The effect of the density ratio on the nonlinear dynamics of the unstable fluid interface

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1. Motivation and objectives

When a light fluid accelerates a heavy fluid, the misalignment of the pressure and density gradients gives rise to the instability of the interface, and produces eventually the turbulent mixing of the fluids (Rayleigh 1892; Davies & Taylor 1950; Richtmyer 1960; Meshkov 1969). This phenomenon is called the Rayleigh-Taylor instability (RTI) if the acceleration is sustained, and the Richtmyer-Meshkov instability (RMI) if the acceleration is driven by a shock or if it is impulsive. The RT/RM turbulent mixing is of extreme importance in astrophysics, inertial confinement fusion, and many other applications (Sharp 1984). To obtain a reliable description of the mixing process, the evolution of a large-scale coherent structure, the dynamics of small-scale structures, and the cascades of energy should be understood.

The large-scale coherent structure is an array of bubbles and spikes periodic in the plane normal to the direction of gravity or the initial shock (Rayleigh 1892; Davies & Taylor 1950; Meshkov 1969; Schneider *et al.* 1998). It appears in the nonlinear regime of RTI and RMI and has a spatial period determined by the mode of fastest growth (Chandrasekhar 1961). The light (heavy) fluid penetrates the heavy (light) fluid in bubbles (spikes). The density ratio is a determining factor of the instability dynamics (Sharp 1984; Schneider *et al.* 1998). Singular aspects of the interface evolution (such as the generation of vorticity and secondary instabilities, resulting in the direct and inverse cascades of energy, a finite contrast of the fluid densities and the non-linearity of the dynamics) cause theoretical difficulties and preclude elementary methods of solution (Dalziel *et al.* 1999; He *et al.* 1999; Gardner *et al.* 1988; Jacobs & Sheeley 1996; Kucherenko *et al.* 2000; Holmes *et al.* 1999; Volkov *et al.* 2001).

For fluids with highly contrasting densities (fluid-vacuum), the effect of singularities on the interplay of harmonics and on the nonlinear motion in RTI/RMI has been studied intensively over the decades (Layzer 1955; Garabedian 1957; Tanveer 1993). A new approach based on group theory has been developed recently by Abarzhi (1998, 2002). The asymptotic theories of Layzer (1955); Garabedian (1957); Abarzhi (1998, 2002) agreed with experiments and simulations. For fluids with a finite density contrast, the influence of singularities on the cascades of energy and the large-scale coherent dynamics in RTI/RMI has yet to be elucidated (Baker *et al.* 1982; Moore 1979; Hou *et al.* 1997; Cowley *et al.* 1999; Matsuoka *et al.* 2003). The empiric models proposed by Sharp (1984) and Oron *et al.* (2001) could not explain observations completely and were a subject for controversy. These models disregarded the conservation of mass and introduced adjustable parameters to balance drag, buoyancy, and inertia in the flow, (Dimonte 2000). In a recent attempt of Goncharov (2002) to reproduce the results of the drag model of Oron *et al.* (2001) in a single-mode approximation, a complete set of the boundary conditions were not satisfied (see below), and the conservation laws were thus violated.

Here we report multiple harmonic theoretical solutions for a complete system of conservation laws, which describe the large-scale coherent dynamics in RTI and RMI for fluids with a finite density ratio in the general three-dimensional case. The analysis yields new properties of the bubble front dynamics. In either RTI or RMI, the obtained dependencies of the bubble velocity and curvature on the density ratio differ qualitatively and quantitatively from those suggested by the models of Sharp (1984), Oron *et al.* (2001), and Goncharov (2002). We show explicitly that these models violate the conservation laws. For the first time, our theory reveals an important qualitative distinction between the dynamics of the RT and RM bubbles. Asymptotically, the RT bubble is curved, and its curvature has a strong dependence on the density ratio, while the RM bubble flattens independently of the density ratio. The velocity of the RT bubble depends has a power-law dependence on the bubble curvature with exponent 3/2 and a universal coefficient independent of the density ratio, while the RM bubble decelerates. The bubble curvature and velocity depend mutually on one another, as do the differences between the RM and RT cases for these quantities. Our theory explains existing data, formulates the universal properties of the RT and RM nonlinear dynamics, and identifies sensitive diagnostic parameter for future observations.

