Dynamic analysis of coherent processes in transitional and turbulent boundary layers

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1. Motivation and objectives

Visualizations of turbulent boundary layers show an abundance of characteristic, arc-shaped hairpin structures whose similar appearance suggests a common origin in a coherent dynamical process. While their formation and regeneration has been the subject of a number of theoretical models, a definite explanation of the underlying flow physics has remained elusive. Part of the difficulty of verifying any hypothesis is the chaotic environment of turbulence which renders the isolated analysis of the evolution of individual structures a challenging task. The present work introduces an algorithm, based on morphological operations, which enables the time-resolved identification and analysis of coherent processes. Application to a database of direct simulations data of transitional and turbulent boundary-layers sheds light on the flow physics of the hairpin process.

The concept of characteristic vortices that take the form of hairpins, earlier also often referred to as horseshoes, was introduced by Theodorsen (1955) more than six decades ago. The work postulated an arc-shaped vortical structure which rises from a vortex sheet following a localized velocity fluctuation in the normal direction. Conditional analyses of direct and large eddy simulations data of turbulent channel flow by Kim & Moin (1986) supported this model and showed that so-called bursting events during which fluid is ejected away from the wall coincide with the formation of hairpins. Adrian (2007) credited a hierarchy of aligned hairpin vortices with the generation of streaks in turbulent shear flow. Visualizations of transitional and turbulent boundary layers by Wu & Moin (2009) and Sayadi et al. (2013) show a profusion of hairpin structures at the outer edge of the boundary layer. While the hairpins get gradually more distorted and less organized with increasing Reynolds number as noted for instance by Schlatter et al. (2014), the flow statistics remain largely the same (Sayadi et al. 2013). Results from the tracking of vortical structures by Lozano-Durán & Jiménez (2014) in channel flow suggested that the hairpins may be the outcome rather than the cause of ejections of fluid away from the wall. In terms of their topology, the turbulent hairpins bear resemblance with the flow structures observed in the late-stages of natural transition to turbulence, which raises the question whether the underlying mechanism is as well equivalent.

Recently, evidence has emerged for a possible link between classical linear stability theory and the generation of hairpin vortices. Hack & Zaki (2014) showed that the varicose breakdown of streaks following inviscid secondary instability is accompanied by the formation of a hairpin structure. Resolvent analyses by Sharma & McKeon (2013) demonstrated a localization of the modes associated with very large structures in the vicinity of the critical layer based on the turbulent mean profile.

Several studies examined coherent structures in turbulence using methodologies such as proper orthogonal decomposition (POD) (e.g. Bakewell & Lumley 1967; Aubry et al. 1988). The method generates a set of linearly independent modes which are ranked by the fluctuation energy and hierarchically describe the spatio-temporal evolution of the
flow and was, for instance, applied by Moin & Moser (1989) to examine channel flow. Owing to the energy ranking and the orthogonality of the modes, a large number of modes may be necessary to represent flow structures that are only weakly energetic but nonetheless contribute significantly to the overall dynamics. The dynamic mode decomposition (DMD) proposed by Schmid (2010) alleviates this issue by allowing non-orthogonality of the modes and extracting flow features based on their single-frequency content. While DMD has proven to be an excellent tool in the study of periodic flows, including transition due to harmonic forcing, Sayadi et al. (2014) note that it may be less well suited for chaotic flows such as fully developed turbulence which lack a well-defined peak in the frequency spectrum.

The present work aims to shed light on the mechanism that drives the generation of hairpins in late-stage transitional and turbulent boundary layers. We present an approach for the investigation of dynamic processes based on topological criteria. The method allows the algorithmic identification of structures in broadband environments such as turbulent flows and preserves the temporal evolution of coherent processes. Furthermore, it is independent of the scale of the structures assuming they are supported by the underlying computational grid.

2. Dynamic analysis of flow processes

The methodology for the analysis of flow structures presented in the following makes use of concepts borrowed from the field of computer vision. Through a set of morphological operations, the structure of the flow field is converted into a discrete graph. The approach enables the identification and time-resolved sampling of coherent processes based on topological criteria.

