# Study of the turbulence modulation in particle-laden flows using LES

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## 1. Objective and motivation

One of the most interesting problems in fluid dynamics is the prediction of particle-laden turbulent flows. These flows are as diverse as pollutant dispersion in the atmosphere and contaminant transport in industrial applications. An issue of primary importance for moderately dense suspensions concerns how particles affect the turbulent flow itself, the so-called two-way coupling. It is known that the addition of particles to a turbulent flow may change the intensity significantly, even at very low volume fraction. The principal difficulty in the prediction of particle-laden turbulent flows is that traditional approaches model particle transport using the Reynolds-averaged Navier-Stokes (RANS) equations. The RANS methods do not accurately predict the Eulerian turbulence field, and it is known that accurate prediction of particle transport is strongly dependent upon providing a realistic description of the velocity field encountered along particle trajectories.

Although the most accurate approach to representing the structure of turbulence – including particle transport – is direct numerical simulation (DNS), is not practical for use as a predictive tool because it remains restricted to relatively low Reynolds numbers. An approach which is not as severely restricted in the range of Reynolds number as DNS is large eddy simulation (LES). LES predictions are less sensitive to modeling errors than RANS calculations and, since the subgrid scales are more universal than large scales, it is also possible to represent the effect of the subgrid scales using relatively simple models. A significant advantage of LES over RANS methods is that it permits a much more accurate accounting of particle-turbulence interactions. If modulation of the turbulence by particles is negligible and if the particle relaxation time is of the order of the turbulent time macro-scales, LES of gas-particle flows can be expected to be as accurate as in single-phase flow. In contrast, if two-way coupling effects are important, then the subgrid turbulence model might require modification. The principal objective of this work is application of large eddy simulation to computation of a well defined turbulent shear flow, fully developed channel flow, for which experimental results – Kulick *et al.* (1994) – exist for comparison. Also, a subgrid model which takes into account the presence of particles is proposed and evaluated.

## 2. Physical and numerical model

### 2.1. *Fluid motion*

The space-filtered continuity and time-dependent Navier-Stokes equations were used to model the continuous gas phase.

$$\frac{\partial v_{gi}}{\partial x_i} = 0 \qquad (2.1)$$

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$$\frac{\partial v_{gi}}{\partial t} + \frac{\partial v_{gi} v_{gj}}{\partial x_j} = -\frac{\partial}{\partial x_i}\left(\frac{p}{\rho_g}\right) + \frac{1}{Re}\frac{\partial^2 v_{gi}}{\partial x_j \partial x_j} - \frac{\partial \tau_{ij}}{\partial x_j} + \frac{f_{pi}}{\rho_g} \tag{2.2}$$

In order to take into account the particle effect on the fluid (two-way coupling) an additional term was included in the momentum equation $f_p$. This term is given by the sum of all forces on all particles in a fluid computational cell.

$$f_p = -\frac{1}{V_{cell}} \sum_{j=1}^{N_{cell}} f_{gj} \tag{2.3}$$

where $V_{cell}$ is the volume of a fluid computational cell, $N_{cell}$ is the number of particles in that cell and $f_{gj}$ is the fluid force on the $j$th particle in that cell.

### 2.2. *Particle motion*

A Lagrangian approach is employed to predict the properties of each particle directly from the equation of motion. The particle equation of motion used in the simulations describes the motion of particles with densities substantially greater than that of the surrounding fluid, and diameters small compared to the Kolmogorov scale:

$$\frac{dv_{pi}}{dt} = -\frac{\rho_g}{\rho_p}\frac{3}{4}\frac{C_D}{d}|\mathbf{v}_p - \mathbf{v}_g|(v_{pi} - v_{gi}) \tag{2.4}$$

where $v_{pi}$ is the velocity of the particle and $v_{gi}$ is the velocity of the gas at the particle position. An empirical relation for $C_D$ from Clift *et al.* (1978), valid for particle Reynolds numbers up to about 40, was employed:

$$C_D = \frac{24}{Re_p}(1 + 0.15 Re_p^{0.687}). \tag{2.5}$$

### 2.3. *SGS model*

The following assumptions have been made to derive the new model:

(*a*) The density of the particles is much larger than the gas density, and Basset forces and virtual mass can be neglected.

(*b*) The particles are spherical.

