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From Semantic Restrictions to Reciprocal Meanings

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Abstract

This paper proposes a new approach to the interpretation of reciprocal expressions using the Strongest Meaning Hypothesis of Dalrymple et al. (1998). We propose a system in which reciprocal meanings are derived directly from semantic restrictions using the SMH, and characterize this derivation process. We present methods to construct a linguistic test for the availability of a reciprocal meaning, or otherwise to prove that a specific meaning is not available for reciprocals. These methods are then used to analyze two controversial reciprocal meanings.

Keywords STRONGEST MEANING HYPOTHESIS, RECIPROCAL EXPRESSIONS, SEMANTIC RESTRICTIONS

2.1 Introduction

The interpretation of reciprocal expressions (*each other*, *one another*) exhibits a remarkably wide variation, which is affected in intricate ways by the predicate in the scope of the reciprocal. For example, sentence (1) entails that each person in the group likes every other person in the group, while sentences (2) and (3) do not entail an analogous claim.

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(1) These three people like each other.

(2) The three planks are stacked on top of each other.

(3) The 3rd grade students gave each other measles.

In an attempt to explain this phenomenon, Dalrymple et al. (1998) (henceforth DKKMP) introduced the *Strongest Meaning Hypothesis* (SMH). According to this principle, the reading associated with the reciprocal in a given sentence is *the strongest available reading which is consistent with relevant information supplied by the context*. This allows sentence (2) to be felicitous even though it is impossible for each of the three planks to be stacked on top each of the other planks. A similar weakening occurs in (3), since one cannot get measles from more than one person.

DKKMP postulate an array of reciprocal meanings which the SMH has to choose from, independently of the SMH itself and the semantic properties of predicates. This paper proposes a new system for predicting the interpretation of reciprocals in a given sentence. In this system, the SMH is implemented as a mapping from semantic restrictions on the predicate's denotation into the interpretation of the reciprocal, with no independent assumptions about available reciprocal meanings. We present methods to construct a test for the availability of a reciprocal meaning, or otherwise to prove that a specific meaning is not available for reciprocals. These methods are then used to analyze two previously suggested reciprocal meanings.

2.2 Semantic Restrictions and Reciprocal Meanings

In this section we define the notion of *semantic restriction* and show its relevance in delimiting the range of interpretations available for a reciprocal in a given sentence. Then we define the notion of *reciprocal meaning*, imposing on it natural restrictions from generalized quantifier theory. We subsequently show that for every reciprocal interpretation there is exactly one minimal meaning that extends it, thereby proposing a method for attesting reciprocal meanings using natural language sentences. The implications of this method are studied in the next sections.

2.2.1 Notation

Let $R, R' \subseteq E^2$ be binary relations over E, let $A \subseteq E$ be a subset of E, and let $\alpha, \beta \subseteq \wp(E^2)$ be sets of binary relations over E. We use the following notation:

- ▷ The identity relation: $I \stackrel{def}{=} \{(x, x) \mid x \in E\}$.

- ▷ R restricted to A: $R|_A \stackrel{def}{=} R \cap A^2$.
- ▷ R restricted to A and disregarding identities: $R{\downarrow_A} \stackrel{def}{=} R|_A \setminus I$.
- ▷ $R \subseteq_A R' \iff R{\downarrow_A} \subseteq R'{\downarrow_A}$, and similarly for $R =_A R'$, $R \neq_A R'$, etc.
- ▷ $\alpha \subseteq_A \beta \iff \{R{\downarrow_A} \mid R \in \alpha\} \subseteq \{R{\downarrow_A} \mid R \in \beta\}$, and similarly for $\alpha =_A \beta$, $\alpha \neq_A \beta$, etc.
- ▷ $min(\alpha) \stackrel{def}{=} \{R \in \alpha : \forall R' \in \alpha\, [R' \subseteq R \Rightarrow R' = R]\}$
- ▷ Let X and Y be sets, and let $D \subseteq X \times Y$ be a binary relation. For any $x \in X, y \in Y$:
  - $D(x, y)$ holds if and only if $(x, y) \in D$, and
  - $D(x)$ is the image of x under D: $D(x) \stackrel{def}{=} \{y \in Y \mid D(x, y)\}$

2.2.2 Semantic Restrictions

We first take a closer look at the informal concept of 'relevant information' which is used by DKKMP in their formulation of the SMH. Clearly, not all contextual information allows weakening of the reciprocal meaning. Otherwise, according to the SMH by DKKMP, the two sentences in (4) below would not be contradictory, since the information given in the first sentence would cause the reciprocal in the second sentence to require weaker truth conditions.

