at the least, open-ended interviews in which children are verbally interrogated about their understanding of concepts and relevant cognitive tasks. This criticism is less valid than it was a decade ago, because much of the emphasis, especially on the part of American investigators following the Piaget line of development, has been on the careful design of experiments to test Piagetian concepts. There now exists a rather substantial literature of an experimentally sound character in the Piagetian tradition, and the reader will find current issues of journals like *Developmental Psychology* and the *Journal of Experimental Child Psychology* full of carefully designed experiments that clearly grow out of this tradition.

The third line of criticism of the Piagetian approach is the lack of clarity in the development of key concepts and the absence of sharply defined experimental tests of the key concepts. To illustrate the problem and to provide a comparison with the earlier discussion of all-or-none conditioning as a behavioral approach, I paraphrase and present briefly an analysis I have given elsewhere of Piaget's concept of stages (Suppes, 1972).

I select Piaget's concept of stages, because it is central to much of his work in development, and because it also has become increasingly important in developmental psycholinguistics. I hasten to add, however, that a similar analysis could be given of other key concepts. An instance of how Piaget uses the concept of stages can be gained from the following quotation, in which the analysis of three stages of multiple seriation is discussed in Inhelder and Piaget (1964, p. 270).

> We shall distinguish three stages, corresponding to the usual three levels. During stage I, there are no seriations in the strict sense. The child's constructions are intermediate between classification and seriation. . . . During stage II, there is seriation, but only according to one of the criteria, or else the child switches from one criterion to the other. . . . Finally, during stage III (starting at 7–8 years), the child reaches a multiplicative arrangement based on the twofold seriation of the set of elements.

There is in this passage, as elsewhere in the writings of Piaget, little indication that matters could be otherwise—that development could be incremental and continuous and that stages may be an artificial device with no real scientific content. No one denies that children develop in some sequential fashion as they acquire new capacities and skills. The problem is in determining whether they proceed in stages or continuously. We could of course artificially and conventionally divide any period of incremental development and label it as a particular "stage." In principle, the issue about stages versus incremental acquisition of concepts is exactly the issue faced by the behavioral approach in comparing the all-or-none conditioning model with the ordinary incremental model.

In other places Piaget does comment on the question of the actual existence of stages, but he does not address the matter in ways that seem scientifically sound. Piaget (1960, p. 121) writes as follows:

> I now come to the big problem: the problem of the very existence of stages; do there exist steps in development or is complete continuity observed? . . . when we are faced macroscopically with a certain discontinuity we never know whether there do not exist small transformations which would be continuous but which we do not manage to measure on our scale of approximation. In other words, continuity would depend fundamentally on a question of scale; for a certain scale of measurement we obtain discontinuity when with a finer scale we should get continuity. Of course this argument is quite valid, because the very manner of defining continuity and discontinuity implies that these ideas remain fundamentally relative to the scale of measurement or observation. This, then, is the alternative which confronts us: either a basic continuity or else development by steps, which would allow us to speak of stages at least to our scale of approximation.

The confusion in this passage is in the introduction of the spurious issue of the scale of measurement. Obviously this is an issue to be discussed in a refined analysis but, as the literature on all-or-none conditioning models versus incremental models shows, a perfectly good and sound prior investigation exists at a given level of measurement, namely, the level of standard experimental studies. What Piaget does not seem to recognize is the existence of a clear alternative and the necessity of testing for the presence or absence of this alternative in providing a more correct account of the sequential development that occurs in a child.

This discussion of stages is meant to indicate the tension that exists in any fair evaluation of the work of Piaget and his collaborators. On the one hand they have without doubt contributed enormously to the current intense interest in cognition, especially in the cognitive development of children. Piaget and his collaborators have put the problem in a proper perspective by insisting on investigating not just a few skills and concepts, but the entire range that we intuitively expect and believe are part of the child's developing competence. On the other hand, both the theory and experimentation have often been loose and more suggestive than definitive. Methodological and theoretical criticisms are easy to formulate. Certainly, deeper clarification of both the experimental methodology and the theory is required before widespread applications to the critical problems of development in handicapped children are extensively pursued.

Information-Processing Approach

The information-processing approach to cognition has been deeply influenced by related developments in computer science and the widespread impact of computers themselves since the early 1950s. A good example of any early influential article in this approach to cognition is Newell, Shaw, and Simon (1958). An influential book of the early 1960s was that edited by Feigenbaum and Feldman (1963). Perhaps the most impressive recent example of this general approach to cognition is Newell and Simon's treatise (1972) on human problem solving.

In broad terms, the difference between the information-processing approach and the developmental approach of Piaget is that Piaget has primarily been concerned with the characterization of tasks and the sequence in which the child learns to solve these tasks; in contrast, the information-processing approach has been concerned with the processing apparatus necessary to handle even the most elementary forms of cognition.

