# Effect of scale-dependent corrections to flow intensity on turbulent burning rate

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## 1. Motivation and objectives

Calculation of the burning rate is one of the basic problems in modern combustion science. The problem of flame propagation is practically solved for a planar flame front, which occurs infrequently in reality. Flame shape is usually curved due to the intrinsic flame instabilities, an external turbulent flow, flame interaction with shock and sound waves, flame interaction with the chamber walls, the interplay of all these effects and other features of the burning setup configuration (Williams 1985; Zeldovich *et al.* 1985; Poinsot & Veynante 2005). As a result, a real burning rate may exceed the planar flame speed $S_L$ considerably. Over the years, there have been attempts to express the turbulent burning rate $S_T$ as a function of the planar flame speed and the turbulent intensity $u'$ (the root-mean-square flow velocity in one direction). The researchers were looking for a function in the form (Williams 1985)

$$S_T/S_L = f\left(u'/S_L\right). \tag{1.1}$$

Almost all trials to find such a function failed for the following reasons. First, a flame interacts with a turbulent flow in a complicated way, which has not been qualitatively understood (Williams 1985; Driscoll 2007). Second, turbulence itself has not been thoroughly investigated so far (Peters 2000). Third, the universal function like (1.1) might be very complicated (if it exists at all): the burning rate also depends on numerous flame and flow parameters such as the thermal expansion in burning, the flame thickness, the turbulent spectrum and integral turbulent length, the Reynolds number of the flow, etc. (Abdel-Gayed *et al.* 1987; Peters 2000; Pope 2000; Bychkov & Liberman 2000; Lipatnikov & Chomiak 2005; Kadowaki & Hasegawa 2005; Driscoll 2007). Finally, the very possibility of a function like (1.1) is questionable. Various experimental measurements (Abdel-Gayed *et al.* 1987; Aldredge *et al.* 1998; Lee & Lee 2003; Filatyev *et al.* 2005; Savarianandam & Lawn 2006) show completely different flame behaviors in different burning configurations (flames in tubes, combustion bombs, burning in an opening or in a closed chamber, flames in Bunsen-like burners, etc.), even for the same value of $u'/S_L$ in each experiment. The studies (Aldredge *et al.* 1998; Lee & Lee 2003; Filatyev *et al.* 2005) have generally demonstrated that turbulent burning rates (measured at the same $u'/S_L$) may differ by up to an order of magnitude for different experimental setups and/or for different flow conditions within one setup. In that case, no universal law like Eq. 1.1 can be used, since the large-scale flow is specific to any experiment. As a result, any particular example of turbulent burning requires a detailed analysis – experimental, theoretical and numerical.

Unfortunately, direct numerical simulations (DNS) of turbulent combustion are difficult due to many factors including a huge difference in length scales and scalar gradients involved in the problem. Indeed, in DNS of turbulent combustion one has to resolve both the internal structure of the burning zone, $(10^{-3}-10^{-4})cm$, and the characteristic length scale of the flow, $(10 - 10^2)cm$. Meanwhile, there is a standard idea that the flame behaviors on small and large scales may be considered separately. In that case, large eddy simulations (LES) may be performed for a particular combustion configuration (Peters 2000; Pope 2000; Poinsot & Veynante 2005; Pitsch 2005; 2006). LES are typically limited to large scales only, while the internal flame structure and small-scale effects are not resolved. Instead, various subgrid-scale (SGS) models (specified from the flame and flow properties on small scales) are usually used (Smagorinsky 1963; Germano *et al.* 1991; Lilly 1992). There are several methods for moving from the small scales to the large ones in LES of premixed turbulent burning. One method is the level-set approach (Williams 1985; Peters 1999; Pitsch & de Lageneste 2002). According to the approach, the flame front is treated as a scalar field isosurface, which propagates locally (normal to itself) with the planar flame speed $S_L$ and is convected by the flow velocity field. This method works properly only if the inner flame structure is small compared to the smallest turbulent length scale. Another popular approach is the so-called method of thickened flames, where the small scales are removed by increasing properly transport coefficients, and by replacing the laminar flame speed with the "local turbulent" one (Colin *et al.* 2000; Selle *et al.* 2004; Poinsot & Veynante 2005). Then an artificially thickened flame front may be resolved in LES as in DNS, with no SGS model for chemical closure at all. Yet the question remains regarding how to specify the "local" turbulent burning rate on small scales. One possibility is to modify the chemical term in the simulations; an appropriate correction for the chemical term may be obtained from DNS on small scales (Angelberger *et al.* 2000). Unfortunately, intrinsically different hydrodynamic and chemical processes are mixed in that case, which may lead to undetermined consequences. It would be better to perform a rigorous theoretical/numerical analysis of local turbulent flame speed on small scales. Since certain universality is expected for small-scale turbulent combustion, one can review Eq. 1.1 looking for the local average turbulent burning rate.

According to numerous theories and models, the turbulent burning rate (local or total) may be written in the Pythagorean form

$$S_T^2 = A^2 S_L^2 + B^2 u'^2, \tag{1.2}$$

where the factor $A$ describes an increase in the flame velocity due to the laminar effects, and the coefficient $B$ is related to turbulence; basically, the parameters $A$ and $B$ depend on flame and flow properties, including thermal expansion and the length scale of the flow. In the case of weak turbulence, $u' \ll S_L$, Eq. 1.2 is reduced to a quadratic dependence (Clavin & Williams 1979; Aldredge & Williams 1991; Akkerman & Bychkov 2005),

$$\frac{S_T}{S_L} = A + \frac{B^2}{2A}\frac{u'^2}{S_L^2}. \tag{1.3}$$

In the opposite limit of strongly turbulent flow, $u' \gg S_L$, Eq. 1.2 reads as a linear relation $S_T \approx Bu'$ (Pocheau 1994; Bychkov 2003). Note that Eq. 1.2 is not universal. In particular, Kerstein & Ashurst (1992) suggested the 4/3-power law

$$S_T/S_L - 1 = Cu'^{4/3}. \tag{1.4}$$

Typical phenomenological estimations for the average turbulent flame speed are

$$S_T/S_L - 1 \propto Cu', \qquad S_T \propto Cu'. \tag{1.5}$$

The factor $C$ is adjusted from the experiments or numerical simulations.

