Wall corrections in modeling rotating pipe flow

By S. V. Poroseva

1. Motivation and objectives

The presence of a wall significantly influences the turbulence structure of a flow. Wall effects are naturally described by the exact transport equations for turbulence characteristics with appropriate boundary conditions imposed. In practice, however, turbulent flow features are simulated with the averaged Navier-Stokes equations in which some terms must be modeled. This implies that the model expressions used for those terms should respond correctly to the presence of the wall.

The terms to be modeled in the averaged Navier-Stokes equations are dissipative tensors, pressure-containing correlations, and turbulent diffusion terms. Modern turbulence models are mostly based on the second-order weighted averages of the Navier-Stokes equations, the Reynolds-stress transport equations (RSTE), from which simpler, industry-oriented turbulence models are then derived. To date, model expressions for the dissipative tensor ε_{ij}, the pressure-containing correlations Π_{ij} and the turbulent diffusion term have been derived based on assumptions which are not consistent for the different terms. Moreover, some of those assumptions are too simplified to reflect the real physics of a flow: they hold only for some limiting states of turbulence that are rarely (if at all) met in real flows. As a result, turbulence models do not naturally reproduce wall effects, and usually need additional wall corrections. It is important to clarify which model expressions contribute most to model failure and are therefore in most need of such corrections. The present study is not an attempt to solve this problem in general, but only in a particular case of turbulent flow: flow in an axially-rotating cylindrical pipe. This flow is of interest because it relates to phenomena encountered in various engineering systems involving boundary layers on rotating surfaces, *e.g.*, heat exchangers and rotor cooling systems.

It was shown by Kurbatskii & Poroseva (1999) that the commonly-used model of Daly & Harlow (1970) for the turbulent diffusion does not adequately describe the behavior of the third-order velocity moments in a pipe flow. At present, the tensor-invariant model of Hanjalić & Launder (1972) is the best choice for modeling turbulent diffusion in the RSTE. Though it is less accurate in predicting the third-order moments than models based on the transport equations for these moments, the HL model provides results which are quantitatively comparable with those obtained using more accurate models. Also, it is cheaper and more robust in different combinations of Reynolds and rotation numbers. The difference between results obtained with the DH and HL models is large, especially near a wall, and increases with swirl. The tensor-invariant model for the turbulent diffusion significantly improves the description of turbulence structure near a wall, especially with large wall rotation. Nevertheless, in a stationary pipe, changing the DH model to a more adequate turbulent diffusion model does not solve the problem of inaccurate prediction of the turbulent intensities near a wall. The model expressions for the dissipative tensor and the pressure-containing correlations (Launder *et al.* 1975) used in Kurbatskii & Poroseva (1999) still need additional wall corrections (So & Yoo 1986; Gibson & Launder 1978).

Of the other two terms in the RSTE that need to be modeled, the present study focuses on the pressure-containing correlations. One of the reasons for this is the fact that little progress has been made in modeling the dissipative tensor. With the state of art in this field, it is possible to say that including or excluding wall corrections in commonly-used models for ε_{ij} does not make those models more or less physical. To date, their main function is more to compensate for inaccuracies in modeling other terms, rather than to reproduce the dissipative process itself accurately. In contrast, progress in modeling the pressure-containing correlations is more noticeable. New approaches based on more refined physical assumptions (*e.g.* Speziale *et al.* 1991; Kassinos *et al.* 2000, 2001) have been developed in the last decade. It could be expected that their performance should depend less on wall corrections. One of the objectives of this work was to clarify this question.

Rotation can have a crucial role in the development of wall corrections to model expressions for the pressure-containing correlation. It was noticed by Gibson & Launder (1978) that, in a stably-stratified boundary layer, wall effects diminish as stability is increased. Though the physical mechanism of influence of rotation on a pipe flow structure differs from that of stable stratification, the analogy between those two flows is very well known. Thus, it can be expected that wall effects in a rotating pipe flow will diminish with increasing rotation number N, defined as the ratio of the pipe wall velocity W_o to the mean-flow velocity at the pipe center U_o. Indeed, it was found by Poroseva *et al.* 2000, 2001) that including the wall effects (Gibson & Launder 1978; Durbin 1991) in turbulence models, such as the structure-based Q model (Kassinos *et al.* 2000, 2001) and the non-linear SSG model (Speziale *et al.* 1991), does not decisively improve the ability of the models to describe a flow under rotation, though it does improve the description of turbulence structure in a stationary pipe flow. The maximum value of the rotation number N at which a model still appropriately reproduces the turbulence structure is the same in both cases; whether wall effects are included in the model or not. Moreover, the stronger the rotation, the smaller the influence of wall effects on flow characteristics. This conclusion relates in larger degree to the mean-velocity components and shear stresses. Turbulent kinetic energy is more sensitive to the description of wall effects. This question is the second focus of the present work.

