Optimal aeroacoustic shape design using approximation modeling

By Alison L. Marsden, Meng Wang and Petros Koumoutsakos

1. Introduction

Reduction of noise generated by turbulent flow past a trailing edge continues to pose a challenge in many aeronautical and naval applications. Aeroacoustics problems related to such applications necessitate the use of large-eddy simulation (LES) or direct numerical simulation in order to capture a wide range of turbulence scales which are the source of broadband noise. Much previous work has focused on development of accurate computational methods for the prediction of trailing-edge noise. For instance, aeroacoustic calculations of the flow over a model airfoil trailing edge using LES and aeroacoustic theory have been presented by Wang & Moin (2000) and were shown to agree well with experiments. To make the simulations more cost-effective, Wang & Moin (2002) successfully employed wall models in the trailing-edge flow LES, resulting in a drastic reduction in computational cost with minimal degradation of the flow solutions. With the recent progress in simulation capabilities, the focus can now move from noise prediction to noise control. The goal of the present work is to apply shape optimization and control theory to the trailing-edge flow previously studied, in order to minimize aerodynamic noise. In this work approximation modeling techniques are applied for shape optimization, resulting in significant noise reduction in several cases.

1.1. Choice of optimization method

One general distinction among optimization techniques is between gradient-based methods and non-gradient-based methods. The choice of method for a particular problem depends on factors such as the cost of evaluating the function, the level of noise in the function, and the complexity of implementation. Gradient-based methods generally include adjoint solutions and finite-difference methods. Non-gradient-based methods may include pattern-search methods, approximation models, response-surface methods and evolutionary algorithms. In LES-based aeroacoustic shape design, cost-function evaluations are computationally expensive, and hence the efficiency of the optimization routine is crucial. Therefore, a key consideration is cost minimization when choosing an optimization method.

One of the difficulties in gradient-based optimization methods is the calculation of the gradient of the cost function with respect to the control parameters. The most widely-used gradient method is to solve an adjoint equation in addition to the flow equations, as has been successfully demonstrated by Jameson et al. (1998) and Pironneau (1984). However, adjoint methods are difficult to implement for time-accurate calculations, and can present data storage issues. Additionally, adjoint solvers are not portable from one flow solver to another. Because of these factors, the method of “incomplete sensitivities” was initially chosen for the gradient calculation. This method, suggested by Mohammadi & Pironneau (2001), ignores the effects of geometric changes on the flow field when computing the
gradient of a surface-based cost function. This makes it simple to use, and far more cost-effective than solving the full adjoint problem. In fact, only a small additional cost to the flow computation is needed for every iteration. Examples demonstrating the method are given by Mohammadi & Pironneau (2001).

Initially, application of the method of incomplete sensitivities produced seemingly promising results, as presented by Marsden, Wang & Mohammadi (2001). However, on further study the method was found to break down for several important cases. A systematic evaluation of the incomplete-sensitivity method was carried out by comparisons with the “exact” gradient in the case of a single control parameter. It was found that the exact and incomplete gradients do not agree with each other; furthermore, they do not always have the same sign, as shown by Marsden et al. (2002). This finding shows that neglecting the state contribution to the gradient is not valid for the problem of trailing-edge noise. We thus conclude that the incomplete sensitivities approach is not adequate for the present application.

In choosing an alternate optimization method, we are concerned with identifying a method that has robust convergence properties and yet is computationally feasible. To this end, the method of approximation modeling was chosen for exploration. Approximation modeling was developed for use in engineering optimization problems which require the use of expensive numerical codes to obtain cost function values. Gradient information for these problems is often difficult or impossible to obtain. In addition, many optimization problems have associated data sets which have error, or cost functions which are noisy. For all of these reasons, there has recently been considerable interest in using approximation modeling for optimization with large engineering simulations.

1.2. Introduction to approximation modeling

Approximation modeling is a family of non-gradient-based methods which rely on model, or surrogate, functions to approximate the actual function. Optimization is performed, not on the expensive actual function but on the model, which is cheap to evaluate. The use of approximation modeling for expensive functions has been demonstrated by Booker et al. (1999), Serafini (1998), Chung & Alonso (2002) and others. These surrogates can be polynomials, in which case the models are called “response surface” models, or interpolating functions such as splines or more advanced functions.