Unfortunately, controversy exists between different theoretical, numerical and experimental studies related to the dependence of the burning rate versus the Reynolds number (i.e., the length scale of the flow). Numerical studies (Colin *et al.* 2000; Selle *et al.* 2004; Akkerman *et al.* 2007) demonstrated an increase in the burning rate $S_T$ with growth of the integral turbulent length $\lambda_T$, with no visible limit or saturation. This strongly contradicts the basic theoretical ideas about turbulent burning proposed so far (Clavin & Williams 1979; Yakhot 1988; Pocheau 1994; Aldredge & Williams 1991; Kerstein & Ashurst 1992; Akkerman & Bychkov 2005). Indeed, one of the main tools of the theories is computing the burning rate for an infinitely thin flame front (which corresponds to an infinitely large integral turbulent length). In that case, assuming universal character of the simulations (Colin *et al.* 2000), we obtain an infinitely large burning rate for a flame of zero thickness, which makes any analysis meaningless. The domain of length scales attainable in Colin *et al.* (2000) is limited, and the results obtained may be invalid on infinitely large length scales, though they correspond to reality within a certain domain. Meanwhile, the gap between the theoretical and numerical results prevents the very possibility of rigorous studies on turbulent flame speed until the reason for such a discrepancy is clarified.

The main purpose of the present work is to close the gap between the theories/models and the simulation/experimental results. From our perspective, neither theoretical or numerical results, nor experimental measurements, nor both, are wrong. We propose that the problem might be in the determining of the flow intensity. Indeed, calculation of the turbulent rms-velocity is a separate problem, sometimes quite difficult and unclear: typically, flame propagation affected by turbulence, in turn, influences the flow ahead of the front modifying $u'$. Such a positive (or negative) feedback of flame-flow interaction has not yet been properly studied. As a result, it cannot be determined with 100% certainty whether a statistically stationary (averaged in time and space) or time-dependent, global or local, flow rms-velocity does characterize an action of turbulence experienced by the flame front. In addition, the local/total rms-velocity measured in the experiments or calculated numerically may be quite different from the realistic intensity of the turbulent flow. Therefore, the value $u'$ used in a theory may strongly differ from that used in numerical/experimental studies. In that case, the theoretical predictions should be corrected in an appropriate way. In the present work we demonstrate that the corrections to the flow rms-velocity could not be determined empirically. It is shown that the corrections depend strongly on the length scale of the flow and the initial flow intensity. We modify the previous theoretical studies of turbulent burning and show how the corrections influence the turbulent burning rate. Flame interactions with a single turbulent vortex and with a turbulent flow of wide spectrum are considered. It is shown that modified theoretical predictions agree quite well with the numerical results.

## 2. Results

### 2.1. *Flame interaction with a single vortex*

First, let us consider the interaction of a flame front with a single vortex of size $\lambda$ (i.e., of wavelength $k = 2\pi/\lambda$). The characteristic flame thickness is typically defined as $L_f = \nu/\operatorname{Pr} S_L$, where $\nu$ is the kinematical viscosity and $Pr$ is the Prandtl number. Obviously, the ratio $\lambda/L_f$ is the key parameter for the problem. Here we are interested in relatively large vortices, $\lambda \gg L_f$; small wrinkles are assumed to disappear too quickly. The characteristic time of flame propagation through the vortex may be estimated as $\tau_p = \lambda/S_T$. Assume that the vortex has the kinetic energy $K_0$ at the initial time instant when the flame starts propagation through the vortex. Due to viscosity, the turbulent kinetic energy decreases with time; in the first-order expansion in $L_f/\lambda$, turbulence decays as

$$K(t) = K_0 \exp(-4\nu k^2 t) = K_0 \exp\left(-S_L \delta \lambda^{-2} t\right), \tag{2.1}$$

where $\delta = 16\pi^2 \, \mathrm{Pr} \, L_f$. Then the characteristic time of turbulence damping is $\tau_d \approx \lambda^2/S_L\delta$; the kinetic turbulent energy drops $e$ times during $\tau_d$. Basically, the parameter $\delta$ is a cut-off size of vortex damping during interaction with the flame; it plays the role of an effective flame thickness on large scales. The value $\delta$ exceeds $L_f$ by two orders of magnitude and may be considered as a filter size for LES. The average kinetic energy of the vortex during its interaction with a flame is

$$\bar{K} = \frac{1}{\tau_p}\int_0^{\tau_p} K(t)\, dt = K_0 \frac{S_T}{\lambda}\int_0^{\tau_p} \exp\left(-\frac{S_L\delta}{\lambda^2}t\right) dt = K_0 \frac{S_T}{S_L}\frac{\lambda}{\delta}\left[1 - \exp\left(-\frac{S_L}{S_T}\frac{\delta}{\lambda}\right)\right]. \tag{2.2}$$

In the present work we suppose that the researchers (either in simulations or in experiments) measure the value proportional to $K_0$, while a flame "feels" the action of $\bar{K}$ in reality. One can see a considerable difference between these values. (Of course, it is difficult to suggest what people measure and average in reality. Still, the only scalar of energy dimension in the problem is $K_0$. Thus the value measured should be proportional to $K_0$, with a proportionality factor independent of the length scales. Indeed, in reality we have many vortices of different sizes, and cannot average their intensity separately.) As a result, the "realistic" flow rms-velocity in one direction is $u'_0 \propto \sqrt{\bar{K}}$, while a flame has felt an effective rms-velocity $u'_{eff} \propto \sqrt{K_0}$, and these values are related as

$$u'_{eff} \cong u'_0 \left(\frac{S_T}{S_L}\frac{\lambda}{\delta}\right)^{1/2}\left[1 - \exp\left(-\frac{S_L}{S_T}\frac{\delta}{\lambda}\right)\right]^{1/2}. \tag{2.3}$$

