An experimental and computational investigation of flow past cacti

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1. Motivation and objectives

This is an interdisciplinary study motivated by the saguaro cactus and other tall arborescent (treelike) succulents that withstand high wind velocities in their natural habitat. These desert plants have a cylindrical shape, modified by complex surface geometry. Typical diameters are of the order of 0.5 m, and at the highest wind speeds, when the cactus is in danger of being uprooted, the Reynolds number \((Re)\) can be as large as \(10^6\). Because the shape of an object influences the surrounding airflow, natural selection may favor body morphologies that reduce forces exerted by wind gusts in their habitat. We hypothesize that the tall cacti morphology of longitudinal cavities and spines may function to reduce wind forces, including drag and also the fluctuating side-force caused by vortex shedding. We will address this hypothesis by experiments and numerical simulations.

The evolutionary process of random mutations followed by selection for or against those mutations is a continual shaping mechanism on organisms. Being products of their environment, organisms are equipped with adaptations that allow them to cope with the environmental stresses of their habitat. Longitudinal cavities and spines on succulent cylindrical plants evolved independently in two plant families: the Cactaceae of North and South America and the Euphorbiaceae of Southern Africa (Figs. 1a and 1b; Gibson & Nobel 1986). Thus, distantly related plant species living on different continents but in habitats with similar abiotic stresses have converged on a common body morphology. Convergent evolution to a common body shape provides compelling circumstantial evidence for the adaptive significance of this morphology in desert environments.

There has been much speculation on the function of cavities and spines on cacti, and the adaptive significance of the proposed functions is still open to speculation (Geller & Nobel 1984). Natural selection acts on the random mutations of existing structures (traits), resulting in improved structures, novel structures, and/or multiple-functionality of existing structures. Therefore, one function of a trait does not necessarily preclude other functions, and many traits may contribute to a common function. Given that the shape of an object affects the flow, it is surprising that no studies have examined how cavities and spines on desert succulents influence airflow.

Because there are many species of tall arborescent succulents, varying in body size, depth and number of cavities, and spine arrangement, we will focus on one of the most-studied of the tall arborescent succulents, the saguaro cactus, \textit{Carnegiea gigantea} (Fig. 1b, 2, and 3). Saguaro are long-lived and slow to mature. They take 30 to 50 years to reach reproductive maturity and live up to 150 years of age. Adult saguaros have one main cylindrical stem ranging from 0.3 to 0.8 m in diameter (Benson 1981) and over 8 to 15 m in height (Hodge, 1991). Ten to 30 v-shaped cavities span the length of the stem (Hodge 1991). The number of cavities depends on the diameter of the stem, and new cavities can be added or deleted to maintain a cavity depth ratio \((L/D\) – depth of the cavity divided by the diameter of the cylinder) of approximately \(0.07 \pm 0.0015\) (Geller &
Figure 1. Convergence of the external morphology of desert succulents: (a) Euphorbia sp. (Euphorbiaceae) from Southern Africa and (b) young saguaro, *Carnegiea gigantea* (Cactaceae) from North America.

Figure 2. (a) Addition of cavities (ribs) on an adult saguaro trunk (b) Saguaro forest, and (c) Root system of a saguaro toppled by the wind.

Nobel 1984; Fig. 2a). Apices of the cavity junctures are adorned with whorls of 15 to 30 spines 2.5 to 7.6 cm long (Benson 1981).

In order for wind to be a selective agent on saguaros, high wind velocities must occur in saguaro habitats and they must affect their reproductive success. Within the distribution of saguaros, high wind velocities were recorded 15 m above the ground for a nine-year period (Bulk 1984). The maximum wind velocity recorded was 38 m/s, \((Re = 10^6)\), and velocities exceeding 22 m/s \((Re = 7 \times 10^5)\) occurred almost every month. Saguaro habitats contain less vegetation cover than other ecosystems and, consequently, have few if any other tall plants to shelter them from the wind (Fig. 2b). There is substantial circumstantial evidence that wind gusts exert enough force to topple saguaros, and thus, cause
their premature mortality (Fig. 2c; Benson 1981; Alcock 1985; Pierson and Turner 1998), although information on the wind velocities required to topple large desert succulents is lacking. The natural-selection scenario would suggest that some saguaros are toppled by gusts (Fig. 2b), while many others remain standing. Considering that most tall cacti live for 150 years and take 30 to 50 years or more to reach reproduce maturity, strong gusts need only occur only every 30 to 50 years to be important in the natural selection of tall succulent morphology.

