Simulation methodologies and open questions for acoustic combustion instability studies

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1. Motivations and objectives

Combustion instabilities are one well known example of thermoacoustics phenomena. These acoustics/combustion interactions are a major issue for many industrial programs (Krebs *et al.* 2002; Lieuwen & Yang 2005) as well as an important research topic in multiple groups today because the impact of these strong unstable modes on combustion chambers is important and their occurrence still difficult to predict (Figure 1).

From an academic point of view, simulating combustion instabilities constitutes a major challenge: many unstable modes appear only in complicated configurations and can not be reproduced simply in most academic laboratories or numerically because of their complexity (importance of geometrical details, kinetics, two-phase flow...). The example of azimuthal modes in annular chambers (as found in most gas turbine chambers) is useful to demonstrate the evolution of research in this field: these azimuthal modes have been known to be crucial in real gas turbines for years (Krebs *et al.* 2002; Lieuwen & Yang 2005) but their fundamental study has been quite limited for a long time (Pankiewitz & Sattelmayer 2003). After the first LES of these phenomena five years ago demonstrated that this problem contained many exciting fundamental aspects (Staffelbach *et al.* 2009; Wolf *et al.* 2010, 2012*b*), new experiments have appeared where simplified annular chambers with full optical access now permit to investigate these mechanisms with a level of precision that has never been seen before (Worth & Dawson 2012; Bourgouin *et al.* 2013).

In the field of simulation techniques for combustion instabilities, multiple paths are open today and it is interesting to discuss them and compare their capacities and limitations. That is the objective of this report, which presents a summary of present and future research simulation approaches for acoustic/flame instabilities.

2. Methods for combustion instability simulations: why LES is not enough

Figure 2 gives an overview of the methods used to study combustion instabilities. These methods range from analytical techniques to full scale LES on massively parallel systems. Compressible LES is the most accurate method for these phenomena as it can incorporate almost all physical phenomena involved in instabilities. This explains why LES has been developing rapidly for combustion instability problems in the past ten years (Sengissen *et al.* 2007; Fureby 2010; Kuenne *et al.* 2011; Wolf *et al.* 2012*a*).

Even though brute force LES has demonstrated its potential in this field (Moin & Apte 2006; Mahesh *et al.* 2006; Staffelbach *et al.* 2009; Wolf *et al.* 2009) and is the proper tool to reproduce unstable modes observed in real devices (something which is impossible with classical RANS codes), the need to understand combustion instabilities and not only to reproduce them in a simulation has led to the development of other tools.

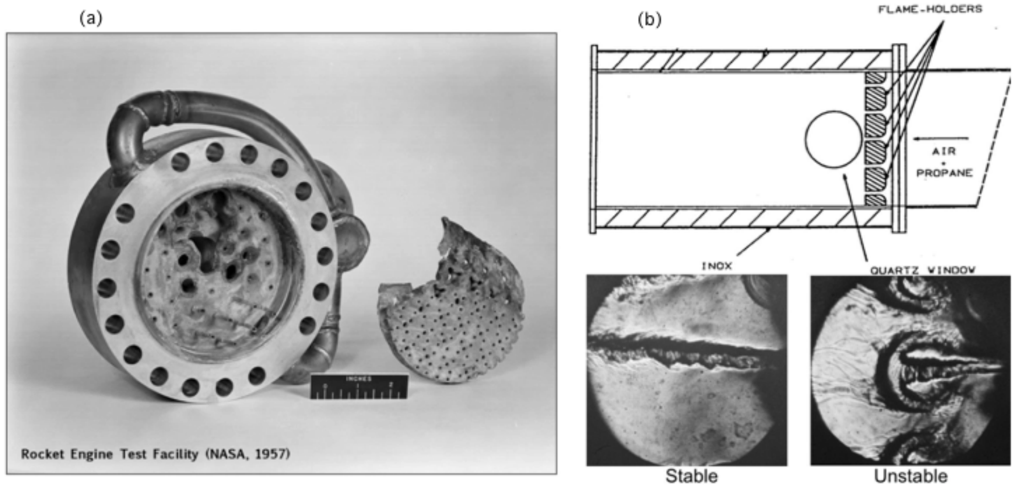

FIGURE 1. Combustion instabilities. (a): a rocket chamber destroyed by an unstable mode, (b): visualization of stable and unstable modes in a laboratory rig (Poinsot *et al.* 1987).

