Unsteady 3D RANS simulations using the v^2-f model

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1. Motivation & background

Recent increased computational power has led to interest in the simulation of time-dependent flows for problems ranging from noise prediction to fluid/structure interactions. The computational cost and the resolution requirements are mainly related to the inviscid flow structures induced by geometry and wall layers; nevertheless, turbulence plays a crucial role in establishing such flow structures.

Several approaches can be used to numerically simulate the behavior of unsteady flows: in the simplest, the Navier-Stokes equations are ensemble-averaged, converting turbulent fluctuations into Reynolds stresses (Reynolds-Averaged Navier-Stokes equations, RANS), while leaving the large scale, rotational motions to be resolved as unsteady phenomena. The large eddy simulation (LES) approach, on the other hand, employs a spatial averaging over a scale sufficient to remove scales not resolved by the particular grid being used. The subgrid scale turbulence is then modeled.

A practical difference is in the degree of mesh resolution required: LES resolves the larger eddies of the turbulence itself, whereas the unsteady RANS approach models the turbulence and resolves only unsteady, mean flow structures – primarily larger than the turbulent eddies. Consequently, LES typically requires much higher grid resolution, at least locally, and is therefore more costly. On the other hand, LES only models the subgrid turbulence structures and is universally applicable to the extent that a universal grid-independent subgrid model can be established.

In addition, LES resolves the complete range of scales of random motion, up to the cut-off frequency, while unsteady RANS aims to capture a single frequency (e.g. corresponding to coherent shedding) and to model the random motions using standard turbulence closures. Therefore, LES requires very long integration time to build a statistically-averaged solution; on the other hand, a few shedding periods are usually enough to obtain accurate phase-averaged solution with RANS, thus limiting its overall computational cost.

The objective of this work is to apply the RANS approach with the $\overline{v^2} - f$ turbulence model (Durbin, 1995) to the solution of the flow around a surface mounted cube. A complete and reliable experimental database is available (Hussein & Martinuzzi, 1996) and, in addition, LES simulations have been carried out with considerable success (Shah, 1998 and Krajnovic, 2000). This problem was one of the test case presented at the "Workshop on Large Eddy Simulation of Flows Past Bluff Bodies"; several LES calculations were compared, showing a high degree of accuracy (Rodi, 1997). RANS results were also included for comparison; they were obtained using simple turbulence models derived from k-ϵ and, generally, gave worse agreement with the measurements than did LES.

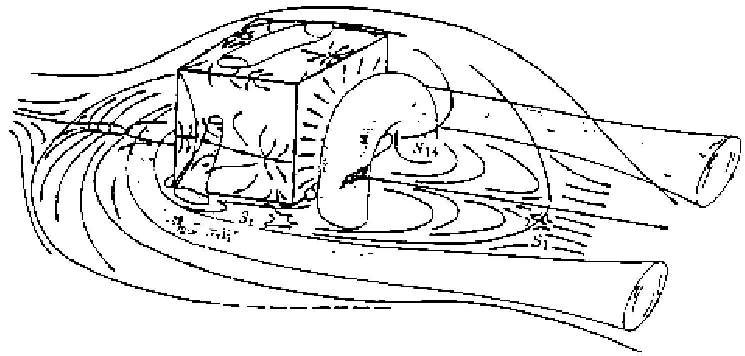

FIGURE 1. Schematic representation of the main flow features (Hussein & Martinuzzi, 1996).

2. Results

The flow around a cube exhibits characteristics common to other flows past three-dimensional obstacles: strong three-dimensionality of the mean flow, separation, and large scale unsteadiness. A schematic representation of the flow features (due to Hussein & Martinuzzi, 1996) is reported in Fig. 1: a strong horseshoe vortex and an arc-shaped vortex in the near wake are inferred from the analysis of the oil-flow patterns on the wind tunnel floor.

The (steady/unsteady) RANS equations are solved on a structured grid made up of 500,000 grid cells with clustering close to the walls to capture the near-wall turbulent regions (y^+ is less than one everywhere). The Reynolds number is 40,000 (based on the inlet bulk velocity, $u_b = 32m/s$ and the height of the cube, $h = 1m$). The domain size is the same as used by Shah (1998): $10h$, $2h$, and $3h$ in the streamwise, normal, and spanwise directions, respectively. The cube is located at $x = 3h$ from the inlet. A fully developed channel flow solution is used at the inlet, and solid walls are considered in the spanwise direction (this is the main difference with respect to the LES simulation by Shah (1998) where periodic conditions were employed).