Another way stationary organisms can cope with high wind velocities is to increase their structural strength; however, investment in structural tissues has opportunity costs (Denny 1994). Saguars have low investment in the structural tissues of the stem and even less in the roots. Succulent stems are 90 to 94% water (Gibson and Nobel 1986), and, therefore, use little structural tissue "wood" to support their massive structures. Saguaro wood is confined to the center of the stem (xylem fibers: Fig. 3a). The composite stem tissue has a density specific stiffness \( \rho/\epsilon \) less than half of that for a solid wood stem (Niklas and Buchman 1994). The ratio of dry-weight investment in root mass to stem mass in cacti (0.08 to 0.14) is considerably less than most other plants forms (0.3 to 7.3), suggesting that saguaros invest comparatively little in root structural tissue (Nobel 1994). The saguaro root system is shallow, having a mean root depth of 25 cm and consisting of thin roots up to 2.5 cm in diameter (Fig. 2c). Their shallow root system, which provides poor root anchorage, has been noted to result in saguaro toppling when exposed to high wind velocities (Hodge 1991). The ability to dampen fluctuating side-force may also be particularly important in keeping these structures upright because large fluctuations in forces may break or dislodge roots. Because there are probably constraints on tissue strength, and evolution occurs by the natural selection of random mutations, it is conceivable that stationary organisms may evolve shapes that reduce drag and diminish fluctuating side-force.

At high Reynolds numbers (\( Re > 10^4 \)) the drag coefficient (\( C_d \)) curves for spheres and cylinders have four distinct flow ranges, characterized by changes in drag caused by boundary-layer separation and by transition from laminar to turbulent flow (Fig. 4a; Roshko 1961; Achenbach 1977; Farell 1981). In the subcritical range, \( C_d \) is almost independent of \( Re \) (separation is laminar). Then, at the beginning of the critical range, \( C_d \) drops rapidly (boundary layer undergoes transition to turbulence). The lowest \( C_d \) on the curve is within the critical range, at what is referred to as the critical \( Re \). The next range is the supercritical range, where \( C_d \) increases with increasing \( Re \) and continues to increase to the fully turbulent transcritical range.

**Figure 3.** (a) Saguaro stem anatomy (Niklas & Buchman 1994). (b) Sketch of the cross-section
When comparing $C_d$ curves of uniformly rough and smooth cylinders, rough cylinders have $C_d$ curves to the left of their smooth analogs and, therefore, experience the critical range at lower $Re$ (Achenbach 1971). Roughness promotes transition, and, generally, the greater the roughness the greater the shift of the $C_d$ curve to the left (the degree of surface roughness is quantified by the parameter $k/D$, the height of the roughness divided by the diameter of the cylinder). Although a greater degree of uniform surface roughness results in a lower critical $Re$, it is accompanied by a smaller drop in $C_d$ and a smaller critical $Re$ range. In addition, rough cylinders often have higher $C_d$ in the postcritical regime.

Experimental evidence shows that the shape of the $C_d$ curve depends not only on the size but also on the shape and distribution of surface roughness. Cylinders with distributed strips of roughness have been shown to experience early transition without a rapid rise in $C_d$ in the supercritical range (Fig. 4b; Nakamura and Tomonari 1982). Complex surface roughness, such as dimples on a golf ball (Bearman and Harvey 1976) and on a cylinder (Bearman and Harvey, 1993), also have a larger $Re$ range of $C_d$ reduction than cylinders with uniform roughness. Other surface modifications have been studied to passively reduce drag and fluctuating lift forces on circular cylinders; however, none have studied spanwise v-shaped cavities with $0.07L/D$.

This project addresses fundamental concepts in evolution by examining whether organisms are optimally shaped through natural selection to reduce drag and fluctuating lift. The fluid mechanics of cacti has not been examined experimentally or numerically. Such investigation would provide information on how longitudinal cavity depth and complex surface roughness can affect flow. There are surprisingly few studies on the fluid mechanics of biological organisms, especially terrestrial organisms with bluff bodies. There are no known bluff organisms that use surface roughness to reduce drag (Vogel 1981). Surface roughness has been argued to be an unlikely adaptation to control drag, because the reduction in $C_d$ afforded by the surface roughness is accompanied by a dramatic increase in $C_d$ at higher $Re$ (Denny 1988 and Vogel 1981). However, if the increase in $C_d$ occurs at Reynolds numbers that are rarely if ever experienced by the organism in question, it should have no effect on the organism’s evolution.