(*c*) The particle volume fraction is small enough that particle-particle interaction can be neglected.

(*d*) In the local movement of the particles, gravity can be neglected compared to inertia.

The equation of motion for the particles can be rewritten as follows:

$$\rho_p V_p \frac{d\mathbf{u}}{dt} = \rho_p V_p \frac{d\mathbf{v}_p}{dt} - \mathbf{F}_D \tag{2.6}$$

where $\mathbf{u}$ stands for the relative velocity between fluid and particle, and

$$\mathbf{F}_D = \frac{1}{2} C_D \rho_g S_g \mathbf{u}|\mathbf{u}|. \tag{2.7}$$

For the different turbulent length scales, $\lambda$, in the interval between the integral and Kolmogorov scales ($L > \lambda > \eta$), it is possible to scale the different terms of the equation of motion:

- For the gas phase

$$\tau_{g\lambda} \sim \frac{\lambda}{v_{g\lambda}} \sim \frac{\lambda^{2/3}}{\epsilon^{1/3}} \tag{2.8}$$

$$v_{g\lambda} \sim (\epsilon\lambda)^{1/3} \tag{2.9}$$

$$\frac{dv_g}{dt} \sim \frac{v_{g\lambda}}{\tau_{g\lambda}} \sim \frac{\lambda}{\tau_{g\lambda}^2} \sim \frac{\epsilon^{2/3}}{\lambda^{1/3}} \tag{2.10}$$

- For the particles

$$\tau_{p\lambda} \sim \frac{\lambda}{u_\lambda} \tag{2.11}$$

$$\frac{du}{dt} \sim \frac{u_\lambda}{\tau_{p\lambda}} \sim \frac{u_\lambda}{\lambda/u_\lambda} \sim \frac{u_\lambda^2}{\lambda} \tag{2.12}$$

- Drag force

$$\mathbf{F}_D \sim \frac{1}{2} C_D \rho_g S_g u_\lambda^2 \tag{2.13}$$

Substituting in the equation of motion

$$\rho_p V_p \frac{d\mathbf{u}}{dt} \sim \rho_p V_p \frac{d\mathbf{v}_g}{dt} - \mathbf{F}_D \tag{2.14}$$

we obtain for the relative velocity:

$$u_\lambda \sim (\rho_p V_p)^{1/2} \frac{\epsilon^{1/3}\lambda^{1/3}}{(\rho_p V_p + \frac{1}{2} C_D \rho_g S_p \lambda)^{1/2}}. \tag{2.15}$$

It can be seen that $u_\lambda$ should have a maximum for a certain value of $\lambda$, given by:

$$\frac{\partial u_\lambda}{\partial \lambda} = 0 \rightarrow \lambda|_{u_\lambda max} = \lambda^* \sim \frac{1}{C_D} \frac{\rho_p}{\rho_g} \frac{V_p}{S_p} \rightarrow \lambda^* >> d \tag{2.16}$$

$$u^* \sim d^{1/3} \epsilon^{1/3} \left(\frac{\rho_p}{\rho_g}\right)^{1/3} \left(\frac{1}{C_D}\right)^{1/3}. \tag{2.17}$$

It is assumed that the turbulence dissipation due to the particles occurs mainly near the scale $\lambda^*$, and it will be represented by:

$$-\tau_{ij}\overline{S}_{ij}|_p \sim \rho_g C_D \left(\frac{\rho_g}{\rho_p}\right) \frac{u^{*3}}{d} \overline{\phi}_p. \tag{2.18}$$

Introducing $u^*$ from the previous equation we obtain:

$$-\tau_{ij}\overline{S}_{ij}|_p \sim \rho_g \epsilon \overline{\phi}_p \Rightarrow -\tau_{ij}\overline{S}_{ij}|_p = C_p \rho_g \epsilon \overline{\phi}_p \tag{2.19}$$

In previous work with RANS models, by García & Crespo (2000) and Crespo *et al.* (2001)) the constant $C_p$ has been estimated by comparison with different experiments. Then the total dissipation can be estimated as:

$$-\tau_{ij}\overline{S}_{ij} = \rho_g \epsilon (1 + C_p \overline{\phi}_p) \tag{2.20}$$