2. Governing equations

Let t be time, (x, y, z) be the Cartesian coordinates, and $\theta(x, y, z, t)$ be the scalar function with $\theta = 0$ at the fluid interface. Locally, $\theta = z^*(x, y, t) - z$ where $z^*(x, y, t)$ is the position of the fluid interface. The fluid density and velocity have the form $\rho = \rho_h H(-\theta) + \rho_l H(\theta)$, and $\boldsymbol{v} = \boldsymbol{v}_h H(-\theta) + \boldsymbol{v}_l H(\theta)$, where H is the Heaviside step-function, and $\rho_{h(l)}$ and $\boldsymbol{v}_{h(l)}$ are the density and velocity of the heavy (light) fluid located in the region $\theta < 0$ $(\theta > 0)$. For incompressible fluids, $\nabla \cdot \boldsymbol{v} = 0$, and the values of $\rho_{h(l)}$ are independent of the coordinates and time. The equation of continuity is reduced then to

$$\left(\dot{\theta} + \boldsymbol{v}_h \cdot \nabla\theta\right)\rho_h\big|_{\theta=0} = \left(\dot{\theta} + \boldsymbol{v}_l \cdot \nabla\theta\right)\rho_l\big|_{\theta=0} \tag{2.1}$$

where the dot indicates a partial time-derivative. If there is no mass flux across the moving interface, the normal component of velocity is continuous at the interface and

$$\boldsymbol{v}_h \cdot \nabla\theta\big|_{\theta=0} = \boldsymbol{v}_l \cdot \nabla\theta\big|_{\theta=0} = -\dot{\theta}\big|_{\theta=0}\,. \tag{2.2}$$

With neglected terms for viscous stress and surface tension, the momentum equation is transformed into the conditions

$$\rho_h\left(\dot{\boldsymbol{v}}_h + (\boldsymbol{v}_h \cdot \nabla)\boldsymbol{v}_h + \boldsymbol{g}\right) + \nabla p_h\big|_{\theta<0} = 0\,, \tag{2.3}$$

$$\rho_l\left(\dot{\boldsymbol{v}}_l + (\boldsymbol{v}_l \cdot \nabla)\boldsymbol{v}_l + \boldsymbol{g}\right) + \nabla p_l\big|_{\theta>0} = 0, \tag{2.4}$$

$$p_h - p_l\big|_{\theta=0} = 0, \tag{2.5}$$

where $p_{h(l)}$ is the pressure of the heavy (light) fluid, and $\boldsymbol{g}$ is the gravity directed from the heavy fluid to the light fluid with $|\boldsymbol{g}| = g$. There are mass sources in the flow, and the boundary conditions at the infinity close the set of the governing equations

$$\boldsymbol{v}_h\big|_{\theta=-\infty} = \boldsymbol{v}_l\big|_{\theta=+\infty} = 0. \tag{2.6}$$

The spatial period, the time-scale, and the symmetry of the motion in (2.1-2.6) are determined from the initial conditions. We choose the spatial period λ in a vicinity of the wavelength of the mode of fastest growth, $\lambda \sim \lambda_{max}$, where λ_{max} is set by surface tension and viscosity (Chandrasekhar 1961). For RTI, the time-scale is $\tau_{RT} \sim \sqrt{\lambda/Ag}$, where $A = (\rho_h - \rho_l)/(\rho_h + \rho_l)$ is the Atwood number with $0 \le A \le 1$. For RMI, $\boldsymbol{g} = 0$ and $\tau_{RM} \sim \lambda/v_0$, where v_0 is the initial velocity value. Based on the experimental observations, we separate scales and divide the fluid interface into active and passive regions, similarly to Aref & Tryggvason (1989). In the active regions (scales $\ll \lambda$) the vorticity is intensive, while the passive regions (scales $\sim \lambda$) are simply advected. In order to be stable under modulations with scales $\gg \lambda$, the large-scale coherent motion must be invariant under one of symmorphic groups with inversion in the plane (Abarzhi 1998, 2002).

To describe the dynamics of the nonlinear bubble, we reduce (2.1-2.6) to a local dynamical system. All calculations are performed in the frame of reference moving with velocity in the z-direction, where $v(t)$ is the velocity at the bubble tip in the laboratory frame of references. For the large-scale coherent motion, $\boldsymbol{v}_{h(l)} = \nabla\Phi_{h(l)}$, and in the case of a 3D flow with hexagonal symmetry

$$\Phi_h = \sum_{m=1}^{\infty} \Phi_m(t)\big(z + \big(\exp(-mkz)/3mk\big)\sum_{i=1}^{3}\cos(m\boldsymbol{k}_i\boldsymbol{r})\big) + \text{cross terms} + f_h(t), \quad (2.7)$$