(4) # John and Bill don't know each other. John, Bill and Dan know each other.

To eliminate such undesired consequences, we propose to only consider *semantic restrictions* of the binary predicate in the scope of the reciprocal, along the lines of Winter (2001). A semantic restriction of a binary predicate P over the domain of entities E is a set Θ_{P} of binary relations over E: $\Theta_P \subseteq \wp(E^2)$. This is the set of relations that are possible as denotations of the predicate. For example, the denotation of the predicate *stare at* is limited to relations that are also (possibly partial) functions, since one cannot stare at more than one person at a time. Therefore $\Theta_{\text{stare at}}$ is the set of binary relations over E which are (possibly partial) functions.

We consider reciprocal sentences of the form *NP P each other*, where *NP* denotes a set of entities and P denotes a binary relation R over entities. The denotation of the reciprocal expression *each other* is accordingly assumed to be a relation between sets of entities and binary relations. Obviously, the denotation of the reciprocal expression in a given sentence cannot be determined for binary relations outside the semantic restriction of P. Thus, given a semantic restriction Θ, the *interpretation* of a reciprocal expression relative to Θ is a binary relation

$\mathcal{I}_\Theta \subseteq \wp(E) \times \Theta$. The *reciprocal interpretation domain* of Θ, denoted RECIP_Θ, is the set of all possible reciprocal interpretations relative to Θ: $\text{RECIP}_\Theta \stackrel{def}{=} \wp(\wp(E) \times \Theta)$.

It is known that the SMH is most easily attested with spatial predicates such as *sit alongside* and *stand on top*. Very often the SMH does not affect kinship relations as well as some other types of relations. The following contrast demonstrates this:

(5) The two chairs are stacked on top of each other.

(6) #Ruth and Beth are each other's mother.

A weakening effect allows sentence (5) to be felicitous, but a similar effect does not occur in sentence (6), although world knowledge precludes both two-way stacking and two-way mothering. We conjecture that semantic restrictions are not always an exact representation of world knowledge, and are more refined for some classes of predicates than for others. The reasons for this differentiation are poorly understood and require further research.

2.2.3 Reciprocal Meanings

The interpretation of a reciprocal relative to a semantic restriction, as defined above, is a novel notion and central to our analysis of reciprocals in general. However, different *meanings* for reciprocals have been suggested and debated upon extensively in the literature. In contrast with a reciprocal interpretation, a reciprocal meaning is defined for *all* binary relations and not only for relations in a given semantic restriction. As a preliminary to our analysis of the meanings available for reciprocals, we propose a formal definition of the notion of reciprocal meaning. The definition captures the properties that a reciprocal meaning must have, though it does not require that the meaning manifest itself in an actual reciprocal expression.

A reciprocal meaning is a relation $\Pi \subseteq \wp(E) \times \wp(E^2)$. Thus, reciprocal meanings are all in the domain RECIP_Θ with $\Theta = \wp(E^2)$. We assume that a reciprocal meaning must be conservative on its first argument,[1] as expected of any natural language determiner (Keenan and Westerståhl, 1996). Furthermore, reciprocal meanings are never sensitive to relations between identical pairs.[2] In addition, all reciprocal meanings suggested so far in the literature are upward monotonic in the second argument,[3] and we expect this to be true in general. These three properties are all subsumed by the following single property of *argument*

[1]Formally, $\forall A \subseteq E, R \subseteq E^2\, [\Pi(A,R) \iff \Pi(A, R \cap A^2)]$

[2]Formally, $\forall A \subseteq E, R \subseteq E^2\, [\Pi(A,R) \iff \Pi(A, R \setminus I)]$

[3]Formally, $\forall R, R' \subseteq E^2\, [(\Pi(A,R) \wedge R \subseteq R') \Rightarrow \Pi(A,R')]$

monotonicity:

Definition 1 A binary relation $D \subseteq \wp(E) \times \beta$, where $\beta \subseteq \wp(E^2)$, is *argument-monotonic* if and only if the following holds:

$$\forall A \subseteq E\,[\forall R, R' \in \beta\,[(D(A,R) \wedge R \subseteq_A R') \Rightarrow D(A,R')]]$$

Argument monotonicity is therefore used as the underlying property of reciprocal meanings:

Definition 2 A *reciprocal meaning* over a domain E is a relation $\Pi \subseteq \wp(E) \times \wp(E^2)$ that is argument-monotonic.

For similar reasons to the ones listed above, we assume that like reciprocal meanings, reciprocal interpretations in natural language are also argument-monotonic.