As the name suggests, the information-processing approach has been much influenced by the organization of information processing in computers. There is concern that the major aspects of information processing that have been the focus of computer organization also be given attention in any conception of human processing. It is important not to be misunderstood on this point. Investigators like Newell and Simon are far too sophisticated to think that the present stage of computer developement provides anything like an adequate model of human processing. Although they do not put it in so many words, it is probably fair to say that they would regard the problems of computer organization as indicating some of the necessary but not sufficient conditions for information processing in humans.

The major feature of the information-processing approach that differs from either the behavioral or developmental approach is the emphasis on the detailed steps a person or child takes in solving a concept, and the detailed analysis of the verbal protocol that can be obtained from him in the process of mastering a problem. The information-processing approach is like the developmental approach in its emphasis on the importance of verbal reports of the subject in an experiment, but it differs from the developmental approach and is more like the behavioral approach in its emphasis on a highly detailed analysis of the structure and content of the protocol.

As is characteristic of other areas of psychology, the different approaches also tend to develop different types of tasks considered typical of cognition. The information-processing approach, especially in the work of Newell and Simon, has been concerned with cryptarithmetic, simple logical inference, and the kind of problem solving that goes into complex games like chess.

The most characteristic and important feature of the information-processing approach has been the attempt to simulate by a computer program the detailed processing in which a human subject engages in problem solving. This has proved to be both a strength and weakness of this approach to cognition. It is a strength because of the effort to capture as much as possible the explicit details of the human subject's thought processes in mastering a cognitive problem; in this ambition it goes far beyond anything that has yet been attempted in the behavioral approach. The weakness of the approach is methodological. It centers around the difficulty of evaluating whether or not the simulation, even at the level of individual subjects, provides a good match or not to the actual ongoing processing in the human subject. The very complexity of the simulation raises new methodological problems that do not arise in the same form in either the behavioral or developmental approaches to cognition.

In spite of some of the reservations one may express about the methodology, the Newell and Simon approach to cognition seems to hold excellent promise of application to the study of cognition in handicapped children. Let me give one example.

An excellent review of the relative efficiency of concept usage by retarded and nonretarded children is found in Zigler and Balla (1971); they reviewed eight major studies, which by and large equated the mental age of retarded and nonretarded subjects. A couple of the studies reported more than one experiment. The 19 experiments, whose results are summarized, included the tasks of selecting 3 pictures that illustrate a concept from a set of 7 pictures, verbalizing a concept common to the 3 pictures, associative clustering, defining all words in an experiment, sorting cards in terms of some concept, and selecting 4 pictures that illustrate a concept from a set of 7 using different types of concepts. The performance of the normal and retarded subjects was about the same in 12 of the experiments, and that of the nonretarded subjects was better in the remaining 7.

Similar results are reported in Blake and Williams (1968). Retarded, normal, and superior groups of students were compared on their attainment of concepts by deduction, induction-discovery, and induction-demonstration. When mental age was held constant, the groups did not differ in level of concept attainment. Also, for all three groups, deduction was the most effective, while the two inductive methods were about equal in effectiveness.

A recent study of Blount (1970) found no significant difference between retarded and normal subjects on a concept-usage task made up from familiar items. The task required choosing the three of five pictures that went together, as well as giving a verbal label for the exemplified concept. The only superior aspect of the nonretarded subjects' performance was in their verbal labeling of the concept. Jones (1971) studied the feasibility of educable mentally retarded children's learning simple schemata exemplified in stimulus patterns on checkerboards. While the results were positive, they were not compared with a control group of normal subjects.

Although many of these studies are well designed from an experimental standpoint and consequently report empirically significant results, what is missing almost uniformly in the studies cited, including the large group analyzed by Zigler and Balla, is a theoretical framework in which to deepen the understanding of the results. For example, we do not currently have a theoretical framework in which to pinpoint more exactly the point at which to differentiate the performance of normal and retarded subjects in a concept-attainment task. The radical difference in language competence discussed in the next section does not in itself provide an explanation of the difference in concept attainment, and we need the kind of meticulous examination of details characteristic of the Newell and Simon approach to give us a deeper theoretical insight into the differential performance of retarded and normal subjects. It would be my conjecture that the application of this theoretical approach would yield significant results beyond those obtained by the kind of primarily experimental studies without an explicit theoretical framework now characteristic of much of the concept-attainment literature mentioned above.

Recently the broad spectrum of problems attacked under the heading of artificial intelligence by computer scientists has provided also a broader based approach to cognition than the particular approach of Newell and Simon. It is not that the approach via artificial intelligence is in contradiction with that of Newell and Simon—it is that new components with a different emphasis have been added. The work of Marvin Minsky and Seymour Papert has been especially influential in this development. They have taken this approach at a mathematical level in their book, *Perceptrons* (1969), and still more explicitly in their recent analysis of the close relation between artificial intelligence and the development of a child's intelligence. Perhaps the most characteristic feature of their recent work is the emphasis on a procedure or program on the one hand, and the process of debugging the procedure or the program on the other. The idea that learning a cognitive skill is primarily a matter of learning a procedure which itself might be broken into separate procedures, and that each of these separate procedures must go through a process of debugging similar to debugging a computer program is an important insight not previously exploited in any detail in the theory of cognition. Although the details are far from clear, it is now a widespread belief that we must be able to conceptualize the internal programs that an organism uses in solving a conceptual or perceptual problem.