Using an eddy-viscosity model we get:

$$\nu_T = (C_s \Delta)^2 |\overline{S}| (1 + C_p \overline{\phi}_p) \tag{2.21}$$

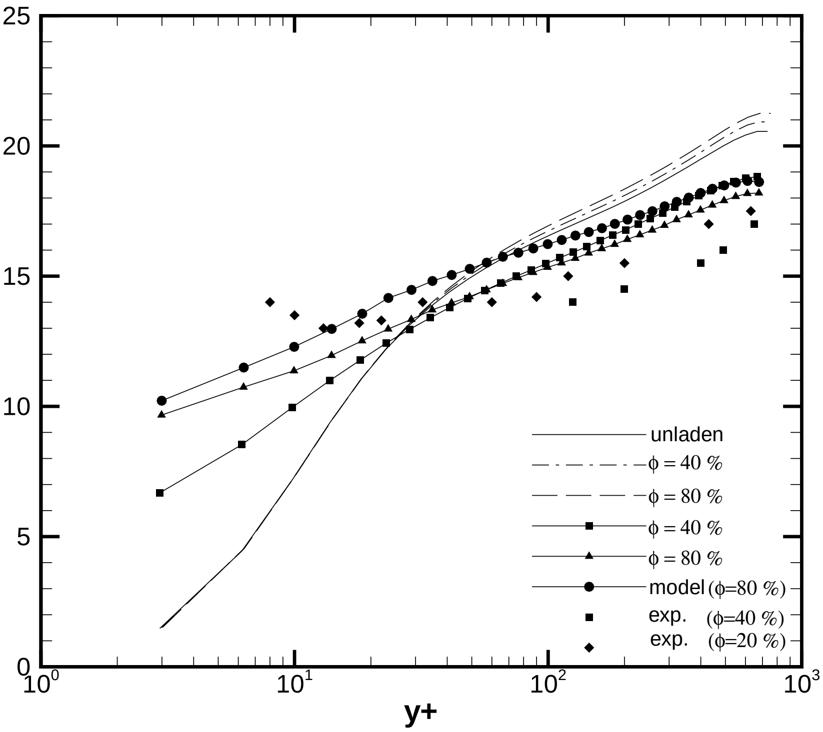


FIGURE 1. Particle streamwise mean velocity profiles, in wall units

Applying the Dynamic Procedure to obtain the model coefficient, we can obtain the Leonard term

$$\mathcal{L}_{ij} = -C \underbrace{\left( \widetilde{\overline{\Delta}}^2 |\widetilde{\overline{S}}| \widetilde{\overline{S}}_{ij} \left(1 + C_p \widetilde{\overline{\phi_p}}\right) - \overline{\Delta}^2 |\overline{S}| \overline{S}_{ij} \left(\widetilde{1 + C_p \overline{\phi_p}}\right) \right)}_{M_{ij}} \tag{2.22}$$

and, using least-squares averaging, the model parameter can be computed as:

$$C = -\frac{L_{kl} M_{kl}}{M_{kl} M_{kl}} \tag{2.23}$$

## 3. Simulation procedure

The calculations have been made with Pierce's code, described in Pierce & Moin (2001) implemented with Oefelein's routines for the simulation of Lagrangian particle dynamics. Large eddy simulations were performed under conditions chosen to match the experiments of Kulick *et al.* (1994). The fluid is air (kinematic viscosity $\nu = 1.5 \times 10^{-5}\,\mathrm{m^2 s^{-1}}$), and the friction velocity $u_\tau$ is $0.49\,\mathrm{ms^{-1}}$. The Reynolds number based on friction velocity and channel half-width is 644 (corresponding to a Reynolds number of 13,800 based on centerline velocity and channel half-width). The flow was resolved using $64 \times 64 \times 64$ grid

