Numerical simulation of turbulent polymer solutions

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1. Introduction

Drag reduction using polymer additives in wall-bounded flows poses many challenges to our understanding of turbulence and polymer dynamics, due to the large spectrum of scales involved. Up to 80% of drag reduction can be obtained with ultra-dilute solutions of polymers with high molecular weight, for which each molecule is several orders of magnitude smaller than the smallest turbulent scale of the flow. Owing to this range of scales, numerical simulations as the ones presented here can tackle such flows only by using a continuum model for the polymer dynamics (Sureshkumar et al. 1997). The validity of the most popular models was assessed by comparing the evolution of polymer stress with Brownian dynamic (BD) simulations in simple shear, extensional or rotational flows (Herrchen & Öttinger 1997) but not in turbulent flows. For the latter, the validation has so far been limited to qualitative comparisons of turbulent statistics with experimental data, and only, to the knowledge of the author, for small drag reductions. Even though the agreement of the existing simulations is good, the extension of this type of comparison to higher drag reduction is therefore necessary, but not sufficient. It remains to be demonstrated that the polymer dynamics predicted by the model is consistent with corresponding BD simulations. The present paper uses, for one of the very first times, the two types of comparison to discuss the limitation of the numerical schemes used to solve the model.

For drag-reduced flows with polymers, turbulent statistics have been extensively characterized by experiments. Thus Warholic et al. (1999) established the existence of two distinct statistical regimes. For a given polymer molecule, the mean-velocity profile experiences an upward shift of its log-law region for the smallest concentration, up to a drag reduction \( DR \) of the order of 40%. This regime is referred to as the Low-Drag Reduction (LDR) regime. A further increase in concentration leads to a change in the slope of the log law which defines the High-Drag Reduction (HDR) regime. As more polymers are added, the flow tends toward an asymptotic state, called the Maximum Drag Reduction (MDR) regime, for which drag is slightly higher than the laminar state (Virk & Mickley 1970). The components of the Reynolds stress tensor \( \overline{u_i u_j} \) decrease in magnitude when scaled with outer variables (here, the centerline mean velocity of the Poiseuille flow \( U_c \) and the channel half-width \( h \)) as \( DR \) increases. Yet the diminution of the rms \( u' \) of the streamwise velocity fluctuations is small compared to \( v' \), \( w' \) or \( \overline{uv} \) and it results in an increase of the maximum of \( u'^+ \) in the wall region \((^+ \text{ denotes the scaling by inner variables based on the skin-friction velocity } u_* \text{ and the viscosity } \nu)\). LDR produces the largest maximum values of \( u'^+ \) whereas the peak seems to reduce back to the \( DR = 0\% \) case at HDR and MDR. In this latter regime, Warholic et al. (1999) measured a vanishing Reynolds stress \( -\overline{uv} \) as MDR is approached. The authors concluded that the polymer stress has to be the only source of energy which prevents relaminarization occurring. A more recent experiment by Ptasinski et al. (2001) supports the decreasing
trend of the Reynolds stress but found it to be non-negligible at MDR. It indicates that turbulence structures did not fully vanish in their experiments, adding more confusion as to how MDR could be defined.

In order to simulate all regimes of drag reductions, it has to be assumed that a viscoelastic model can be used. The first simulation of this kind by Sureshkumar et al. (1997), using the FENE-P model, proved to reproduce the shift of the log law in the mean-velocity profile, the increase of streamwise velocity fluctuations and reduction of transverse fluctuations observed at LDR. Sureshkumar et al. used spectral methods while recent results (Min et al. 2001; Dubief & Lele 2001) have shown that finite differences can produce similar results. By improving the robustness of the temporal scheme used in Dubief & Lele (2001) to solve the FENE-P model, it is shown in this paper that a state very similar to the one observed experimentally at HDR can be achieved. The comparison with experiments is extended to coherent structures in the near-wall region.