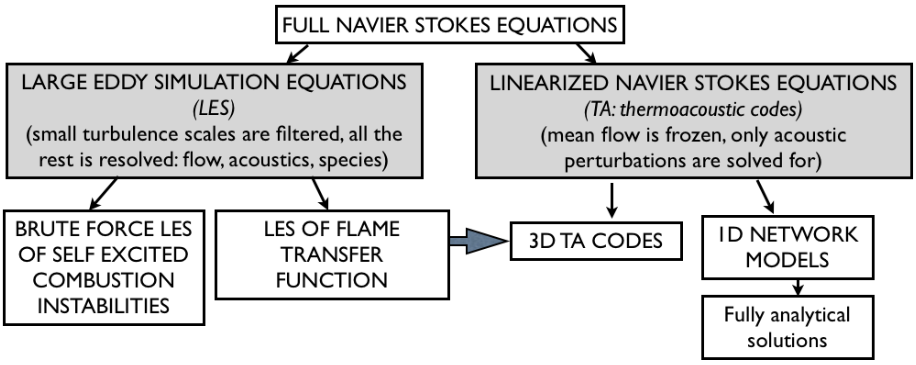

FIGURE 2. Methods to study combustion instabilities.

Some of the reasons that limit the development of a brute force LES strategy in thermoacoustics are listed below:

• Cost is obviously the first issue because LES for turbulent combustion instabilities require very large meshes and CPU resources. But the main question is not that one simulation requires a significant amount of time but that one simulation is usually totally insufficient to understand combustion instabilities. A very high number of regimes and configurations must be studied to analyze mechanisms and this is impossible using LES.

• The first parameters that have to be changed systematically to study instabilities are the fuel and air flow rates (their temperatures as well) which control the regime and the stability map. Most combustors are stable only in a limited parameter range and understanding how the system evolves from stability to instability is a major objective.

• Unfortunately, the stability map itself depends on other parameters and the most important one is the acoustic impedances (Figure 3). Each set of impedances imposed at the inlet and outlet of the chamber will lead to a different stability map. Repeating LES for all possible situations is not only expensive: it becomes also difficult from a numerical point of view because the compressor and turbine impedances are complex

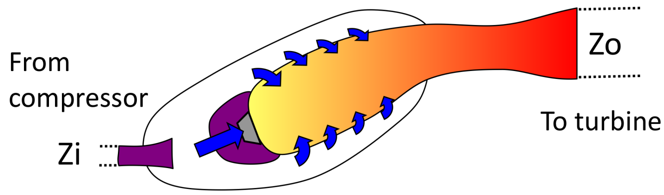

FIGURE 3. Acoustic impedances in a real gas turbine chamber. The unstable modes of the chamber cannot be computed without taking into account the acoustic effects of the compressor (left) and the turbine (right) which are usually quantified by their impedances Z_i and Z_o.

numbers with non-zero imaginary parts corresponding to time delays. This means that waves leaving the chamber are reflected back into the domain after prescribed delay times (depending on frequency), a mechanism that can be mimicked easily in frequency space but not simply in time solvers like LES codes. In a time domain code, impedances with non-zero imaginary parts can be accounted for by (a) storing the values of the wave amplitudes leaving a given outlet over a finite time segment, (b) using FIR (Finite Impulse Recursive) filters to convolute these wave amplitudes in order to evaluate the instantaneous amplitude of the reflected wave, and (c) imposing this incoming wave amplitude using characteristic boundary conditions. The whole procedure is not easy to implement and is often unstable. One alternative solution would be to compute the whole gas turbine (compressor/chamber/turbine) as proposed by Stanford University a few years ago (Schlüter *et al.* 2005) and CERFACS today so that all acoustic phenomena are captured by the LES but this seems out of reach for the moment.