Without entering into full details, I illustrate the use of programs or procedures to study cognitive behavior by some of the recent work in the Institute for Mathematical Studies in the Social Sciences at Stanford. A preliminary account of this work may be found in Suppes (1972). We take as our cognitive task the elementary algorithms of arithmetic. The objective is to give an account in terms of the kind of processing the student must learn in order to solve one of these algorithms in standard format. For purposes of the present discussion we may take as a typical example ordinary column addition, that is, the usual algorithm for finding the sum of numbers when each number is represented in a row, with one row placed vertically over another to yield the standard vertical format for addition exercises.

In considering a processing approach, basically we need to think about two things. One is the characterization of the kind of registers or memory devices available to the student, and the second is the kind of instructions, like the machine instructions for a computer, that must be learned in order to correctly process the exercise. These instructions unlike computer instructions must involve, at least in elementary and schematic form, some perceptual aspects of the problem format. For the present discussion I shall drastically simplify the perceptual situation by conceiving each exercise as being presented on a grid with at most one symbol on each square of the grid. For column addition, we number the coordinates of the grid from the upper right-hand corner, Thus, in the exercise.

$$\begin{array}{r} 26 \\ 17 \\ +34 \\ \hline \end{array}$$

the coordinates of the digit 6 are (1,1), the coordinates of 7 are (2,1), the coordinates of 4 are (3,1), the coordinates of 2 are (1,2) and so forth, with the first coordinate being the row number and the second being the column number.

In terms of registers for memory, we need in general three registers, a stimulus-support register [SS] that holds an encoded representation of a printed symbol to which the student is perceptually attending at a given moment. For the present example the alphabet of such symbols consists of the ordinary 10 digits and the underlined symbol used to terminate the column. As new symbols are attended to, previously stored symbols in this stimulus-support register must be transferred to a nonstimulus-support register [NSS]. It is also convenient to use an operations register [OP] that acts as a short-term store, both for encodings of external stimuli and for results of calculations carried out on the contents of other registers.

The set of instructions needed for column addition are then the following 10, which are formulated in terms of the use made of the three registers.

Attend (a, b):	Direct attention to grid position (a, b).
(±a, ±b):	Shift attention on the grid by (±a, ±b).
Readin [SS]:	Read into the stimulus-supported register the physical symbol in the grid position addressed by Attend.
Lookup [R1] + [R2]:	Look up table of basic addition facts for adding contents of

	register [R1] and [R2] and store the result in [R1].
Copy [R1] in [R2]:	Copy the content of register [R1] in register [R2].
Deleteright [R]:	Delete the rightmost symbol of register [R].
Jump L:	Jump to line labeled L.
Jump (val) R, L:	Jump to line labeled L if content of register [R] is val.
Outright [R]:	Write (output) the rightmost symbol of register [R] at grid position addressed by Attend.
End:	Terminate processing of current exercise.
Exit:	Terminate subroutine processing and return to next line of main program.

A perusal of these instructions shows quickly enough that only the *lookup* instruction does not have an elementary character. In terms of these instructions we can then write subroutines or programs for solving exercises in column addition, and the manner in which we write these programs is similar to the way in which programs are written for a computer in machine or assembly language. Details are omitted, because even in the case of column addition the program written in terms of these instructions requires more than 20 lines.

The particular example chosen for discussion is simple and in certain ways rather special, but it is meant only to illustrate the approach to cognition through procedures or programs. It can be anticipated that this kind of approach will be extensively used in the decade to come, and in all likelihood a wide variety of cognitive tasks will be analyzed in terms of programs or subroutines of elementary processing instructions.

In many ways it seems especially promising to use this kind of approach for the meticulous and detailed analysis of the tasks we want handicapped children to master as part of their education. The application of these ideas to the learning or performance of handicapped children has not yet taken place, but it is a feasible and practical application for research in the years ahead, with considerable significance for practical problems of instruction.

Linguistic Approach

An excellent expression of the linguistic approach to cognition is found in Chomsky (1972). At the outset, an important difference to be noted about the linguistic approach in contrast to the three other approaches discussed already is that the linguistic approach does not in principle propose to be a general theory of cognition, but rather it concentrates on that significant part of cognition that is language dependent or consists of language skills themselves. Linguists like Chomsky consider the phenomenon of language the most important single phenomenon of cognitive psychology and, consequently, believe that a large place should be occupied by the linguistic approach to cognition, even if it is not meant to encompass all cognitive phenomena.

Linguists and psycholinguists with a strong linguistic orientation have been insistent that none of the other approaches to cognition provides anything like an adequate detailed theory of language development or language performance in either children or adults. Indeed, it is customary for linguists like Chomsky to insist that even their own theories offer only the barest beginning of an adequate approach to the analysis of language. Long ago, Aristotle defined man as a *rational* animal, but much is to be said for the viewpoint that man should rather be defined as a *talking* animal. The linguistic approach to cognition insists upon the central place of language in the cognitive behavior of man and rightly denies the adequacy of any theory of cognition that cannot account for major aspects of language behavior.