As an illustration of this method, let us assume we wish to find a minimum of the function $y = f(x)$ within an allowable domain $x_{\text{min}} \leq x \leq x_{\text{max}}$. The basic procedure using an approximation-modeling technique is as follows. First, we begin with a set of initial data points $\mathbf{x} = [x_1, x_2, \ldots, x_n]$ where the function values are known. We then fit a surrogate function through these points to approximate the actual function. We express the surrogate function as $\hat{y} = \hat{f}(x)$. Because the surrogate function is inexpensive to evaluate, its minimum (within the allowable range of $x$) can be easily found using standard optimization methods. When the minimum is found, the actual function is evaluated at this point, the surrogate fit is updated, and the process continues iteratively until convergence to a minimum function value.

Approximation-modeling methods have several possible variations. One is the choice of surrogate function. Others include the choice of initial data distribution, and the use of merit functions to ensure a good distribution of the data. In this work, results are presented using approximation-modeling methodology on the model problem of Marsden et al. (2001). Details of the optimization procedure are discussed in section 3. In section 4, results are presented for a one-parameter case, for which we compare the performance of several surrogate functions. In section 5 we present results of a calculation using two
parameters, and discuss the effects of the initial data set on the final solution. Extension of the method to several parameters is also discussed. Using approximation-modeling methods, a significant reduction in cost function is demonstrated for several one- and two-parameter cases. The method is shown to be robust and computationally affordable. In addition, the method has uncovered a new airfoil shape which gives a greater reduction in noise than previously achieved.

2. Formulation and cost-function definition

We begin by formulating the general optimization problem. Given a partial differential equation $A(U, q, a) = 0$ defined on the domain $\Omega$ with control variables $a$, state variable $U$ and design parameters $q$, we wish to minimize a given cost function $J(U, q, a)$. The control problem can be stated as

$$
\min \{ J(U, q, a) : A(U, q, a) = 0 \ \forall x \in \Omega, \ b(U, q, a) = 0 \ \forall x \in \partial \Omega \} \quad (2.1)
$$

where $b(U, q, a)$ is the boundary condition of the PDE. In our problem, the state equations are the Navier-Stokes equations and the cost function is the acoustic source.

The ultimate goal of this work is to optimize an airfoil shape with fully-turbulent flow at the trailing edge. Because of the cost of LES calculations, the optimization method is first implemented and validated on an unsteady laminar model problem, which is the subject of the present work. The airfoil geometry for the model problem is shown in figure 1 and is a shortened version of the airfoil used in experiments of Blake (1975). The airfoil chord is 10 times its thickness, and the right half of the upper surface is allowed to deform. The flow is from left to right and results presented in this work are at a chord Reynolds number of $Re = 10,000$. Previously, in Marsden et al. (2001), results were also presented for $Re = 2,000$, and it was shown that the cost function was easily reduced to zero. The focus of the present work is therefore on the higher Reynolds number.

Before discussion of the optimization method, we outline the derivation of the cost function for the model problem. For unsteady laminar flow past an airfoil at low Mach number, the acoustic wavelength associated with the vortex shedding is typically long relative to the airfoil chord. Noise generation from an acoustically-compact surface can be expressed as follows, using Curle’s extension to the Lighthill theory (Curle 1955),

$$
rho \approx \frac{M^3}{4\pi} \frac{x_i}{|x|^2} \hat{D}_i(t - M|x|), \quad \hat{D}_i = \frac{\partial}{\partial t} \int_S n_j p_{ij}(y, t) d^2y \quad (2.2)
$$

where $\rho$ is the dimensionless acoustic density at far field position $x$, $p_{ij} = \rho \delta_{ij} - \tau_{ij}$ is the compressive stress tensor, $n_j$ is the direction cosine of the outward normal to the airfoil surface $S$, $M$ is the free-stream Mach number, and $y$ is the source-field position vector. All the variables have been made dimensionless, with airfoil chord $C$ as the length scale, free stream velocity $U_\infty$ as velocity scale, and $C/U_\infty$ as the time scale. The density and pressure are normalized by their ambient values. Note that (2.2) implies the three-
dimensional form of Lighthill’s theory, which is used here to compute the noise radiated from unit span of a two-dimensional airfoil. The radiation is of dipole type, caused by the fluctuating lift and drag forces.

The mean acoustic intensity can be obtained from (2.2), as

\[ I = \frac{M^6}{16\pi^2|x|^2} \left( \overline{D_1 \cos \theta + D_2 \sin \theta} \right)^2 \]  

(2.3)

where the overbar denotes time averaging, and \( \theta = \tan^{-1}(x_2/x_1) \). To minimize the total radiated power, we need to minimize the integrated quantity

\[ \int_0^{2\pi} I(r, \theta) r \, d\theta = \frac{M^6}{16\pi |x|} \left( \overline{D_1 \theta + D_2^2} \right). \]  

(2.4)

Hence, the cost function is defined as

\[ \bar{J} = \left( \frac{\partial}{\partial t} \int_S n_j p_{ij}(y, t) \, d^2y \right)^2 + \left( \frac{\partial}{\partial t} \int_S n_j p_{2j}(y, t) \, d^2y \right)^2 \]  

(2.5)

which corresponds exactly to the acoustic source function.