2. Experimental study

2.1. Wind tunnel

Circular cylinders with diameter $D$ of 9.98 cm were manufactured from Ren Shape 460 Modeling board. Five test cylinders are considered: a smooth cylinder, a uniformly rough cylinder ($k/D = 2.5 \times 10^{-3}$), and three cylinders differing in the depth of the vertical v-shaped cavities ($L/D = 0.035, 0.07, \text{ and } 0.105$, see Fig. 3b). Each $L/D$ cylinder had 24 cavities spanning $15^\circ$; bits were used to cut angles of $124^\circ, 82.5^\circ$ and $60^\circ$ for the 0.035, 0.07, and 0.105 respectively. Roughness on the uniformly rough cylinder was provided by commercial 36 grit sandpaper (hydrodynamic roughness height, $k/D = 2.5 \times 10^{-3}$; Güven, Farell and Patel 1980). Sheets of sandpaper were cut and attached to the smooth cylinder with double-sided adhesive tape, and the thickness added to the cylinder was less than 2 mm.

Experimental measurements were performed at flow velocities from 13 to 29.5 m/s in a low-speed blower wind tunnel with a test section 1.18 m $\times$ 1.18 m in cross section. Cylinders were mounted vertically between two endplates attached to the roof and floor, giving an aspect ratio of $7.06 (h/D)$ and a geometric blockage (cylinder diameter divided by the width of the test section) of 13%. The endplates were $8D$ long by $7D$ wide, and
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**Figure 4.** (a) $C_d$ curve defining the regimes of flow around a smooth cylinder ($Re > 10^4$) and (b) Experimental $C_d$ curves of smooth and rough cylinders cylinders. —— : smooth surface (Achenbach 1971); —— : $k/D = 5.16 \times 10^{-3}$ (Nakamura & Tomonari 1982); —— : smooth cylinder with strips of $k/D = 5.16 \times 10^{-3}$ at $\theta = 50$ degrees.

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**Figure 5.** Smoke flow visualizations at $Re = 13,000$. Flow is right to left (black line behind the cylinders is a fracture in the glass).

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the distance between the cylinder axis and the leading edge was $3.5D$ (Szepessy 1994). The cylinder was rotated about its axis to place the cavities at different orientations to the flow, and then secured with supports.

Simple visualizations were performed using tufts of yarn attached to the cylinders, to a wand, and at the wire intersections of a fine framed grid. Flow was documented
using a Camcorder (Panasonic PV-L857). Visualization experiments were also carried out in a low-speed smoke tunnel (Collins model # 300; Collins Radio Co., Cedar Rapids, Iowa) with a test section 64.3 cm deep, 61 cm high, and 107 cm wide. Vortex streets at Re of approximately 13,000 were observed in all cases but the cylinder with a L/D of 0.07 (Fig. 5). The symmetric vortex shedding of the 0.07 L/D may be an artifact of the test cylinders not spanning the entire width of the test section. All test cylinders were examined in the same way.

Wake velocity profiles were measured with a Pitot-static tube supported by a motorized traversing mechanism. Profiles were measured at 3.2D behind the cylinder. The Pitot-static probe was traversed across the test section to a distance of about D from each wall. A total of 63 points were measured in the wake at a sampling rate 100 Hz for one minute (6000 samples/point).

2.2. Data analysis

In the Re range from 90,000 to 200,000, the cylinders with cavities and the one with uniform roughness had narrower wakes, with smaller velocity defect, than the smooth cylinder. On both counts, this suggests that the cylinders with cavities have a lower C_d than the smooth cylinder.

Velocity profiles were measured at different locations behind the cylinders to determine whether the flow was two-dimensional. Behind the smooth cylinder (Fig. 6a) the profiles are in very good agreement whereas larger discrepancies can be observed behind the cylinder with cavities (Fig. 6b), suggesting that longitudinal cavities may induce strong three-dimensional effects. Additional measurements and flow visualization are required to clarify this issue.
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3. Numerical study

3.1. Numerical method

Preliminary simulations of the flow around a cactus section are carried out by solving the Navier-Stokes equations in two dimensions. Two codes are used: INS2D (Rogers & Kwak 1990) and Fluent (Fluent 1999). INS2D is an upwind based, third-order accurate code for structured (multiblock) grids; the artificial-compressibility approach is used for pressure-velocity coupling and the time integration is second-order accurate. Fluent is an unstructured-mesh solver based on second-order-accurate spatial and time discretization; the SIMPLE technique is used for pressure-velocity coupling. Turbulence modeling is based on the $v^2 - f$ model (Durbin 1995; Iaccarino 2001).