As a result, LES today is viewed as one necessary tool for combustion instabilities that cannot be used alone and requires other simpler and sometimes more powerful tools. These tools are described below before discussing the major limitations and open questions associated with these methods.

3. Analytical techniques

The need to account explicitly for multiple parameters, to explore their effects rapidly on instabilities and to study control strategies has led to a renewed interest in analytical techniques for thermoacoustics. Analytical methods are developed to complement LES and other three-dimensional solvers and investigate instability phenomena using simplified formulations, which can lead to explicit solutions (Moeck *et al.* 2009, 2010; Morgans & Stow 2007). Recent examples show that the nature and the structure of azimuthal modes in annular chambers (Parmentier *et al.* 2012) can be clarified without any CFD simulation and that these results can guide the interpretation of complex simulation results.

4. Thermoacoustic solvers (TA codes)

The idea of using an acoustic formulation in which the mean flow is frozen and only acoustics are explicitly computed (Figure 4) has been exploited for a long time in the combustion community (Crocco 1951; Benoit & Nicoud 2005; Nicoud *et al.* 2007; Culick & Kuentzmann 2006). These thermoacoustic (TA) solvers take into account the complex geometry of combustors (and therefore are three-dimensional), however, avoid the solution of the full Navier Stokes equations, thereby offering an attractive compromise between cost and efficiency. TA codes (Figure 2) solve the wave equation in a reacting

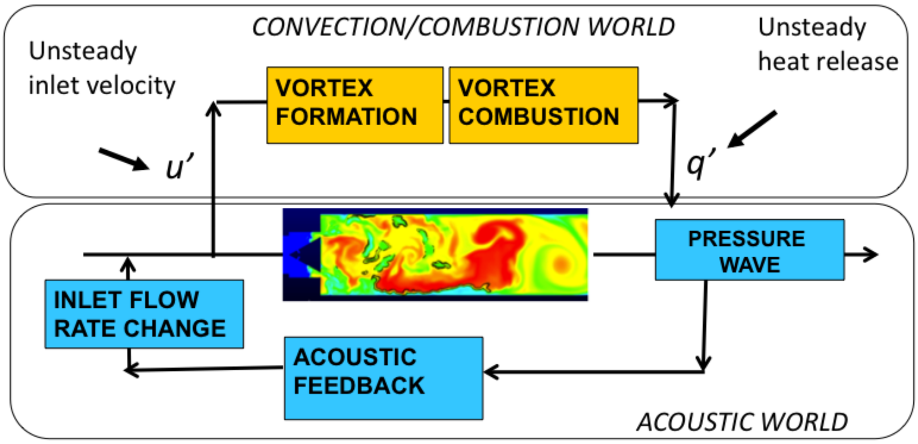

FIGURE 4. The paradigm used in TA codes. The instability loop is closed in two parts: a convective one containing also reaction and an acoustic one.

flow at zero Mach number. This is usually done in frequency space so that complex impedances can easily be accounted for.

In TA codes, flames are replaced by active acoustic elements. The acoustic emission of these elements is controlled by the unsteady reaction rate and is supposed to depend on the acoustic velocity upstream of the flame. The relationship between the unsteady acoustic velocity at a reference point and the unsteady reaction rate is called the Flame Transfer Function (FTF) and is one of the critical points in all TA formulations (Candel 1992; Ducruix *et al.* 2003; Paschereit *et al.* 2002; Truffin & Poinsot 2005; Tay-Wo-Chong & Polifke 2013) because it has to contain all phenomena that are not acoustic: convection, kinetics, heat release, turbulence, etc.