Equation 2.3 specifies a correction, which might reduce the gap between the theories and the simulations/experiments; note that the value $u'_{eff}$ is used in the theories, while $u'_0$ is a numerical/experimental result. In addition, Eq. 2.3 determines a useful dimensionless parameter characterizing a regime of turbulent burning

$$\chi = \frac{S_L}{S_T}\frac{\delta}{\lambda} = 16\pi^2 \, \mathrm{Pr} \, \frac{L_f}{\lambda}\frac{S_L}{S_T}. \tag{2.4}$$

Replacing $u'$ in the theoretical prediction 1.2 with $u'_{eff}$ specified by Eq. 2.3, we find

$$\frac{u'_0}{S_L} = \frac{1}{B}\left(\frac{S_T^2}{S_L^2} - A^2\right)^{1/2}\left(\frac{\delta}{\lambda}\frac{S_L}{S_T}\right)^{1/2}\left[1 - \exp\left(-\frac{S_L}{S_T}\frac{\delta}{\lambda}\right)\right]^{-1/2}. \tag{2.5}$$

Equation 2.5 determines a qualitatively new dependence of the turbulent burning rate versus the flow intensity; unlike Eq. 1.1, Eq. 2.5 presents $u'_0/S_L$ as an explicit function of $S_T/S_L$. The scaled turbulent burning rate specified by Eq. 2.5 obviously depends on the flow intensity, the ratio $\lambda/\delta$, and the parameters $A$ and $B$. We start with the approach of $A = 1$ and $B = \sqrt{2}$ valid, in particular, in the case of zero thermal expansion at the front (Pocheau 1994). The result is shown in Fig. 1 by the solid lines for different vortex sizes: $\lambda/\delta = 0.01;\ 0.04;\ 0.1;\ 1$. The dashed line presents Eq. 1.2 with $A = 1$ and $B = \sqrt{2}$. One can see that turbulence decay strongly reduces the turbulent burning rate on small scales $\lambda \ll \delta$; the curve for $\lambda/\delta = 0.01$ demonstrates $\approx 4$ times smaller flame velocity than the dashed line. Still, the correction becomes weaker on large scales; for $\lambda > \delta$ the difference between the Pocheau approximation (Pocheau 1994) and the present

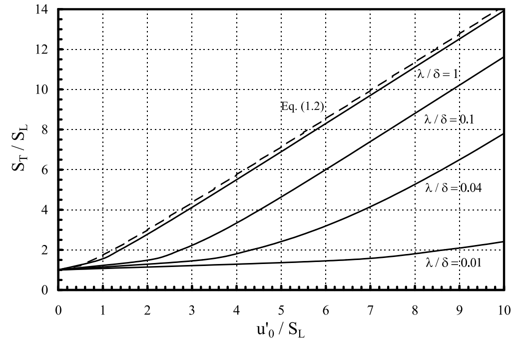


FIGURE 1. The scaled average turbulent burning rate $S_T/S_L$ versus the scaled initial flow rms-velocity $u_0'/S_L$, Eq. 2.5, for $A = 1, B = \sqrt{2}$ and different vortex sizes $\lambda/\delta = 0.01$; 0.04; 0.1; 1 (solid). The theoretical prediction (1.2) is shown by the dashed line.

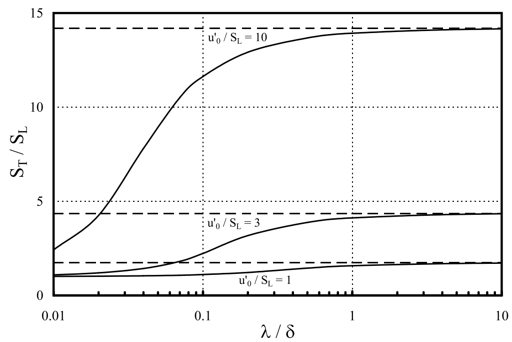


FIGURE 2. The scaled average turbulent burning rate $S_T/S_L$ versus the scaled vortex size $\lambda/\delta$, Eq. 2.5, for $A = 1, B = \sqrt{2}$ and different initial flow rms-velocities $u_0'/S_L = 1$; 3; 10 (solid). The respective theoretical predictions (1.2) are shown by the dashed lines.

results is hardly detectable. One more interesting feature observed in Fig. 1 is that, in agreement with the previous analysis, all the curves reproduce a linear dependence of the burning rate versus the flow intensity for sufficiently large $u_0'$. In Fig. 2, where the scaled turbulent burning rate is plotted versus the scaled vortex size $\lambda/\delta$ at $A = 1$ and $B = \sqrt{2}$, we observe a tendency similar to Fig. 1. Solid lines are related to different scaled initial flow rms-velocities $u_0'/S_L = 1$; 3; 10. Figure 2 demonstrates a noticeable increase in the burning rate with the vortex size on small scales. On large scales, the dependence of the burning rate versus the vortex size is much weaker; the flame velocity tends slowly to a saturation value specified by Eq. 1.2. Note that the plots of Fig. 2 are presented in a semi-logarithmic scale; in usual scale the increase in the flame velocity with the vortex size would look smoother. Figures 1 and 2 show that the corrections to the flow intensity (and the burning rate) due to turbulence decay are meaningless and may be omitted for $\lambda > \delta$. Also, we cannot use the results of the present work on small scales, $\lambda \approx L_f$, when turbulence interplays with the chemical processes inside the burning zone. Thus the calculations above are applicable for the length scales within the domain, say, $\lambda/\delta \approx 0.1 - 1$, when the corrections to the flow rms-velocity are of primary importance. Of course, an approach of zero thermal expansion used in Figs. 1 and 2 is unrealistic; realistic $A$ and $B$ may involve additional scale-dependence into Eq. 2.5.