3. Optimization procedure

In this section, we outline the steps in the algorithm used to optimize the airfoil shape. Our aim is to find the minimum of the cost function defined by (2.5). The cost function, \( J \), depends on control parameters corresponding to the surface deformation. To start the optimization process, the cost function is evaluated for several initial points in the parameter space. The subsequent steps are as follows:

1. Fit a surrogate function through the set of known data points
2. Estimate the function minimum using the surrogate function
3. Evaluate the true function value at the estimated minimum
4. Check for convergence
5. Add new data point to list of known points and go back to 1.

Iterations continue in this way until the parameters have converged to give a final airfoil shape.

The control parameters are defined as follows. Each parameter corresponds to a deformation point on the airfoil surface which must be within the deformation region. The value of each parameter is defined as the displacement of this point relative to the original airfoil shape, in the direction normal to the surface. A positive parameter value corresponds to displacement in the outward normal direction, and a negative value corresponds to the inward normal direction. A spline connects all the deformation points to the trailing-edge point and the left side of the deformation region to give a continuous airfoil surface. Both ends of the spline are fixed. While the surface must be continuous and smooth on the left side, the trailing-edge angle is free to change.

For a given set of parameter values, there is a unique corresponding airfoil shape. To calculate the cost-function value for a given shape, a mesh is generated and the flow simulation is performed until the solution is statistically converged. Because the flow has unsteady vortex shedding, the cost function is oscillatory. In the optimization procedure, the mean cost function \( \bar{J} \) (cf. (2.5)) is used, and is obtained by integrating in time until convergence. An example of the oscillatory cost function, and the time-averaged value is shown in figure 2. The case shown corresponds to the original airfoil shape.
4. One-parameter results

Results using a single parameter, $a$, are presented for several surrogate function choices. With a polynomial surrogate function, at least three initial data points are needed. In all one parameter cases presented here, the allowable range of $a$ is $-0.05 \leq a \leq 0.02$. The thickness of the airfoil is 0.1, and the chord length is unity.

Figure 3 shows the evolution of the response surface using a third-order polynomial as the surrogate function. The upper left plot shows the three initial points, and following plots show three iterations on the value of $a$ corresponding to the minimum. With each iteration, the surrogate function evolves to include all known cumulative data. Convergence is reached when the function minimum does not change from one iteration to the next. A least-squares fit of the polynomial is used, so the polynomial does not go exactly through the data points. A total of six function evaluations is required to reach convergence, and a 19% reduction in cost function is achieved.

As expected, an improvement in the function fit is obtained by using a fourth order polynomial, as shown in figure 4. The optimization procedure is the same as for the third order case. A more significant reduction in cost function, 26%, is achieved. However, there is a trade-off in computational cost, since the higher-order polynomial picks up more detail in the function but requires eight function evaluations.

Figure 5 shows results for a single parameter case using a cubic spline as the surrogate function. The optimum airfoil shape corresponding to the minimum cost-function value achieved with the spline fit is shown in figure 6. It is qualitatively similar to the shapes obtained using the polynomial surrogates and these are not shown. Because the surrogate spline is piecewise and fits exactly through the data points, it captures more detail in the function than either polynomial case for the present problem. Similar to the fourth-order polynomial case, the cost function reduction is 27% with 12 function evaluations.

Using only one parameter, we have demonstrated that the approximation modeling
method is robust and converges to a minimum with a modest number of function evaluations. A significant reduction in cost function has been achieved and the results for all surrogate functions were qualitatively similar. Comparatively, the spline surrogate function resulted in the greatest cost-function reduction. The cases using polynomials emphasize that it is undesirable to use low-order polynomials as global models, since they are unable to capture details of the function such as multiple local minima. However, there is also danger in increasing the order of the polynomial, due to oscillations between the data points known as the “Runge” phenomenon. To avoid these problems, one may wish to use a “trust region” method, in which the polynomial model is restricted to a region near the minimum, where the function is approximately quadratic.
5. Two parameters

Although the results using one parameter are very promising, the true test is whether the computational cost remains reasonable when the method is extended to more parameters. In this section, results are presented using two parameters, \(a\) and \(b\), for which a biharmonic spline was the surrogate function. The deformation points for parameters \(a\) and \(b\) are evenly spaced in the deformation region of the airfoil surface. A spline is chosen as the surrogate function, based on the results for the one-parameter test case, and the optimization procedure is the same as in the one-parameter case. We also study the effect of choice of initial data on the final solution. Three sets of initial data were used, which we call A, B and C. For all cases, the parameters are limited by \(-0.05 \leq a \leq 0.02\) and \(-0.035 \leq b \leq 0.02\). The lower limit corresponds to a straight line connecting the left edge of the deformation region and the trailing edge.