2.2.4 When is a Reciprocal Meaning Attested?

When presented with a potential reciprocal meaning, we would like to find out in which settings we can test whether this meaning is indeed available. In other words: what semantic restrictions of binary predicates would allow us to attest a given reciprocal meaning? Formally, we define the notion of *congruence* between a reciprocal meaning and a reciprocal interpretation $\mathcal{I}_\Theta \in \text{RECIP}_\Theta$, for a semantic restriction Θ:

Definition 3 Let Θ be a semantic restriction over E. A reciprocal meaning Π over E is *congruent* with a reciprocal interpretation $\mathcal{I}_\Theta \in \text{RECIP}_\Theta$ if Π is a minimal reciprocal meaning that extends $\mathcal{I}_\Theta$. Formally, Π satisfies:

1. $\forall A \subseteq E, R \in \Theta\,[\mathcal{I}_\Theta(A,R) \iff \Pi(A,R)]$, and
2. Any reciprocal meaning Π' that satisfies 1, also satisfies $\Pi \subseteq \Pi'$.

Because of the semantic restrictions on the denotation of two-place predicates in natural language, we cannot always directly extract a meaning for a reciprocal expression using the truth-conditions of reciprocal sentences. Consider for instance the following sentence:

(7) Proposals 1 through n are similar to each other.

Given that the predicate *be similar* is symmetric, the interpretation of the reciprocal in (7) is in RECIP_{SYM} where SYM is defined by: $SYM = \{R \subseteq E^2 \mid \forall x,y \in E[R(x,y) \Rightarrow R(y,x)]\}$. Since (7) is true only if every proposal is similar to every other proposal, the interpretation of *each other* in (7) is the relation $\mathcal{I}^0_{SYM} \in \text{RECIP}_{SYM}$ defined by: $\mathcal{I}^0_{SYM} \stackrel{def}{=} \{(A,R) \in \wp(E) \times SYM \mid \forall x,y \in A[x \neq y \Rightarrow R(x,y)]\}$. This interpretation can be extended by at least two reciprocal meanings proposed in the literature: Both *Strong Reciprocity*[4] (SR) from Langendoen (1978) and *Strong Alternative Reciprocity*[5] (SAR) from DKKMP match. But SR is congruent with $\mathcal{I}^0_{SYM}$ while SAR is not. More generally, we claim that any meaning associated with reciprocals should be congruent with the interpretation of the reciprocal in at least one natural language sentence. In this case we say that this sentence *attests* the meaning in question.

According to the following two propositions, if $\mathcal{I}_\Theta$ is argument-monotonic, it is congruent with exactly one reciprocal meaning.

Proposition 1 *For every semantic restriction* Θ *over* E *and a reciprocal interpretation* $\mathcal{I}_\Theta \in \text{RECIP}_\Theta$*, there is at most one reciprocal meaning* Π *over* E *that is congruent with* $\mathcal{I}_\Theta$*.*

Proposition 2 *For every semantic restriction* Θ *over* E *and an argument-monotonic reciprocal interpretation* $\mathcal{I}_\Theta \in \text{RECIP}_\Theta$*, there exists a reciprocal meaning* Π *over* E *that is congruent with* $\mathcal{I}_\Theta$*.*

Here and henceforth, proofs are omitted in the body of the paper. Selected proofs can be found in the appendix.

By Propositions 1 and 2, for any semantic restriction Θ over E and an argument-monotonic reciprocal interpretation $\mathcal{I}_\Theta \in \text{RECIP}_\Theta$, there is a unique reciprocal meaning that is congruent with $\mathcal{I}_\Theta$. On the empirical side, this result means that when given a sentence with a reciprocal expression, such as sentences (1)-(3), when Θ is the semantic restriction of the predicate in the sentence, the important semantic decision concerns the *interpretation* of the reciprocal chosen from the domain RECIP_Θ. The meaning of the reciprocal can be uniquely determined by this choice. In the following section we propose a new way of choosing a reciprocal interpretation according to the SMH.

2.3 The Interpretation of the Reciprocal

We propose that the SMH is realized as a *local maximality* principle: a reciprocal sentence is consistent with models in which no pairs in the antecedent set can be added to the denotation of the predicate within its semantic restriction. Formally:

Definition 4 Let Θ be a semantic restriction over E. The *SMH-based interpretation* of the reciprocal is the relation $\mathsf{R}_\Theta \in \text{RECIP}_\Theta$, defined as follows:

$$\forall A \subseteq E, R \in \Theta \, [\mathsf{R}_\Theta(A, R) \iff \forall R' \in \Theta \, [(R \subseteq_A R') \Rightarrow (R =_A R')]]$$