2. Numerical method

2.1. Formalism

Polymer dynamics has typical length scales much smaller than the smallest turbulent flow scales. Using traditional numerical schemes for flow simulation makes the explicit resolution of molecules unfeasible with current computer facilities; therefore the polymer field has to be modeled. The evolution of polymers is predicted from bead-spring (dumbbell) models. Each dumbbell is subject to the hydrodynamic forces exerted by the flow on the beads, the spring force and Brownian forces. The balance of forces gives an evolution equation for the end to end dumbbell vector \( \mathbf{q} \), known as the FENE (Finitely Extensible Nonlinear Elastic) model. A constitutive approach is obtained by taking the Brownian motion into account, using a phase average of the product of the \( \mathbf{q} \)-components, which defines the conformation tensor \( c_{ij} = \langle q_i q_j \rangle \). The hydrodynamic and relaxation (spring) forces are explicitly simulated; the latter force can be estimated with various models. The model used here is the FENE-P model, where P stands for the Peterlin function, \( f \), defining the following set of equations

\[
\partial_t c_{ij} + u_k \partial_k c_{ij} = c_{kj} \partial_k u_i + c_{ik} \partial_k u_j - \frac{1}{We} (f c_{ij} - \delta_{ij}), \tag{2.1}
\]

\[
f = \frac{1}{1 - c_{kk}/L^2}, \tag{2.2}
\]

The parameter \( L \) is the maximum polymer extension and the Weissenberg number, \( We \), the ratio of the polymer to the flow time scales and ensures the non-dimensionality of (2.1). Finally the contribution of polymers to the flow is brought in the momentum equations via the divergence of the polymeric stress tensor \( \tau_{ij} \),

\[
\tau_{ij} = \frac{1}{We} (f c_{ij} - \delta_{ij}) , \tag{2.3}
\]

yielding the viscoelastic momentum equations,

\[
\partial_t u_i + u_j \partial_j u_i = -\partial_i p + \frac{\beta}{Re} \partial_j \partial_j u_i + \frac{1 - \beta}{Re} \partial_i \tau_{ij}, \tag{2.4}
\]

where \( \beta \) is the ratio of the solvent viscosity \( \eta_s \) to the total viscosity \( \eta \). The last term in the r.h.s. of (2.4) is the contribution of the viscoelastic stress to the flow.
2.2. Spatial derivatives

The numerical code was described by Dubief & Lele (2001). The numerical method follows that of Min et al. (2001). Velocities are discretized on a staggered grid while the pressure and the polymeric tensors $c_{ij}$ and $\tau_{ij}$ are located at the cell-center. Velocity derivatives are computed using second-order finite-difference schemes. To maintain good resolution, the polymeric stress derivatives are calculated with a non-dissipative fourth-order compact scheme. The advection terms of (2.1) are solved using a compact upwind scheme similar to Min et al. (2001), modified to guarantee at least third-order accuracy. Using the following upwinding coefficient,

$$\varepsilon = \frac{1}{2} (s^- + s^+),$$

(2.5)

where $s^-$ and $s^+$ are the sign of the velocity at the interface of the cell, the compact scheme writes

$$(2 + 3\varepsilon)\phi_{i-1}^{'} + 8\phi_{i}^{'} + (2 - 3\varepsilon)\phi_{i+1}^{'} = \frac{1}{6\Delta} [(-1 - \varepsilon)\phi_{i-1} + 2\varepsilon\phi_{i} + (1 - \varepsilon)\phi_{i+1}],$$

(2.6)

Like any upwind scheme, it introduces numerical dissipation at small scales, which proves to stabilize the solution of (2.1). However, as mentioned by Min et al. (2001), an extra dissipation has to be locally added, wherever the tensor $c_{ij}$ is not positive-definite, i.e. when det($c_{ij}$) < 0. The number of nodes affected by the local artificial dissipation (defined in Min et al. 2001; Dubief & Lele 2001) depends on the strength of the flow, the length and Weissenberg number of the polymers and the coefficient of local artificial dissipation (LAD). The worst case is the uncoupled case as will be shown later; as much as 20% of grid points may have det($c_{ij}$) < 0 for high $L$ and $We$. In drag-reduced flows, the number of points requiring LAD drops to significantly smaller fractions, of the order of 5% and less for LDR, and less than 1% for HDR. Further insight on this issue will be given in section 3.

