Doubly-conditional moment closure modeling of
turbulent nonpremixed combustion

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1. Motivation and objectives

The accurate treatment of extinction and reignition phenomena is thought to be an important factor in determining how flame stabilization occurs in practical, nonpremixed combustion systems (Peters (2000)). In the joint probability density function (PDF) approach (Pope (1985), Dopazo (1994)), extinction/reignition can be treated with improved mixing models (Xu & Pope (1999)). However, joint PDF methods employ a Monte-Carlo type solution procedure and thus become computationally expensive when many species increase the dimensionality of the modeled joint PDF evolution equation. Presumed PDF approaches such as flamelet modeling (Peters (1983), Peters (2000)) and conditional moment closure modeling (Klimenko (1990), Bilger (1993), Klimenko & Bilger (1999)) can be computationally tractable when many species are to be described, as is generally the case in problems of practical importance. First-order flamelet modeling and conditional moment closure modeling with singly-conditional, first-moment closure breaks down when extinction and/or reignition phenomena are present (Peters (2000), Klimenko & Bilger (1999)). The focus of the present paper concerns extensions of first-order conditional moment closure modeling to describe extinction/reignition.

Currently, a fundamental closure approximation in singly-conditional moment closure modeling is first-order closure of the average nonlinear chemical source terms, $\bar{w}$, conditioned on the mixture fraction, $\xi(t, x) = \eta$:

$$\langle \bar{w}(Y(t, x), T(t, x), \rho(t, x)) | \xi(t, x) = \eta \rangle \approx \bar{w}(<Y|\eta>, <T|\eta>, <\rho|\eta>)$$

where $Y$ are the vector of mass fractions of the reacting species, $T$ and $\rho$ are the temperature and density of the mixture, respectively, and $\eta$ is the sample space variable of $\xi$. The first-order closure approximation is not valid when fluctuations about the conditional means become significant. Figure 1 shows local extinction/reignition events in (i) $\xi$ space from experiments of a turbulent methane/air reacting jet (Barlow & Frank (1998)) and (ii) in scalar dissipation rate space at the stoichiometric value of $\xi$ from a direct numerical simulation (DNS) of a single-step chemical reaction in grid turbulence (Sripakagorn (2000)). The scalar dissipation rate is $\chi \equiv 2D(\nabla \xi)^2$, where $D$ is the molecular diffusion coefficient. $\theta$ is the reduced temperature,$$

\theta \equiv \frac{T - T_\infty}{T_f - T_\infty},

with $T_f$ the adiabatic flame temperature and $T_\infty$ the reference temperature. The figure illustrates how extinction/reignition events appear as large fluctuations about $\langle \theta | \eta \rangle$.

Two strategies have been proposed to improve singly-conditional moment closure modeling (Klimenko & Bilger (1999)):

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(a) higher-order closures and/or
(b) use of additional conditioning variables.

Testing of (a) have already begun (Swaminathan & Bilger (1998), Mastorakos & Bilger (1998)). This paper introduces the use of a second conditioning variable as a more promising alternative to a second-order closure strategy. The presumption is that the extinction/reignition events, seen in Fig. 1 for example, can be described in the space of \( \chi \) (in addition to \( \xi \) and \( x \)).

The paper is organized as follows: In the next section, an existing second-order closure remedy is reviewed. This extension is compared to a new conditional moment closure model where the scalar dissipation rate is introduced as a second conditioning variable. In Sec. 3, a priori modeling comparisons are made with DNS experiments of a single-step, second-order, reversible reaction in grid turbulence. Finally, the two conditional moment closure modeling extensions are assessed and future directions outlined.

2. Combustion model

2.1. DNS experiment

Presently, the a priori combustion modeling comparisons are made with DNS experiments performed by Sripakagorn (2000). Briefly, fuel (F), oxidizer (O), and product (P) involved in a one-step, second-order, reversible reaction,

\[ F + O \xrightarrow{k/k} 2P \]