Figure 3 is a counterpart of Fig. 1 for $A = 3$ and $B = 0.3$. Despite a quantitative discrepancy between Figs. 1 and 3, the main tendencies observed in both figures are qualitatively the same. The changes induced by the parameters $A$ and $B$ are only quantitative. The question of how corrections to the flow intensity depend on $A$, or $B$, or both, in different combustion configurations requires a separate detailed study; it will be discussed elsewhere. It is also interesting to see how the corrections due to turbulence decay change the 4/3-power law suggested by Kerstein & Ashurst (1992). The modified Eq. 1.4 is

$$\frac{u_0'}{S_L} = \tilde{C}\left(\frac{S_T}{S_L} - 1\right)^{3/4}\left(\frac{\delta}{\lambda}\frac{S_L}{S_T}\right)^{1/2}\left[1 - \exp\left(-\frac{S_L}{S_T}\frac{\delta}{\lambda}\right)\right]^{-1/2}, \tag{2.6}$$

where the factors $C$ and $\tilde{C} \equiv C^{-3/4}S_L^{-1}$ should be adjusted. Figure 4 presents Eq. 2.6 for $\tilde{C} = 1$. Solid lines show the average burning rate versus the initial flow rms-velocity for

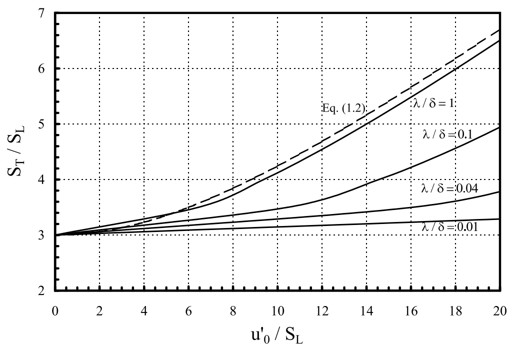


FIGURE 3. The scaled average turbulent burning rate $S_T/S_L$ versus the scaled initial flow rms-velocity $u_0'/S_L$, Eq. 2.5, for $A = 3, B = 0.3$ and different vortex sizes $\lambda/\delta = 0.01;\ 0.04;\ 0.1;\ 1$ (solid). The theoretical prediction (1.2) is shown by the dashed line.

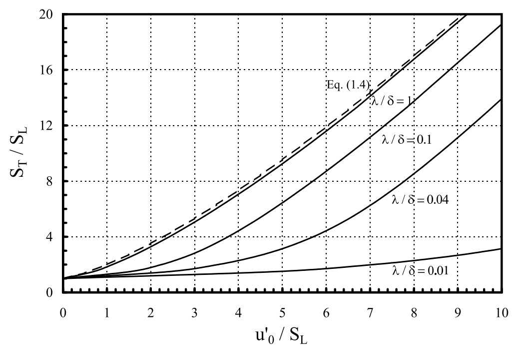


FIGURE 4. The scaled average turbulent burning rate $S_T/S_L$, specified by Eq. 2.6 with $\tilde{C} = 1$, versus the scaled initial rms-velocity of the flow $u_0'/S_L$ for different vortex sizes $\lambda/\delta = 0.01;\ 0.04;\ 0.1;\ 1$ (solid). The theoretical prediction (1.4) is shown by the dashed line.

different vortex sizes $\lambda/\delta = 0.01;\ 0.04;\ 0.1;\ 1$. The previous theoretical prediction 1.4 is shown by the dashed line. In general, the plots in Fig. 4 strongly resemble that of Fig. 1. Again, we observe a noticeable role of the corrections on small scales, while for large vortices the effect of the corrections is hardly detectable. Due to the 4/3-power law, the burning rate in Fig. 4 grows with the increase in the vortex size much faster than in Fig. 1. Still, all curves in Fig. 4 share this universal property; it does not influence the effect of the corrections itself.

Finally, we compare the present calculations to the numerical studies (Colin *et al.* 2000) where flame interaction with a vortex (vortex pair) has been simulated. Numerical fit of Colin *et al.* (2000) suggests the following expression for the turbulent burning rate

$$\frac{S_T}{S_L} - 1 = \zeta \frac{u_0'}{S_L} \left(\frac{\lambda}{\kappa L_f}\right)^{2/3} \exp\left[-\frac{1.2}{(u_0'/S_L)^{0.3}}\right]. \tag{2.7}$$

The coefficient $\kappa$ in Eq. 2.7 reflects the fact that the real flame thickness is larger than $L_f$. In the case of $\mathrm{Pr} = 0.68$ used in Colin *et al.* (2000) we obtain $\kappa = 4\,\mathrm{Pr} = 2.72$. The situation with the parameter $\zeta$ is significantly more complicated. Colin *et al.* (2000) tried to express this value as a function of the Reynolds number and various adjusted coefficients, the origin of which was not completely clarified in a general case. Basically, the dependence of $\zeta$ versus $Re$ resembles the dependence of $B$ from Eq. 1.2 versus $Re$ in (Akkerman & Bychkov 2005). There is a considerable difference between the studies (Colin *et al.* 2000) and the paper (Akkerman & Bychkov 2005). Taking into account $Re$ - dependencies from both works we cannot state with 100% certainty whether our comparison is fruitful. The situation becomes understandable if we take both the values $\zeta$ and $B$ to be a constant. The scaled average turbulent burning rate $S_T/S_L$ specified by Eq. 2.7 is shown in Fig. 5 versus the scaled initial flow intensity $u_0'/S_L$ by the dashed lines. Two curves are related to different vortex sizes $\lambda/\delta = 0.05;\ 0.2$. The solid lines in Fig. 5 show the result (2.5). Due to thermal conduction, a flame front is absolutely stable on small scales. Thus we took $A = 1$ for $\lambda/\delta = 0.05$. On large scales, the intrinsic flame instability develops. We used $A = 1.3$ at $\lambda/\delta = 0.2$ (Bychkov & Liberman 2000) for this reason. In agreement with Pocheau (1994), we kept $B = \sqrt{2}$ in both cases.