2.1. Morphological thinning

Morphological thinning compresses the geometric information contained in a general binary image by extracting the medial axes of three-dimensional objects. In three-dimensional Euclidean space, the medial axis of a geometry is formally defined as the locus of the centers of all inscribed maximal spheres of the object where these spheres touch the boundary at more than one point (see e.g. Lam et al. 1992). Our method adopts the general concept presented by Lee & Kashyap (1994) and is briefly outlined below. The underlying theory is rooted in a field known as digital topology. An arbitrary geometry is represented in a discrete three-dimensional binary space $Z = [0, i_{\text{max}}] \times [0, j_{\text{max}}] \times [0, k_{\text{max}}]$. Objects are described by the subset $S \in Z$, which consists of all points $v$, also referred to as voxels, with value 1. The empty environment is represented by the complement $\bar{S}$, which contains all voxels with value 0.

The medial axis of an object is generated by successively removing voxels located at the object surface by setting their values to zero. Surface points are removed only if their absence does not change the topology. In formal terms, this objective can be represented using the Euler characteristic,

$$G(S) \equiv O(S) - H(S) + C(S),$$

where $O(S)$ is the number of objects, $H(S)$ is the number of holes and $C(S)$ is the number of cavities in $S$. In contrast to cavities, holes are not entirely surrounded by $S$. A necessary but insufficient criterion for topographical preservation is the invariance of the local Euler characteristic, $\delta G(S \cap N(v)) = 0$. Here, $N(v)$ describes the 26-point neighborhood of the voxel considered for removal, $v$ (see Figure 1). Following Morgenthaler (1981), a complete
characterization of topological preservation is achieved by additionally requiring that locally $\delta H(S \cap N(v))$ or $\delta O(S \cap N(v)) = 0$.

Additional constraints are necessary to avoid the elimination of an object when performing the thinning operation in parallel. Each thinning step is divided into six sub-cycles based on the type of a border point (N,S,W,E,U or B), corresponding to cubic indices of 13, 12, 10, 15, 4 and 21 (see Figure 1). Doing so, however, does not entirely prevent the removal or splitting of objects. In the present implementation, the topology is preserved by sequentially verifying the connectivity of each point. Let $R$ denote the voxels labeled for removal during a sub-step and $Q = S - R$, then $v \in R$ is removed if points in $R \cap N(v)$ and $Q \cap N(v)$ are still connected, which is equivalent to requiring that $O (\{R \cap N(v)\} \cup \{Q \cup N(v)\}) = 1$.

To demonstrate the outcome of the thinning procedure, the algorithm is applied to a test geometry. A trefoil knot is generated on a grid of size $[0,1] \times [0,1] \times [0,1]$ using a resolution of $128 \times 128 \times 128$ points. Voxels within a distance 0.05 from the curve are set to one with the remainder initialized to zero. An isosurface at $Z = 0.5$ is shown in Figure 2(a). The thinning procedure substantially compresses the geometric information by removing more than 99% of the object’s voxels. The resulting medial line is shown in Figure 2(b). It is worth pointing out that while the sample geometry is generated on a uniform grid, the thinning algorithm does not rely on grid isotropy. As such, it can be immediately applied to flow fields on anisotropic computational grids as commonly used in the numerical simulation of shear layers.
2.2. Graph generation and capturing of flow structures

While the medial axis representation appreciably reduces the information required to describe the topology, the binary voxel image is unsuited for the algorithmic identification of structures. The starting point towards this objective is the generation of a parametric description of the geometry by transforming the network of medial axes into a discrete graph. In a first step, the connectivity of the voxels representing the objects is evaluated as indicated in Figure 3 for a two-dimensional case. Numbers within each of the axis-voxels represent the total of nonzero points within its 26-point neighborhood. Voxels with more than two neighbors, i.e., with more than two nonzero voxels in their neighborhood, are defined as nodes.