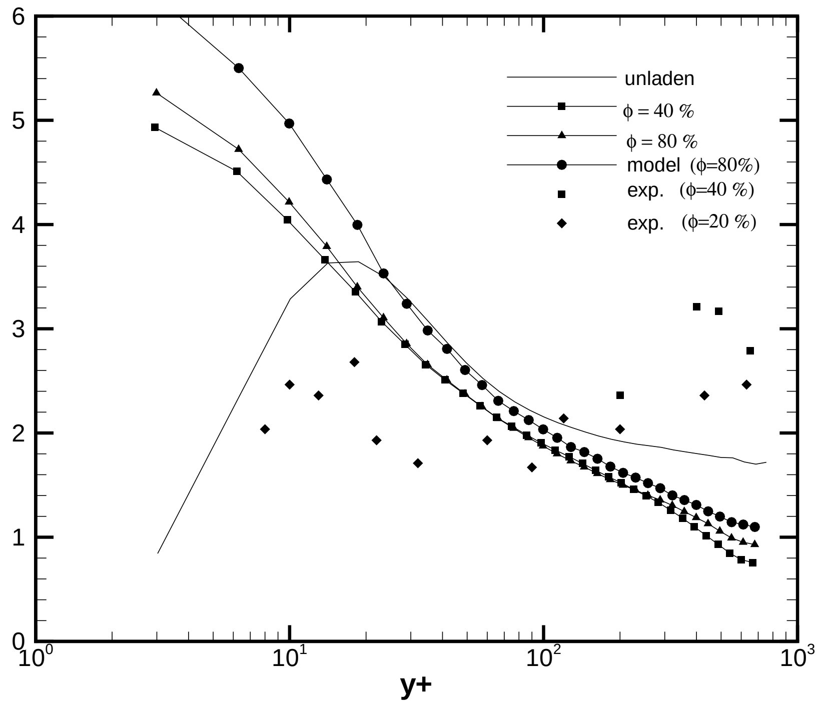


FIGURE 2. Particle streamwise fluctuations: mean-square intensity profiles, in wall units

points in the $x$, $y$, and $z$ directions, respectively. The channel domain for the calculation was the same used by Wang and Squires (1996), $5\pi\delta/2 \times 2\delta \times \pi\delta/2$. The channel half width is $\delta = 0.02\,\mathrm{m}$. The grid spacing in wall coordinates in the $x$ and $z$ directions was $\Delta x^+ = 83$ and $\Delta z^+ = 17$. A stretched grid was used in the wall normal direction and the minimum grid spacing (close to the wall) was $\Delta y^+ = 1.5$.

Different values for the constant $C_p$ have been used. At present a value of 0.9 produces the best agreement with the experimental results.

## 4. Results

The results obtained to date are presented in Figs. 1 to 4. In all cases, the particles used in the simulations are copper particles with density $\rho_p = 8800$ kgm$^{-3}$ and diameter $d = 70\,\mu$m. Copper particles have been chosen for the first calculations because the effect of turbulence modulation is more intense and therefore is more sensitive to new models. The Figures show the results obtained for mass loading of 40 % and 80 %. The experimental data are taken from Kulick *et al.* (1994).

Figure 1 shows mean streamwise gas-velocity profiles. The calculated results show that mean gas-velocity profiles change slightly in the logarithmic region: this result agrees with numerical results from Yamamoto it et al. (2001). This Figure also shows the mean streamwise particle-velocity profiles. The profiles for particles are flatter than those for

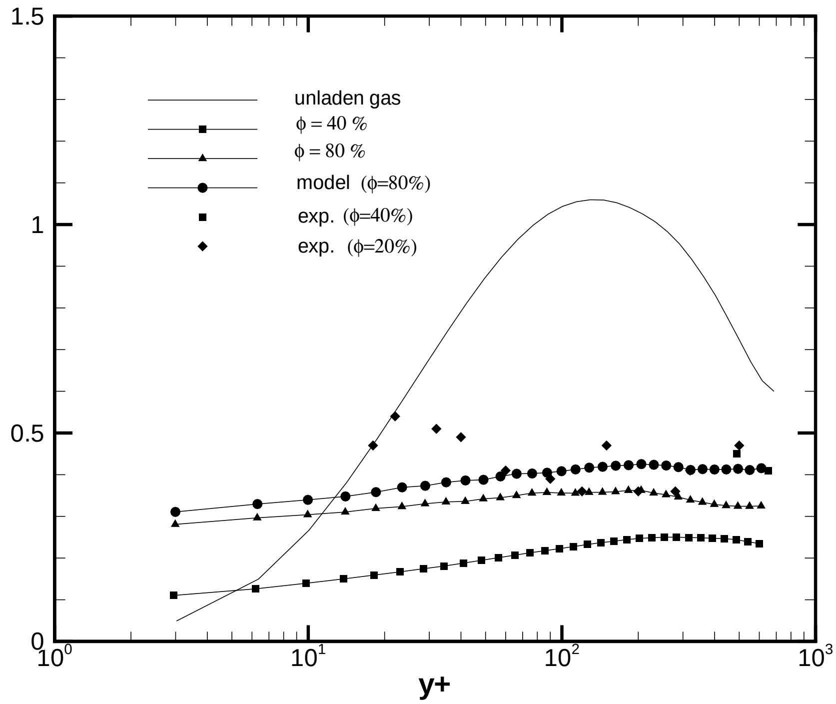


FIGURE 3. Particle wall-normal fluctuations: mean-square intensity profiles, in wall units