The linguistic viewpoint has emphasized understanding the complex and sometimes bewildering grammar of spoken language. There is, however, another aspect of language with a long tradition of analysis, which is equally important from a cognitive standpoint. I have in mind the theory of meaning and reference, or what is usually termed the *semantics* of a language. This semantics tradition derives more from philosophy and logic than from linguistics. The current approaches run back in a continuous line to the magnificent work of Gottlob Frege in the 19th century. (A good introduction to Frege's work may be found in the volume edited by Geach and Black, 1966.) The emphasis in this century has derived from the pioneering work of Alfred Tarski, beginning with his classic monograph on the concept of truth in formalized languages (1935). In the last decade, important semantic ideas of Tarski, including above all the important concept of a semantic model of a language, have been applied to natural languages, especially by Tarski's former student, Richard Montague (1970), and also in recent work of my own (Suppes, 1971).

The purpose of these theories that derive from the work of Frege and Tarski is to give a detailed and explicit theory of the meaning of utterances in ordinary language. Just as in the case of the grammatical analysis derived from the work of Chomsky and others, so in the case of these semantical efforts, it would certainly be incorrect to claim at this time that they have been entirely successful. However, a solid beginning has been made, and, perhaps more importantly, it is now clear how the extensive conceptual developments that have arisen from the earlier work of Tarski, and that have been exceedingly fruitful in the analysis of formal languages, can also be applied to the semantics of natural language.

In principle, we should like to be able to give a detailed account of the grammar and semantics of spoken speech, and especially to trace the development of both grammar and semantics in the speech of young children beginning at an age earlier than 2 years. We are yet far from being able to achieve these objectives and from having an understanding of the mechanisms used by the child in the rapid development of his verbal abilities between the ages of 2 and 5 years.

I want to conclude this section by saying something more about the theoretical problems of developing a completely adequate and detailed theory. Before doing so, however, it will be useful to examine some of the work done in studying the language development of handicapped children. In the case of deaf or retarded children the problem of language development seems to be the most serious cognitive problem faced in training and educating these children and in understanding how explicit and carefully designed approaches to their instruction can result in maximum benefit to their cognitive development.

In presenting these examples of studies of language development in handicapped children, I have not attempted to provide anything like a systematic survey of the literature. Rather, I have selected certain studies that present results of interest to the theoretical issues characteristic of linguistic approaches to cognition. The analysis is organized around three subheadings: linguistic development of retarded children; the role of language in concept formation of deaf children; and, finally, the use of regression models in the analysis of language comprehension by deaf children. The emphasis on deaf children is partly fortuitous, due to the Institute's own concern with the teaching of deaf children over the past 3 years and the fact that my own research on handicapped children has been entirely concerned with deaf children.

Linguistic Development of Retarded Children. A major study by Lenneberg, Nichols, and Rosenberger (1964) examined over a period of 3 years the language development of Mongoloid children ranging in age from 3 to 22 years. The IQ's of the children ranged from the 20's to the 70's. Their major findings were: IQ does not predict the stage of language development but chronological age does; a significant relation exists between motor development and the onset of language; although the rate is much slower, language development in Mongoloid children is similar to that in normal children; some Mongoloid children are able to process syntactically complex sentences. These authors used their results to defend the general proposition that language development is not closely related to intellectual ability, but rather it is more closely related to general biological processes of maturation. As with most general hypotheses of this kind, the data are not presented in a fashion that permits a sharp statistical evaluation or quantitative assessment of the degree to which the hypothesis is actually supported.

A number of highly specific linguistic studies of the language of retarded children are to be found in the literature. Lovell and Bradbury (1967) studied 160 children of ages 8 to 15, inclusive. Their three hypotheses were: (i) the ability of these children to inflect, derive, and analyze compound words improves little between 8 and 15 years of age and is generally below that of normal first graders; (ii) there is a significant relationship between reading level and the ability to inflect lexicon words; (iii) there is a significant relationship between IQ and the ability to inflect nonsense words, but little relationship between reading attainment and the inflection of such words. The data confirmed all three hypotheses.

Graham and Graham (1971) studied the syntactic characteristics of the speech of nine retarded children with chronological ages ranging from 10 to 18 years and mental ages ranging from 3 years 6 months to 10 years. Their data supported the hypothesis that non-Mongoloid retardates develop language at a different rate, but in approximately the same way as normal children.

Semmel, Barritt, Bennett, and Perfetti (1968) undertook a grammatical analysis of word associations of educable mentally retarded and normal children. In studies of the language development of normal children it has been found that as they get older they tend to increasingly give associations to stimuli falling within the same grammatical form class as the stimulus. These investigators found the highest level of such form-class responses in the older normal children and the lowest incidence of such responses in the institutionalized retardates.