2.3. Time-stepping technique

The numerical method used to solve (2.4) is based on a semi-implicit, fractional-step method (Le & Moin 1991). The Newtonian viscous stress in the wall-normal direction is advanced in time with the Crank-Nicolson scheme, while all other terms in (2.1) and (2.4) are advanced with a third-order Runge-Kutta (RK3) method. After solving (2.1) at time (l), the resulting algorithm is

$$\frac{u_i^{(*)} - u_i^{(l-1)}}{\Delta t} = -\gamma_i N_i^{(l-1)} - \zeta_i N_i^{(l-2)} + \alpha_i \left( L_i^{(l)} + L_i^{(l-1)} + T_i^{(l)} + T_i^{(l-1)} \right)$$

(2.7)

$$\partial_k \partial_k \phi = \frac{1}{\alpha_i \Delta t} \partial_k u_k^{(*)}$$

(2.8)

$$u_i^{(l)} = u_i^{(*)} - \alpha_i \Delta t \partial_k \phi$$

(2.9)
In (2.7), \( N, L \) and \( T \) denote the non-linear, viscous and polymeric terms, respectively. The index \( l \) is the substep of the RK3 and \( \gamma_l, \zeta_l \) and \( \alpha_l \) the corresponding coefficients:

\[
\begin{align*}
\gamma_1 &= \frac{8}{15}; \quad \zeta_1 = 0; \quad \alpha_1 = \frac{4}{15} \\
\gamma_2 &= \frac{12}{5}; \quad \zeta_2 = -\frac{17}{60}; \quad \alpha_2 = \frac{1}{15} \\
\gamma_3 &= \frac{3}{4}; \quad \zeta_3 = -\frac{5}{12}; \quad \alpha_3 = \frac{1}{6}
\end{align*}
\]

The use of a fully-explicit scheme for the the time derivative in (2.1) appeared to be unstable under strong magnitude of turbulence or for large \( We \). The relaxation force is extremely stiff when the trace \( c_{ii} \) approaches \( L^2 \). Small time steps are then required to diminish numerical errors that might induce some local extensions to become larger than \( L^2 \) (Dubief & Lele 2001). From (2.1), it can be inferred that the simulation diverges to infinity when the Peterlin function (2.2) turns negative. Previous studies have used fully or semi-implicit scheme in order to avoid this issue. For FENE dumbbell simulations, Herrchen & Ottinger (1997) implemented a second order semi-implicit predictor-corrector scheme. The implicit part of this scheme yields a cubic equation for the length of the polymer which has a unique root within the physical bounds \( \eta^2 \in [0; L^2] \). A similar method can be applied to the trace of (2.1) in which the relaxation term is solved implicitly and the stretching terms explicitly. Following the same numerical scheme as for the momentum equations (2.4), the time advancement of (2.1) is

\[
\frac{c_{ij}^{(l)}}{\Delta t} - \frac{c_{ij}^{(l-1)}}{\Delta t} = \gamma_l R_{ij}^{(l-1)} + \zeta_l R_{ij}^{(l-2)} - \alpha_l \left[ \frac{1}{We} \left( \frac{c_{ij}^{(l)}}{1 - c_{kk}^{(l-1)} / L^2} - \delta_{ij} \right) + \frac{1}{We} \left( \frac{c_{ij}^{(l-1)}}{1 - c_{kk}^{(l-1)} / L^2} - \delta_{ij} \right) \right],
\]

(2.10)

where

\[
R_{ij}^{(l)} = -u_k^{(l)} \partial_k c_{ij}^{(l)} + (c_{ik}^{(l)} \partial_k u_j^{(l)} + c_{kj}^{(l)} \partial_k u_i^{(l)})
\]

(2.11)

By summing the equations for the diagonal components and using the variable \( \psi^{(l)} = 1 - c_{kk}^{(l)} / L^2 \), equation (2.10) can be simply recast into a second-order polynomial,

\[
(\psi^{(l)})^2 + \frac{\alpha_l \Delta t}{We} \left( 2 + \frac{1}{\psi^{(l-1)} - \frac{6}{L^2}} \right) + \frac{\Delta t}{L^2} \left( \gamma_l R_{ii}^{(l-1)} + \zeta_l R_{ii}^{(l-2)} - \psi^{(l-1)} \right) - \frac{\alpha_l \Delta t}{We} \psi^{(l)} = 0,
\]

(2.12)

whose roots are real and of opposite sign. It can be shown that the unique positive root

\[
\psi^{(l)} = \frac{1}{2} \left( -b + \sqrt{b^2 + 4 \frac{\alpha_l \Delta t}{We}} \right),
\]