$$\Phi_l = \sum_{m=1}^{\infty} \tilde{\Phi}_m(t)\big(-z\big(\exp(-mkz)/3mk\big)\sum_{i=1}^{3}\cos(m\boldsymbol{k}_i\boldsymbol{r})\big) + \text{cross terms} + f_l(t), \quad (2.8)$$

where $\boldsymbol{k}_i$ are the vectors of the reciprocal lattice, $\boldsymbol{r} = (x, y)$, and $f_{h(l)}$ are time-dependent functions. For $x \approx 0$ and $y \approx 0$ the interface can be expanded as a power series, $z^* = \sum_{N=1}^{\infty}\zeta_N(t)(x^{2N} + y^{2N})$ + cross terms, where $\zeta_1(t)$ is the principal curvature at the bubble tip, and $\zeta_1(t) < 0$.

Substituting these expressions in (2.1-2.6), taking the first integral of (2.5), and re-expanding (2.1-2.6) $x, y \approx 0$, we derive a dynamical system of ordinary differential equations for the variables ζ_N and the moments $M_n(t) = \sum_{m=1}^{\infty}\Phi_m(t)(km)^n$+cross terms and $\tilde{M}_n(t) = \sum_{m=1}^{\infty}\tilde{\Phi}_m(t)(km)^n$+cross terms, where n is an integer. The moments are correlations functions by their physical meaning. For $N = 1$, the conditions in (2.2-2.6) take respectively the form

$$\dot{\zeta}_1 = 2\zeta_1 M_1 + M_2/4 = 2\zeta_1\tilde{M}_1 - \tilde{M}_2/4, \quad (2.9)$$

$$\big(\dot{M}_1/4 + \zeta_1\dot{M}_0 - M_1^2/8 - \zeta_1 g\big)\rho_h = \big(\dot{\tilde{M}}_1/4 - \zeta_1\dot{\tilde{M}}_0 - \tilde{M}_1^2/8 - \zeta_1 g\big)\rho_l, \quad (2.10)$$

$$M_0(t) = -\tilde{M}_0(t) = -v(t)\ . \quad (2.11)$$

The local dynamical system (2.11) describes the dynamics of the bubble in a vicinity of its tip as long as the spatial period λ of the coherent structure is invariable. The presentation in terms of moments M_n and $\tilde{M}_n$ allows one to perform a multiple harmonic analysis and find the regular asymptotic solutions with a desired accuracy.

3. Regular asymptotic solutions

Retaining only the first order amplitudes in the expressions for the moments, we derive a nonlinear solution of the Layzer-type, which conserves mass, momentum, and has no mass sources. In RTI, for $t/\tau_{RT} \gg 1$ the curvature and velocity of the Layzer-type bubble are $\zeta_1 = \zeta_L = -Ak/8$, and $v = v_{L,RT} = \sqrt{Ag/k}$, in agreement with the empiric approach of Sharp (1984). In RMI, for $t/\tau \gg 1$, the curvature and velocity are $\zeta_1 = \zeta_L = -Ak/8$, and $v = v_{L,RM} = (2 - A^2)/Akt$. These single-mode solutions however do not satisfy (2.2) and (2.9) and permits mass flux across the interface. To avoid this difficulty Goncharov (2002) has violated the boundary conditions in (2.6) and (2.11), and introduced an artificial time-dependent mass flux of the light fluid in the flow. For the solutions of Goncharov (2002), $\zeta_1 = \zeta_D = -k/8$, and $v = v_{D,RT} = \sqrt{2Ag/(1+A)k}$ in RTI, and $\zeta_1 = \zeta_D = -k/8$, and $v = v_{D,RM} = 2/(1+a)kt$ in RMI, in agreement with drag model of Oron *et al.* (2001). We conclude that Layzer-type approach (either in our version or in the models Oron *et al.* (2001) and Goncharov (2002)) does not satisfy the complete set of the conservation laws, and the single-mode solutions are not therefore physical. The reason for this difficulty lies in a non-local character of the non-linearity in (2.1-2.6) and (2.9-2.11).

To find regular asymptotic solutions, describing the nonlinear evolution of the bubble front in RTI or RMI, one should account for non-local properties of the flow that has singularities (Abarzhi *et al.* 2003). The singularities determine the interplay of harmonics in the global flow as well as in the local dynamics system. They transfer the fluid energy to smaller and larger scales and generate higher order harmonics. If the energy transports are not extensive, so the symmetry and the spatial period of the coherent structure do not change, the singularities affect the shape and velocity of the regular bubble. Assuming the bubble shape, parameterized by the principal curvature(s) at its tip, is free, we find a continuous family of regular asymptotic solutions for the local system. The family involves all solutions allows by the symmetry of the global flow. For the regular asymptotic solutions the interplay of harmonics is well captured. We perform a stability analysis and choose the fastest stable solution in the family as being physically significant. The reader is referred to Abarzhi *et al.* (2003) for more details.