\( (\xi_{st} = 0.5) \) evolve in isotropic, homogeneous, and decaying turbulence. The flow is incompressible. The forward reaction rate, \( k \), has an Arrhenius temperature dependence
with the equilibrium constant, $K$, held fixed. The reaction-rates are given by

$$\dot{w}(Y_F, Y_O, Y_P, \theta) = -A \exp \left( -\frac{Ze}{\alpha} \right) \exp \left[ -\frac{Ze(1 - \theta)}{1 - \alpha(1 - \theta)} \right] \left( Y_F Y_O - \frac{1}{K} Y_P^2 \right),$$

where $\dot{w}_F = \dot{w}$, $\dot{w}_O = \dot{w}$, and $\dot{w}_P = -2\dot{w}$. $A$ is the frequency factor (multiplied by density and divided by molecular weight, assumed equal for all species), $\alpha \equiv (T_f - T_{\infty})/T_f$ is the heat release parameter, $Ze \equiv \alpha T_a/T_f$ is the Zeldovich number, and $T_a$ is the activation temperature. The Schmidt number is 0.7 and Lewis numbers unity. Figure 2 shows the level of extinction/reignition from the DNS experiment with $A = 8 \times 10^4$/sec, $\alpha = 0.87$, $Ze = 4$, and $K = 100$ (cf. Sripakagorn (2000) for details of the simulation).

### 2.2. Second-order closure

A strong nonlinearity of the chemical source terms is due to the exponential temperature dependence of the Arrhenius function, $k = A \exp(-T_a/T)$. A second-order closure strategy in singly-conditional moment closure results from conditionally averaging a series expansion of the Arrhenius function (Mastorakos & Bilger (1998)):

$$\langle k | \eta \rangle \approx -A \exp \left( -\frac{\beta}{\alpha} \right) \exp \left[ -\frac{Ze(1 - \langle \theta | \eta \rangle)}{1 - \alpha(1 - \langle \theta | \eta \rangle)} \right] (1 + B) \quad (2.1a)$$

$$B = \frac{\alpha Ze \langle \theta^2 | \eta \rangle}{[1 - \alpha(1 - \langle \theta | \eta \rangle)]^3} \left[ \frac{1}{2} \left( \frac{\alpha/Ze}{[1 - \alpha(1 - \langle \theta | \eta \rangle)]} - 1 \right) \right] \quad (2.1b)$$

valid for $\theta'/(\langle \theta | \eta \rangle) < 1$. For the present case of a single-step reaction, the conditional averages of all species and temperature can be obtained from the single equation for the singly-conditional average of $Y_P$:

$$\frac{d}{dt} Q_P = \frac{\langle \chi | \eta \rangle}{2} \frac{\partial^2}{\partial \eta^2} Q_P - 2\dot{w}(Q_F, Q_O, Q_P, \langle \theta | \eta \rangle)(1 + B), \quad (2.2)$$

where $Q_F = \eta - Q_P/2$, $Q_O = 1 - \eta - Q_P/2$, and $\langle \theta | \eta \rangle = Q_P$. $B$ is given by Eq. (2.1b), where $\langle \theta^2 | \eta \rangle$ is to be taken directly from the DNS experiment. Details of the derivation are found in Klimenko & Bilger (1999). Briefly,

(a) Terms representing the diffusion of the conditional averages in physical space in Eq. (2.2) have been ignored ($\epsilon_Q$ closure).

(b) A closure hypothesis involving contributions to the balance of the conditional average by fluctuations about singly-conditional means has been invoked ($\epsilon_Y$ closure), allowing the resulting terms to be ignored for the present fully-homogeneous case (Klimenko & Bilger (1999)).

Assumption (a) is strictly valid only at high Reynolds numbers in non-homogeneous (shear) flows (Klimenko & Bilger (1999)), has been shown to be valid at low Reynolds numbers for a semi-homogeneous flow (Bushe et al. (1999)), and is exact for the present spatially degenerate (homogeneous, isotropic) case. The closure hypothesis invoked in (b) has been validated against a DNS database of a temporal mixing layer (Bushe & Bilger (1999)). Thus, we offer both singly-conditional moment closure with and without a second-order closure of the chemical reaction-rate as baseline comparisons to the strategy of multiply-conditioning. We refer to Eq. (2.2) as the \texttt{cmc1} model throughout the remainder of the paper. Setting $B = 0$ gives the standard (singly-) conditional moment closure model and is referred to as model \texttt{cmc0}.
Figure 2. DNS experiment of spatially isotropic, homogeneous, and decaying turbulence performed by Sripakagorn (2000). The reduced temperature is shown at $t^* = 0.5$ as a function of the local scalar dissipation rate, $\chi \equiv 2D(\nabla \xi)^2$. 
2.3. Second conditioning variable

In Klimenko & Bilger (1999), the derivation of the multiply-conditional moment closure equations assumes a multi-stream mixing problem which is characterized by multiple conserved mixing variables. Two distinctions are to be made with the present formulation using a second conditioning variable:

(a) Only a two-stream mixing process is considered, where the advantage of multiply-conditioning expectations (in addition to the mixture fraction) is to account for additional fluctuations about the singly-conditional means.