The FTF paradigm was first introduced by Crocco (1965) who expressed the unsteady reaction rate q' of a flame as a function of the upstream acoustic velocity u' at a reference point P:

$$q' = nu'(t - \tau), \tag{4.1}$$

where n is an interaction index and τ is a time delay. The parameters n and τ must include all non-acoustic mechanisms (for example vortex formation caused by the acoustic velocity surge, vortex convection followed by vortex combustion). In general, n and τ are functions of frequencies and, in many recent theories, of the forcing amplitude (Palies *et al.* 2011) so that Eq. 4.1 actually defines a FTF between u' and q' (when it is independent of amplitude) or a Flame Describing Function (when it also depends on amplitude).

Linking q' to u' in Eq. 4.1 is an incomplete formulation from an acoustic point of view: if the objective is to characterize the unsteady reaction rate as a function of the acoustic field, this can not be achieved by using only the velocity perturbation. Using pressure perturbations as well in Eq. 4.1 or using acoustic velocities at two reference points would allow for capture of all longitudinal acoustic waves. The choice of the unsteady acoustic quantities that should appear in the model of Eq. 4.1 is actually a critical issue that is further discussed in the next section.

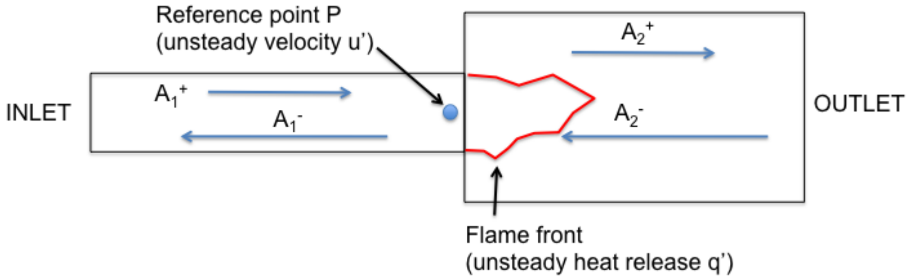

FIGURE 5. An illustration of wave propagation in a combustor. The A_1^- and A_2+ waves cannot influence the unsteady reaction for causality reasons.

5. Flame Transfer Function (FTF) models and causality

Interestingly, while the FTF (or FDF) paradigms have been used in numerous approaches, they have multiple limitations. The first one is readily observed in rocket engines where flames are often submitted to transverse instability modes that obviously cannot follow an equation similar to that of Eq. 4.1: axial velocity fluctuations play a minor role in these instabilities and other models must be introduced to capture these phenomena. A second and less known important limitation which often remains unquestioned is linked to causality or to the well-posedness of the whole concept: linking q' to the velocity fluctuations upstream of the combustor seems a plausible method because the combustor inlet velocity certainly affects the combustor unsteady reaction rate. This question, however, changes nature when it is considered from an acoustic point of view (Figure 5): in a combustion chamber, assuming one dimensional acoustic wave propagation, the pressure and velocity perturbations at each point of a duct (numbered $i = 1$ to 2) can be written as the real part of:

$$u'(x,t) = \left[A_i^+ \exp(jkx) + A_i^- \exp(-jkx)\right] \exp(-j\omega t). \tag{5.1}$$

If one considers the inlet duct, the quantity which affects combustion is the wave propagating from the inlet duct toward the chamber A_1^+. The wave leaving the chamber to propagate upstream (A_1^-) cannot affect the chamber for simple causality reasons: it is going in the wrong direction. In the standard FTF model (4.1), the quantity controlling q' is supposed to be $u'(P,t)$. This velocity signal at a reference point P in the inlet duct is the superposition of the two waves A_1^+ and A_1^-:

$$u'(P,t) = u'(x=0,t) = (A_1^+ + A_1^-) \exp(-j\omega t). \tag{5.2}$$

Therefore, the velocity signal $u'(P)$ contains information (A_1^+) that has the right causality link with q', and one (A_1^-) that has the wrong causality link with q': an acoustic A_1^- wave propagating upstream and away from the combustion chamber cannot affect the unsteady reaction rate and therefore its contribution should not be included in a FTF model. This argument is often used to justify the use of scattering matrices in two-port models for combustion instabilities: scattering matrices link outgoing waves (A_1^-, A_2^+) to the ingoing waves (A_2^-, A_1^+). In a scattering matrix, causality is naturally satistifed (Paschereit *et al.* 2002). It is not in the Crocco model (Eq. 4.1).

Despite the above mentioned shortcomings in the construction of FTF and FDF models, they have been widely used in the past and have produced successful results in multiple cases (Krebs *et al.* 2002; Dowling 1995; Nicoud *et al.* 2007; Motheau *et al.* 2013; Polifke *et al.* 2003). FTF can be obtained experimentally by using velocity measurements

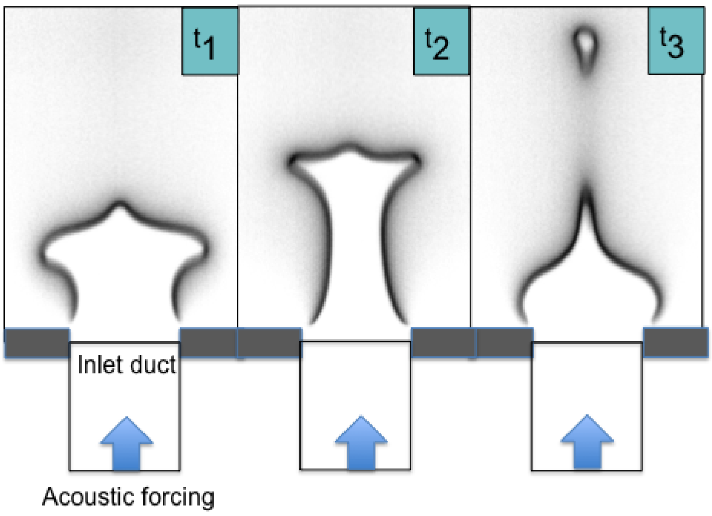

FIGURE 6. FTF measurement in a laminar configuration. Flame position (experimental result) during a forcing cycle at three instants (PhD thesis of Y. Dhue, IMFT).

(LDA or hot wire) and heat release data (usually through radical emission). They can also be computed and this is usually done using DNS (for small flames) or LES (for turbulent large scale systems). A typical example of FTF determination using DNS is given in Figure 6 where a laminar burner developed at IMFT is submitted to inlet forcing. The flame reacts to this forcing by producing the flame shapes observed in Figure 6. The transfer function is measured by correlating the local inlet velocity signal and the global unsteady reaction rate.

In a causal model, the same simulation would be used but processed differently, correlating the unsteady heat release with the incoming acoustic wave and not with the inlet velocity. These causal FTF approaches will require measuring the flame response when it is forced from upstream (through the A_1^+ wave) but also when it is forced from downstream using the A_2^- wave since these two waves are propagating toward the flame and lead to different flame responses.

6. Non linearity and non normality in thermoacoustics

The basic idea driving most thermoacoustic codes discussed in the previous sections is that combustion instability phenomena are linear mechanisms that can be described using linear acoustics equation and a FTF model. There are many indications that this approach could be to simplistic.

For example non-linear mechanisms (Culick 1976, 1994) are certainly present in flames and may also appear in thermoacoustics. In most cases, however, non linearity is not considered for the following reasons:

- Since the first objective is to completely suppress combustion oscillations, studying limit cycles is not as interesting as trying to suppress the linear growth of these modes. Moreover, the simplicity of linear approaches is a strong incentive to avoid injecting more complexity into the problem.
- Except in rocket engines where pressure oscillations can reach very large values,

acoustic waves remain within the linear regime in most combustors and there is no need to account for any non-linear effect for acoustics.