Cartwright (1968) studied the written language abilities of educable mentally retarded in comparison with normal children. His subjects were 80 12- through 15-year-old educable mentally retarded and 160 8- through 15-year-old normal children. Comparisons were made on the following language measures: composition length, sentence length, type-token ratio, percentage of usage of different parts of speech, grammar and spelling. The normal children of the same age had significantly higher scores on all these measures. Younger normal children, ages 8 through 11, obtained significantly higher scores than the educable mentally retarded group on three of the measures; namely, type-token ratio, grammar, and spelling. The absence of difference in sentence length is significant, considering the extent to which mean utterance length is currently used as a measure of language development by a number of psycholinguists.

One of the more extensive studies of the spoken vocabulary of retarded children has been made by Beier, Starkweather, and Lambert (1969). They interviewed 30 retarded children and recorded 2,700 words from each. The approximately 80,000 words of output were analyzed and compared with the output of normal children. While they found differences in the word lists, they also found a large number of similarities in performance of the retarded and normal groups. They interpreted their overall findings as supporting the assumption that mentally retarded children suffer from a conceptual and organizational deficit in their language usage.

These various studies show that even if the sequence of language development is similar in normal and retarded children, most cognitive functions of language are less developed in retarded children. However, it is not yet clear whether the deficit is most pronounced in the primarily cognitive aspects of language. Much better and more detailed data on the impact of training would also be most desirable; for example, the rate of acquisition of new words, the rate of improvement in spoken and written grammar.

Language and Concept Formation in Deaf Children. Excellent reviews of the literature on concept formation in deaf children have been provided by Furth (1964, 1966, 1971). In the most recent of these reviews (Furth, 1971), 39 studies are listed and summarized. The fundamental issue raised by Furth and many of the investigators whose experiments he summarized is the question of whether deaf children show a deficit in concept formation once verbal aspects of the task are removed. Put another way, in experiments that require no verbal comprehension are there significant differences in performance between deaf and normal children? Even more than in the case of concept formation or identification by retarded children, Furth has presented persuasive evidence from a number of experiments that there are often not significant differences. As he admits, the situation is not simple, and contrary evidence can be cited. The important issue, however, is the role of language in concept formation. Here, it seems to me, Furth does not really make a strong theoretical point, because his analysis is concerned entirely with command of a standard natural language. As he points out, in letter recognition tasks and others, the processes deaf children use are not clear. Process-oriented approaches to cognitive skills seem to argue strongly that some sort of language is being used internally, even if the language is not that of the society in which the children live.

Apart from the issue of the necessity of an internal processing language, two other remarks may be made about Furth's position. The first is that it would be interesting to see what the performance of deaf children who understand sign language would be if sign language were used to provide equivalent verbal instructions, or in the case of responses, to provide a medium for response by the child. There are of course some difficult problems of methodology. If comparison with normal children is desired, as in most cases it is, then comparability of the two media of communication is needed to judge whether a communication deficit exists. The methodological problem is rather similar to the study of concept formation in blind children when concepts are transferred from the visual to some other sensory modality.

The second remark concerns Furth's discussion of logical reasoning and the claim from some of his own experiments that deaf children exhibit capacities that show only small deficits at most. The point is that the experiments on logical reasoning are all extremely elementary. More complex kinds of inference, even of the kind that can be given young normal children (ages 6 and 7 years, for example), are difficult to test outside a verbal context. For example, in Suppes (1965), data on the intuitive inference capacities of young children are cited for the classical forms of inference running from modus ponendo ponens to quantificational logic using universal and existential quantifiers and two-place predicates. The experimental items are all verbal in form, and it would not be possible to give an exact parallel in nonverbal form.

When we turn to still more complex material requiring logical inference, the situation is even more completely and more throughly imbedded in a verbal context. I mention, for example, recent studies of the kinds of mathematical proofs given by college students in introductory logic courses (Kane, 1972; Moloney, 1972; Goldberg & Suppes, 1972). Here again, more sophisticated forms of reasoning can scarcely be investigated in a nonverbal context.

Language Comprehension in Deaf Children. The most salient missing aspect of the analyses of the language of either deaf or retarded children is the absence of serious attention to the semantics of their language and the identification of defects in semantics, either in terms of comprehension or production. The problems of identifying difficulties of comprehension may be approached at many different levels of detail.

An example of a medium level of detail, without a really satisfactory underlying theory, may be given from some research conducted in the institute on the written language comprehension of deaf students. This example applies the kind of regression methods we have used extensively for the analysis of relative difficulty of exercises in elementary mathematics (Suppes, Hyman, & Jerman, 1967; Suppes, Jerman, & Brian, 1968; Suppes & Morningstar, 1972). The regression models considered were developed and tested by Mrs. Jamesine Friend, who was coordinator of the project in computer-assisted instruction for deaf students in the institute from 1968 to 1971. This example deals with the analysis of difficulties deaf students encounter in reading and following written directions. The directions occur at the beginning of the computer-assisted instruction course "Language Arts for the Deaf," which was delivered to deaf students in residential schools and also to deaf students in day classes using teletype terminals connected by telephone lines to the institute's computer at Stanford. Some examples of the directions are the following. I show in capital letters the question and the example to which the question must be applied.