(b) The scalar dissipation rate, used as the second conditioning variable presently, is not conserved. The source term for $\chi$ is (Ruetsch & Maxey (1992))

$$\frac{L(\chi)}{\rho} \equiv \left( \frac{\partial}{\partial t} + \mathbf{u} \cdot \nabla - D \nabla^2 \right) \chi = -4D(\nabla \xi \otimes \nabla \xi) : \nabla \mathbf{u} - 4D^2(\nabla \xi \otimes \nabla \xi) : (\nabla \xi \otimes \nabla \xi) \quad (2.3)$$

for the present case of constant $\rho$ and $D$.

The choice of the scalar dissipation rate as the second conditioning variable follows, most recently, from the line of research pursued by Bushe & Steiner (1999). There, conditional source-term estimation is used to better approximate the chemical reaction-rates in physical space. In contrast, the present model performs combustion calculations in the space of $\xi$ (and $\chi$) and would convolve the conditionally-averaged solutions into physical space through the joint pdf of $\xi$ and $\chi$. Presently, we choose $\chi_0 \equiv \chi / F(\xi)$ (with $F(\xi) \equiv \exp\{-2[\text{erf}^{-1}(2\xi - 1)]^2\}$) as the second conditioning variable, further specializing the combustion modeling to the present DNS experiment where $\xi$ and $\chi_0$ have been found to be statistically independent (cf. Fig. 3).

Following the decomposition procedure (Klimenko & Bilger (1999)), the doubly-conditional moment closure equations with $\xi$ and $\chi_0$ as conditioning variables can be derived.
using the differentiation rules: 

\[
\frac{\partial Y_P}{\partial t} = \frac{\partial Q_P}{\partial t} + \frac{\partial Q_P}{\partial \eta} \frac{\partial \xi}{\partial t} + \frac{\partial Q_P}{\partial \chi_0} \frac{\partial \eta}{\partial t} + \frac{\partial Y_P'}{\partial t}
\]

\[
\nabla Y_P = \nabla Q_P + \frac{\partial Q_P}{\partial \eta} \nabla \xi + \frac{\partial Q_P}{\partial \chi_0} \nabla \chi_0 + \nabla Y_P',
\]

where \(Y_P'\) represents fluctuations about the doubly-conditional mean \(\langle Y_P|\eta, X \rangle = Q_P\) (with \(X\) the corresponding sample space variable for \(\chi_0\)). Substitution into the local, instantaneous equation for \(Y_P\) and conditionally averaging yields

\[
\left( \frac{d}{dt} + \alpha_0 \frac{\partial}{\partial X} \right) Q_P = \left( \frac{XF_0}{2} \frac{\partial^2}{\partial \eta^2} + \beta_0 \frac{\partial^2}{\partial X^2} + \gamma_0 \frac{\partial^2}{\partial \eta \partial X} \right) Q_P
\]

\[
-2\dot{\omega}(Q_F, Q_O, Q_P, \langle \theta|\eta, X \rangle)
\]

\[
\alpha_0 \equiv \left( \frac{L_{\chi_0}|\eta, X}{F} \right) + \frac{1}{F} \frac{\partial F}{\partial \eta} \gamma_0 + \frac{1}{2} X^2 \frac{\partial^2 F}{\partial \eta^2}
\]

\[
\beta_0 \equiv \langle D(\nabla \chi_0)^2|\eta, X \rangle
\]

\[
\gamma_0 \equiv \langle 2D \nabla \xi \cdot \nabla \chi_0|\eta, X \rangle
\]