Figure 5 demonstrates good agreement between the solid and dashed lines when $\lambda/\delta =$

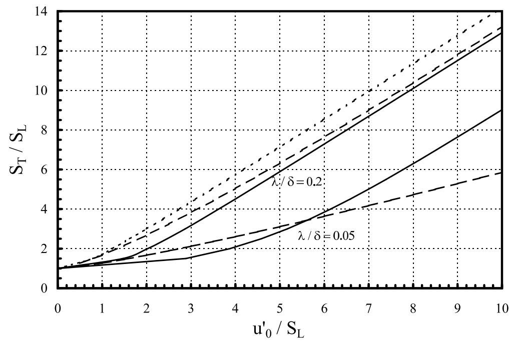


FIGURE 5. The scaled average turbulent burning rate $S_T/S_L$ versus the scaled initial flow rms-velocity $u'_0/S_L$ for different vortex sizes $\lambda/\delta = 0.05$; 0.2. The solid lines show Eq. 2.5. The dotted line presents Eq. 1.2. Formula (2.7) is shown by the dashed lines.

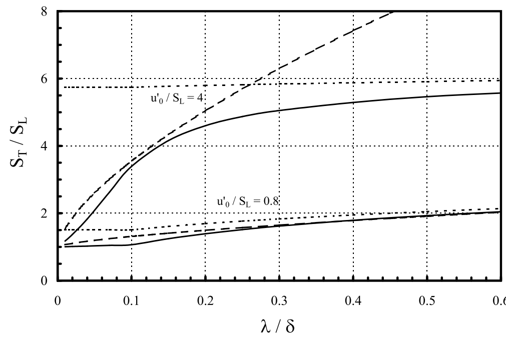


FIGURE 6. The scaled average turbulent burning rate $S_T/S_L$ versus the scaled vortex size $\lambda/\delta$ for different initial flow rms-velocities $u'_0/S_L = 0.8; 4$. The solid lines show Eq. 2.5. The dotted lines present Eq. 1.2. Formula (2.7) is shown by the dashed lines.

0.2. The result of the present work is significantly closer to the numerical fit (Colin *et al.* 2000) than the dotted line related to the previous analysis (Pocheau 1994). In the case of $\lambda/\delta = 0.05$, the agreement (though reasonable) is not as good as for $\lambda/\delta = 0.2$ (especially for large intensities of the flow). This discrepancy may be explained as follows. The vortex size $\lambda/\delta = 0.05$ is actually quite small; it corresponds to $\lambda \approx 5L_f$. Taking such a small vortex with large rms-velocity (say, $u' > 7S_L$), we most likely break the approach of the flamelet regime of burning coming into the thickened flames regime. Then the turbulence penetrates the burning zone and increases the planar flame speed $S_L$. As a result, the "real" $S_T$ to $S_L$ ratio decreases, and we may again have agreement between the present studies and the simulations. It is much more interesting to determine the turbulent burning rate as a function of the length scale of the flow. Figure 6 shows the flame speed versus the vortex size for different initial flow rms-velocities $u'_0/S_L = 0.8;\ 4$. The dashed lines describe Eq. 2.7, (Colin *et al.* 2000). The result of the present studies, Eq. 2.5, is shown by the solid lines. The respective dotted lines are related to Eq. 1.2. For $u'_0/S_L = 4$, we observe good agreement between the present work and the simulations (Colin *et al.* 2000) on small scales, $\lambda/\delta < 0.25$, while for larger scales the curves disagree Still, the numerical fit (2.7) has been specified on small scales, $\lambda/\delta < 0.25 - 0.3$, only. As a result, Colin *et al.* (2000) suggested the monotonic increase in the burning rate. They could not see the saturation in the dependence, which starts on larger scales, when the solid line tends to the dotted one. Thus our result (2.5) reproduces the estimation (2.7) within the domain where Eq. 2.7 was obtained. The present results are more comparable to those of Colin *et al.* (2000) than the previous formula (1.2) shown by the dotted line. In the case of weakly turbulent flow, $u'_0/S_L = 0.8$, we observe encouraging agreement between the present work and the simulations (Colin *et al.* 2000). Such agreement is explained as follows. For $u'_0/S_L = 0.8$, the role of turbulence is meaningless, while an increase in the flame velocity due to the Darrieus-Landau instability becomes of primary importance. In that case, all the curves for $u'_0/S_L = 0.8$ in Fig. 6 show the effect of the instability, while corrections to the flow intensity as well as turbulence itself do not play any role.

### 2.2. *Flame interaction with a turbulent flow of wide spectrum*

Although the situation of flame interaction with a single vortex (or a set of coherent vortices) is typical for numerical simulations (Colin *et al.* 2000; Meneveau & Poinsot 1991; Selle *et al.* 2004; Akkerman *et al.* 2007), it is quite unrealistic. A real turbulent flow resembles a continuum of vortices of different scales. Respectively, vortices of different sizes make different contributions into the total turbulent burning rate. To extrapolate the results above to multi-scale turbulence, we introduce a spectral density of the turbulent kinetic energy $\omega(\lambda, t)$ such as

$$K(t) = \int \omega(\lambda, t)\, d\lambda, \qquad \frac{dK}{dt} = \int \frac{d\omega}{dt} d\lambda. \tag{2.8}$$

In general, the limits o the integral in Eq. 2.8 should be the Kolmogorov (dissipation) length scale $\lambda_v$ and the integral turbulent length scale (maximal vortex size) $\lambda_T$. The change in the spectral density during a small time interval $dt$ is determined by turbulence decay due to viscosity and the energy transfer from large to small scales,

$$\omega(\lambda, t + dt) = \omega(\lambda, t)\, exp\left(-S_L \lambda^{-2} \delta dt\right) + \varphi(\lambda, t, dt), \tag{2.9}$$

where the function $\varphi$ describes the energy exchange between the modes. Then

$$\frac{d\omega}{dt} \approx -\omega(\lambda, t) \frac{S_L \delta}{\lambda^2} + \frac{d\varphi}{dt}, \tag{2.10}$$

and

$$\frac{dK}{dt} \approx -\int \omega(\lambda, t) \frac{S_L \delta}{\lambda^2} d\lambda + \frac{d}{dt} \int \varphi(\lambda, t, dt)\, d\lambda = -S_L \delta \int \frac{\omega}{\lambda^2} d\lambda. \tag{2.11}$$