2. Models

There are two ways to evaluate the performance of different model expressions for the pressure-strain correlations. The most physical way would be to use direct numerical simulation (DNS). This gives complete information in the case of low-Reynolds-number flows. However, with increasing Reynolds number, the contribution of the different physical mechanisms to turbulence processes changes. Therefore, some conclusions valid at low Reynolds numbers will not necessarily hold for high Reynolds numbers. Such a situation could be expected for wall effects, which are the focus of the present study. The flow in the current work was calculated at two Reynolds numbers, 2×10^4 and 4×10^4, which are too high to apply DNS. For such Reynolds numbers, another way to evaluate model expressions for Π_{ij} can be used. Namely, to calculate the flow with a turbulence model, in which model expressions for the turbulent diffusion and the dissipative tensor are kept the same, but the model for the pressure-containing correlations is adjusted.

The transport equation governing the evolution of the Reynolds stresses has the following form

$$\frac{\bar{D} < u_i u_j >}{Dt} = P_{ij} + D_{ij} + \Pi_{ij} - \varepsilon_{ij}.$$

Here,

$$\frac{\bar{D}}{Dt} = U_j \frac{\partial}{\partial x_j} + \frac{\partial}{\partial t},$$

U_i are the mean velocity components, and u_i are the fluctuating velocity components. Cartesian tensor notation is used throughout the text for the sake of simplicity. The production term $P_{ij} = - < u_i u_k > U_{k,j} - < u_j u_k > U_{k,i}$ does not need modeling. The diffusion term D_{ij} includes the molecular diffusion $\nu(< u_i u_j >_{,kk})$ (ν is the kinematic viscosity), which also does not need modeling, and the turbulent diffusion $- < u_i u_j u_m >_{,m}$. The DH model (Daly & Harlow 1970):

$$< u_i u_j u_k > = -C_{s1} \tau (< u^m u_k > < u_i u_j >_{,m})$$

has been chosen to model the turbulent diffusion in the present study. Here, $C_{s1} = 0.18$ and the time scale τ is modeled as $\tau = k/\varepsilon$, where ε is the dissipation rate of the turbulent kinetic energy k. Though the HL model (Hanjalić & Launder, 1972) proved to give better results in a pipe flow (Kurbatskii & Poroseva, 1999), the main focus of the present study is evaluation of the performance of the pressure-containing correlation models, not suggestions for a complete RSTE model, and the simplest turbulent-diffusion model serves for this aim.

The pressure-containing correlation term $\Pi_{ij} = -(< u_i p_{,j} > + < u_j p_{,i} >)/\rho$ (p is the pressure fluctuation, ρ is the flow density) is usually split into the pressure-strain correlations $< (u_{i,j} + u_{j,i}) p > /\rho$ and the pressure-velocity correlations $-(< u_i p >_{,j} + < u_j p >_{,i})/\rho$. Then the correlations $-(< u_i p >_{,j} + < u_j p >_{,i})/\rho$ are either combined with the turbulent diffusion term or neglected. Neither practice has solid justification, but at the moment this issue will be skipped and the same approach will be used throughout this work. Thus, progress in modeling the pressure-containing correlations mostly relates to modeling the pressure-strain correlations. Based on previous work on modeling a pipe flow (for a review see, *e.g.*, Poroseva *et al.* 2000), the following model expressions for the pressure-strain correlations were chosen for evaluation: the IP model and the full LRR (Launder *et al.* 1975), the linearized SSG (Gatski & Speziale 1993) and the non-linear SSG (Speziale *et al.* 1991). All models can be represented by the same expression:

$$\Pi_{ij} = -(C_1' \varepsilon + 2C_1'^* P) b_{ij} + C_2' \varepsilon \left(b_{ik} b_{kj} - \frac{1}{3} b_{kl} b_{kl} \delta_{ij} \right) + \left(C_3' - C_3'^* \sqrt{b_{kl} b_{kl}} \right) k S_{ij}$$
$$+ C_4' k \left(b_{ik} S_{jk} + b_{jk} S_{ik} - \frac{2}{3} b_{kl} S_{kl} \delta_{ij} \right) + C_5' k (b_{ik} \Omega_{jk} + b_{jk} \Omega_{ik})$$

with different coefficients:

$$\text{IP} : (C_1',\ C_1'^*,\ C_2',\ C_3',\ C_3'^*,\ C_4',\ C_5') = (3.6,\ 0,\ 0,\ 0.8,\ 0,\ 1.2,\ 1.2)$$
$$\text{LRR} : (C_1',\ C_1'^*,\ C_2',\ C_3',\ C_3'^*,\ C_4',\ C_5') = (3.6,\ 0,\ 0,\ 0.8,\ 0,\ 1.75,\ 1.31)$$
$$\text{LSSG} : (C_1',\ C_1'^*,\ C_2',\ C_3',\ C_3'^*,\ C_4',\ C_5') = (3.4,\ 1.8,\ 0,\ 0.36,\ 0,\ 1.25,\ 0.4)$$
$$\text{SSG} : (C_1',\ C_1'^*,\ C_2',\ C_3',\ C_3'^*,\ C_4',\ C_5') = (3.4,\ 1.8,\ 4.2,\ 0.8,\ 1.3,\ 1.25,\ 0.4)$$

Here, $b_{ij} = 1/2(< u_i u_j > /k - 2\delta_{ij}/3)$, $S_{ij} = (U_{i,j} + U_{j,i})/2$ and $\Omega_{ij} = (U_{i,j} - U_{j,i})/2$. Note that in the present study the value of the coefficient C_1' in the LRR model is 3.6 instead of the usual 3.0.