The eddy viscosity is computed according to the $\overline{v^2} - f$ turbulence model (Durbin, 1995); the model requires the solution of three transport equations for turbulent quantities plus the solution of an elliptic equation. The model has a built-in realizability limiter (Durbin, 1996) and does not require any damping or wall function to correctly capture the near-wall turbulence (Durbin, 1991).

A second order discretization scheme (both in time and space) is used. Initially, a very large time step is employed leading to a steady solution; later the time step is decreased (to $\Delta t = 0.00156$ seconds) and a time accurate simulation is carried out. A coherent vortex shedding is obtained after ≈ 3 seconds, corresponding to ≈ 10 flow through times. Then, an averaged solution is computed on a period of ≈ 3 seconds, and in what follows it is referred to as unsteady solution.

Comparisons of the available experimental data and the steady and the unsteady solutions are presented; the LES results by Shah (1998) are also reported.

In Fig. 2 (left column) the streamwise velocity component (u/u_b) on the symmetry plane is reported. The experimental data (confirmed by the LES results) indicate a separation length of $\approx 1.6h$ downstream of the block; the steady solution overpredicts

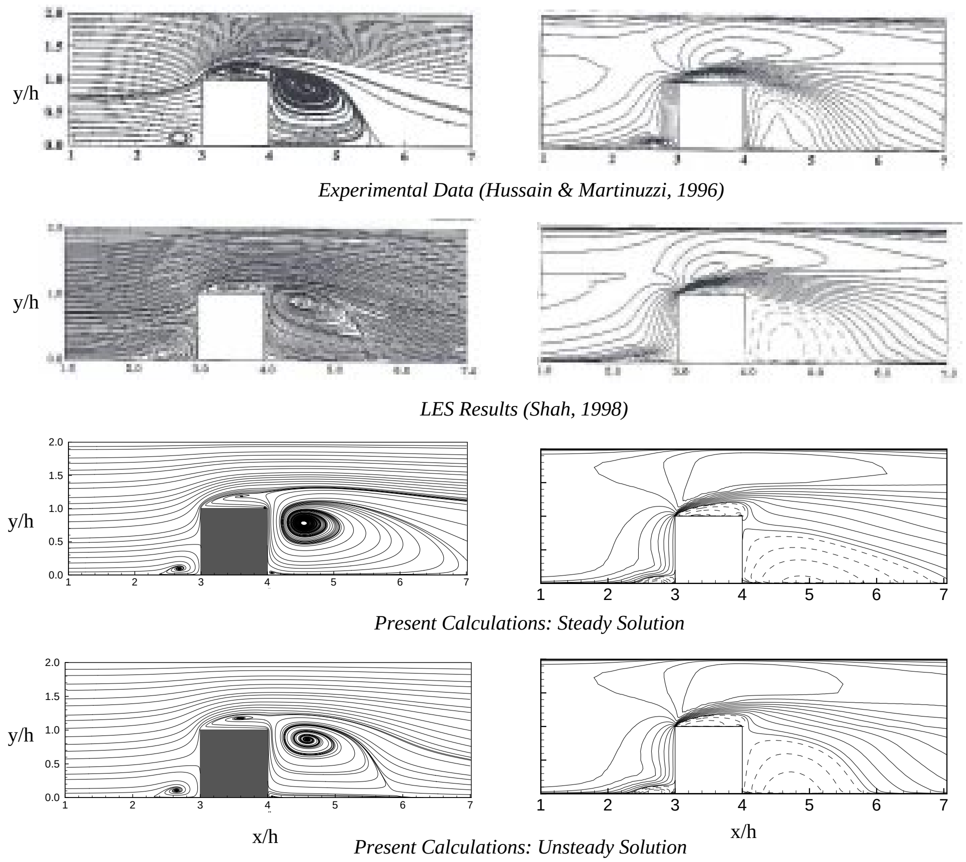

FIGURE 2. Streamlines (left) and streamwise velocity (right) on the symmetry plane. Dashed lines indicate negative velocity.

the extent by more than 100% ($3.3h$). On the other hand, the unsteady solution predicts a reattachment length of $\approx 1.9h$ in reasonable agreement with the measurements.