For the Rayleigh-Taylor instability, the bubble velocity is the function on the bubble curvature and density ratio, $v = v(\sqrt{g/k}, A, (\zeta_1/k))$. In the interval $\zeta_{cr} < \zeta \leq 0$, the Fourier amplitudes Φ_m and $\tilde{\Phi}_m$ decay exponentially with increase in their m, the lowest-order amplitudes are dominant, and higher order corrections for family solutions are small. For $\zeta \sim \zeta_{cr}$ the convergence is broken, where $\zeta_{cr} \approx -k/6$ for $A \approx 1$ and $\zeta_{cr} \approx 0$ for $A \approx 0$. For fluids with highly contrasting densities, $A \sim 1$, the magnitudes of the Fourier amplitudes of the light fluids are much larger that those of the heavy fluid, so $|\Phi_m| \sim |\tilde{\Phi}_{m+1}|$. For fluids with similar densities, $A \sim 0$, this difference is insignificant, and $|\Phi_m| \sim |\tilde{\Phi}_m|$.

The fastest solution in the family has the curvature and velocity

$$\zeta_1 = \zeta_{A,RT}, \quad v = v_{A,RT} \tag{3.1}$$

Explicit analytical expressions for $\zeta_{A,RT}$ and $v_{A,RT}$ are cumbersome and not presented here. Remarkably, the bubble velocity and the curvature obey a universal dependence

$$v_{A,RT} = \sqrt{g/k}(8|\zeta_{A,RT}|/k)^{3/2}. \tag{3.2}$$

In the limiting case of fluids with highly contrasting densities, $A \approx 1$, the solution (3.1) takes the form

$$\zeta_{A,RT} \approx -(k/8)\big(1 - (1-A)/8\big), \quad v_{A,RT} \approx \sqrt{g/k}\big(1 - 3(1-A)/16\big). \tag{3.3}$$

In the other limiting case, $A \approx 0$, it can be expanded as

$$\zeta_{A,RT} \approx -(k/2)A^{1/3}, \quad v_{A,RT} \approx (3/2)^{3/2}\sqrt{Ag/3k}, \tag{3.4}$$

Stability analysis shows that solutions with $\zeta \sim \zeta_{cr}$ and $\zeta \sim 0$ are unstable, while solutions with $\zeta \sim \zeta_{A,RT}$ are stable. Therefore, the physically significant solution in the RT family is the solution (3.1).

For the Richtmyer-Meshkov instability, the multiple harmonic regular asymptotic solutions in (2.9-2.11) with $M_n \sim 1/t$, $\tilde{M}_n \sim 1/t$ and $v \sim 1/t$, and time-independent ζ_N are found in a similar way. The RM and RT families have a number of common properties such as the dominance of the lowest order amplitudes, convergence, and critical solutions. However the asymptotic dynamics of the bubble front in RMI appears quite different from that in RTI. In the RM family, the fastest solution corresponds to a bubble with a flattened shape:

$$\zeta_1 = \zeta_{A,RM}, = 0 \quad v = v_{A,RM} = 3/Akt \tag{3.5}$$

Higher order corrections for the solution (3.5) are reasonably small. The stability analysis shows that solutions with a finite curvature $\zeta \sim 1/\lambda$ are unstable, while flattened bubbles with $\zeta = \zeta_{A,RM}$ are stable. Therefore, the physically significant solution in the RM family is the solution (3.5).

The foregoing analysis can be applied for 3D flows with other symmetries. In either RTI or RMI, the nonlinear dynamics of 3D highly symmetric flows (Abarzhi 1998, 2002)) coincide except for the difference in the normalization factor k. A nearly isotropic shape of the bubble is the reason of this universality. The 3D and 2D results are similar qualitatively. In main order, in RTI $v_{A,RT,3D}/v_{A,RT,2D} \sim \sqrt{3}$ and $\zeta_{A,RT,3D}/\zeta_{A,RT,2D} \sim 3/4$, while in RMI $v_{A,RM,3D}/v_{A,RM,2D} \sim 2$ and $\zeta_{A,RM,3D}/\zeta_{A,RM,2D} \sim 1$, similarly to Abarzhi (1998) and Abarzhi (2002) for $A = 1$. For 3D low-symmetric flows, the asymptotic analysis shows a tendency of 3D bubbles to conserve isotropy in the plane, and a discontinuity of the 3D-2D dimensional crossover.