(2.13)

approaches zero as

\[
\psi^{(l)} \sim \frac{2 \alpha_l \Delta t}{b^2 We},
\]

(2.14)

when \( b \gg 1 \) (\( b \) is the coefficient of \( \psi^{(l)} \) in 2.12). Note that this scheme can ensure only
Numerical simulation of turbulent polymer solutions

that the trace is upper bounded ($\psi > 0$) but may allow negative values of $c_{kk}$ ($\psi > 1$). The latter situation occurs wherever

$$\alpha_i \Delta t \left( 1 + \frac{1}{\psi^{(l-1)}} - \frac{6}{L^2} \right) - \psi^{(l-1)} + 1 + \frac{\Delta t}{L^2} \left( \gamma_i R_{ii}^{(l-1)} + \zeta_i R_{ii}^{(l-2)} \right) > 0$$

(2.15)

is not satisfied. In this equation, the term $I$ is strictly positive for any $\psi^{(l-1)} \in [0; 1[$ when $L^2 > 3$, while $II$ is subject to fluctuations in both the advection and the stretching terms of $c_{ij}$, which render the value of $II$ with respect to $I$ difficult to predict in a turbulent flow. Even though a positive $\Delta t$ can always be found to ensure that (2.15) is satisfied, we are interested in time steps of the order of the time step of the flow when the CFL number is of the order of unity. The time constraint of the compact upwind scheme (2.6) requires $CFL = 0.5$ (Min et al. 2001), which turns out to be too high for the computation of (2.10). At this CFL number, the solution of the polymer field exhibit strong oscillations at high wavenumbers. For our flow conditions, $CFL = 0.25$ was enough to get rid of the high frequency oscillations, and marginal differences were found with results calculated at $CFL = 0.025$. These two simulations gave approximately the same fraction of grid points at which $c_{ij}$ was not positive-definite. In order to get more insight in the behavior of the term $II$ in (2.15), a simulation was performed without the advection term, so that the polymers do not move with the flow; it was found that the determinant of $c_{ij}$ was negative for only $\sim 10^{-3}\%$ of the nodes for $CFL = 0.25$ and that (2.15) was always satisfied. In this numerical experiment, it can be argued that there is not enough stretching to create a breakdown of (2.15), since the small-scale structures are advected and the polymers are not. The simulation discussed in the next section, where the Eulerian advection is replaced by a Lagrangian, does not experience any loss of positiveness of the conformation tensor. The Eulerian advection can therefore be identified as the major cause of instabilities and unphysical solutions in the computation of the discrete evolution equations of $c_{ij}$, (2.10). It should be noted that the conditions (2.15) and $\det(c_{ij}) > 0$ are necessary but not sufficient. We are currently investigating a modification of the procedure of Min et al. (2001) for which the lost of positiveness is defined as $(\lambda_k(x) \geq 0; k = 1, 3$, where $\lambda_k(x)$ is the $k^{th}$ eigenvalue of $c_{ij}(x)$.

3. ‘Eulerian vs. Lagrangian’ or the problem of pure advection

The FENE-P equations are derived from molecular theories, which predicts the evolution of a single molecule as a function of hydrodynamic, entropic and Brownian forces. In this particular framework, particles move with the flow and the advection term in (2.1) reproduces this motion in the macroscopic formalism. In the Stanford group working on polymers, Mr. V. Terrapon is in charge of Brownian Dynamic simulation of polymer molecules represented as particles moving with the flow. This method allows the study of the effects of the flow on polymers without back-coupling, due to the inadequate number of particles ($\sim 10^5$ in a minimal channel flow unit). The comparison of this microscopic approach with our macroscopic formalism can be made simply by solving (2.1) without the advection terms on the particles which are advected in a Lagrangian manner (called hereafter Particle-Tracking, PT):