where \(F(\eta) \equiv \exp\{-2[\text{erf}^{-1}(2\eta - 1)]^2\}\). Again, for the present simplified chemistry case, \(Q_F = \eta - Q_P/2, Q_O = 1 - \eta - Q_P/2, and \langle \theta|\eta, X \rangle = Q_P\). The relation \(\chi \equiv \chi_0 F(\xi)\) has been used in Eq. (2.3) to obtain the source term of \(L(\chi_0) \equiv \rho \chi_0\). \(Q_P\) in Eq. (2.4), henceforth termed model \(\text{cmc}2\), is to be distinguished from the singly-conditional means of models \(\text{cmc}0\) and \(\text{cmc}1\). With the exception of the convection term in \(X\) space (the second term on the left-hand side of Eq. (2.4a)), the same unclosed terms involving fluctuations about the conditional means result as detailed in the derivation in Klimenko & Bilger (1999). These terms are neglected following analogous arguments used in relation to Eq. (2.2). With doubly-conditioned moments, fluctuations are presumably reduced and their neglect a more accurate approximation. In particular, first-moment closure of the chemical reaction-rate in Eq. (2.4a) is expected to yield improved results over the analogous closure with singly-conditioned means (the \(\text{cmc}0\) model). It is to be shown that first-order doubly-conditioned moment closure modeling yields further improvements to \(\text{cmc}1\) as well.

### 2.4. Implementation issues

All coefficients in Eq. (2.2) and (2.4) are calculated directly from the DNS experiment. Binning of the DNS experimental data is done with equal points per bin as opposed to using equally incremented bin sizes. Cubic interpolation is used in \(\xi\) and \(\chi_0\) space, and cubic splines are used to interpolate the one-dimensional profiles \(\langle \chi|\eta \rangle\) and \(\langle Y_P'^2|\eta \rangle\) for the \(\text{cmc}0\) and \(\text{cmc}1\) models and the two-dimensional profiles \(\langle \alpha_0, \beta_0, \gamma_0 \rangle\) for the \(\text{cmc}2\) model in time where the DNS data are not available.

Boundary conditions for the singly-conditioned case (at \(\eta = 0, 1\)) are \(Q_P(t, 0) = Q_P(t, 1) = 0\). For the doubly-conditioned case, boundary conditions must be specified for \(\eta \in [0, 1]\) at \(X = X_{\text{min}}, X_{\text{max}}\) and for \(X \in [X_{\text{min}}, X_{\text{max}}]\) at \(\eta = 0, 1\). \(Q_P(t, 0, X) = Q_P(t, 1, X) = 0\); at \(X_{\text{min}} \approx 0\), equilibrium solutions are used while pure mixing solutions are used at some \(X_{\text{max}} \gg \chi_q\). \(\chi_q\) is the quenching value of the scalar dissipation rate. A value of \(X_{\text{max}} = \exp(3)\) was found to be large enough to not influence solutions. Cubic interpolation is also used to fit the DNS data at the initial time in \((\eta, X)\) space for the initial conditions.
Figure 4. Scalar-dissipation rate dependence of the coefficients in doubly-conditional moment closure modeling. $\alpha_0$ and $\beta_0$ (circles) and $\langle L(\chi) / \rho | \xi = \xi_{st}, \chi = X \rangle$ (note conditioning on $\chi$, not $\chi_0$) and $\langle D(\nabla \chi)^2 | \xi = \xi_{st}, \chi = X \rangle$ (squares). Filled symbols are corresponding rms information.

Figure 3 (i) shows a typical profile illustrating the $\eta$ dependence of $\langle \chi | \eta \rangle$ and $\langle \chi^2 | \eta \rangle$. $\langle Y^2 | \eta \rangle$ behaves similarly in $\eta$ space: zero at the boundaries of $\eta$ and maximum at $\eta = \xi_{st}$. Both $\langle \chi | \eta \rangle$ and $\langle Y^2 | \eta \rangle$ are taken directly from the DNS for the singly-conditional moment modeling calculations. Singly-conditional moment modeling calculations show no appreciable effect on the solutions with $\langle \chi | \eta \rangle$ modeled as $\langle \chi_0 \rangle F(\eta)$ (cf. Fig. 3). This modeling has been used in all cmc0 and cmc1 results to follow.