Example 1 (from Directions Lesson 1):

// WHICH IS THE FIRST WORD?
SOME DOGS ARE FRIENDLY.

Example 2 (from Directions Lesson 2):
// WHICH WORD COMES AFTER "VERY"?
MY TYPEWRITER IS VERY BIG AND HEAVY.

Example 3 (from Directions Lesson 9):
// WHICH LETTER COMES BEFORE "E"?
SILVER

Example 4 (from Directions Lesson 16):
// TYPE THE LAST TWO LETTERS.
MILLION

Example 5 (from Directions Lesson 25):
// TYPE THE NUMBER BELOW 4.

2	7
6	4
8	3

A number of structural features in these exercises affect their difficulty. In this kind of analysis we identify the structural features independent of any response data from the students, so that typical structural features are syntax, number of words, number of characters, and so forth. Variables of this kind have been used as structural features to predict the relative difficulty of arithmetic word problems (Jerman, 1971; Loftus, & Suppes, 1972; Suppes, Loftus, & Jerman, 1969). Mrs. Friend identified 14 such variables in the context of the language arts exercises on following directions. The variables she tested are the following.

Variable X_1: 0 if the direction is imperative.
1 if interrogative.

Variable X_2: 0 if the direction is a simple sentence or a transform of a simple sentence.
1 if compound.

Variable X_3: Number of key words in direction. ("Key words" distinguish one direction from another within the same lesson. In Example 1 above, there is only one key word, "FIRST," whereas in Example 5, there are two key words, "LAST" and "TWO.")

Variable X_4: 0 if the position cue is named (as in WHICH LETTER COMES BEFORE "E"?).
1 if the position cue is described (as in WHICH LETTER COMES BEFORE THE LAST LETTER?).

Variable X_5: Number of words in the instruction.

Variable X_6: 0 if direction does not contain "above," "below," "under," "before" or "after."
1 if it contains "above," "below" or "under."
2 if it contains "before" or "after."

Variable X_7: Lesson number.

Variable X_8: Ordinal position of the exercise within the lesson.

Variable X_9: 0 if preceding exercise involved the same task.
1 if otherwise.

Variable X_{10}: Number of elements (words, letters, numbers) in the stimulus display.

Variable X_{11}: 0 if there are no critical distractors, i.e., distractors that would be correct responses if the direction from the preceding exercise were used.
1 if otherwise.

Variable X_{12}: Length of correct response (in characters).

Variable X_{13}: Number of distractors preceding the correct response.

Variable X_{14}: Number of characters in the stimulus display (spaces not included).

These 14 variables were applied to predict the mean probability of a correct response to each of 125 exercises in lesson pretests for a sample of some 300 students. To be explicit, the regression equation is first transformed, because in an ordinary additive regression probability is not necessarily preserved, and we can

TABLE 1.—Step-Wise Linear Regression for 125 Exercises on Following Directions

Step number	Variable number	Multiple R	Multiple R^2	Increase in R^2	F value for del.	Last regression coefficients
1	6	0.37960	0.14410	0.14410	20.7108	—0.01019
2	13	0.56850	0.32319	0.17910	32.2826	—0.01903
3	7	0.59690	0.35629	0.03310	6.2309	—0.04448
4	9	0.61200	0.37454	0.01825	3.4883	0.00387
5	10	0.61880	0.38291	0.00837	1.6261	—0.02949
6	14	0.62590	0.39175	0.00884	1.7131	0.00180
7	2	0.62880	0.39539	0.00364	0.6907	0.09375
8	8	0.65500	0.42903	0.03364	6.8437	0.00531
9	4	0.66430	0.44129	0.01227	2.5144	0.03370
10	15	0.66930	0.44796	0.00667	1.3822	0.00102
11	5	0.67070	0.44984	0.00188	0.3970	—0.01215
12	11	0.67130	0.45064	0.00081	0.1502	—0.00182
13	12	0.67160	0.45105	0.00040	0.0735	—0.00588
14	3	0.67190	0.45145	0.00040	0.0820	0.00952

get predictions of negative probabilities or probabilities greater than one. We have therefore customarily used the transformation

$$z_i = \frac{\log 1 - p_i}{p_i}.$$

The regression equation then assumes the following form in terms of the dependent variable z_i

$$z_i = \Sigma a_i x_i + a_0.$$

The results of the stepwise linear regression are shown in table 1. Nine of the variables account for 44 percent of the variance and the remaining five contribute little. (The square of the multiple correlation (R^2) is a measure of the percentage of variance accounted for by the model.) The most powerful variable is X_6, which deals with the inclusion or exclusion of certain prepositions. The relative difficulty deaf students have with prepositions is well known and familiar in the literature. The second most important variable is X_{13}, which deals with the number of distractors preceding the correct response. This variable corresponds closely to a serial position variable for the correct response. The other variables entering during the first nine steps of the regression, namely, variable X_7, X_9, X_{10}, X_{14}, X_2, X_8, and X_4, each contribute something, but do not make the dramatic contribution of variables X_6 and X_{13}.