$$\frac{d{x}_p}{dt} = u(x_p),$$

(3.1)
where $x_p$ is the location of a particle in the flow. The simulations are run with identical polymer and flow parameters: $L = 60$ and $30$, $We = 3.0$, $\beta = 1$. In the PT simulation, the average is performed over small bins in the wall-normal direction. The mean normal stress is plotted as a function of $y^+$ in figure 1. For both polymer lengths, the overestimation of the Eulerian method is obvious, and the averaging due to the bins for the PT statistics has a very small effect compared to the magnitude of the discrepancy. For stiff polymers ($L = 30$), the difference is slightly smaller than for $L = 60$. In the latter case, increasing the number of grid points brings the macroscopic solution closer to the microscopic one, yet the error remains large. Figure 2 displays snapshots of $c_{kk}/L^2$ in cross-planes for three different resolutions. The finer the resolution, the thinner the regions of highly-stretched polymer. For the coarsest grid, the discretized solution does predict high stretch of the vortices; however this phenomenon is spread over several grid points, yielding these large dark regions. The diffusion is not only due to the use of artificial dissipation. Obviously, the conformation tensor equations generate small scales and sharp gradients that are of energetic importance to the polymer dynamics. As the discretized Navier-Stokes equations behave in LES by increasing the streak dimensions to fit a coarse grid, the discretized equation (2.1) seems to adjust its physics to the grid.

As was implied in the previous section, the advection term cannot be handled without dissipation and was found to cause unphysical solutions. Taken alone as a pure advection equation, such an equation is known to produce extremely sharp gradients, impossible to
Figure 3. Spectral distributions of the streamwise velocity and polymer stress fluctuations. 
**Left:** $DR = 0\%$.  $\circ : u_x$; $\cdots : \tau_{xx}$; $\cdot : \tau_{yy}$; $\ldots : \tau_{zz}$; $\cdots : \tau_{xy}$. **Right:** Symbols and lines show spectra of wall-normal velocity and polymer stress, respectively. $\circ$, $\cdot$: $DR = 0\%$, $\triangle$, $\ldots$: $DR = 20\%$; $\Box$, $\ldots$: $DR = 65\%$. The spectra are normalized by their respective variance.

capture numerically without any artifacts such as artificial diffusion. In the conformation tensor equations, the stretching and entropic terms relate to scales imposed by the flow, from the largest to the Kolmogorov scale ($\eta = \nu^{3/4}/\varepsilon^{1/4}$, where $\varepsilon$ is the dissipation rate), and the relaxation time controlled by the Weissenberg number. Intuitively, it could be assumed that small eddies would create thin regions of polymer stretch and, in such a case, advection is likely to act on these regions to produce smaller scales. How small depends merely on a diffusive phenomenon, which is not explicit in (2.1), since the diffusion of polymer molecules in the solvent is extremely low. A rough estimation of a Schmidt number would be $10^5 \sim 10^6$. The stretching and entropic terms indicate that the behavior of a polymer molecule is not related to its neighbors but depends on the flow scales it experiences locally. Therefore, as a first approximation, it is tempting to draw an analogy between (2.1) and a passive-scalar equation at very high Schmidt number ($Sc = \nu/\kappa$, where $\kappa$ is the scalar diffusion), with the addition of a source term containing the hydrodynamic and entropic forces.

The spectral properties of the passive-scalar equation,

$$\partial_t \theta + u_j \partial_j \theta = \frac{1}{ScRe} \partial_j \partial_j \theta, \quad (3.2)$$

were first studied by Batchelor (1959) and Batchelor et al. (1959). At small scales, their analysis predicts two distinct behaviors of the passive-scalar spectrum depending on the value of $Sc$ relative to unity. For $Sc < 1$, they found that the spectrum of the variance of scalar fluctuations advected by the velocity scales from the inertial subrange should decay as $E_\theta(k) \sim k^{-17/3}$, for wavenumbers greater than $k_B = Sc^{3/4}/\eta$. For $Sc \gg 1$, the theory predicts the so-called Batchelor spectrum,