• The flame response itself, however, is known to be strongly non linear. This effect can be incorporated into TA codes using FDF as introduced by Palies *et al.* (2011) and Noiray *et al.* (2008). These non-linear formulations can be included in full three-dimensional TA codes (Silva *et al.* 2013). They permit capture of phenomena such as mode hopping, triggering, and limit cycles.

Non normality is a second issue: recent research in this field suggests that with a sufficiently large initial impulse (in combustion, such an impulse can come from a regime change for example), a thermoacoustic system can be submitted to self-sustained oscillations even when all modes are linearly stable (Juniper 2011; Balasubramanian & Sujith 2008). This process, known as triggering, is out of reach of linear thermoacoustics methodologies. One source of this problem is that the linear operator associated to the wave equation in a reacting flow can be non-normal (Nicoud *et al.* 2007), leading to a transient growth of modes that otherwise would be damped. Another difficulty is that turbulent fluctuations can feed energy into initial perturbations leading to non linear phenomena. This field of research is explored actively today (see for example Wieczorek *et al.* (2011)) but not in complex situations: in most cases, only laminar flames are considered.

7. MISO versus SISO descriptions

Most TA codes work on a SISO (Single Input Single Output) mode: the input is the air flow acoustic velocity and the output is the unsteady reaction rate. This is the basis of all FTF approaches, and it is well suited to perfectly premixed systems where only one stream containing both fuel and air feeds the chamber. In real flames, however, a MISO (Multiple Inputs Single Output) should be used: these flames are fed by (at least) two streams. Air is usually injected alone and fuel is added into the chamber using a separate line that has its own acoustic characteristics. From an FTF point of view, this implies that the unsteady reaction rate q' should be a function not only of the acoustic velocity in the air stream but also of the velocity in the fuel stream (which can be liquid). This is a complication that is seldom taken into account (Huber & Polifke 2009) but has been shown using LES to be quite important because independent fluctuations of fuel and air flow rates in a chamber lead to variations of local equivalence ratios (Lieuwen *et al.* 2001) that can excite unstable modes (Hermeth *et al.* 2013).

8. Uncertainty Quantification and combustion instabilities

The future evolution of thermoacoustic codes must include Uncertainty Quantification (UQ) capabilities. The combustion community (researchers and industries alike) is facing considerable difficulties predicting instabilities in a reliable way at the design stage: they are usually discovered once the engine is fired leading to costly modifications. Scientifically speaking, predicting instabilities is not a work in which a precision of a few percent are sought in terms of drag or lift. Instabilities correspond to bifurcations: either the engine is stable or it is not and will not pass the certification tests. This is a challenging field for UQ: parameters, that affect the bifurcation predictions, must be understood and controlled to make TA codes results useful. This type of problem is innovative for UQ methods: using UQ analysis near bifurcation points raises new and specific questions.

The uncertain parameters affecting an acoustic solver for combustion stability are similar to those found for classical CFD: geometrical details, boundary conditions (acoustic impedances on compressor and turbine sides, description of chemical kinetics, and flame response). Quantifying the effects of these uncertain parameters on the stability prediction will be a required feature of thermoacoustic codes.

9. Conclusions

LES has changed the simulation approaches used to study combustion instabilities. It is today the reference high-fidelity method used to explain mechanisms controlling instabilities. It is, however, insufficient to address all questions linked to the development, growth, and control of these mechanisms, so that thermoacoustic codes have to be developed at the same time. These codes split the problem in two parts: acoustics on one hand and combustion/convection on the other side. Despite the numerous fundamental questions linked to these simplifications, thermoacoustic codes provide information that cannot be obtained using LES because they are able to vary all parameters (regime, impedances, geometry) much faster than LES. More importantly, they allow investigation of the physics of the instability mechanisms, something which is often difficult using LES results.

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