[4] $\forall A \subseteq E, R \subseteq E^2 \, [SR(A,R) \iff \forall x, y \in A[x \neq y \Rightarrow R(x,y)]]$
[5] $\forall A \subseteq E, R \subseteq E^2 \, [SAR(A,R) \iff \forall x, y \in A[x \neq y \Rightarrow (R(x,y) \vee R(y,x))]]$

This definition allows a correct prediction of the meaning of sentences presented in DKKMP and analyzed there using their system. Let us review examples (1)-(3). According to our system, the interpretation of the reciprocal in each sentence is determined by the semantic restrictions of the predicate. In sentence (1), the predicate *like* has no restrictions: $\Theta_{\text{like}} = \wp(E^2)$. Hence, $\mathsf{R}_{\Theta_{\text{like}}}(A, R) \iff R \supseteq A^2 \setminus I$, i.e. the sentence is deemed true only if each person in the antecedent set likes each of the others. In sentence (2), we assume that the semantic restriction of the predicate *stack on top* is the set $\Theta_{\text{stack on top}}$ that includes all the relations $R \subseteq E^2$ such that R and R^{-1} are (possibly partial) functions, and R is acyclic. Consequently, $\mathsf{R}_{\Theta_{\text{stack on top}}}(A, R)$ holds if and only if the elements of A are arranged into one sequential stack, as expected. In sentence (3), the predicate *give measles* may only denote acyclic relations which are the inverse of a function: one cannot get measles twice or give measles before getting measles. Using this semantic restriction, we find that the sentence is predicted to be true if and only if each 3rd grade student is connected to each other 3rd grade student by the transitive and symmetric closure of the denotation of *give measles*. This is in fact the expected meaning of this sentence. Unlike DKKMP, this proposal also gives a correct prediction of the truth conditions for the following sentence:

(8) The pirates are staring at each other.

The system proposed by DKKMP expects this sentence to be consistent with *Intermediate Reciprocity* (Langendoen, 1978), which requires all pirates to be connected via the transitive closure of the *stare at* relation. However, as they observe, the actual truth conditions of this sentence match the weaker *One-way Weak Reciprocity*, which only requires that each pirate stares at some other pirate. In the present proposal, we derive this interpretation of the reciprocal assuming that $\Theta_{\text{stare at}}$ is the set of (possibly partial) functions over E.

From Definition 4, it is clear that R_{Θ} is argument-monotonic for any semantic restriction Θ. Therefore, by Propositions 1 and 2, for each semantic restriction Θ there is exactly one reciprocal meaning congruent with R_{Θ}. In the following section we use the proposed framework and the definition of R_{Θ} to examine the possibility of attesting two controversial meanings that have been suggested for reciprocals.

2.4 Predicting the Existence of Reciprocal Meanings

In this section we study the implications of our method for two meanings of reciprocals that were proposed in the literature. Section 2.4.1 shows two general lemmas that are useful in characterizing the semantic restriction Θ for which R_Θ is congruent with a given reciprocal meaning Π. In sections 2.4.2 and 2.4.3 we apply these lemmas in studying congruence with the reciprocal meanings *Weak Reciprocity* (Langendoen, 1978) and *Inclusive Alternative Ordering* (Kański, 1987).

2.4.1 Characterizing the Congruence Relation

In this section we present two lemmas which provide general methods for analyzing the possibility of attesting a given reciprocal meaning. Though presented here for finite domains, these lemmas are also provable for infinite domains, as long as the reciprocal meaning conforms to an additional (reasonable) requirement, which we do not elaborate upon here.

Lemma 3 below provides a characterization of the congruence relation between the interpretation R_Θ of a given semantic restriction Θ, and a given reciprocal meaning. This characterization may then be used to check which semantic restrictions attest a reciprocal meaning in question. If a natural language predicate with one of these semantic restrictions is found, it is then possible to devise a reciprocal sentence which attests the given meaning.

Lemma 3 *Let Θ be a semantic restriction over a finite domain E, and let Π be a reciprocal meaning over E. Then R_Θ is congruent with Π if and only if $\forall A \subseteq E\,[\mathsf{R}_\Theta(A) =_A min(\Pi(A))]$.*

The following lemma shows that in order to check whether there is *any* semantic restriction that attests a given reciprocal meaning Π, it is enough to check one semantic restriction determined by Π, which we denote M_Π: $M_\Pi \stackrel{def}{=} \bigcup_{A\subseteq E} min(\Pi(A))$.