3.2. Computational grids

Cylinders with v-shaped cavities (with cavity ranging from $L/D = 0.0$ to $L/D = 0.105$) are considered. Several meshes have been generated to assess the sensitivity of the solution. In Fig. 7, examples of the grids are shown. Simulations using the structured grids (Fig. 7a and 7b) have been performed using both Fluent and INS2D. The structured grid is generated as an O-type mesh wrapped around the cylinder. The cavities are slightly smoothed to improve the orthogonality of the grid lines at the cylinder surface. The height of the first cell is adjusted according to $Re$; the distance from the far field boundary is $25D$ as used in Rogers & Kwak 1990. The unstructured meshes are generated using a quadrilateral paving technique (Blacker et al. 1991); this approach allows flexibility in clustering the grid cells in the wake region and close to the surface.

Table I shows results for the computations performed on different grids at a very low Reynolds number. The flow is unsteady and exhibits a periodic vortex shedding from the cylinder, but only the averaged drag coefficient is reported. Grid independence is achieved.
for the smooth cylinder \( L/D = 0 \) using both the structured and the unstructured grids, and the corresponding values are extremely close.

The results for the flow around the cylinders with cavities show that grid independence is achieved only using the unstructured grids. An increase in cavity depth requires a finer resolution to capture accurately the in-cavity flow; in addition, the quality of the structured grid degrades as the cavity depth increases. It is worth noting that the results obtained using the finest structured grid (761 \( \times \) 201) are in good agreement with the grid-independent results for the unstructured mesh.

In the following Sections only results computed using the unstructured grids are reported.

<table>
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<tr>
<th>Grid</th>
<th>Elements</th>
<th>( L/D )</th>
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Structured grids

<table>
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<th>( L/D )</th>
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<td>1.338</td>
<td>1.309</td>
<td>1.310</td>
<td>1.317</td>
</tr>
</tbody>
</table>

Unstructured grids

Table I. Computed time-averaged \( C_d \) for different computational grids – \( Re = 100 \)

3.3. Laminar simulations

Flow simulations at low Reynolds number \((Re = 100 \text{ and } Re = 200)\) are carried out to evaluate the effect of cavity depth (and the accuracy of the predictions) without uncertainties related to the turbulence modeling. Two-dimensional simulations have been performed with unstructured grids using 6,000 to 42,000 elements (only the fine mesh results are presented). The calculations are carried out using a timestep \( \Delta t U/D = 0.01 \) (corresponding to approximately 35 time steps per vortex shedding period) and for a total time of \( TU/D = 150 \). The time history of drag and lift coefficients at \( Re = 100 \) are reported in Fig. 8a and 8b respectively. The statistics (time-averaged values, Strouhal number \( St \) etc.) are computed over a period \( T_{av} = 50D/U \) and are reported in Table II: here \( C_l \) is the coefficient of fluctuating side force (peak values shown).

<table>
<thead>
<tr>
<th>( L/D )</th>
<th>( C_d )</th>
<th>( C_l )</th>
<th>( St )</th>
<th>( L/D )</th>
<th>( C_d )</th>
<th>( C_l )</th>
<th>( St )</th>
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<tbody>
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<td>0</td>
<td>1.339 ± 0.010</td>
<td>± 0.330</td>
<td>0.160</td>
<td>0</td>
<td>1.365 ± 0.037</td>
<td>± 0.664</td>
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<td>0.035</td>
<td>1.304 ± 0.011</td>
<td>± 0.325</td>
<td>0.161</td>
<td>0.035</td>
<td>1.361 ± 0.045</td>
<td>± 0.713</td>
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<td>0.070</td>
<td>1.309 ± 0.010</td>
<td>± 0.334</td>
<td>0.162</td>
<td>0.070</td>
<td>1.364 ± 0.057</td>
<td>± 0.742</td>
<td>0.172</td>
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<tr>
<td>0.105</td>
<td>1.318 ± 0.012</td>
<td>± 0.336</td>
<td>0.161</td>
<td>0.105</td>
<td>1.381 ± 0.049</td>
<td>± 0.740</td>
<td>0.170</td>
</tr>
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</table>

Table II. Statistics for low Reynolds number flow around cacti

The results indicate a small drag reduction \((\leq 10\%)\) associated with the presence of the cavities. The cavity depth \( L/D = 0.05 \) is nearly optimal. The change in the unsteady side-force is also small, showing that the effect of the cavity is limited.
The results presented for the smooth cylinder at $Re = 200$ are in good agreement with the numerical simulations and the experimental data reported in Rogers & Kwak (1990). It is worth noting that $Re = 190$ represent the onset of three-dimensional flow in the wake of the cylinder.

### 3.4. Turbulent simulations

Calculations at $Re = 20,000$ and $Re = 100,000$ (subcritical regime, Fig. 4a) are performed using the $u^2 – f$ turbulence model. The time step, the simulated time and the averaging time are the same as before; the time history of lift and drag is shown in Fig. 9.