(Due to the energy conservation, the integral over the entire spectrum $\int \varphi d\lambda = 0$ for $\forall\, t, dt$.) Searching for a general solution to Eqs. 2.10 – 2.11 may be complicated, because the function $\varphi$ is not clearly specified. Meanwhile, the renormalization analysis used below assumes self-similar flame properties, i.e., certain regularity of flame-flow interaction. As a result, we should introduce an additional limitation: temporal fluctuations occur much faster than the flame propagates through vortices. Thus the spectrum is assumed to be time-independent, and the spectral density may be expressed as

$$\omega(\lambda, t) = \alpha(\lambda)\, \beta(t), \quad with \quad \beta(0) = 1,\ \omega(\lambda, 0) = \alpha(\lambda),\ K_0 = K/\beta = \int \alpha d\lambda. \tag{2.12}$$

Then, independent of the function $\varphi$, Eq. 2.11 may be easily solved as

$$\frac{K(t)}{K_0} = \frac{\omega(\lambda, t)}{\alpha(\lambda)} = \beta(t) = \exp(-\Gamma \delta S_L t), \qquad \Gamma = \frac{1}{K_0} \int \frac{\alpha(\lambda)}{\lambda^2} d\lambda. \tag{2.13}$$

The value $\Gamma^{-2}$ plays the role of an "effective" integral length scale for the entire spectrum; compare Eqs. 2.1 and 2.13. Similar to Eq. (2.2), we find an effective spectral density "felt" by the flame $\omega_{eff}$ and then, integrating over the whole spectrum, an effective flow rms-velocity as

$$\omega_{eff}(\lambda) = \frac{1}{\tau_p} \int_0^{\tau_p} \omega(\lambda, t)\, dt = \alpha(\lambda)\, \Psi, \qquad u'_{eff} = u'_0 \Psi^{1/2}, \tag{2.14}$$

where

$$\Psi = \widetilde{\chi}^{-1} \left[1 - \exp(-\widetilde{\chi})\right], \qquad \widetilde{\chi} = \Gamma \delta \lambda_T S_L / S_T. \tag{2.15}$$

The result (2.13) – (2.15) depends on the turbulent spectrum. In the case of the Kolmogorov spectrum

$$\alpha\left(\lambda\right)=\frac{2}{3}\frac{K_0\lambda^{-1/3}}{\lambda_T^{2/3}-\lambda_v^{2/3}}\approx\frac{2}{3}K_0\lambda_T^{-2/3}\lambda^{-1/3} \tag{2.16}$$

we obtain

$$\Gamma=\frac{1}{2\lambda_T^2}\frac{\left(\lambda_T/\lambda_v\right)^{4/3}-1}{1-\left(\lambda_v/\lambda_T\right)^{2/3}}\approx\frac{1}{2\lambda_T^{2/3}\lambda_v^{4/3}},\qquad\qquad \widetilde{\chi}=\frac{1}{2}\frac{S_L}{S_T}\frac{\delta\lambda_T^{1/3}}{\lambda_v^{4/3}}. \tag{2.17}$$

If $\lambda_T\gg\lambda v$, then $\Gamma\gg\lambda_T^{-2}$, $\widetilde{\chi}\gg\chi$, and the flow intensity decreases much faster than in the case of a single vortex of size $\lambda_T$, which makes a noticeable difference between $u_0'$ and $u_{eff}'$. Equation 2.14 describes corrections to the flow intensity in the case of a wide spectrum. To specify corrections to the burning rate, one more equation (coupling the turbulent burning rate and the turbulent kinetic energy) is required. We will use the so-called renormalization analysis (Yakhot 1988; Pocheau 1994; Bychkov 2003), which assumes self-similar (scale-invariant) properties of the corrugated flame front. Following Pocheau (1994) and Bychkov (2003), we decompose the turbulent flame wrinkles into components ("bands") with different wavenumbers, each one providing a similar small increase in the burning rate. Then a differential analogue of Eq. 1.2 takes the form

$$S^{-1}dS=a\left(\lambda\right)d\lambda+S^{-2}b\left(\lambda\right)\omega_{eff}\left(\lambda\right)d\lambda. \tag{2.18}$$

Integrating Eq. 2.18 over the entire scale diversity, one finds a counterpart of Eq. 2.5 for the case of a wide, continuous spectrum. The solution to such an equation presents a dependence like (1.1), which may be used as an SGM model for LES on large scales. Of course, the result of integration of Eq. 2.18 depends on the spectral law for the values $a$, $b$, $\omega_{eff}$. Here we assume the Kolmogorov spectrum, Eq. 2.16, and approximate the spectral densities $a$ and $b$ as

$$a=0,\ \ \lambda<\lambda_c;\qquad a=a_1\widetilde{\delta}\left(\lambda-\lambda_c\right),\ \ \lambda_c\le\lambda\le\lambda_w;\qquad a=1/3\lambda,\ \ \lambda>\lambda_w, \tag{2.19}$$

$$b=0,\ \ \lambda<\lambda_c;\qquad b=b_1/\lambda,\ \ \lambda_c\le\lambda\le\lambda_w;\qquad b=b_2,\ \ \lambda>\lambda_w, \tag{2.20}$$

where $\widetilde{\delta}$ is the delta-function, the constants $a_1$, $b_1$, $b_2$ depend on the thermal-chemical parameters, and $\lambda_c$ and $\lambda_w$ are the primary and secondary cut-off wavelengths of the Darrieus-Landau instability. Then Eq. 2.18 may be integrated analytically as

$$\frac{u_0'}{S_L}=\frac{1}{2}\left(\frac{S_T^2}{S_L^2}-a_1\right)^{1/2}\left(\frac{\lambda_T}{b_1\Psi}\right)^{1/2}\left[\left(\frac{\lambda_T}{\lambda_c}\right)^{1/3}-1\right]^{-1/2} \tag{2.21}$$

if $\lambda_c\le\lambda_T\le\lambda_w$, and

$$\frac{u_0'}{S_L}=\frac{1}{2\sqrt{\Psi}}\left[\frac{S_T^2}{S_L^2}-a_1\left(\frac{\lambda_T}{\lambda_w}\right)^{2/3}\right]^{1/2}\left\{\frac{b_1}{\lambda_w}\left[\left(\frac{\lambda_w}{\lambda_c}\right)^{1/3}-1\right]\left(\frac{\lambda_T}{\lambda_w}\right)^{2/3}+\frac{b_2}{3}\ln\left(\frac{\lambda_T}{\lambda_w}\right)\right\}^{-1/2} \tag{2.22}$$

if $\lambda_T>\lambda_w$. (Not considered is the situation when the largest vortex size is compatible to the chemical length scales, $\lambda_T<\lambda_c$; the scale-invariant renormalization analysis would be inoperable in that case.)