The left side of figure 7 shows the initial data points used for the two-parameter case with data set A. Contours of the mean cost function value, \(\bar{J}\), are shown with parameters \(a\) and \(b\) plotted on the axes. In this case, the initial data points are not chosen to lie in a particular pattern. The final surrogate-function fit is shown on the right side of figure 7. The cluster of points near the minimum shows the convergence of the solution, and the surface has one minimum valley. The cost-function reduction for this case was
Figure 7. Case A: Contours of mean cost function, $\bar{J}$, vs. parameters $a$ and $b$ for two-parameter case with biharmonic spline as surrogate. Data points marked with \times. Initial data points are shown on left, final converged solution is shown on right.

Figure 8. Initial (black) and final (gray) airfoil shapes using two parameters with biharmonic spline as surrogate function, data set A.

29% which is a slight improvement over the best one-parameter case. As expected, the two-parameter case requires more function evaluations; 17 evaluations were required for case A. Figure 8 shows the initial and final airfoil shapes for this case.

Contours of the initial surrogate-function fit using data set B are shown on the left of figure 9. In this case, the initial data were chosen in a small star pattern centered around the origin. Like case A, the final surrogate fit, shown in the right of figure 9, has one minimum valley. However, it gives a solution qualitatively very different from case A, suggesting that the actual function has at least two local minima. The optimum airfoil shape for case B is shown in figure 10. In contrast to case A (Fig. 8), the trailing-edge angle in case B has increased instead of decreased. Although the magnitude of the shape deformation is relatively small, the reduction in cost function is significant at 52%. This solution was not previously expected.

Cases A and B show that the initial data set can dramatically impact the final solution, causing the solutions to converge to two distinct local minima. In both cases a viable airfoil shape was found which resulted in a significant cost-function reduction. However, ideally, we desire the solution to converge to the global minimum, and to give the same result independent of the initial data choice. It is, of course, impossible to guarantee convergence to the global minimum, and the cost-function reduction is always limited by the parameter space. However, there are ways to increase our chances of converging to a global minimum and improve robustness. For instance, by choosing an initial data set which covers the entire allowable range of $a$ and $b$, the solution is not biased toward a minimum in a particular area. To demonstrate this, initial data set C is chosen as a
large star pattern centered around the origin and shown by the surrogate fit in the left of figure 11. The final surrogate fit is shown on the right of figure 11 and this solution gives a cost function reduction of 45%. We notice that the final fit of case C captures two local minima. The final airfoil shape is qualitatively similar to that of case B but the minimum is located in a slightly different position, suggesting that solutions B and C may not be well converged. The total number of function evaluations has been reduced dramatically, from 23 evaluations required for case B to 9 evaluations required in case C.

By introducing a second parameter to the problem, it has been demonstrated that results improve dramatically and the cost of the optimization problem remains manageable. A second parameter also gave the flexibility to find new solutions, as in cases B and C, which are not admissible in the one-parameter space. The importance of choosing an initial data set which spans the parameter space has also been confirmed. It can increase the chance of finding the global minimum and reduce the number of iterations.

The physical reasons for the reduction in cost function can be explained by the vortex-shedding characteristics and associated unsteady forcing on the airfoil. Figure 12 compares vortex-shedding characteristics for the original shape and the final shapes of cases A and C in terms of the instantaneous streamwise velocity. We see that the vortex shedding strength has decreased for both cases A and C compared to the original. As a result, the amplitude of lift fluctuations, which dominate the acoustic dipole source, has been reduced by 12% for case A and 24% for case C. The larger decrease in lift amplitude

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**Figure 9.** Case B: Contours of mean cost function, $\bar{J}$, vs. parameters $a$ and $b$ for two-parameter case with biharmonic spline as surrogate. Data points marked with $\times$. Initial data points are shown on left, final converged solution is shown on right.

**Figure 10.** Initial (black) and final (gray) airfoil shapes using two parameters with biharmonic spline as surrogate function, data set B.
Figure 11. Case C: Contours of mean cost function, $\bar{J}$, vs. parameters $a$ and $b$ for two-parameter case with biharmonic spline as surrogate. Data points marked with $\times$. Initial data points are shown on left, final converged solution is shown on right.