An analysis of the streamlines (Fig. 2, right column) suggests that the vortex cores are in the same locations, and the reattachment lengths (both upstream and downstream of the cube) are in remarkable agreement. The steady solution yields a too large recirculation area in the wake of the cube but a fairly good agreement elsewhere.

In Table I a summary of the length of the downstream and upstream recirculation bubbles is reported together with the experimental data and the numerical results presented in Rodi (1997), Shah (1998), and Krajnovic (2000).

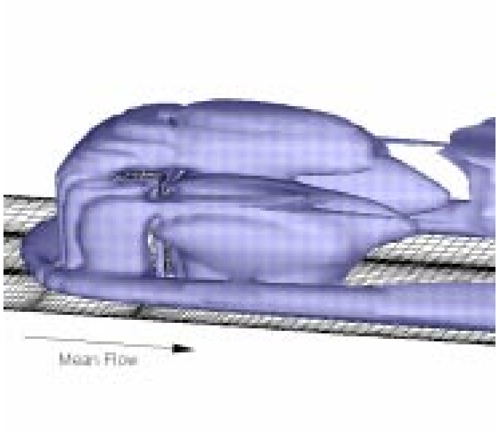

FIGURE 3. Visualization of the Horseshoe Vortex Using λ_2 (Jeong & Hussain, 1994)

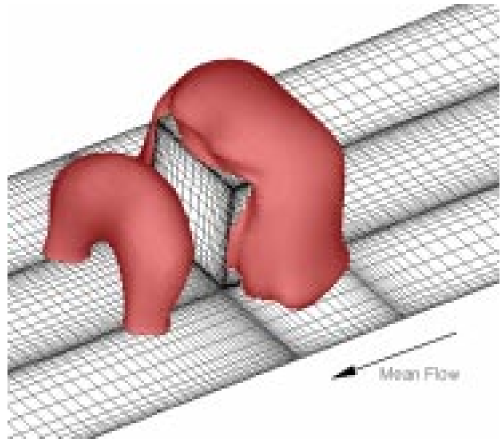

FIGURE 4. Visualization of the arch-vortex in the wake using an isobar surface.

Contribution	Model	x_F/h	x_{R_1}/h	x_{R_2}/h
Martinuzzi & Tropea, 1993	Experiments	1.040	1.612	-
Rodi *et al.*, 1997	LES	0.998	1.432	0.134
	RANS	0.950	2.731	0.252
Shah, 1998	LES	1.080	1.690	0.160
Krajnovic & Davidson, 1999	LES	1.060	1.380	-
Present Calculations	RANS Steady	0.640	3.315	0.310
	RANS Unsteady	0.732	1.876	0.204

x_F/h: upstream separation;
x_{R_1}/h and x_{R_2}/h: downstream primary and secondary recirculation.
Table I. Recirculation bubble lengths.

The LES calculations reported in Rodi (1997) used either the wall function approach in conjunction with the Smagorinsky sub-grid scale (SGS) model, or the dynamic SGS model; the latter gave better results, as reported in Table I. The calculations showed a very good overall agreement with the measurements. The steady RANS computations, on the other hand, were not accurate in capturing the length of the wake bubble; even the best results (reported in Table I under Rodi, *et al.*), obtained using a two-layer approach for the near-wall turbulence, severely overestimate the recirculation region. The LES results of Shah (1998) and of Krajnovic (1999), both obtained with the dynamic SGS model, confirms the reliability of LES in reproducing the mean flow behavior. The