Voxels of connectivity one and two make up the elements of links. By running along the \( j \)-th link, the grid coordinates \( x_j(k) \) of the constituting voxels are evaluated and stored as a function of the integer parameter \( k \). Finally, the start and end nodes of each link are evaluated and stored. Isolated voxels that are not connected to any other point are discarded. Furthermore, links which are only connected to one node are discarded when their length is below a prescribed threshold set at the expected minimum extent of the flow structures of interest.

The application of the above-described algorithm to a three-dimensional flow field yields databases of nodes and links representative of the structure of the flow field. Specific classes of flow features can be identified and isolated based on their topology by filtering the databases using topological criteria which can be specified in absolute or relative terms. An absolute criterion could, for instance, require the extent of structures in a certain dimension to be within a certain range while a relative topological criterion could require that the extent in a specific dimension be larger than that in another dimension. The possibility of specifying relative criteria makes the approach scale-independent and allows the identification of topologically similar structures across multiple scales.

An isolated hairpin structure is visualized in Figure 4 using an isosurface of \( Q = -\frac{1}{2}u_{i,j}u_{j,i} \). The link generated from the medial axis line is shown in black.

The analysis of dynamic processes builds on the static identification described so far. The evolution of a specific flow structure can be traced as it travels through the flow field by applying the identification method to successive snapshots of a time series. The outcome of this process is a database which contains the coordinates of each structure as a function of the solution time of the simulation.
3. Direct numerical simulations

The flow fields used in the analysis are obtained from the direct numerical simulations of complete K-type transition by Sayadi et al. (2013). The simulation code solves the compressible Navier-Stokes equations in conserved variables. At the considered free-stream Mach number, $Ma = 0.2$, compressibility effects are assumed to be negligible. In the following, length scales are normalized with the distance of the inflow location to the leading edge, $x_0 = 1$, and velocities are normalized with the free-stream convective velocity, $U_\infty = 1$. The viscosity is $\nu = 1 \times 10^{-5}$ and the Reynolds number at the inflow location is $Re_x = 10^5$. A detailed account of the computational setup is provided in Sayadi et al. (2013).

The simulations follow the experiments by Kachanov & Levchenko (1984) and the computations by Fasel & Konzelmann (1990) in triggering natural transition to turbulence through a controlled blowing- and suction-boundary condition at the wall at $1.36 \leq x \leq 1.56$. The forcing introduces a primary Tollmien-Schlichting (TS) wave with zero spanwise wavenumber and frequency $F = 10^6 \times \omega \nu / U_\infty^2 = 110$. The three-dimensionality required for breakdown to turbulence is provided by a temporally fundamental mode with spanwise wavenumber $\beta = 2\pi/\lambda = 41.88$, which is also introduced through the wall forcing. The TS wave is exponentially unstable at the considered Reynolds number and amplifies during the initial stage of the transition process.

The skin friction coefficient is presented in Figure 5. The fundamental K-type case is characterized by a relatively rapid transition process, and the skin friction curve begins to depart from the Blasius solution at $x \approx 2.5$, corresponding to a Reynolds number of $2.5 \times 10^5$. Also shown for reference is the subharmonic H-type breakdown which is considerably slower with transition initiated at $x \approx 4.5$. Both cases show a characteristic overshoot of the skin friction curves beyond the turbulent correlation. Statistical results show that the properties of the flow at this point are largely the same as in the turbulent flow farther downstream, see Sayadi et al. (2013). In the following, the structure-identification algorithm is applied to time series of approximately 250 fully resolved snapshots from the K-type case.
4. Transitional flow