$$E_\theta \sim k^{-1}, \quad (3.3)$$

valid from the Kolmogorov wavenumber up to the Batchelor wavenumber,

$$k_B = \frac{Sc^{1/2}}{\eta}. \quad (3.4)$$

If the conformation tensor equations (2.1) were to be derived including the actual diffusion of polymers, the length scale would be $10^{-2}$ to $10^{-3}$ smaller than $\eta$, assuming that the effect of the source term could be disregarded. The analogy with the passive scalar
finds some support in the comparison of spectral distributions of the polymer stress to those of velocity. Figure 3 displays spectra, which are normalized by their respective variance to make comparisons easier. It is striking that the polymer stress contains considerable energy at the highest wavenumbers resolved. This result is not surprising since polymers are expected to be mostly affected by small scales. However, this plot demonstrates that our resolution \((\Delta z^+ = 4.5)\) is far from sufficient in the uncoupled case \((DR = 0\%)\). The local character of the artificial dissipation used to stabilize the advection term is equivalent to a MILES approach, which could be defined as a large-eddy simulation where the subgrid-scale model is embedded in the numerical scheme. All the energy contained at the highest wavenumbers is contaminated by the dispersion of our finite difference schemes. An energy backscatter could be a solid assumption to explain the discrepancy with PT simulations (Fig 1). Also plotted in figure 3 is the evolution of the spectra of \(v\) and \(\tau_{yy}\) with increasing \(DR\). The reduction in the small scale energy of the velocity field allows polymer stress spectra to drop faster at high wavenumbers, although the decay is obviously very different from that of the velocity. At HDR, the decay of spectral energy or \(\tau_{ij}\) is sufficient that a fairly coarse grid can be used. An ongoing resolution study for this regime suggests that the coarsest grid used in figure 1 gives results in good agreement with simulations at higher resolution.

The implementation of a subgrid-scale model for the advection term only is currently being investigated, and it has so far been found to improve the solution regarding the loss of positiveness of \(c_{ij}\). Statistics are being gathered to assess the effect on polymer-stress statistics.

4. Results

In this section, three simulations are discussed. The Reynolds number based on the channel half-width and the centerline velocity is 7500, which translates into \(h^+ = 295\) when \(DR = 0\%). This study aims at comparing the solution obtained with a minimal channel flow and the one computed from a domain four times the length and span of a minimal channel flow. These simulations respectively predict \(DR = 67\%)\) and \(DR = 47\%)\) with \(L = 60, \beta = 0.9\) and \(We_r = 84\). A third simulation using the large computational domain is also presented, for which the length and Weissenberg number are increased \((L = 100, We = 120\)\). The drag reduction is 60\%. All simulations are performed on grid with a constant resolution of \(\Delta x^+ = 15, \Delta y^+ = 0.1 - 8\) and \(\Delta z^+ = 9\), when normalized by the skin friction at \(DR = 0\%\).

4.1. Velocity statistics

The effect of the domain size on the mean velocity is quite dramatic (figure 4). While the minimal channel predicts an MDR regime, the larger domain produces a regime which appears to be close to the lower bound of the HDR regime. It is obvious that the HDR regime is populated by very-large-scale structures that have an impact on the drag and therefore need to be resolved. The fact that turbulence does not vanish in the minimal channel flow demonstrates that these very large scales are yet not of the highest importance for the dynamics of the flow. Our hope to achieve MDR by increasing the polymer length and elasticity has not been rewarded, but the trend is encouraging. Also plotted in figure 4 are the data obtained by Dr. White and Mr. Somendapalli, who take care of the experimental study at Stanford. The agreement is very good for \(DR = 47\%)\) and a departure is observed for the MDR case near the centerline. The experiment has been shown to have strong inhomogeneities in the polymer distribution across the
Numerical simulation of turbulent polymer solutions

boundary layer, resulting in the existence of turbulent structures in the outer region of the flow. The near-collapse of the mean velocity profile in the minimal channel with Virk & Mickley (1970)’s asymptote can be attributed to the insufficient length and span of the outer region. As seen in figure 4, mean-velocity profiles at HDR retain a shape typical of turbulence rather than tending toward the laminar profile.