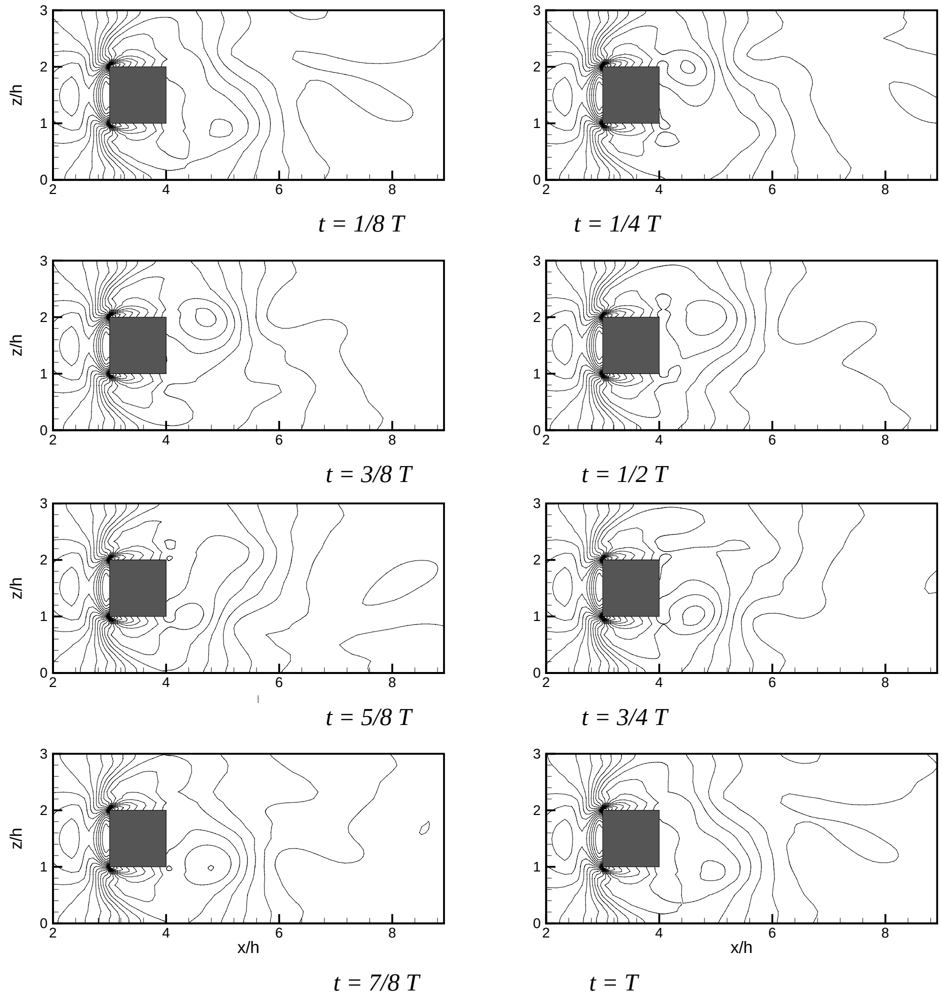

FIGURE 5. Time-history of the pressure distribution on the floor.

present time-averaged, unsteady computations are also in very good agreement with the experimental data, with only a slight overprediction of the wake reattachment length (x_{R_1}).

Two flow visualizations of the unsteady solution are reported. In the first (Fig. 3), the horseshoe vortex is represented by using the vortex-detection criterion of Jeong and Hussain (1995). The visualization of the arch vortex in the wake is performed using an isosurface of the pressure (Fig. 4).

In addition, the time evolution (during a period) of the pressure distribution on the wind tunnel floor is reported in Fig. 5. The flow is indeed periodic. The vortex shedding from the side of the cube induces an oscillation (yaw) of the arch vortex. One leg becomes stronger because of the vorticity contribution coming from the detached boundary layer on the cube side; it moves downward while a new vortex is shed from the opposite side. The shedding period is $T = 0.184$ seconds, corresponding to a Strouhal number of 0.17