A new approach, which offers an alternative to the standard method based on the modeling of the RSTE, is the structure-based Q model (Kassinos *et al.* 2000, 2001). A preliminary formulation of this approach has been evaluated here also. The pressure effects are treated in the Q model in a more profound way: non-local effects are taken into account by incorporating additional structure tensors into the model. Details of the transport equation for the third-rank **Q**–tensor can be found in Kassinos *et al.* (2000) and Poroseva *et al.* (2000, 2001).

The dissipation tensor ε_{ij} is modeled by the isotropic expression, with a correction for low Reynolds numbers near a solid wall, as

$$\varepsilon_{ij} = \frac{2}{3}\delta_{ij}\varepsilon + 2\nu\frac{< u_i u_j >}{x_n^2} \tag{2.1}$$

(So & Yoo 1986), where x_n is the normal distance to a pipe wall and δ_{ij} is the Kronecker delta tensor. The equation for the dissipation rate of the turbulent kinetic energy is used in the following form

$$\frac{\bar{D}\varepsilon}{Dt} = \left[\left(\nu\delta_{jk} + C_\varepsilon\tau < u_j u_k >\right)\varepsilon_{,j}\right]_{,k} + \frac{1}{\tau}\left(C_{\varepsilon 1}P - C^*_{\varepsilon 2}\varepsilon\right) - \frac{2\nu\varepsilon}{x_n^2}f_1. \tag{2.2}$$

Here, $P = P_{ii}/2$. The model coefficients are: $C_\varepsilon = 0.18$ (0.22 for the Q model), $C_{\varepsilon 1} = 1.54$ (1.5 for the Q model), $C^*_{\varepsilon 2} = C_{\varepsilon 2} f_2$ in a stationary pipe flow and $C_{\varepsilon 2} = 11/6$. The functions f_1 and f_2 are damping functions (So & Yoo 1986): $f_1 = \exp\ (-0.5 x_n u_{*0}/\nu)$, where u_{*0} is the friction velocity, and $f_2 = 1 - 2/9\ exp\left[-\left(k^2/(6\nu\varepsilon)\right)^2\right]$. In rotating flows the model coefficient $C^*_{\varepsilon 2}$ is a function of the "Richardson number"

$$C^*_{\varepsilon 2} = \max(1.4, C_{\varepsilon 2} f_2(1 - C_R Ri)), \quad C_R = 2, \tag{2.3}$$

$$Ri = \frac{\frac{\partial W}{\partial r}\frac{W}{r}}{\left(\frac{\partial W}{\partial r}\right)^2 + \left(\frac{\partial U}{\partial r}\right)^2}.$$

The restrictive condition on the value of $C^*_{\varepsilon 2}$ is imposed to avoid excessive values of the dissipation rate close to a wall, which would be predicted in rotating flows.
Modification of the $C^*_{\varepsilon 2}$ coefficient by the Richardson number is necessary to reproduce correctly the effects of strong turbulence suppression observed in an initial pipe section. It was found by Kurbatskii & Poroseva (1999) and Poroseva *et al.* (2000, 2001) that the choice of a model for the pressure-strain correlation, and the presence of additional wall corrections to that model, do not solve the problem, and an Ri modification is still necessary. In this study, more models are examined to clarify the question.

When the fully-developed part of the flow is simulated with the same $C_R = 2$, turbulence is suppressed very soon, at $N = 0.5$. This contradicts the experimental observations (Kikuyama *et al.*, 1983; Nishibori *et al.* 1987; Imao *et al.*, 1996) which show decreasing turbulence level with increasing rotation, but not full suppression. On the contrary, they show the existence of some limit state, which is rather insensitive to further increase of rotation and to Reynolds number. The level of turbulence is lower than in a stationary pipe, but not by much. Therefore, the present RANS calculations of a fully-developed rotating pipe flow are made with $C_R = 0$. In (2.3), U and W are the axial and the circumferential mean velocity components respectively, r is the radial coordinate.

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The presence of damping functions in Eqs. (2.2)-(2.3) is also not influenced by the pressure-strain correlation model. It relates to formulation of a model for the dissipative tensor. Though it is evident that a more physical alternative to Eqs. (2.1)-(2.3) should be sought in the future, in a pipe flow this formulation provides good, stable results with different combinations of Reynolds and rotation numbers. Moreover, with such a formulation, the step size in the axial direction is significantly larger than, for instance, that in the formulation used by Kassinos *et al.* (2000) and Poroseva *et al.* (2000, 2001). The issue of computing time becomes very important when complex model expressions for Π_{ij} are incorporated into the model.