Lemma 4 *Let Π be a reciprocal meaning over a finite domain E that is congruent with R_Θ for some semantic restriction Θ. Then Π is congruent with R_{M_Π}, where M_Π is the semantic restriction defined above.*

2.4.2 Weak Reciprocity

Weak Reciprocity (Langendoen, 1978) defines for any given domain E the reciprocal meaning WR specified by:

$$\forall A \subseteq E, R \subseteq E^2[WR(A,R) \iff \forall x \in A[\exists y \in A\,[y \neq x \wedge R(x,y)] \wedge \exists y \in A\,[y \neq x \wedge R(y,x)]]]$$

In words, WR requires that each member of the set A participates in the relation both as the first and as the second argument. WR was suggested in Langendoen (1978) as a possible reciprocal meaning. However, this view is rejected on empirical grounds by DKKMP, where it is claimed that all the examples in the literature that had been claimed to demonstrate WR are in fact consistent with other known reciprocal meanings as well. DKKMP point out that the predicates used in those examples are all symmetric. We show that according to the current system, it is in fact impossible to attest WR with *any* semantic restriction except for very small domains.

Proposition 5 *For a domain E such that $|E| \geq 6$, there is no semantic restriction Θ over E such that WR is congruent with R_Θ.*

Since WR is defined for any given domain E, showing that the reciprocal meanings it provides for some domains are unattestable disqualifies Weak Reciprocity as a generator of reciprocal meanings.

2.4.3 Inclusive Alternative Ordering

DKKMP include in their system the operator *Inclusive Alternative Ordering* (IAO) (Kański, 1987), defined by:

$$\forall A \subseteq E, R \subseteq E^2[IAO(A,R) \iff \\ \forall x \in A\,[\exists y \in A\,[x \neq y \wedge (R(x,y) \vee R(y,x))]]]$$

IAO is proposed in DKKMP as the weakest meaning available for reciprocal expressions. It requires that each member of the antecedent set participate in the relation as either the first or the second argument. IAO thus allows a "partitioning" of the antecedent set into subsets not connected by R. The following sentence is claimed by DKKMP to exemplify IAO:

(9) He and scores of other inmates slept on foot-wide planks stacked atop each-other.

This sentence is true if there are several disjoint stacks of planks, a configuration that is allowed by IAO but not by other reciprocal meanings in the system of DKKMP.

Using Lemma 3, we can characterize the semantic restrictions attesting IAO. Let Θ_{IAO} be the set of binary relations $R \subseteq E^2$ such that (1) R is anti-symmetric; and (2) there are no paths longer than 2 edges in the underlying undirected graph induced by R.

Proposition 6 *IAO is congruent with $\mathsf{R}_{\Theta_{IAO}}$.*

Θ_{IAO} is not the only semantic restriction Θ for which IAO is congruent with R_Θ. However, by Lemma 3, for any semantic restriction Θ such that IAO is congruent with R_Θ, $\forall A \subseteq E\,[\mathsf{R}_\Theta(A) =_A \mathsf{R}_{\Theta_{\mathrm{IAO}}}(A)]$. Consequently, for any semantic restriction Θ such that IAO is congruent with R_Θ, all relations in Θ must satisfy the conditions given for relations in Θ_{IAO}. In addition, Θ must allow any element in E to stand in the relation with any given number of other elements in E. We submit that although it is theoretically possible to construct a case for attesting IAO, a binary predicate with the sort of semantic restriction required for such a test is unlikely to be found in natural language.

We propose a different explanation to the truth condition of (9). We claim that the "partitioning" effect in (9) is external to the reciprocal and not part of its meaning. The following contrast exemplifies the effect of such "external partitioning":

(10) The planks are stacked atop each other.

(11) Planks 1, 2, 3, and 4 are stacked atop each other.

Sentence (10) is felicitous if there are four planks arranged in two stacks of two planks each. This is in contrast with the infelicity of (11) in the same situation. Winter (2000) observes that partitioning effects occur with plural definites, but not with proper name conjunction. We follow this line and claim that partitions in reciprocal sentences are external and not inherent to the reciprocal interpretation.

2.5 Summary

This paper presents a novel approach to the systematic analysis of reciprocal meanings according to the Strongest Meaning Hypothesis. The system we propose derives reciprocal interpretations directly from the operation of the SMH on the semantic restrictions of the predicate. The logical restrictions affecting reciprocal meanings were spelled out, and it was shown that they uniquely determine a meaning from an interpretation of the reciprocal. Principles for the examination of meanings and the construction of appropriate linguistic tests for attesting them were defined and exemplified, and some negative and positive conclusions on the availability of previously suggested reciprocal meanings were shown to follow from these criteria.