The late stage of natural transition to turbulence in boundary layers is characterized by the formation of characteristic hairpin vortices. The structures can be visualized, for instance, through vortex identification criteria such as the \( Q \) criterion (Hunt et al. 1988) or the vortex surface field method (see e.g., Yang et al. 2016). In the following, the evolution of an individual hairpin is analyzed by applying the method described above to the late transitional region of the K-type case, \( 2.4 \times 10^5 \lesssim Re_x \lesssim 3.0 \times 10^5 \). In this context, it should be noted that while the precise shape of the flow structures is related to the chosen threshold of \( Q \), the former is of minor relevance for the following analyses. Throughout the work, the actual hairpin is used only as an indicator. Quantitative evaluations focus on physical quantities such as velocity fluctuations. To this aim, the instantaneous velocity field, \( \mathbf{u}(x, y, z, t) = (u, v, w)^T \), is separated into a temporal and spanwise mean, \( \overline{\mathbf{u}}(x, y) \) and a fluctuation component, \( \mathbf{u}'(x, y, z, t) \),

\[
\mathbf{u} = \overline{\mathbf{u}} + \mathbf{u}'.
\]

A time sequence showing the temporal evolution of a hairpin in late-stage transition is presented in Figure 6. Black and white isosurfaces respectively represent negative and positive streamwise velocity fluctuations. The visualization shows that the legs of the hairpin delimit an area of low-speed fluid whose intensity increases as the structure evolves in time. In the fourth frame, a secondary hairpin has formed upstream of the primary structure. The streamwise extent of the region of low-speed fluid shrinks and is in the last two frames confined to the center of the hairpin head. Visualizations of the normal fluctuations in Figure 7 indicate that the intensity is highest at the head of the hairpin. The shape of the area of positive \( v' \) fluctuations coincides with the hairpin structure in the \( Q \) field.

The normal instantaneous streamwise velocity profile at the center of the hairpin at \( t = t_0 + 5.14 \) along the vertical line indicated in the third frame of Figure 6 is compared in Figure 8(a) to the local mean flow. The high intensity of the distortion locally reduces the instantaneous normal velocity gradient to zero. Comparison of the fluctuations in all three dimensions provided in Figure 8(b) indicates that the magnitude of the streamwise component peaks at more than 50% of the free-stream velocity whereas the normal fluctuation reaches a maximum of approximately 15%.

The temporal evolution of the peak streamwise and normal fluctuations is presented in Figure 9(a). At \( t = t_0 \), \( u' \) already has an appreciable amplitude, while \( v' \) is on the order of 1%. At \( t - t_0 \approx 3.6 \), the amplitude of the \( u' \) fluctuation has increased to 0.25, and \( v' \)
begins to amplify rapidly until it reaches a maximum of about 28% of the free-stream speed. Insight into the nature of the growth mechanism is gained by repeating the plot using a logarithmic scaling for the ordinate, see Figure 9(b). The strong growth of $v'$ between $t - t_0 = 3.6$ and $t - t_0 \approx 5$ is represented by a straight line, showing that the initial stage of the hairpin process that generates the extremely high fluctuation levels is driven by an exponential instability.
Further evidence for the exponential nature of the growth mechanism is provided in Figure 10(a) which shows the hairpin with its critical layer, i.e., the set of locations where the streamwise instantaneous velocity is equal to the propagation speed of the patches of high-intensity fluctuations at the center of the hairpin. A cut-plane through the tip of the hairpin at $t = t_0 + 5.14$ is presented in Figure 10(b). In line with observations of inviscid streak instabilities in bypass transition (e.g., Andersson et al. 2001; Hack & Zaki 2014), the patches of positive and negative $v'$ are centered around the critical layer.

5. Turbulent flow

As the flow progresses from the transitional regime towards a turbulent state, visualizations show an abundance of arc-shaped vortical structures which resemble the hairpins observed in the late stages of transition (see e.g. Figure 7 in Sayadi et al. 2013). In the following, we apply the structure-identification algorithm to the turbulent portion of the flow field from the K-type simulation. The focus is on arc-shaped hairpin structures which are identified in isosurfaces of the $Q$ criterion. Since a single sample of a dynamic process obtained from the chaotic turbulent flow is of limited relevance, we introduce a dynamic averaging process which normalizes the structures with respect to their spatial extent.
but also with respect to their lifetime,

$$\langle u' \rangle (\tilde{x}, \tilde{y}, \tilde{z}, \tau) = \frac{1}{N} \sum_{n=1}^{N} G_5 (u', x, y, z, t) u'(x, y, z, t). \quad (5.1)$$