3. Results

All approaches to modeling the pressure-strain correlation have been evaluated at two Reynolds numbers, 2×10^4 and 4×10^4. Note again that all parts of the turbulence model except the pressure-strain model were kept the same for all computations. The same flow conditions were applied as in the experiments (Kikuyama *et al.*, 1983; Zaets *et al.* 1985): a rotating flow was obtained by conveying a fully-developed turbulent flow from a stationary cylindrical pipe into a rotating cylindrical section of the same diameter. As experiments demonstrate (Kikuyama *et al.*, 1983; Zaets *et al.* 1985; Nishibori *et al.* 1987; Imao *et al.* 1996), it is possible to distinguish two regions in a rotating pipe flow, with different turbulence structure. In the initial pipe section, with a length of about $30D$ ($D = 2R$ is the pipe diameter), strong suppression of turbulence characteristics is observed (Zaets *et al.* 1985). After suppression, however, statistical quantities increase in value and eventually stabilize at some level which is lower than in a stationary flow, but not by much (Nishibori *et al.* 1987). This is the region of fully-developed turbulence, which is observed beyond about $170D$ for any Reynolds number (Kikuyama *et al.*, 1983; Imao *et al.* 1996).

In the calculations, the grid was non-uniform in r, the total number of nodes being 128 (64 for the Q-model) at $Re = 4\times10^4$ and 68 at $Re = 2\times10^4$.

The RSTE model with the IP model expression for the pressure-strain correlation does crucially need additional wall corrections. The turbulent diffusion model has practically no influence on mean-velocity components (Kurbatskii & Poroseva, 1999). Results for different turbulence characteristics at $Re = 2\times10^4$ are in strong disagreement with experimental data (Imao *et al.* 1996). The disagreement increases significantly with increasing rotation number. Profiles of the axial mean velocity are shown on Fig.1a. With increasing Reynolds number ($Re = 4\times10^4$), the dependence of the model results on additional wall modifications decreases for the mean velocity components (Fig. 1b) and the shear stress (Fig. 2a). However, the turbulent kinetic energy does need additional wall corrections. The higher the value of the rotation number, the less critical the wall corrections become. This is especially true at higher Reynolds number.

The mean velocity profiles calculated with the LRR, LSSG, SSG, and Q models are given in Figs. 3-4.

Nearly all models predict the axial mean velocity well, in a stationary pipe and with increasing rotation number (up to $N = 1$). Only the non-linear SSG model already fails at $N = 1, Re = 2\times10^4$, predicting complete turbulence suppression. This contradicts the experimental data (Kikuyama *et al.*, 1983; Nishibori *et al.* 1987; Imao *et al.* 1996). The other models do not fail at $N = 1$, but they show the same incorrect tendency to turbulence suppression and fail at larger N. One must conclude that modern turbulence models provide trustworthy results in a rotating pipe flow only at moderate rotation

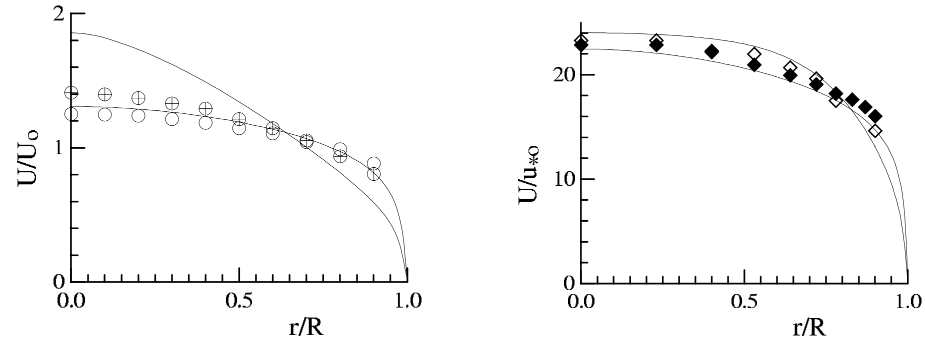

FIGURE 1. Axial mean velocity at **a)** $Re = 2 \times 10^4$, **b)** $Re = 4 \times 10^4$. Calculations: (———) IP model. Experiments: **a)** (○) $N = 0.$, (⊕) $N = 0.5$ (Imao *et al.* 1996); **b)** (♦) $N = 0.$, (◊) $N = 0.6$ (Zaets *et al.* 1985)