2.6 Appendix: Selected Proofs

Proposition 1 *For every semantic restriction* Θ *over* E *and a reciprocal interpretation* $\mathcal{I}_\Theta \in \text{RECIP}_\Theta$*, there is at most one reciprocal meaning* Π *over* E *that is congruent with* $\mathcal{I}_\Theta$*.*

Proof Assume for contradiction that there are two reciprocal meanings Π_1 and Π_2 such that Π_1 and Π_2 are both congruent with $\mathcal{I}_\Theta$. Then the relation Π_3, defined by $\Pi_3 \stackrel{def}{=} \Pi_1 \cap \Pi_2$ is also a reciprocal meaning. It extends $\mathcal{I}_\Theta$, and it is stronger than at least one of Π_1 and Π_2. Therefore at least one of Π_1 and Π_2 is not congruent with $\mathcal{I}_\Theta$, a contradiction. □

Proposition 2 *For every semantic restriction* Θ *over* E *and an argument-monotonic reciprocal interpretation* $\mathcal{I}_\Theta \in \textsc{recip}_\Theta$*, there exists a reciprocal meaning* Π *over* E *that is congruent with* $\mathcal{I}_\Theta$*.*

Proof Let Ω be the set of reciprocal meanings that extend $\mathcal{I}_\Theta$. First, we show that $\Omega \neq \emptyset$: Let $\Pi \subseteq \wp(E) \times \wp(E^2)$ be the relation such that

$$\forall A \subseteq E, R \subseteq E^2 \, [\Pi(A,R) \iff \exists S \in \mathcal{I}_\Theta(A) \, [S \subseteq_A R]]$$

Π is a clearly argument monotonic, and is therefore a reciprocal meaning. Π also extends $\mathcal{I}_\Theta$: $\forall A \subseteq E, R \in \Theta \, [\mathcal{I}_\Theta(A,R) \iff \Pi(A,R)]$. The left-to-right implication trivially follows from the definition of Π, and the right-to-left implication follows from the definition of Π and the argument-monotonicity of $\mathcal{I}_\Theta$. Hence $\Omega \neq \emptyset$.

Let $\Pi_\cap \subseteq \wp(E) \times \wp(E^2)$ be the relation defined by:

$$\forall A \subseteq E, R \subseteq E^2 \, [\Pi_\cap(A,R) \iff \forall \Pi \in \Omega[\Pi(A,R)]]$$

$\Pi_\cap$ is argument monotonic, therefore it is a reciprocal meaning. By the definition of $\Pi_\cap$, there is no reciprocal meaning stronger than $\Pi_\cap$ that extends $\mathcal{I}_\Theta$. Therefore $\Pi_\cap$ is congruent with $\mathcal{I}_\Theta$. □

Lemma 3 *Let* Θ *be a semantic restriction over a finite domain* E*, and let* Π *be a reciprocal meaning over* E*. Then* R_Θ *is congruent with* Π *if and only if* $\forall A \subseteq E \, [\mathsf{R}_\Theta(A) =_A min(\Pi(A))]$.

Proof *"Only If"*: Suppose Π is congruent with R_Θ. We first prove that $\forall A \subseteq E \, [\mathsf{R}_\Theta(A) \supseteq_A min(\Pi(A))]$. Assume for the sake of contradiction that there is a set $B \subseteq E$ such that $\mathsf{R}_\Theta(B) \not\supseteq_B min(\Pi(B))$, and let R_0 be a relation such that $R_0 \in min(\Pi(B)) \setminus \{R\!\downarrow_B \mid \mathsf{R}_\Theta(B,R)\}$. We define the reciprocal meaning Π_1 as follows:

$$\Pi_1 \stackrel{def}{=} \Pi \setminus \{(B,R) \mid R\!\downarrow_B = R_0\}$$

Π_1 is indeed argument-monotonic: Let R, R' be relations such that $\Pi_1(B,R)$ and $R \subseteq_B R'$ hold. We need to show that $\Pi_1(B,R')$ holds. $\Pi(B,R)$ holds, hence by argument-monotonicity $\Pi(B,R')$ and $\Pi(B,R\!\downarrow_B)$ hold. In addition, $R_0 \in min(\Pi(B))$. Therefore $R\!\downarrow_B \not\subseteq R_0$, and consequently $R'\!\downarrow_B \neq R_0$. Hence $\Pi_1(B,R')$ holds.