Note that in the present work we are not interested in the exact expressions for $a$ and $b$, but in the relative role of the corrections to the flow intensity. In particular, below we take $a_1=b_2=1$, $b_1=\delta$, $\lambda_w=4\lambda_c=16\lambda_v=2\delta/3$. The result is shown in Figs. 7 and

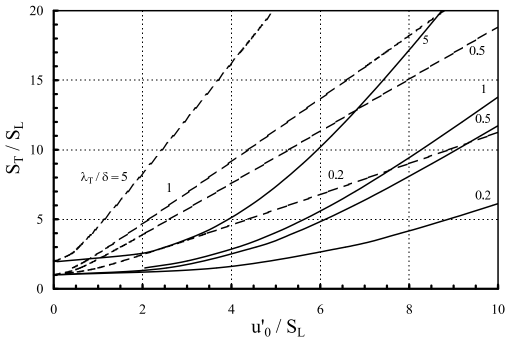


FIGURE 7. The scaled average turbulent burning rate $S_T/S_L$ versus the scaled initial flow rms-velocity $u'_0/S_L$, Eqs. 2.21 – 2.22, for different integral turbulent lengths $\lambda_T/\delta = 0.2; 0.5; 1; 5$. The solid and dashed lines are related to $\Psi$ specified by Eq. 2.15 and $\Psi = 1$, respectively.

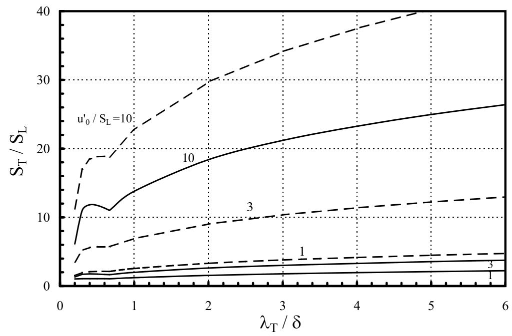


FIGURE 8. The scaled average turbulent burning rate $S_T/S_L$ versus the scaled integral turbulent length $\lambda/\delta$, Eqs. 2.21 – 2.22, for different initial flow rms-velocities $u'_0/S_L = 1;\ 3;\ 10$. The solid and dashed lines are related to $\Psi$ specified by Eq. 2.15 and $\Psi = 1$, respectively.

8. Figure 7 presents the scaled turbulent burning rate versus the scaled flow intensity for different integral turbulent lengths: $\lambda_T/\delta = 0.2; 0.5; 1; 5$. The solid lines describe Eqs. 2.21 - 2.22 with $\Psi$ determined by Eq. 2.15. Basically, the factor $\Psi$ determines the role of the corrections; taking $\Psi = 1$ we go back to the previous predictions (Pocheau 1994; Bychkov 2003). The result (2.21) – (2.22) with $\Psi = 1$ is shown in Fig. 7 by the dashed lines. Similar to the case of flame interaction with a single vortex, we observe a noticeable role of the corrections in the case of a wide spectrum. Figure 8 presents $S_T/S_L$ versus the scaled integral length $\lambda_T/\delta$ for different $u'_0/S_L = 1; 3; 10$. Again, solid and dashed lines are related to $\Psi$ specified by Eq. 2.15 and $\Psi = 1$, respectively. Similar to Fig. 2, in Fig. 8 we observe an increase in the burning rate with the length scale (except for the region close to $\lambda_T \approx \lambda w$). Still, there is no saturation in the case of a wide spectrum. This might happen due to the DL instability on large scales, the role of which increases monotonically; in the case of a single vortex, we did not consider the DL instability on different length scales. Discrepancy between the solid and the dashed lines in Figs. 7 and 8 show that the corrections to the flow intensity (and the burning rate) are of primary importance and should be taken into account either for flame interaction with a single vortex or in the case of a wide spectrum of vortices. Still the result of Figs. 7 and 8 may strongly depend on ratios between the parameters $a_1$, $b_1$, $b_2$, $\delta$, $\lambda_v$ etc. This question requires a separate detailed study.

## 3. Summary

In the present work we derived a set of new relations between the turbulent burning rate and the flow intensity. Instead of looking for $u'$, we have investigated how the burning rate depends on an instantaneous flow rms-velocity $u'_0$ taken, say, at the initial instant of flame-vortex interaction. Unlike $u'$, such a value can be easily determined. Due to viscous decay of turbulence, the flow intensity decreases with time; and the smaller the turbulent vortex, the faster the reduction in the rms-velocity. We specify how the "effective" flow intensity felt by the flame depends on $u'_0$. We substitute the result obtained into the previous theories/models (Clavin & Williams 1979; Aldredge & Williams 1991; Kerstein & Ashurst 1992; Pocheau 1994; Bychkov 2003; Akkerman & Bychkov 2005) and find the mean speed of flame propagation $S_T$ as a function of $u'_0$. The results of the modified theories agree with the simulations (Colin *et al.* 2000) much better than the theories themselves. Thus we suggest that the discrepancy between the theories and the simulations was caused by uncertainties in determining the flow intensity, and that the corrections to the turbulent burning rate studied above allow to reduce the gap between the theories and the simulations.

## 4. Future plans

The present study is only an initial (ideological) stage of the work. Indeed, Eq. 1.2 is written in the very common form; one should specify how thermal expansion and other thermal-chemical flame-flow parameters influence the corrections to the burning rate studied here. The effect of flame propagation on the flow rms-velocity should also be considered. Finally, a universal formula like (1.1) may work only in the opening and/or on small scales, where certain universality of flame dynamics is expected. Otherwise, the burning rate depends mainly on the setup's configuration and on the flow/boundary conditions. Even in that case, the results of the present work, obtained on small scales, may still be useful for construction of a new SGS model for LES of turbulent burning.