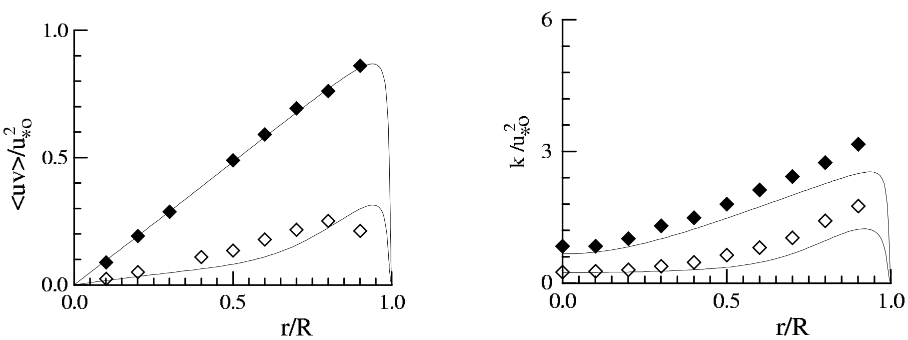

FIGURE 2. Calculations by the IP model at $Re = 4 \times 10^4$: **a)** shear stress, **b)** turbulent kinetic energy (see notation in Fig. 1b).

numbers ($N \leq 1$). The additional wall corrections developed to date do not improve this situation (Poroseva *et al.* 2000, 2001).

At higher Reynolds number, $Re = 4 \times 10^4$, solutions obtained with the different models become very close to each other.

At $Re = 4 \times 10^4$ results for the circumferential mean velocity are more sensitive to model choice than results for the axial mean velocity (Fig. 4a). However, with increasing Re, profiles calculated with different models again become close to each other (Fig. 4b). The LSSG gives slightly better profiles at $Re = 2 \times 10^4$ than the other models. The LRR and SSG models generally give very similar results, but the LRR does not fail at $N = 1$.

The second-order statistics are shown in Figs. 5-9. All models overpredict the normalized shear-stress profile $< uv >$ at $Re = 2 \times 10^4$ (Fig. 5a: U_m is the area-mean axial flow velocity). At $Re = 4 \times 10^4$ the results are in very good agreement with the data (Fig. 5b). In a stationary pipe flow no additional wall correction is necessary. With increasing rotation, some difference between calculated and experimental profiles is observed in the near-wall region. However, this difference can easily be corrected by changing the turbulent diffusion model from the DH model to the tensor-invariant HL, as was shown by Kurbatskii & Poroseva (1999) (see Figs. 7 and 8 in that work).

The choice of a turbulent diffusion model is also important for correct prediction of the turbulent kinetic energy in a fully-developed rotating pipe flow. If the IP model expression is combined with the DH turbulent diffusion model in the RSTE turbulence

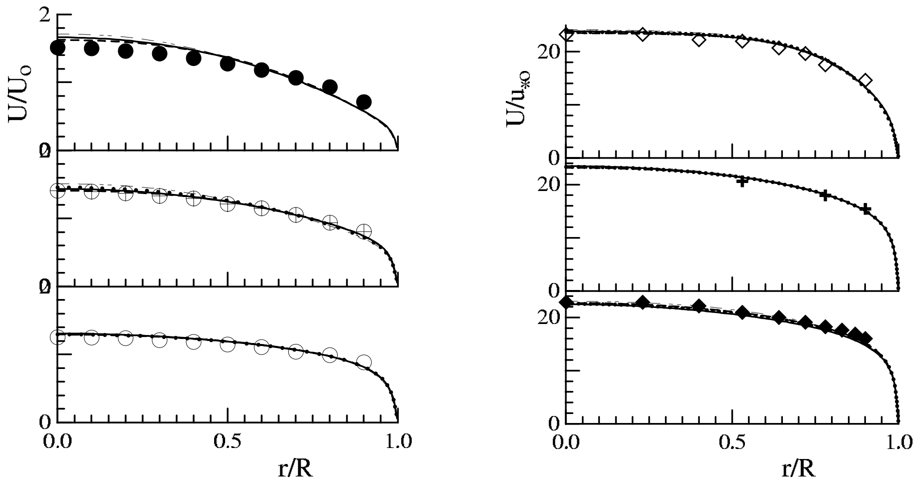

FIGURE 3. Axial mean velocity at **a)** $Re = 2 \times 10^4$, **b)** $Re = 4 \times 10^4$. Calculations: (———) LRR, (- - - -) LSSG; (········) SSG; (—·-—) Q. Experiments: **a)** (○) $N = 0.$, (⊕) $N = 0.5$, (●) $N = 1.$; **b)** (♦) $N = 0.$, (+) $N = 0.15$, (**x**) $N = 0.3$, (◊) $N = 0.6$

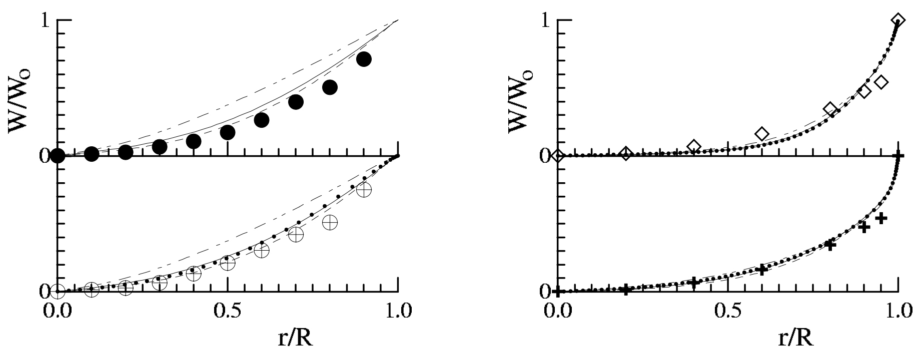

FIGURE 4. Circumferential mean velocity at **a)** $Re = 2 \times 10^4$, **b)** $Re = 4 \times 10^4$ (see notation in Fig. 3).