Regression models of the kind just described are by no means a final answer to the theoretical problems of language production or recognition on the part of deaf students. They do provide a good first entry into the detailed study of comprehension. From the standpoint of constructing curriculum they can be especially useful in providing a practical technique for creating items of a given desired level of difficulty, for new items—questions or exercises—can be written such that they have specified values of the structural variables, and thus a predicted probability correct for a given reference population of students.

I return to theoretical remarks about the linguistic approach to cognition in the final section on the problem of synthesis.

Can There Be a Synthesis?

The four approaches to cognition sketched in this article would seem on the surface to be so diverse, both in their methodology and concepts, as well as in their range of actual application, as to offer little hope for a synthesis and the development of a unified theory of cognition in the immediate future. It would be a mistake to be too sanguine about the development of such a theory, but I do think there has been an increasing tendency in the past few years for the four approaches to come together.

This seems to be occurring under two headings. One is the emphasis in all approaches on the problems of language learning and behavior. An increasingly central interest in language is not the exclusive option of the linguistic approach, but is becoming dominant among behaviorists and Piagetians alike. The second unifying theme is the emphasis on process. Behaviorists have from the start been interested in the mechanisms of learning and the study of how these mechanisms work in the acquisition of cognitive concepts. The developmental approach of Piaget and others has not been overly successful in spelling out specific mechanisms of concept acquisition, but on the other hand the many structures they have attempted to describe that are essential to the development of a child's cognitive powers have naturally suggested the nature of some of the mechanisms involved. The information-processing approach is of course primarily process-oriented, and these remarks about process lead me to my final remarks about reaching for a detailed understanding of the grammar and semantics of children's speech.

Methods that provide detailed descriptions of the grammatical and semantic structure of children's speech are needed and will continue to be developed. What will have an even more powerful effect in deepening our grasp of cognition is the development of procedural grammars and semantics that yield not only a proper analysis of the structure of children's speech, but that also provide in first approximation the necessary mechanisms for generating the speech, both in its grammatical and semantical features. There is currently throughout the world an increasingly intensive focus on these matters on the part not only of psychologists interested in cognition and psycholinguistics, but also on the part of computer scientists concerned mainly with artificial intelligence, linguists concerned mainly with language, and philosophers and logicians concerned mainly with the theory of meaning and reference. It is perhaps not too optimistic to anticipate considerable scientific progress in the next decade. At the same time we can also hope that aspects of that progress will enable us to understand better the cognitive development of handicapped children and the ways in which we may facilitate their development, as well as to map the limitations of what we may hope to do in the best of possible circumstances.

References

Beier, E. G., Starkweather, J. A., & Lambert, M. J. Vocabulary usage of mentally retarded children. *American Journal of Mental Deficiency*, 1969, *73*, 927–934.

Blake, K. A., & Williams, C. L. Induction and deduction and retarded, normal, and superior subjects' concept attainment. *American Journal of Mental Deficiency*, 1968, *73*, 226–231.

Blount, W. R. *Retardate and non-retardate concept usage performance: Abstraction ability, number of referents and item familiarity. Papers and reports.* Vol. 1. Tampa, Fla.: University of South Florida, Institute III, Exceptional Children and Adults, 1970.

Bower, G. H. Application of a model to paired-associate learning. *Psychometrika*, 1961, *26*, 255–280.

Cartwright, G. P. Written language abilities of educable mentally retarded and normal children. *American Journal of Mental Deficiency*, 1968, *72*, 499–505.

Chomsky, N. *Language and mind.* New York: Harcourt Brace Jovanovich, 1972.

Estes, W. K. Learning theory and mental development. New York: Academic Press, 1970.

———. New developments in statistical behavior theory: Differential tests of axioms for associative learning. *Psychometrika*, 1961, *26*, 73–84.

Feigenbaum, E. A., & Feldman, J. (eds.). *Computers and thought.* New York: McGraw-Hill, 1963.

Furth, H. G. Linguistic deficiency and thinking: Research with deaf subjects 1964–1969. *Psychological Bulletin*, 1971, *76*, 58–72.

———. *Thinking without language: Psychological implications of deafness.* New York: The Free Press, 1966.

———. Research with the deaf: Implications for language and cognition. *Psychological Bulletin*, 1964, *62*, 145–164.

Geach, P., & Black, M. (eds.). *Translations from the philosophical writings of Gottlob Frege.* Oxford: Blackwell, 1966.

Goldberg, A., & Suppes, P. A computer-assisted instruction program for exercises on finding axioms. Technical Rept. No. 186, 1972, Stanford University, Institute for Mathematical Studies in the Social Sciences. *Educational Studies in Mathematics*, 1973, in press.

Graham, J. T., & Graham, L. W. Language behavior of the mentally retarded: Syntactic characters. *American Journal of Mental Deficiency*, 1971, *75*, 623–629.