Figure 12. Instantaneous streamwise velocity contours for original (top), case C (middle) and case A (lower). Contour levels are the same for all cases, between $-0.4 < u < 1.53$.

for case C explains why the reduction in cost function is greater for cases B and C than for case A. The shedding frequencies for all cases are similar. The optimal shape found in case A also shows an increase in mean lift of 40% over the original value, whereas in case C a slight 3% decrease in mean lift is observed. In practical application it is often necessary to ensure that the mean lift is not reduced by the shape optimization. To this end, aerodynamic properties will need to be included using multi-objective optimization methods, which may slightly compromise the large reduction in cost function found in case C.

To extend the method to several parameters, it will be desirable to use a surrogate function which is easy to implement and has good behavior in high dimensions. Many model functions exhibit undesirable behavior such as excessive “wiggles” between data
points when extended to high order. An alternative is the use of kriging functions, a type of interpolating model first introduced in the geostatistics community by South African geologist D. G. Krige. Kriging is based on statistics and random-function theory, and is easy to implement in an arbitrary number of dimensions. The underlying idea is to use a weighted linear combination of values at the sampled data locations to interpolate the function. The best linear estimator is then found by minimizing the error of the estimation. A detailed derivation of the method can be found in Isaaks & Srivastava (1989) and Guinta (2002). Kriging has since been adopted by the optimization community and is now used in many engineering problems.

6. Discussion and future work

A summary of results using several surrogate functions with one and two parameters is presented in table 1. The table clearly shows an improved reduction in cost function with the addition of a second shape parameter. Generally, the cases with greater cost function reduction also require more function evaluations, although a judicious selection of the initial data points can speed up convergence dramatically. It is advantageous to chose initial data which span the entire parameter space. Results using approximation modeling are a nearly two-fold improvement over the results of previous methods presented in Marsden et al. (2001).

In future work, the use of kriging functions with several optimization parameters will be explored. Extension to multiple parameters will determine the scalability of this method in terms of the number of function evaluations. It will also determine whether further reductions in cost function are possible, and if the trade-off in computational cost is significant. Additionally, there are several variations on the method used here which could improve convergence and robustness. As shown by Booker et al. (1999) and Serafini (1998), the use of a polling step in the algorithm rigorously guarantees convergence to a local minimum. The use of merit functions, as discussed in Torczon & Trosset (1998), to ensure good data distribution will be explored. With this method, a weighting function is employed to determine whether the data points are well distributed throughout the domain. If the data points are clustered together, additional points may be evaluated to improve the surrogate function fit in areas lacking data. To speed up the optimization process, it may be desirable to evaluate the minimum point in parallel with several other points which improve the function fit. These variations of the method will explore the possible cost trade-off between fast convergence to a local minimum and increased chance of reaching a global minimum. The use of multi-objective optimization to include aerodynamic properties (lift and drag) and thickness will also be required in the design of practical trailing-edge shapes.

Once confidence has been gained in the specifics of the approximation-modeling method, optimization of the turbulent trailing-edge flow will be performed. In the turbulent case, the airfoil is not acoustically compact for all the frequencies of interest, and the cost function may need to be reconsidered. Alternatively, an approximation of the cost function can be used so long as it is well correlated with the true acoustic source function. The choice of cost function will be influenced to some degree by whether the objective is to reduce noise in a band of frequencies of primary interest, or to reduce the total radiated power.
surrogate function | parameters | $\bar{J}_{\text{orig}}$ | $\bar{J}_{\text{optimum}}$ | % reduction | function evaluations
--- | --- | --- | --- | --- | ---
$3^{rd}$ order polynomial | 1 | 0.166 | 0.1348 | 19% | 4
$4^{th}$ order polynomial | 1 | 0.166 | 0.1223 | 26% | 8
Cubic spline | 1 | 0.166 | 0.1215 | 27% | 12
Biharmonic spline, set A | 2 | 0.166 | 0.1174 | 29% | 17
Biharmonic spline, set B | 2 | 0.166 | 0.0794 | 52% | 23
Biharmonic spline, set C | 2 | 0.166 | 0.0912 | 45% | 9

Table 1. Summary of results for several surrogate functions choices with one and two parameters. Cost function reduction and number of function evaluations needed for convergence are compared for all cases.

Acknowledgments

This work was supported by the Office of Naval Research under grant No. N00014-01-1-0423. Computer time was provided by NAS at NASA Ames Research Center and the DoD’s HPCMP through ARL/MSRC and NRL-DC.

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