model, the turbulent kinetic energy is significantly overpredicted (Fig. 9 in Kurbatskii & Poroseva, 1999): the calculated level is even higher than in a stationary pipe flow, in strong contradiction of the experimental data. As has been found in the present study, the use of more refined model expressions for the pressure-strain correlation does not solve this problem. All models predict the turbulent kinetic energy in a fully-developed rotating pipe flow to be greater than in a stationary pipe flow. In Fig. 6a only the profile for the LRR model is shown at $N = 0.5$. The other models give qualitatively similar results, which are omitted from the plot to reduce the number of curves. The rest of the curves on Fig. 6a correspond to $N = 0$. Wall modifications do not improve these results either (Kurbatskii & Poroseva, 1999; Poroseva *et al.* 2000, 2001). However, using the HL turbulent-diffusion model instead of the DH model did help to correct results in Kurbatskii & Poroseva (1999). Possibly, this will also solve the problem when other model expressions for the pressure-strain correlation are incorporated into the RSTE model. Additional calculations at $Re = 2 \times 10^4$ and lower should be made to clarify the role of the turbulent diffusion model in RSTE model performance.

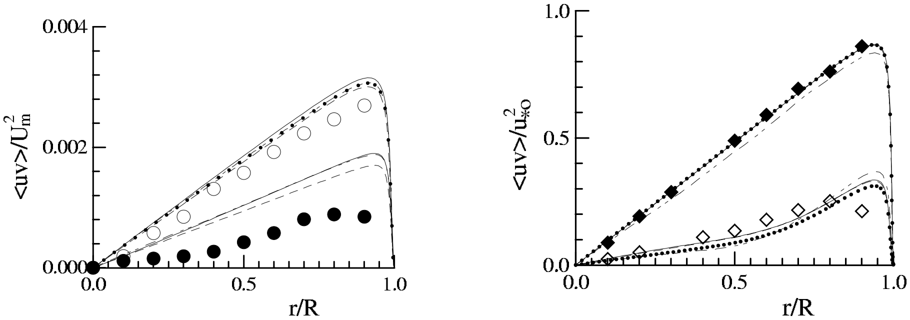

FIGURE 5. Shear stress at **a)** $Re = 2 \times 10^4$, **b)** $Re = 4 \times 10^4$ (see notation in Figure 3).

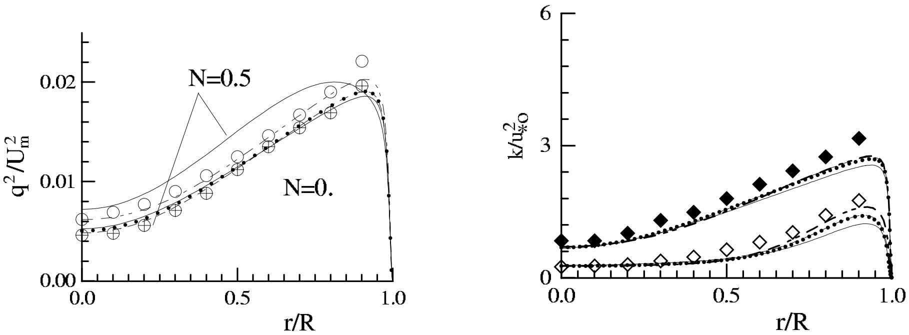

FIGURE 6. Turbulent kinetic energy at **a)** $Re = 2 \times 10^4$, **b)** $Re = 4 \times 10^4$ (see notation in Figure3).

In a stationary pipe flow at $Re = 2 \times 10^4$, the Q model predicts the turbulent kinetic energy better (Fig. 6a). Wall corrections are necessary only in the region ($0.8 \le r/R \le 1$). Figure 6b shows profiles for the turbulent kinetic energy at $Re = 4 \times 10^4$. They are in better agreement with the experimental data than for $Re = 2 \times 10^4$. Also, the results obtained by different models are close to each other, though the Q-model becomes better, relative to the other models, with increasing rotation. It is seen that in a stationary pipe flow all models need additional wall corrections. With increasing rotation, the necessity for such corrections significantly decreases.

The partition of turbulent kinetic energy between different components in a stationary pipe flow is shown in Figs. 7 and 8. For more detailed information in the near-wall area, the data of Laufer (1954) are also given in Fig. 7. Those data were obtained at $Re = 5 \times 10^4$ and therefore can be used only for qualitative comparison.