4. Discussion

Based on the foregoing results, we expect the following dynamics of the bubble front in the Rayleigh-Taylor and Richtmyer-Meshkov instabilities for fluids with finite density differences in the case of a small initial perturbation. In RTI, the bubble curvature ζ_1 and velocity v grow as $\sim \exp(t/\tau_{RT})$ in the linear regime, $t/\tau_{RT} \ll 1$, and reach finite values $\zeta_1 \approx \zeta_{A,RT}$ and $v \approx v_{A,RT}$ asymptotically for $t/\tau_{RT} \gg 1$. The parameters $\zeta_{A,RT}$ and $v_{A,RT}$ depend strongly on the Atwood number. However, the value $v^2_{A,RT}/\zeta^3_{A,RT}$ is a parameter universal for all A. This universality suggests that the bubble front evolution in RTI is quite complicated and cannot be approximated by the motion of a spherical bubble, in contrast to suggestions of the drag models of Oron *et al.* (2001) and Goncharov (2002). For $A < 1$ the bubbles are slower and less curved compared to the case of $A = 1$; for $A \approx 0$ the bubbles flatten and their velocity approaches zero.

In RMI, the bubble curvature and velocity change as $\zeta_1 \sim -kt/\tau_{RM}$ and $v(t) - v_0 \sim -v_0 t/\tau_{RM}$ in the linear regime, $t/\tau_{RM} \ll 1$; for $t \sim \tau_{RM}$ the curvature ζ_1 reaches an extreme value, dependent on the initial conditions; as $t/\tau_{RM} \gg 1$ the bubble flattens, $\zeta_1 \sim \zeta_{A,RM}$ and decelerates $v \sim v_{A,RM}$. For $A < 1$ the bubbles move faster than those for $A = 1$, and for $A \leq 1$ the bubbles are flat. The flattening of the bubble front is a

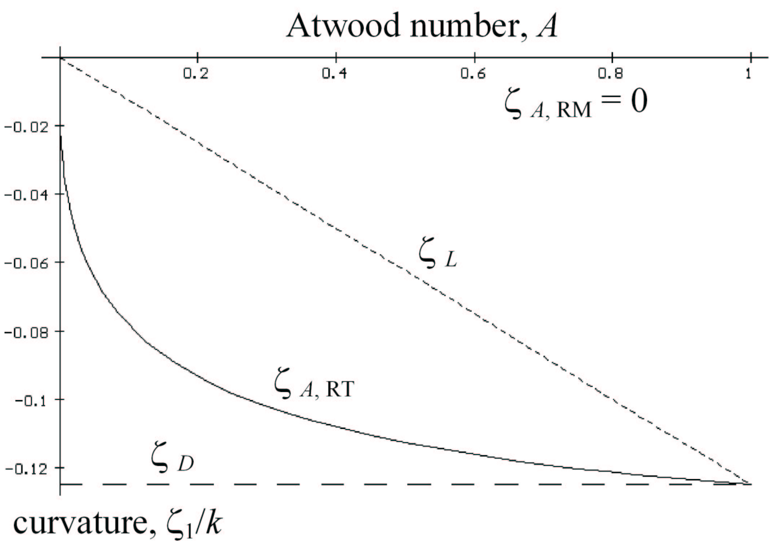

FIGURE 1. Dependence of the bubble curvature on the Atwood number A for 3D highly symmetric flows in the Rayleigh-Taylor and Richtmyer-Meshkov instabilities; k is the wavevector, $\zeta_L = -Ak/8$ is the Layzer-type solution, $\zeta_D = -k/8$ is given by the drag models, $\zeta_{A,RT}$ and $\zeta_{A,RM}$ are given by the non-local multiple harmonic solutions in RTI and RMI respectively.

distinct feature of RMI universal for all A. It follows from the fact that RM bubbles decelerate. The equation (3.5) suggests that for for fluids with very similar densities, $A \sim 0$, the bubble velocity has a much faster time-dependence, as t^a with $-1 < a < 0$, in a qualitative agreement with experiments of Jacobs & Sheeley (1996).