Compared to the results presented at low $Re$, the drag reduction is now larger ($\approx 25\%$). The strength of the unsteady motion is also greatly reduced, as seen in Table III.

<table>
<thead>
<tr>
<th>$L/D$</th>
<th>$C_d$</th>
<th>$C_l$</th>
<th>$St$</th>
<th>$L/D$</th>
<th>$C_d$</th>
<th>$C_l$</th>
<th>$St$</th>
</tr>
</thead>
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<td>0</td>
<td>1.683 ± 0.164</td>
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<td>0.217</td>
<td>0</td>
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<td>0.035</td>
<td>1.452 ± 0.076</td>
<td>± 1.562</td>
<td>0.221</td>
<td>0.035</td>
<td>1.464 ± 0.120</td>
<td>± 1.462</td>
<td>0.224</td>
</tr>
<tr>
<td>0.070</td>
<td>1.419 ± 0.083</td>
<td>± 1.245</td>
<td>0.224</td>
<td>0.070</td>
<td>1.401 ± 0.131</td>
<td>± 1.128</td>
<td>0.221</td>
</tr>
<tr>
<td>0.105</td>
<td>1.359 ± 0.052</td>
<td>± 0.987</td>
<td>0.223</td>
<td>0.105</td>
<td>1.325 ± 0.079</td>
<td>± 0.864</td>
<td>0.221</td>
</tr>
</tbody>
</table>

$Re = 20,000$  

$Re = 100,000$  

Table III. Statistics for high Reynolds number flow around cacti
From the results presented in Table III, it appears that the cavity depth has a relatively strong effect on the drag and a substantial dampening effect on the unsteady motion.

The time averaged turbulent kinetic energy for the four geometries considered is reported in Fig. 10; the intensity very close to the cylinder decreases with the cavity depth, but higher values are observed in the near wake.

The comparison of the computed $C_d$ with the experimental values for the smooth cylinder (Achenbach 1971) shows an overprediction of about 20%. The flow over the smooth cylinder in the subcritical regime is characterized by a laminar boundary layer separation; turbulence is generated in the separated shear layer and is sustained in the near wake. The smooth cylinder calculations ($L/D = 0$) are carried out with the $v^2 - f$ turbulence model switched off for $\theta \leq 90^\circ$. This is necessary, especially at the higher Reynolds numbers, because turbulence models typically anticipate transition. The simulations with cavities are carried out with the model switched on from the stagnation point ($\theta = 0^\circ$) because it is expected that transition occurs immediately after the first cavity. The exact location of transition has an impact on the accuracy of the drag calculation. In addition, in the subcritical range three-dimensional effects in the real-life wake are substantial.

Experimental and computed velocity profiles in the wake are compared in Fig. 11. The results for the smooth cylinder confirm that the calculation overestimate the drag (corresponding to the larger velocity defect in the wake); on the other hand, the data for the cylinder with cavities show remarkable agreement. It must be pointed out that the measurements exhibit three-dimensional effects that are not accounted for in the present two-dimensional simulations.
4. Conclusions and future plans

The preliminary numerical results presented suggest that the v-shaped cavities provide a damping effect of the fluctuating forces and a drag reduction. Further work is required to assess the effect of the cavities in the range of $Re$ relevant for the cacti.

4.1. Experimental work

Future experiments should focus on obtaining $C_d$ curves over a range of $Re$ from $2 \times 10^4$ (for computational comparisons) to $10^6$ (limit of wind velocities in the saguaro habitat). We will measure drag directly (using a multi-component force transducer - MC3A-X1000, Advanced Mechanical Technology, Inc, Watertown, MA), the pressure distribution around the test cylinders (using 16 static ports attached to a scanivalve), and vortex shedding frequency (using hot-wire anemometry). If there are interesting flow phenomena, the effect of spines on flow around the test cylinders will be evaluated (using 3-D PIV). Finally, experimental measurements will be performed on live cactus specimens.

4.2. Numerical calculations

Two-dimensional RANS calculations will be carried out up to $Re = 10^6$. The pressure and skin friction distributions on the surface will be examined for various cavity depth to evaluate the effect on the local flow characteristics.

The effect of the location of the laminar/turbulent transition must be investigated, together with the impact of the turbulence modeling.
Figure 11. Velocity profiles in the wake of cylinders. simulations (Re = 100,000); experiments (Re = 125,000). (a) smooth cylinder (b) cylinder with L/D 0.07.

In addition, three-dimensional direct simulations will be required to perform a fair comparison with the experimental measurements in the subcritical and transcritical range.

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