## REFERENCES


Abdel-Gayed, R. G., Bradley, D. & Lawes, M. 1987 Turbulent burning velocity: a general correlation in terms of straining rates. *Proc. R. Soc. London Ser.* A **414**, 389 – 413.

Akkerman, V. & Bychkov, V. 2005 Velocity of weakly turbulent flames of finite thickness. *Combust. Theory Modelling* **9**, 323 – 351.

Akkerman, V., Bychkov, V. & Eriksson, L. E. 2007 Numerical study of turbulent flame velocity. *Combust. Flame* **151**, 452 – 471.

Aldredge, R. & Williams, F. 1991 Influence of wrinkled premixed-flame dynamics on large-scale low-intensity turbulent flow. *J. Fluid Mech.* **228**, 487 – 511.

Aldredge, R. C., Vaezi, V. & Ronney, P. D. 1998 Premixed-flame propagation in turbulent Taylor-Couette flow. *Combust. Flame* **115**, 395 – 405.

Angelberger, C., Veynante, D. & Egolfopoulos, F. 2000 LES of chemical and acoustic forcing of a premixed dump combustor. *Flow Turb. Combust.* **65**, 205 – 222.

Bychkov, V. & Liberman, M. 2000 Dynamics and stability of premixed flames. *Phys. Rep.* **325**, 115 – 237.

Bychkov V. 2003 Importance of the Darrieus-Landau instability for strongly corrugated turbulent flames. *Phys. Rev.* E **68**, paper 066304, 1 – 12.

Clavin, P. & Williams, F. A. 1979 Theory of premixed-flame propagation in large-scale turbulence. *J. Fluid Mech.* **90**, 589 – 604.

Colin, O., Ducros, F., Veynante, D. & Poinsot, T. 2000 A thickened flame model for large eddy simulations of turbulent premixed combustion. *Phys. Fluids* **12**, 1843 – 1863.

Germano, M., Piomelli, U., Moin, P. & Cabot, W. H. 1991 A dynamic subgrid-scale eddy viscosity model. *Phys. Fluids* A **3**, 1760 – 1765.

Driscoll, J. F. 2007 Turbulent premixed combustion: flamelet structure and its effect on turbulent burning velocities. *Prog. En. Combust. Sci.*, in press, doi:10.1016/j.pecs.2007.04.002.

Filatyev, S. A., Driscoll, J. F., Carter, C. D. & Donbar, J. M. 2005 Measured properties of turbulent premixed flames for model assessment, including burning velocities, stretch rates, and surface densities. *Combust. Flame* **141**, 1 – 21.

Kadowaki, S. & Hasegawa, T. 2005 Numerical simulation of dynamics of premixed flames: flame instability and vortex-flame interaction. *Prog. En. Combust. Sci.* **31**, 193 – 241.

Kerstein, A. R. & Ashurst, W. T. 1992 Propagation rate of growing interfaces in stirred fluids. *Phys. Rev. Lett.* **68**, 934 – 937.

Lee, T. & Lee, S. 2003 Direct ccmparison of turbulent burning velocity and flame surface properties in turbulent premixed flames. *Combust. Flame* **132**, 492 – 502.

Lilly, D. K. 1992 A proposed modification of the Germano subgrid-scale closure Mmethod. *Phys. Fluids* A **4**,633 – 635.

Lipatnikov, A. N. & Chomiak, J. 2005 Molecular transport effects on turbulent flame propagation and structure. *Prog. En. Combust. Sci.* **31**, 1 – 73.

Meneveau, C. & Poinsot, T. 1991 Stretching and quenching of flamelets in premixed turbulent combustion *Combust. Flame* **86**, 311 – 332.

Peters, N. 1999 The turbulent burning velocity for large-scale and small-scale turbulence. *J. Fluid Mech.* **384**, 107 – 132.

Peters, N. 2000 *Turbulent combustion.* Cambridge, England, Cambridge University Press.

Pitsch, H. & de Lageneste, L. D. 2002 Large-eddy simulation of premixed turbulent combustion using a level-set approach. *Proc. Combust. Inst.* **29**, 2001 – 2008.

Pitsch, H. 2005 A consistent level set formulation for large-eddy simulation of premixed turbulent combustion. *Combust. Flame* **143**, 587 – 598.

Pitsch, H. 2006 Large-eddy simulation of turbulent combustion. *Ann. Rev. Fluid Mech.* **38**, 453 – 483.

Pocheau, A. 1994 Scale invariance in turbulent front propagation. *Phys. Rev.* E **49**, 1109 – 1122.

Poinsot, T. & Veynante, D. 2005 *Theoretical and numerical combustion.* 2nd ed., Ann Arbor, Michigan, Edwards.

Pope, S. B. 2000 *Turbulent flows.* Cambridge, England, Cambridge University Press.

Savarianandam, V. R. & Lawn, C. J. 2006 Burning velocity of premixed turbulent flames in the weakly wrinkled regime. *Combust. Flame* **146**, 1 – 18.

Selle, L., Lartigue, G., Poinsot, T., Koch, R., Schildmacher, K. U., Krebs, W., Prade, B., Kaufmann, P. & Veynante, D. 2004 Compressible large eddy simulation of turbulent combustion in complex geometry on unstructured meshes. *Combust. Flame* **137**, 489 –505.

Smagorinsky, J. 1963 General circulation experimnets with the primitive equations: I. The basic equations. *Mon. Weather Rev.* **91**, 99 – 164.

Williams, F. A. 1985 *Combustion theory.* 2nd ed., Redwood City, California, Addison-Wesley Publish. Company.

Yakhot, V. 1988 Propagation velocity of premixed turbulent flames. *Combust. Sci. Technol.* **60**, 191 – 214.

Zeldovich, Ya., Barenblatt, G., Librovich, V. & Makhviladze, G. 1985 *Mathematical theory of combustion and explosion*, New York, Consultants Bureau.