The operator $G_5$ applies a five-dimensional normalization to each sample of the coherent process. The peak amplitude of the streamwise velocity component is normalized such that $\max (|\langle u' \rangle|) \equiv 1$. Similarly, the spatial extent in the streamwise, normal and spanwise dimensions is regularized, $\tilde{x} \in [0, 1]$, $\tilde{y} \in [0, 1]$ and $\tilde{z} \in [0, 1]$. Finally, the lifetime of each individual hairpin structure is normalized, $\tau \in [0, 1]$. The outcome of the procedure is a dynamic average of the sampled process that preserves its temporal causality.

Preliminary results based on a limited number of samples and showing the dynamically averaged evolution of a turbulent hairpin are presented in Figure 11. Insight into the origin of the hairpin process is gained by choosing an initial time that precedes the formation of the structures. At $\tau = -0.3$, i.e. 0.3 normalized lifetimes before the hairpins become discernible in the flow, a patch of negative streamwise velocity fluctuations is observed at the center of the sampling domain. As the hairpins form and evolve, this patch is amplified and increases in extent. Normal fluctuations, visualized in Figure 12, are initially largely absent. During the lifetime of the hairpin, a localized ejection, represented by a patch of positive $v'$, is generated at the center.

The absolute extrema within the normalized sampling domain as a function of the normalized lifetime, $\tau$, are presented in Figure 13. Similar to the mechanism governing the hairpins in the late transitional region, the turbulent hairpins are associated with an amplification of negative $u'$ and negative $v'$. In contrast to classical transient growth, which is characterized by a delayed amplification of $u'$ following $v'$ (see e.g. Hultgren & Gustavsson 1981), the two components amplify simultaneously, suggesting the presence of an exponential growth mechanism.

The analysis also provides insight into the spatial organization of the structures. While Adrian (2007) postulated that the boundary layer would be populated by a hierarchy of hairpin processes, visual inspection of turbulent boundary layers by Schlatter et al. (2014) appeared to provide little evidence for their occurrence in regions closer to the wall. Traces of the streamwise and normal positions of identified structures are presented in Figure 14. Circles mark the initial and final positions where the structure is identified with lines indicating the path its center takes. Only structures which persist for
6. Conclusion and outlook

An algorithm based on computer vision for the identification and analysis of coherent structures was presented. The approach distills the topological information of the flow into a graph described by a set of discrete links and nodes. Application of topological filtering criteria allows the isolation of arbitrary flow structures. A main advantage of the method lies in its ability to identify equivalent flow processes across multiple scales. Application of the scheme to the late stages of K-type natural transition showed that the characteristic hairpin structures are the outcome of a powerful exponential instability mechanism. In the transitional region, the process generates very high fluctuation amplitudes on the order of 50% of the free-stream convective speed. The streamwise extent of the generated perturbations is relatively short and below the local boundary-layer thickness.

Preliminary results for turbulent flow using a dynamic averaging procedure that pre-
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Figure 14. Traces of identified hairpin structures in the transitional and turbulent boundary layer versus $x$ and $y$ (top frame) and $y^+$ (bottom frame). The gray line indicates the local boundary layer thickness.

serves the time-dependence of the process showed trends consistent with the transitional hairpins. The turbulent hairpin process is as well preceded by a patch of negative streamwise velocity fluctuations and causes the amplification of normal and streamwise perturbations on a relatively short streamwise length scale. With the exception of the near-wall region, $y^+ \lesssim 10$, the hairpin process was identified throughout the largest part of the turbulent boundary layer. Overall, the present results indicate that the instability mechanism which gives rise to late-stage transitional hairpins can indeed be considered a prototype for the hairpin process in turbulent shear flows.

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REFERENCES