Figures 4 and 5 show the $\xi$ and $\chi_0$ dependence of the coefficients $\alpha_0$, $\beta_0$, and $\gamma_0$ (round symbols). Also shown are the analogous coefficients if $\chi$ were used as the second conditioning variable (square symbols): $\langle L(\chi) / \rho | \xi = \xi_{st}, \chi = X \rangle$ (analogous to $\alpha_0$, but with second conditioning on $\chi$ instead of $\chi_0$), $\langle D(\nabla \chi)^2 | \xi = \xi_{st}, \chi = X \rangle$ (analogous to $\beta_0$), and $\langle 2D \nabla \xi \cdot \nabla \chi | \xi = \eta, \chi = X \rangle$ (analogous to $\gamma_0$). With regard to Fig. 4, $\gamma_0$ and $\langle 2D \nabla \xi \cdot \nabla \chi | \xi = \eta, \chi = X \rangle$ are independent of $X$ and are not shown. With regard to Fig. 5, $\alpha_0$, $\beta_0$, and $\gamma_0$ are approximately independent of $\eta$, further corroborating the
Figure 5. Mixture fraction dependence of the coefficients in doubly-conditional moment closure modeling: $\alpha_0$, $\beta_0$, and $\gamma_0$ (circles) and $\langle L(\chi)/\rho \xi = \eta, \chi = X_1 \rangle$ (note conditioning on $\chi$, not $\chi_0$), $\langle D(\nabla \chi)^2 \xi = \eta, \chi = X_1 \rangle$, and $\langle 2D \nabla \xi \cdot \nabla \chi | \xi = \eta, \chi = X_1 \rangle$ (squares). Filled symbols are corresponding rms information.
independence of $\chi_0$ and $\xi$. Doubly-conditional moment modeling calculations with and without the cross derivative term, $\partial^2 Q_p/\partial \eta \partial X$ (corresponding to the assumption of $\gamma_0 = 0$), shows no appreciable effect on the solutions. Hence, only values for $\alpha_0$ and $\beta_0$ need to be specified on the $X$ boundaries of the computational mesh. At $X_{\text{min}}$, both are set to zero; at $X_{\text{max}}$, $\alpha_0 = 0$ and a value for $\beta_0$ is extrapolated from a power law fit of the data (cf. Fig. 4).

3. Results and discussion

Figure 6 shows $\text{cmc}0$ (-----), $\text{cmc}1$ (-----), and $\text{cmc}2$ (-----) modeling results against the DNS data (symbols). Comparisons are made for increasing times from $t^* = 1/4$ (top row of subplots) to $t^* = 3/2$ (bottom row) and conditional on three representative values of the scalar dissipation rate: a relatively low value (left column of subplots), moderate (center column), and a relatively high value (right column). More precisely, the center column of subplots are conditioned on $X = \langle \chi_0 \rangle$, $X = \langle \chi_0 \rangle - \chi_0'$ in the left column (with $\chi_0' \equiv \sqrt{\langle \chi_0'^2 \rangle}$), and $X = \langle \chi_0 \rangle + \chi_0'$ in the right column of subplots. Filled symbols are doubly-conditioned rms information of $Y_P$ taken from the DNS experiment. As can also be seen from Fig. 2, the rms of $Y_P$ (filled circles in Fig. 6) increases with time (moving from the top row down in Fig. 6) and higher scalar dissipation rates (moving from the left column to the right in Fig. 6).

Comparing overall modeling performance:

(a) The doubly-conditional moment approach ($\text{cmc}2$) yields the most significant improvement over both singly-conditional moment closure approaches ($\text{cmc}0$ and $\text{cmc}1$) for times $t^* \lesssim 1/2$ and at relatively high scalar dissipation rate values (subplots (iii) and (vi) in Fig. 6). For all times, all three models are comparable at relatively low to moderate values of $\chi_0$, where the effects of extinction are not significant in the mean, and represent the data well.

(b) For increasing times, $t^* \gtrsim 3/4$, when reignition occurs in the mean (globally), the doubly-conditional model slowly converges to the $\text{cmc}0$ and $\text{cmc}1$ results and diverge from the data (witness subplots (ix) and (xii) in Fig. 6). This implies that the $\text{cmc}2$ model predicts reignition too soon (or, alternatively, the level of extinction is underpredicted).