At $Re = 4 \times 10^4$(Fig. 7), all models reproduce each component of the turbulent kinetic energy in the core of the flow very well. The LRR model yields a $< u^2 >$ profile which lies rather far from the experimental data in the remainder of the flow. The LSSG provides slightly better results than the other models for all components. The SSG needs corrections for $< w^2 >$ more than other models. All models need corrections near the wall, in regions which are roughly ($0.8 \le r/R \le 1$) for normalized $< u^2 >$ and even smaller, ($0.9 \le r/R \le 1$), for $< v^2 >$ and $< w^2 >$. It is seen in the plots that it is the axial component of the turbulent kinetic energy that mostly needs additional treatment. Radial $< v^2 >$, and to a lesser degree circumferential $< w^2 >$ components, though they

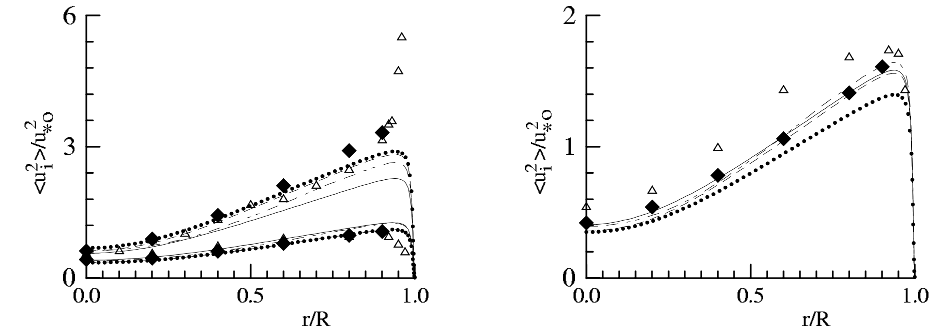

FIGURE 7. Turbulent kinetic energy components in a stationary pipe flow at $Re = 4 \times 10^4$ **a)** $< u^2 > /u_{*o}^2$ (upper), $< v^2 > /u_{*o}^2$ (lower); **b)** $< w^2 > /u_{*o}^2$. Notation is in Figure 3, (△) correspond to experimental data of Laufer (1954).

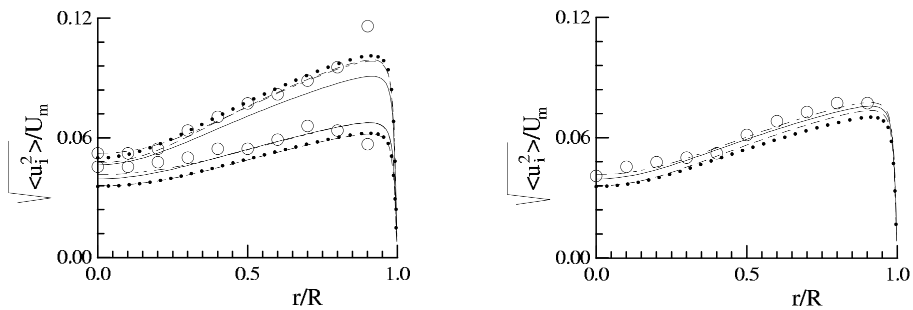

FIGURE 8. Turbulent kinetic energy components in a stationary pipe flow at $Re = 2 \times 10^4$ **a)** $\sqrt{< u^2 >}/U_m$ (upper), $\sqrt{< v^2 >}/U_m$ (lower); **b)** $\sqrt{< w^2 >}/U_m$ (see notation in Figure g3).

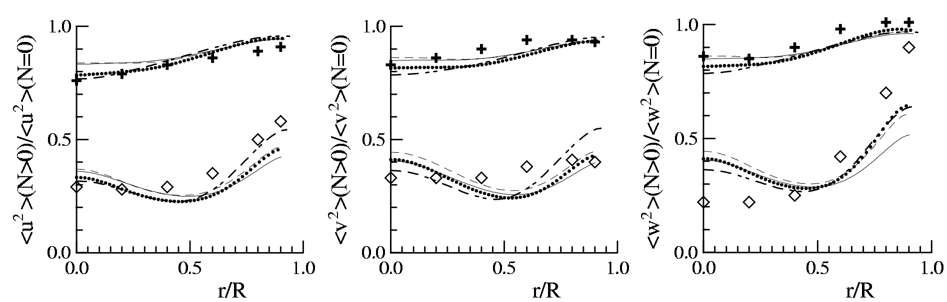

FIGURE 9. Effect of rotation on the turbulent kinetic energy components ($Re = 4 \times 10^4$: see notation in Figure 3).

could be predicted better, contribute less than $< u^2 >$ to the turbulent kinetic energy near a wall.

At $Re = 2 \times 10^4$, the partition of the turbulent kinetic energy between components is shown in Fig. 8. The Q model gives the best agreement with the experimental data. Wall corrections are again necessary only in the region $(0.8 \leq r/R \leq 1)$. It seems that the size of this region is not significantly influenced by Reynolds number, at least for Re values in the range considered here. In rotating flow all three components are significantly overpredicted (not shown here) as in the case of the turbulent kinetic energy.