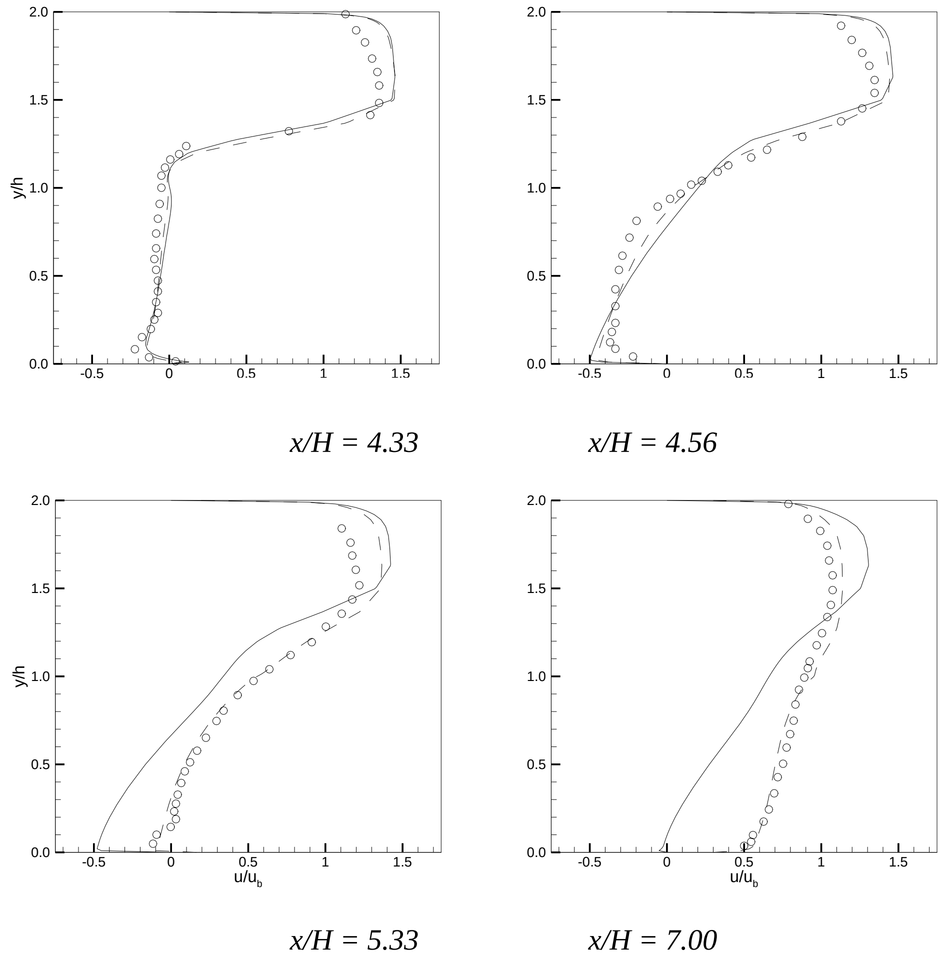

FIGURE 6. Streamwise velocity profiles in the symmetry plane. Symbols: ——— : steady solution; – – – – : unsteady solution; o: experiments.

(experimental value 0.145). No vortex shedding is observed for the boundary on the roof of the cube, and no unsteadiness related to the horseshoe vortex is seen.

The quantitative comparison between the computed results and the experimental data is reported in Fig. 6. The streamwise velocity profiles at four stations on the symmetry plane are shown. The agreement between the experimental data and the unsteady solution is satisfactory. The main discrepancy is due to the inadequate prediction of the high-speed flow between the cube and the wind tunnel roof. The underprediction of the velocity in that region may be due to incorrect capturing of the separated boundary layer on the cube roof (in conjunction with an unsteadiness not detected by the simulation). As expected, the steady solution significantly overpredicts the strength of the recirculation velocity in the wake.

In conclusion, good quantitative and qualitative agreement with the experimental results shows the ability of the present approach (unsteady RANS with the $\overline{v^2}-f$ turbulence model) to correctly reproduce the essential physics associated with massively separated flows.

The presence of the horseshoe vortex and the arch-shaped vortex in the near wake, inferred by the analysis of the experimental oil-flow patterns on the wind tunnel floor, has been demonstrated by means of two vortex-detection criteria. In addition, the inadequacy of steady RANS calculations for flows over bluff bodies has been confirmed by the present results.

3. Future work

The results presented in this report show that the simulation of three-dimensional unsteady flows can be carried out accurately using RANS; this confirms the results obtained by Durbin (1995) for two-dimensional flows. The configurations analyzed, however, are characterized by flow separation fixed at sharp edges (the cube herein, and a triangular cylinder in Durbin, 1995). It remains to verify the ability of the model to accurately predict the unsteady wake development for separation occurring on a smooth surface. Future work will be focusing on the simulation of the flow around a sphere; in this case, to eliminate uncertainties associated with the prediction of the laminar/turbulent transition, high Reynolds number will be considered.

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