Figures 1, 2 and 3 compare our multiple harmonic non-local solutions with the models of Oron *et al.* (2001) and Goncharov (2002), and with the Layzer-type solution, which agrees with Sharp (1984). For $0 < A \leq 1$ the asymptotic dynamics is different in RTI and RMI: the bubble velocity reaches a constant value in RTI and decreases with time in RMI. Therefore the distribution of pressure around the bubble is distinct in RTI and RMI. This should lead to a different shape for the bubble front. Our theory adequately describes the fact that in RTI the curvature $\zeta_{A,RT}$ has a strong dependence on the Atwood number, while in RMI $\zeta_{A,RM} = 0$ for all A, Figure 1. The models of Sharp (1984), Oron *et al.* (2001), and Goncharov (2002) do not predict any difference between the shape of the RT and RM bubbles. In RTI, for $A = 1$ the values of $\zeta_{A,RT} = \zeta_L = \zeta_D$. However, for finite A the difference among the values $\zeta_{A,RT}$, ζ_L and ζ_D is significant, Fig. 1; in the limit $A \to 0$, $\zeta_{A,RT}/\zeta_D \to 0$, while $\zeta_{A,RT}/\zeta_L \to \infty$. We emphasize that for $A = 0$ the RT instability does not develop and the bubble curvature should remain zero for all t, in agreement with (3.4). In RMI, the bubble in (3.5) flattens asymptotically for all A, while the models of Oron *et al.* (2001) and Goncharov (2002) suggest an A-independent finite value of the bubble curvature $\zeta_D = \zeta_L\big|_{A=1}$, Fig. 1.

We conclude that in either RTI or RMI the bubble curvature is sensitive diagnostic parameter, which tracks the conservation of mass in the flow. The value $\lambda|\zeta_1|$ defines how flat or narrow the bubble is for a given lengths scale λ. This dimensionless shape parameter is related to the bubble velocity and determines the flow drag. The value of the drag force is still a subject for controversy in the chaotic RTI and RMI (Dimonte

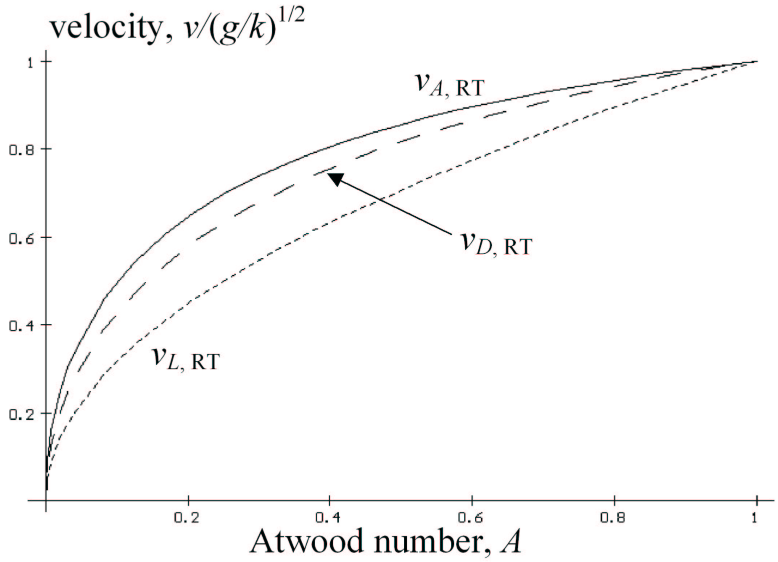

FIGURE 2. Dependence of bubble velocity v on the Atwood number A for 3D highly symmetric flows in the Rayleigh-Taylor instability; k is the wavevector and g is gravity, $v_{L,RT} = \sqrt{Ag/k}$ is the Layzer-type solution, $v_{D,RT} = \sqrt{2Ag/(1+A)k}$ is given by the drag models, and $v_{A,RT}$ is given by the RT multiple harmonic non-local solution.

2000). Our theory resolves this issue for highly nonlinear Rayleigh-Taylor and Richtmyer-Meshkov instabilities. The foregoing analysis suggests that the empiric models of Sharp (1984), Oron *et al.* (2001) and Goncharov (2002) may not be applicable in RTI and RMI.

Our results are in good agreement with available data (Kucherenko *et al.* 2000; He *et al.* 1999; Gardner *et al.* 1988; Jacobs & Sheeley 1996; Holmes *et al.* 1995; Schneider *et al.* 1998; Holmes *et al.* 1999; Dimonte 2000; Volkov *et al.* 2001). However, in most of existing experiments, the measurement of the bubble curvature requires an improvement of diagnostics of the interface dynamics. The major issues to check by future observations are the following: Does the curvature of the RT bubble have a strong dependence on the Atwood number or reach an A-independent value? Does the curvature of the RM bubble vanish asymptotically or approach a finite value?