The turbulent intensities decrease as drag reduction increases, as shown in figure 5 (left). The peak of $u'$ shifts away from the wall, but its magnitude decreases slowly compared to $v'$ ($w'$ behaves as $v'$ and consequently is omitted from the plot for clarity). This difference is clear when inner scaling is used. In drag reduced flow, the maximum of $u'^+$ is actually higher or comparable with the $DR = 0\%$ case, as found in experiments (Warholic et al. 1999; Ptasinski et al. 2001). These plots suggest that extremely long and elastic polymers are needed to damp the turbulence, according to the prediction of the FENE-P model. At HDR, the polymer stress (not shown here) is low and therefore the inability of the FENE-P model to predict MDR for polymers, which are more elastic and concentrated than a typical PEO solution, indicates that internal modes may no longer be ignored. The major interest of the minimal channel flow experiment is to understand where the energy which sustains the turbulence comes from. The Reynolds stress, when normalized by outer length scales, diminishes by an order of magnitude in the upper HDR regime (figure 6). In inner variables, the Reynolds stress appears to be
only reduced by a factor of 4 for $DR = 67\%$, which is consistent with Ptasinski et al. (2001) but disagrees with Warholic et al. (1999) who found a near-zero Reynolds stress at MDR. The contribution of the polymer stress in the balance,

$$\frac{-\overline{\tau y}}{u_z^2} \left(1 - \frac{y}{h}\right) + \frac{\beta}{u_z^2 Re} \frac{\partial U}{\partial y} + \frac{1 - \beta}{u_z^2 Re} \tau_{xy} = 0,$$

is of the same order as the Reynolds stress in our simulations of HDR, even larger in the case of the minimal channel. In spite of the incomplete physics simulated in the minimal channel, such simulation provides extremely valuable information about what MDR could be. Thus it can be surmised that MDR is purely a near-wall phenomenon, since $\tau_{xy}$ is larger than $-\overline{\tau y^+}$ from the wall up to $y^+ = 70$. Considering the weak magnitude of the turbulent intensities, the polymer dynamics is likely to be driven by shear-dominant event. It should be noted that Warholic et al. (1999) and Ptasinski et al. (2001) suggested also that MDR is sustained by polymer stress alone.

4.2. Structure of the HDR turbulence

Using an appropriate vortex-identification scheme (the $Q$-criterion in the present case, $Q = (\Omega_{ij} \Omega_{ij} - S_{ij} S_{ij})/2$, Dubief & Delcayre 2000), figure 7 illustrates the dramatic modification of coherent vortical structures between the Newtonian flow and the HDR regime. At HDR, near-wall vortices are weaker and more horseshoe-type vortices can be found. The weakening of the vortices has also been observed by our experimental group, together with an increase of streak dimensions as depicted by the contours of the polymer stretch $c_{kk}$ at the wall. Due to the no-slip condition, the dynamics of $c_{kk}$ at the wall is driven by $\partial u/\partial y$. So far, the streamwise coherence of the streaks has not been determined either experimentally or numerically. Experimental data indicate that they could be several thousands of wall units long. Also shown in figure 7, the polymer stretch $\sqrt{c_{kk}}/L$ in the channel flow (contours on the side walls) exhibits completely different patterns for the Newtonian simulation and the HDR regime. In the former, regions of high stretch are observed to extend in the wall-normal direction and they are found to correlate with upwash and downwash motions of the flow (not shown here). For the HDR regime the polymer field is organized in thin layers which seem to denote the energetic activity of the polymers in the near-wall region.
Figure 7. Snapshot of vortices (top) and polymer stretch (bottom) for the Newtonian case (left) and HDR (right). The vortices are identified with the $Q$-criterion: $Q = 0.5k^2/U_c^2$ for the Newtonian flow and $Q = 0.08k^2/U_c^2$ for HDR.

5. Perspectives

The simulation of turbulent viscoelastic flows requires overcoming many obstacles, and the present work only partially resolves the major issues. Although desirable, the implementation of a subgrid scale for the advection in the conformation tensor equation is not expected to resolve the discrepancy between the Eulerian and Lagrangian approaches. This study has at least shown the importance of solving scales close to Kolmogorov with as little dissipation for the polymer stress as possible. The size of the computational domain is another issue addressed by this paper. It is not clear how long the channel needs to be, since experimental observations indicate that streaks might be coherent over several thousands wall units. Nevertheless, it is remarkable that, in spite of a simplistic model and unresolved (large and small) scales, our simulations reproduce many features observed experimentally in the HDR regime.

Based on our results obtained experimentally and numerically at the macroscopic and microscopic levels, some advances have been made regarding the understanding of the phenomenon of drag reduction with polymer additives. In publications to appear, it will...
be demonstrated that polymer drag reduction is purely a near-wall phenomenon, which affects almost exclusively quasi-streamwise vortices.

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REFERENCES