With regard to (a), the mean value of $\chi_0$ never exceeds $\chi_q$ in the DNS experiment, hence the singly-conditional modeling results could never predict any extinction events. $\text{cmc}2$ modeling accounts for the fluctuations of $\chi_0$, which can exceed $\chi_q$ by over a factor of 5 (cf. $t^* = 1/4$ in Fig. 2), and thus can yield the improved predictions seen in subplots (iii) and (vi) of Fig. 6 at the relatively large $X > \langle \chi_0 \rangle$. With regard to (b) ($t^* \gtrsim 3/4$ = bottom half of Fig. 6), when extinction begins to occur at even moderate values of $X = \langle \chi_0 \rangle$, the required diffusion in $X$ space is evidently insufficient to yield the reduction in the conditional means (seen in subplot (ix), for example). This may be due to the neglect of the influence of the fluctuations of the scalar dissipation rate, $\beta_0$, the coefficient of the diffusion term in $X$ space (cf. Eq. (2.4)). The present doubly-conditional moment closure model also neglects fluctuations about $\alpha_0$ and $\gamma_0$, which could also contribute to diffusion in scalar dissipation rate space.

Shortcomings of the $\text{cmc}2$ model are due to the fluctuations (rms of $Y_P$) seen in Fig. 6 not accounted by the fluctuations of $\xi$ and $\chi_0$. Accounting for fluctuations about $\alpha_0$, $\beta_0$, and $\gamma_0$ may represent a fruitful course of action to improve results further. However, a second-order closure strategy (in conjunction with second-conditioning) cannot be used to improve the doubly-conditional moment closure modeling results where required. This
Figure 6. Modeling comparisons: Symbols, DNS data; \( \cdot \cdot \cdot \), cmc0; \( \cdot \cdot \cdot \cdot \), cmc1; \( \cdot \cdot \cdot \cdot \), cmc2. Left column: conditioning on relatively low values of \( \chi_0 \) \((X = \langle \chi_0 \rangle - \sqrt{\langle \chi_0^2 \rangle})\), center column = conditioning at \( X = \langle \chi_0 \rangle \), right column: conditioning on relatively high values of \( \chi_0 \) \((X = \langle \chi_0 \rangle + \sqrt{\langle \chi_0^2 \rangle})\). Filled symbols are corresponding rms information.
can readily be seen from comparisons between \texttt{cmc0} and \texttt{cmc1} modeling results, where the second-order closure strategy for \( \bar{w} \) (used in the \texttt{cmc1} model) yields larger \( Q_P \) predictions. The reason that the \texttt{cmc1} model gives larger conditional averages than \texttt{cmc0} can be seen from Eq. (2.2), where \( B \) for the present case is greater than zero. In fact, \( B \) will always be greater than zero at \( \xi_\text{m} \) if \( Ze > O(1) \), a realistic constraint on the Zeldovich number. Hence, the analogous strategy in doubly-conditional moment closure modeling will actually cause results to deviate more quickly from the data.

Within the framework of the current modeling then, this leaves \( \epsilon_Q \) and \( \epsilon_Y \) closure to be reexamined (in both the singly- and doubly-conditional approaches). These terms represent laminar (\( \epsilon_Q \)) and turbulent (\( \epsilon_Y \)) diffusion of the conditional averages in physical space (Klimenko & Bilger (1999)). These effects can only be incorporated into the present homogeneous, isotropic case if global averaging is abandoned. For example,

\[
\langle D \nabla^2 Q_P | \eta \rangle - D \nabla^2 \langle Q_P | \eta \rangle \neq 0
\]

under conditional spatial-filtering. Terms of this form appear in both the \( \epsilon_Q \) term and the closed form of \( \epsilon_Y \) (Klimenko & Bilger (1999)). This again means a more local treatment of the modeling may be required (even for the present homogeneous, isotropic case).

4. Conclusions and future work

Singly-conditional moment closure (with and without a second-order extension of the chemical source-term) is unable to describe the extinction and reignition seen on average in the current DNS experiment. A new conditional moment closure strategy using the scalar dissipation rate as a second conditioning variable describes extinction in the mean well, but predicts the onset of reignition too early. Shortcomings of the model are thought to be due to the neglect of “local effects” such as the influence of the fluctuations of the dissipation rate of the scalar dissipation rate and/or the neglect of the spatial diffusion of the conditional means due to the global averaging procedure.

Continued \textit{a priori} studies to test these extensions are in progress. With regard to practical application, the doubly-conditional moment closure strategy in its present form may be able to account for a significant fraction of the extinction/reignition events seen in the jet-flame of Fig. 1 (a current benchmark data set for competing combustion models). A mapping of these events in scalar dissipation rate space would readily determine whether the present doubly-conditional moment closure strategy would yield good predictions. However, the necessary experimental data do not currently exist and so \textit{a posteriori} doubly-conditional modeling studies are also underway to predict only the existing singly-conditioned data.

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