For the bubble velocity, in the case of $A \sim 1$ the experiments and simulations have been in a reasonable agreement with the dependencies $v_{L,RT}$ and $v_{D,RT}$ in RTI, and with the dependence $v_{D,RM}$ in RMI. On the other hand, as evinced in the foregoing, the single-mode solutions $v_{L,RT}$ and $v_{D,RT}$ as well as $v_{L,RM}$ and $v_{D,RM}$ violate the conservation laws. This apparent paradox is easily explained. Figure 2 shows that in RTI for finite A and for $\zeta_1 \sim 1/\lambda$, the bubble velocity is relatively insensitive to details of the interface dynamics, and the quantitative distinction among the values of $v_{A,RT}$, $v_{D,RT}$ and $v_{L,RT}$ is $10-20\%$. In RMI, the velocities $v_{L,RM}$ and $v_{D,RM}$ differ significantly from $v_{A,RM}$, Figure 3. The difference is however hard to distinguish in experiments. Since the velocity decays as $1/t$, the bubble displacement, $\sim C\log(t/\tau_{RM})/k$, is comparable with an experimental error, and the coefficient C is in this dependence is impossible to evaluate. In contrast, the bubble curvature is a reliable diagnostic parameter as discussed in the foregoing.

Our theory describes the principal influence of the density ratio on the large-scale

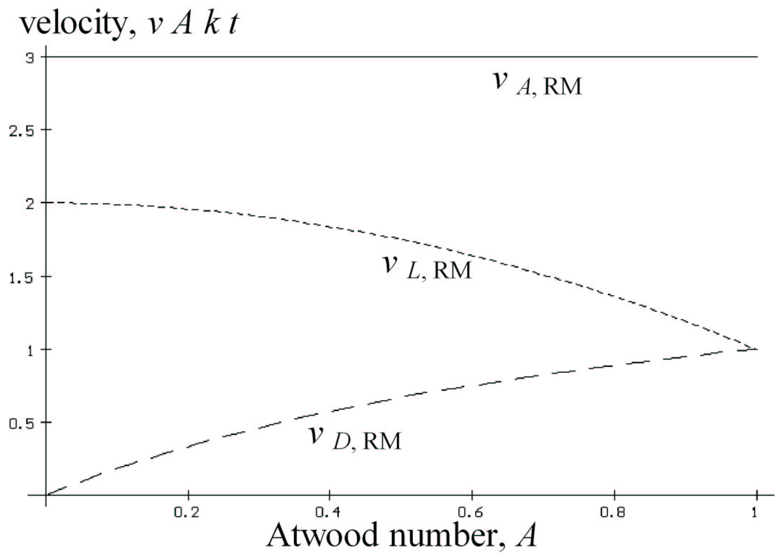

FIGURE 3. Dependence of the bubble velocity v on the Atwood number A for 3D highly symmetric flows in the Richtmyer-Meshkov instability; k is the wavevector, t is time, $v_{L,RM} = (2 - A^2)/Akt$ is the Layzer-type solution, $v_{D,RM} = 2/(1 + A)kt$ is given by the drag models, $v_{A,RM} = 3/Akt$ is given by the RM multiple harmonic non-local solution.

nonlinear dynamics of the RT and RM bubbles. The analysis is based on the assumptions that the flow dynamics is governed by a dominant mode, the transfers of energy to smaller or larger scales are not extensive, and the vorticity does not change the time-dependence of the large-scale coherent motion. If these conditions are broken (for example, for fluids with similar densities, $A \approx 0$), the potential approximation may not give a correct time-dependence for the asymptotic bubble motion. We address these issues in the future.

5. Conclusion

We have found a multiple harmonic solution for a complete system of conservation laws describing the large-scale coherent dynamics in RTI/RMI in general three-dimensional case. The theory yields new dependencies of the bubble curvature and velocity on the density ratio, formulates the universal properties of the nonlinear dynamics in RTI and RMI, reveals an important difference between the dynamics of RT and RM bubbles, and shows the significance of mass conservation for the buoyancy-drag balance.

REFERENCES

ABARZHI, S. I. 1998 Stable steady flows in Rayleigh-Taylor instability. *Phys. Rev. Lett.* **81** (2), 337–340.

ABARZHI, S. I. 2002 A new type of the evolution of the bubble front in the Richtmyer-Meshkov instability. *Physics Letters A* **294** (2), 95–100.

ABARZHI, S. I., NISHIHARA, K. & GLIMM, J. 2003 Rayleigh-Taylor and Richtmyer-Meshkov instabilities for fluids with a finite density ratio. *Phys Letters A* **317**, 470–476.

Aref, H. & Tryggvason, G. 1989 Model of Rayleigh-Taylor instability. *Phys. Rev. Lett.* **62**, 749–752.