the gas. This trend is also observed in the experiments, where the particle velocity profiles are even flatter. The results calculated with the proposed SGS model have the same trend, but the velocities are greater than the previous results with the unmodified SGS model, and therefore greater than the experimental values.

Figure 2 shows profiles of particle streamwise fluctuation intensity profiles. The model proposed produces a profile shifted upward, mainly close to the wall. In any case, the numerical results obtained with or without the modified SGS model are similar to those obtained by Yamamoto it et al. (2001) without inter-particle collision, but the agreement between numerical calculations and experimental results is not good. The results obtained by Yamamoto it et al. (2001) with an inter-particle collision model suggest that collisions could play an important role.

Figure 3 shows fluctuation intensity profiles of the wall-normal particle velocity. In this case the numerical results agree well with the experiments. The fluctuation are larger with the proposed model, but the the opposite trend would be needed to agree with the experiments.

Figure 4 shows streamwise turbulence intensity profiles. It can be seen, in the numerical results obtained, that an increase in the mass loading produces an slight decrease in the turbulence intensity profile. But not so intense than that measured in the experiments. In this figure the proposed model does not produce a significant change compared to the SGS model without modification.

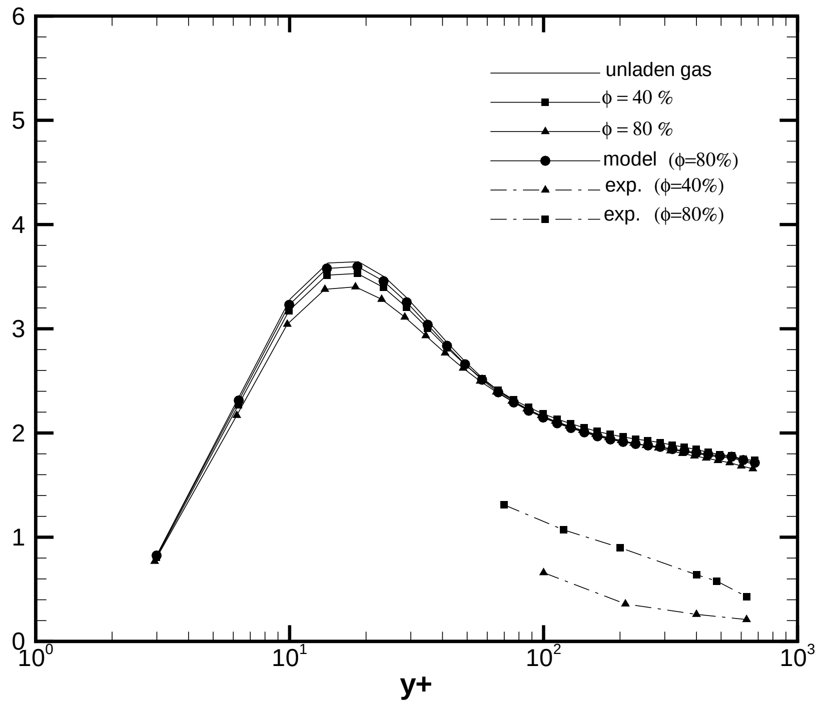


FIGURE 4. Streamwise turbulence mean-square intensity profiles, in wall units

## 5. Conclusions

A subgrid model that takes into account the presence of particles has been investigated. Simulations of gas-solid turbulent flow in a vertical downward channel flow at $Re_\tau = 644$ using LES were performed in order to study the proposed model. The results obtained so far indicate that it will be necessary to use a more sophisticated model to capture the complex phenomena involved in this type of flows. A recent paper suggests that inter-particle collisions could play an important role. It is also thought that an anisotropic model could improve the numerical results. These ways will be investigated in future works.

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