The influence of rotation on profiles of turbulent kinetic energy components at $Re =$

4×10^4 is shown in Fig. 9. It is seen that the Q model describes the change in the $< u^2 >$ profile with increasing rotation very well, and does not need an additional wall correction. Also, the SSG model describes the core of a flow well, whereas the LRR and the LSSG models overpredict the value of the normalized axial component in the core but underpredict it in the near-wall area. Profiles of $< v^2 >$ and $< w^2 >$ at $N = 0.15$ are better described in the flow core by the LSSG and LRR models. At this value of N no model needs wall corrections. At $N = 0.6$ results obtained by different models are comparable. All models overpredict the core values of $< w^2 >$ but underpredict in the near-wall region, though they show qualitatively correct behavior in the latter region. In this case LRR gives worse agreement with the experimental data. As for calculations of $< v^2 >$ at $N = 0.6$, it is difficult to choose which model gives the best results. All profiles are comparable and in satisfactory agreement with the experiments. Problems observed in the near-wall region can be corrected, at least in some degree (Kurbatskii & Poroseva, 1999), by choosing a turbulent diffusion model more suitable to the flow. However, the cause of the strong overprediction of $< w^2 >$ in the core of the flow is not clear at the moment.

4. Conclusions and future plans

Being the simplest, the IP pressure-strain model is widely used in practical applications. However, as we have seen, this model is too simple to describe complex physics in a pipe flow. The simplicity in such a case is actually a disadvantage, to compensate for which the RSTE model should include additional wall corrections, such as damping functions or the elliptic relaxation scheme. This makes the final model more complex, not necessarily more physical or cheaper than more refined basic approaches to pressure-strain correlation modeling.

The other four pressure-strain models tested in the present study capture the physics of the pipe flow significantly better. Profiles of the mean velocity components, calculated without wall corrections, are in good agreement with the experimental data. The models give results close to each other, but at $Re = 2 \times 10^4$ the circumferential mean velocity profiles calculated by the Q model are rather far from the experimental data. With increasing Reynolds number, the difference between profiles calculated with the different models becomes negligible.

Practically no model needs additional corrections for the $< uv >$ shear stress at high Reynolds number ($Re = 4 \times 10^4$). Disagreement between calculations and experiments observed for $< uv >$ in the near-wall region of rotating pipe flow at $N = 0.6$ can be reduced significantly by choosing a more adequate model for the turbulent diffusion, as was shown by Kurbatskii & Poroseva (1999). The popular DH model does not correctly describe the third-order moments anywhere in a pipe flow. The influence of these moments, which appear in the exact turbulent diffusion terms, is significant, especially in the near-wall region. The tensor-invariant HL model was recommended by Kurbatskii & Poroseva (1999): it will be applied in the future to simulate a pipe flow at low Reynolds number, to clarify in more detail the role of the turbulent diffusion model in such a flow. It is expected that predictions ofthe effect of rotation on the second-order statistics at $Re = 2 \times 10^4$ (which are all overpredicted) will also be improved in this way.

The prediction of the turbulent kinetic energy and its partition between components near a wall in a stationary pipe flow is not strongly influenced by the turbulent diffusion model. Therefore, all models need additional wall corrections for the turbulent kinetic energy, and especially for its axial component, in the region $0.8 \leq r/R \leq 1$ approx. It seems that the size of this region is not much influenced by Reynolds number, at least in the range of values considered here. The radial component needs less correction and the circumferential one needs only minor corrections, if any. Any wall modifications applied to a turbulence model should reflect the observed anisotropy.

In general, the LRR model still seems to miss some important physics. This results mostly in poor predictions of the turbulent kinetic energy and its axial component in a stationary pipe flow. The SSG fails at $N = 1$, predicting flow relaminarization, which is in complete disagreement with the experiments. Other models also predict flow relaminarization, but at least at higher values of the rotation number. The Q model gives better results for turbulent kinetic energy in a stationary pipe flow at both Reynolds numbers, and with increasing rotation at $Re = 4 \times 10^4$. Also, it better describes the partition of the turbulent kinetic energy between its components at $Re = 2 \times 10^4$. However, the circumferential mean velocity profiles calculated with the Q model are in poorer agreement with the experimental data than the profiles obtained with the other models. Note however that unlike the other models tested here, the Q model formulation used in this work is incomplete, in the sense that only the transport equation for the Q–tensor incorporates the new structure tensors. The Q-model formulation for the dissipative process that was used here does not include those tensors, even though there is good reason to expect a significant effect of turbulence structure on the dissipative process. Work by the structure-based modeling group at Stanford to include structure information in the transport equation for a second turbulence scale is currently in progress. A full evaluation of the potential of the structure-based model will become possible once this work is completed: therefore the results reported here for the Q model are preliminary. Finally, the LSSG is the recommended choice for practical calculations at the moment. The model provides a reasonable balance between complexity and quality of performance at all Reynolds and rotation numbers considered here.

The influence of Reynolds and rotation numbers is similar. With increases in either parameter, predictions without additional wall corrections in the pressure-strain models improve significantly.

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