Π_1 also extends R_Θ: By our choice of R_0, for any relation R such that $\mathsf{R}_\Theta(B,R)$ holds, $R\!\downarrow_B \neq R_0$. Hence, for all $R \in \Theta$:

$$\Pi_1(B,R) \iff \Pi(B,R) \wedge R\!\downarrow_B \neq R_0 \iff$$
$$\mathsf{R}_\Theta(B,R) \wedge R\!\downarrow_B \neq R_0 \iff \mathsf{R}_\Theta(B,R)$$

We conclude that Π_1 is stronger than Π and extends R_Θ. Therefore Π is not congruent with R_Θ, contradicting the assumption. This concludes the proof that $\forall A \subseteq E\,[\mathsf{R}_\Theta(A) \supseteq_A min(\Pi(A))]$.

Let us now show that also $\forall A \subseteq E\,[\mathsf{R}_\Theta(A) \subseteq_A min(\Pi(A))]$. Let $A \subseteq E$ be a set and R be a relation such that $\mathsf{R}_\Theta(A,R)$ holds. Π extends R_Θ, therefore $\Pi(A,R)$ holds. Let S be a relation such that $S \in min(\Pi(A))$ and $S \subseteq R$. S surely exists since the domain is finite. $\mathsf{R}_\Theta(A) \supseteq_A min(\Pi(A))$, therefore $\mathsf{R}_\Theta(A,S)$ holds. Hence, by the definition of R_Θ, $R \subseteq_A S$ holds. Therefore $S =_A R$, and thus indeed the inclusion holds. This concludes the proof of the "only if" direction.

"If": Suppose the right-hand-side holds. We show that the two conditions for congruence with R_Θ hold for Π.

1. Π extends R_Θ:
   (a) $\forall R \in \Theta\,[\mathsf{R}_\Theta(A,R) \Rightarrow \Pi(A,R)]$: Let $R \in \Theta$ be a relation such that $\mathsf{R}_\Theta(A,R)$ holds. Then by the supposition, there is a relation S such that $S \in min(\Pi(A))$ and $R =_A S$. By the argument-monotonicity of Π, $\Pi(A,R)$ holds.
   (b) $\forall R \in \Theta\,[\Pi(A,R) \Rightarrow \mathsf{R}_\Theta(A,R)]$: Let $R \in \Theta$ be a relation such that $\Pi(A,R)$ holds. Let S be a relation such that $S \in min(\Pi(A))$ and $S \subseteq R$. S surely exists since the domain is finite. By the supposition, there is a relation $T \in \Theta$ such that $T =_A S$ and $\mathsf{R}_\Theta(A,T)$ holds. $T \subseteq_A R$, therefore by argument-monotonicity of R_Θ over Θ, $\mathsf{R}_\Theta(A,R)$ holds.
2. Let Π_1 be a reciprocal meaning that extends R_Θ. We show that $\Pi \subseteq \Pi_1$ holds: Let $A \subseteq E$ be a set and R be a relation such that $\Pi(A,R)$ holds. Let S be a relation such that $S \in min(\Pi(A))$ and $S \subseteq R$. S surely exists since the domain is finite. By the supposition, there is a relation $T \in \Theta$ such that $S =_A T$ and $\mathsf{R}_\Theta(A,T)$ holds. Π_1 extends R_Θ, therefore $\Pi_1(A,T)$ holds. By argument-monotonicity of Π_1, $\Pi_1(A,R)$ holds, hence $\Pi \subseteq \Pi_1$.

□

Lemma 4 *Let Π be a reciprocal meaning over a finite domain E that is congruent with R_Θ for some semantic restriction Θ. Then Π is congruent with R_{M_Π}, where M_Π is the semantic restriction defined by $M_\Pi \stackrel{def}{=} \bigcup_{A \subseteq E} min(\Pi(A))$.*

Proof Assume for contradiction that Π is not congruent with R_{M_Π}. By Lemma 3, there is a set $A \subseteq E$ such that $\mathsf{R}_{M_\Pi}(A) \neq_A min(\Pi(A))$. We use the same lemma to contradict the congruence of Π with R_Θ. Consider the following two cases:

1. If there is a relation $S \in min(\Pi(A))$ such that $\mathsf{R}_{M_\Pi}(A, S)$ does not hold, then by the definition of M_Π, $S \in M_\Pi$. Hence by the definition of R_{M_Π}, there is a relation $R \in M_\Pi$ such that $S \subsetneq_A R$. Let $B \subseteq E$ be a set such that $R \in min(\Pi(B))$. Since Π is congruent with R_Θ, by Lemma 3 $\mathsf{R}_\Theta(B) =_B min(\Pi(B))$. Therefore there is a relation $R' \in \Theta$ such that $\mathsf{R}_\Theta(B, R')$ holds and $R' =_B R$. Since $R \in min(\Pi(B))$, $R \downarrow_B = R$. Therefore $R \subseteq R'$. It follows that $S \subsetneq_A R'$. Consequently, $\forall S' \, [S' =_A S \Rightarrow \neg \mathsf{R}_\Theta(A, S')]$. $S \in min(\Pi(A))$, therefore $\mathsf{R}_\Theta(A) \neq_A min(\Pi(A))$.
2. Otherwise, there is a relation $R \in M_\Pi$ such that $\mathsf{R}_{M_\Pi}(A, R)$ holds and $\forall S \in min(\Pi(A)) \, [R \neq_A S]$. Let S be a relation such that $S \in min(\Pi(A))$. Let $B \subseteq E$ be a set such that $R \in min(\Pi(B))$. Π is congruent with R_Θ, therefore $\mathsf{R}_\Theta(B) =_B min(\Pi(B))$. Hence there is a relation $R' \in \Theta$ such that $R' =_B R$ and $\mathsf{R}_\Theta(B, R')$ holds. As above, $R \subseteq R'$. Since $\mathsf{R}_{M_\Pi}(A, R)$ holds, $R \nsubseteq_A S$ and thus $R' \nsubseteq_A S$. Since the domain is finite, there exists a relation $T \in \Theta$ such that $R' \subseteq_A T$ and $\mathsf{R}_\Theta(A, T)$. In addition, $\forall S \in min(\Pi(A)) \, [T \neq_A S]$. Hence $\mathsf{R}_\Theta(A) \neq_A min(\Pi(A))$.

In both cases, the conditions of Lemma 3 do not hold for Θ, and therefore Π is not congruent with R_Θ, contradicting the assumption. $\square$

Proposition 5 *For a domain E such that $|E| \geq 6$, there is no semantic restriction Θ over E such that WR is congruent with R_Θ.*

Proof According to Lemma 4, it suffices to show that WR is not congruent with $\mathsf{R}_{M_{WR}}$. We show a set $A \subseteq E$ and a relation $R \in M_{WR}$ such that $\mathsf{R}_{M_{WR}}(A, R)$ holds but $WR(A, R)$ does not hold. It follows that WR does not extend M_{WR}, hence it is not congruent with $\mathsf{R}_{M_{WR}}$.

Let $B \subseteq E$ be a set such that $|B| = 6$. We denote the elements of B by $\{a, b, c, d, e, f\}$. Let R be the following relation (See figure 1):

$$R \stackrel{def}{=} \{(a, b), (b, a), (c, a), (d, b), (e, c), (e, d), (e, f), (f, e)$$

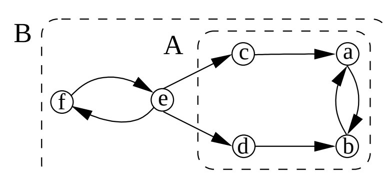

FIGURE 1: The relation R

It is easily verified that $R \in min(WR(B))$. Hence $R \in M_{WR}$. Let A be the set $A \stackrel{def}{=} \{a, b, c, d\}$. $WR(A, R)$ does not hold. We show that $\mathsf{R}_{M_{WR}}(A, R)$ holds.

Assume for contradiction that $\mathsf{R}_{M_{WR}}(A, R)$ does not hold. Then there is some relation $R_1 \in M_{WR}$ such that $R \subsetneq_A R_1$. Let $(z, w) \in A^2 \backslash I$ be a pair in $(R_1 \backslash R){\downarrow_A}$. By the definition of R, there is an element $t \in A$ such that $(z, t) \in R$. $(z, w) \notin R$, therefore $w \neq t$. By definition of M_{WR}, there is a set $C \subseteq E$ such that $R_1 \in min(WR(C))$. Let us define the relation $R_2 \stackrel{def}{=} R_1 \setminus \{(z, t)\}$. We show that $WR(C, R_2)$ holds, contradicting $R_1 \in min(WR(C))$. $WR(C, R_1)$ holds, therefore:

$$\forall x \in C \setminus \{z\} \, [\exists y \in C \, [y \neq x \wedge (x, y) \in R_2]] \wedge$$
$$\forall x \in C \setminus \{t\} \, [\exists y \in A \, [y \neq x \wedge (y, x) \in R_2]]$$

We only have left to prove that:

1. $[\exists y \in C \, [y \neq z \wedge (z, y) \in R_2]]$ and
2. $[\exists y \in A \, [y \neq t \wedge (y, t) \in R_2]]$

$(z, w) \in R_1$, therefore $(z, w) \in R_2$, hence formula 1 holds. $(z, t) \in R$, therefore $t = a$ or $t = b$. In both cases there is another pair (v, t) left in R. Therefore formula 2 holds. We conclude that $WR(C, R_2)$ holds, a contradiction. Thus the proof is complete. □

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