Baker, G. R., Meiron, D. I. & Orszag, S. A. 1982 Generalized vortex methods for free-surface flow problems. *J. Fluid Mech.* **123**, 477–501.

Chandrasekhar, S. 1961 *Hydrodynamic and Hydromagnetic Stability*. London: Oxford University Press.

Cowley, S., Baker, G. & Tanveer, S. 1999 On the formation of Moore curvature singularities in vortex sheets. *J. Fluid Mech.* **378**, 233–267.

Dalziel, S., Linden, P. & Youngs, D. 1999 Self-similarity and internal structure of turbulence induced by the Rayleigh-Taylor instability. *J. Fluid Mech.* **399**, 1–48.

Davies, R. & Taylor, G. 1950 The mechanics of large bubble rising through extended liquids and through liquids in tubes. *Proc. Roy Soc. A London* **200**, 375–390.

Dimonte, G. 2000 Spanwise homogeneous buoyancy-drag model for Rayleigh-Taylor mixing and experimental evaluation. *Phys. Plasmas* **7**, 2255–2269.

Garabedian, P. 1957 *Proc. Roy. Soc. London A* **241**, 423–430.

Gardner, C., Glimm, J. & McBryan, O. 1988 The dynamics of bubble growth for Rayleigh-Taylor instability. *Phys. Fluids* **31**, 447–465.

Goncharov, V. N. 2002 Analytical model of nonlinear, single-mode, classical Rayleigh-Taylor instability at arbitrary Atwood numbers. *Phys. Rev. Lett.* **88**.

He, X., Xhang, R., Chen, S. & Doolen, G. 1999 On three-dimensional Rayleigh-Taylor instability. *Phys. Fluids* **11**, 1143–1152.

Holmes, R., Grove, J. & Sharp, D. 1995 *J. Fluid Mech.* **301**, 51.

Holmes, R. L., Dimonte, G., Fryxell, B., Gittings, M. L., Grove, J. W., Schneider, M., Sharp, D. H., Velkovich, A. L., Weaver, R. P. & Zhang, Q. 1999 Richtmyer-Meshkov instability growth: experiment, simulation and theory. *J. Fluid Mech.* **389**, 55–79.

Hou, T. Y., Lowengrub, J. S. & Shelley, M. J. 1997 The long-time motion of vortex sheets with surface tension. *Phys. Fluids* **9** (7), 1933–1954.

Jacobs, J. & Sheeley, J. 1996 Experimental study of incompressible Richtmyer- .

Kucherenko, Y. A., Pylaev, A., Balabin, S., Murzakov, V., Ardashova, R., Popov, V., Komarov, O., Savel'ev, V., Kozelkov, O., Romanov, I., R, R. C. & Haas, J. 2000 Behavior of turbulized mixtures at the stage of inertial motion for different Atwood numbers. *Laser Part. Beams* **18**, 163–169.

Layzer, D. 1955 *Astrophys. Jour.* **122**, 1.

Matsuoka, C., Nishihara, K. & Fukuda, Y. 2003 Nonlinear evolution of an interface in the Richtmyer-Meshkov instability. *Phys. Rev. E* **67** (art. 036301), 1–12.

Meshkov, E. 1969 *Sov. Fluid Dyn.* **4**, 101–104.

Moore, D. 1979 Spontaneous appearance of a singularity in the shape of an evolving vortex sheet. *Proc. Roy. Soc. London A* **365**, 105–119.

Oron, D., Alon, U., Offer, D. & Shvarts, D. 2001 Dimensionality dependence of the Rayleigh-Taylor and Richtmyer-Meshkov instability late-time scaling laws. *Phys. Plasmas* **8**, 2883–2889.

Rayleigh, L. 1892 On the instability of a cylinder of viscous liquid under capillary force. *Pilos. Mag.* **34**, 145.

Richtmyer, R. 1960 Taylor instability in shock acceleration of compressible fluids. *Commun. Pure Appl. Math.* **13**, 297–319.

Schneider, M., Dimonte, G. & Remington, B. 1998 Large and small scale structure in Rayleigh-Taylor mixing. *Phys. Rev. Lett.* **80**, 3507–3510.

Sharp 1984 An overview of the Rayleigh-Taylor instability. *Physica D* **12**, 3–18.

Tanveer, S. 1993 Singularities in the classical Rayleigh-Taylor flow - formation and subsequent motion. *Proc. Roy. Soc. London A* **441**, 501–525.

Volkov, N. V., Maier, A. E. & Yalovets, A. P. 2001 The nonlinear dynamics of the interface between media possessing different densities and symmetries. *Tech. Phys. Lett.* **27**, 20–24.