Inhelder, B., & Piaget, J. *The early growth of logic in the child.* New York: Harper and Row, 1964. English translation by E. A. Lunzer and D. Papert.

Jerman, M. Instruction in problem solving and an analysis of structural variables that contribute to problem-solving difficulty. Technical Rept. No. 180, 1971, Stanford University, Institute for Mathematical Studies in the Social Sciences.

Jones, E. C. The feasibility of training mental retardates to use available channel capacity more economically. Final report, 1971, Frostburg State College, Maryland.

Kane, M. T. Variability in the proof behavior of college students in a CAI course in logic as a function of problem characteristics. Technical Rept. No. 192, 1972, Stanford University, Institute for Mathematical Studies in the Social Sciences.

Lenneberg, E. H., Nichols, I. A., & Rosenberger, E. F. Primitive stages of language development in mongolism. In *Disorders of communication.* Vol. 42. Research Publications ARNMD, 1964.

Loftus, E. F., & Suppes, P. Structural variables that determine problem-solving difficulty in computer-assisted instruction. *Journal of Educational Psychology*, 1972, *63*, 531–542.

Lovell, K., & Bradbury, B. The learning of English morphology in educationally subnormal special school children. *American Journal of Mental Deficiency*, 1971, *71*, 609–615.

Minsky, M., & Papert, S. *Perceptrons.* Cambridge, Mass.: MIT Press, 1969.

Moloney, J. M. An investigation of college student performance on a logic curriculum in a computer-assisted instruction setting. Technical Rept. No. 183, 1972, Stanford University, Institute for Mathematical Studies in the Social Sciences.

Montague, R. English as a formal language. In B. Visentini (ed.), *Linguaggi nella società e nella tecnica.* Milan, 1970.

Newell, A., Shaw, J. C., & Simon, H. A. Elements of a theory of human problem solving. *Psychological Review*, 1958, *65*, 151–166.

———, & Simon, H. A. *Human problem solving.* Englewood Cliffs, N.J.: Prentice-Hall, 1972.

Piaget, J. Discussion in J. M. Tanner and B. Inhelder (eds.), *Discussions on child development.* Vol. 4. New York: International Universities Press, 1960.

Semmel, M. I., Barritt, S., Bennett, S. W., & Perfetti, C. A. A grammatical analysis of word associations of educable mentally retarded and normal children. *American Journal of Mental Deficiency*, 1968, *72*, 567–576.

Sigel, I., & Hooper, F. H. (eds.). *Logical thinking in children based on Piaget's theory.* New York: Holt, Rinehart and Winston, 1968.

Stoll, E. Geometric concept formation in kindergarten children. Unpublished doctoral dissertation, Stanford University, 1962.

Suppes, P. Facts and fantasies of education. Technical Rept. No. 193, 1972, Stanford University, Institute for Mathematical Studies in the Social Sciences.

———. On the behavioral foundations of mathematical concepts. *Monographs of the Society for Research in Child Development*, 1965, *30*, 60–96.

———. Semantics of context-free fragments of natural languages. Technical Rept. No. 171, 1971, Stanford University, Institute for Mathematical Studies in the Social Sciences. In K. J. J. Hintikka, J. M. Moravcsik, & P. Suppes (eds.), *Approaches to natural language.* Dordrecht: Reidel, 1973, in press.

———. Stimulus-response theory of finite automata. *Journal of Mathematical Psychology*, 1969, *6*, 327–355.

———, Jerman, M., & Brian, D. *Computer-assisted instruction: Stanford's 1965–66 arithmetic program.* New York: Academic Press, 1968.

———, Hyman, L., & Jerman, M. Linear structural models for response and latency performance in arithmetic on computer-controlled terminals. In J. P. Hill (ed.), *Minnesota symposia on child psychology.* Minneapolis: University of Minnesota Press, 1967.

———, & Ginsberg, R. A fundamental property of all-or-none models, binomial distribution of responses prior to conditioning with application to concept formation in children. *Psychological Review*, 1963, *70*, 139–161.

———, ———. Experimental studies of mathematical concept formation in young children. *Science Education*, 1962, *46*, 230–240.

———, Loftus, E. F., & Jerman, M. Problem-solving on a computer-based teletype. *Educational Studies in Mathematics*, 1969, *2*, 1–15.

———, & Morningstar, M. *Computer-assisted instruction at Stanford, 1966–68: Data, models, and evaluation of the arithmetic programs.* New York: Academic Press, 1972.

Tarski, A. Der Wahrheitsbegriff in den Formalisierten Sprachen. *Studia Philosophica*, 1935, *1*, 261–405. English translation in A. Tarski, *Logic, semantics, metamathematics.* Oxford: Clarendon Press, 1956.

Woodward, M. Concepts of number of the mentally subnormal studied by Piaget's method. *Journal of Child Psychology and Psychiatry*, 1961, *2*, 249–259.

Zigler, E., & Balla, D. Critique of Luria's verbal deficiency theory. *American Journal of Mental Deficiency*, 1971, *75*, 400–413.